Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Fault Diagnosis based on the Enclosure of Parameters Estimated with an Adaptive Observer

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Abstract: The proposed fault diagnosis approach associates an adaptive observer for residual generation with set-membership computations based on zonotopes for residual evaluation. The main advantage of this approach is its rigorous propagation of pre-specified modeling uncertainty bounds to the computed residuals. Within the assumed modeling uncertainty bounds, fault detection is guaranteed to be free of false alarm, while the efforts made with set-membership computation minimize the conservativeness of fault detection decisions. The novelty compared to earlier works mainly resides in a guaranteed robustness to bounded parameter variations and in a method for dealing with occasional lack of input excitation.

1. INTRODUCTION

In model based fault diagnosis approaches, the mathematical model, which is at the basis of such approaches, is generally affected by modeling uncertainties. To achieve robust diagnosis decisions, it is often necessary to explicitly take into account such uncertainties. The robustness issue has been dealt with in the literature of fault diagnosis in different ways: statistical rejection (Basseville and Nikiforov, 1993), deterministic structured disturbance rejection (Chen and Patton, 1999), fuzzy logic reasoning, adaptive threshold decision (Zhang et al., 2001), and interval analysis or set membership approaches (Adrot et al., 2002; Stancu, 2003). The assumed form of modeling uncertainties has strong implications to the design of fault diagnosis methods. When bounded uncertainties are assumed, it is in principle possible make decisions free of any false alarm. However, due to the mathematical complexity in the processing of bounded uncertainties, such approaches usually lead to conservative decisions and may result in an unacceptable rate of misdetections. In order to reduce the conservativeness in such approaches, a method associating adaptive estimation and set-membership computation has been proposed in (Combastel and Zhang, 2006). In this method, the designed residual is related to the assumed uncertainty bounds through a dynamic model, and, for residual evaluation, the uncertainty bounds are propagated to the residual by set-membership computation. It can be seen as an adaptive threshold approach, but the application of set-membership computation makes it less conservative than traditional methods generating adaptive thresholds.

The present paper further develops the results of (Combastel and Zhang, 2006) by enlarging the assumption about modeling uncertainties and by dealing with non persistently exciting inputs: As in the previous work, parametric faults are considered, but instead of assuming constant parameters in the fault-free case, the parameters related to faults may now vary slowly within some bounds and robust adaptive thresholds are computed accordingly. This first extension makes the model more realistic, thus improving the applicability of the method. A second contribution of the present paper consists in dealing with non persistently exciting inputs. The resulting divergence of the estimated parameters enclosure is controlled by the intersection with an a priori fixed (and possibly large) domain corresponding to some physical limits. Such limits represent values out of which the model is clearly no more valid. As a result, the estimated parameter bounds can never diverge, even though they may become very large (i.e. the precision of the parameter estimation becomes very low) when the inputs do not sufficiently excite the system. Though no fault diagnosis decision can be reliably taken when the excitation condition is not satisfied, the a priori parameter limits prevent the divergence of the fault diagnosis algorithm, so that it can easily recover its decision ability when the input excitation comes back.

The paper is organized as follows: After the problem formulation in section 2, a residual generator based on an adaptive observer is given in section 3. The proposed fault diagnosis scheme based on set-membership computations is described in section 4, before its application to the simplified model of a satellite in section 5.

2. PROBLEM FORMULATION

Let Λ be a set of physical and/or identified parameters used to express a continuous vector field f modeling the system under study in the fault-free case: $dx/dt = f(x(t), u(t), \omega(t), \Lambda)$ ($\omega(t)$ is a vector of bounded uncertainties). In the proposed scheme, each fault is modeled by a scalar element in the time-varying vector $\theta(t)$ and the influence of faults on the system dynamical behavior is assumed to be modeled by the mapping $\Lambda(\theta(t))$. $\Lambda(.)$ can be obtained either from some physical knowledge (knowledge model) and/or from several estimations of Λ under a representative set of faulty operating conditions (set of behavioral models interpolated by θ). The proposed problem formulation thus covers both the case of multiple faults (when $\theta(t)$ is a non scalar vector) and the case of variation of several parameters due to a single fault (the variation of a single scalar element of $\theta(t)$ may change the value of any element of $\Lambda(\theta(t))$). Finally, integrating $\Lambda(.)$ into the expression of the vector field f leads to a model of the system including faults: $dx/dt = g(x(t), u(t), \omega(t), \theta(t))$. An approximate discretization scheme allows to derive a discrete time model from the original continuous time formulation: $x_{k+1} = h(x_k, u_k, \varpi_k, \theta_k)$ where ϖ_k is a vector of bounded uncertainties. The guarantee of inclusion may be lost if the discretization error is not properly enclosed. A linearization of h with an inclusion of the linearization error (only valid within a pre-specified domain) such as the one proposed in (Combastel, 2005) can be used to obtain the problem formulated in this paper which is similar to those of (Zhang et al., 2001; Zhang et al., 2002). Let $x_k \in \Re^n$, $u_k \in \Re^l$ and $y_k \in \Re^m$ be respectively the state, input and output at the discrete time instant k of a system described by the equations:

$$x_{k+1} = A_k x_k + B_k u_k + E_k w_k + f_k \tag{1}$$

$$y_k = C_k x_k + F_k v_k \tag{2}$$

where A_k, B_k, C_k, E_k, F_k are time varying matrices of appropriate sizes, $w_k \in [-1;+1]^n$ and $v_k \in [-1;+1]^m$ are normalized bounded (state and measurement) errors, and $f_k \in \Re^n$ represents the influence of faults possibly affecting the system. The faults influence vector $f_k \in \Re^n$ is assumed to satisfy the time varying regression model:

$$f_k = \Psi_k \theta_k \tag{3}$$

with $\Psi_k \in \Re^{n \times p}$ and $\theta_k \in \Re^p$. This regression model either comes from some physical knowledge about the fault, or consists of a general approximating estimator. In (Combastel and Zhang, 2006), the fault model was assumed to be $f_k = \Psi_k \theta$ with a constant parameter vector θ. Though the constant fault model is often reasonable, it cannot ensure a guaranteed robustness to small variations that can be considered as normal. One contribution of this paper consists in proposing a fault diagnosis scheme which is robust to such pre-specified small variations:

$$\theta_{k+1} = \theta_k + G_k e_k \tag{4}$$

The error e_k is assumed bounded: $e_k \in [-1;+1]^p$ and G_k is a matrix of appropriate size. For instance, G_k can be chosen constant and diagonal to express that the difference between two consecutive values of θ_k in the fault-free case belongs to a box, the size of which is defined by the diagonal elements of G_k. Remark that, though the modeling error e_k is assumed bounded, the parameter vector θ_k and its long term variation are not necessarily bounded by this model description. Ideally, the fault-free case should correspond to f_k=0 or θ_k=0.

For robustness, small deviations of θ_k from zero should be tolerated in the fault-free case. Let the logical flag *SysOK* indicate the status of the monitored system, with the values "true" and "false" respectively meaning "fault-free" and "faulty". It is assumed that:

$$SysOK \Rightarrow \theta_k \in [-\varepsilon;+\varepsilon] \tag{5}$$

where $\varepsilon \in \Re^p$ defines a small domain (box) around the ideal fault-free case (θ_k = 0). According to such a model, a fault has occurred if the value of θ_k gets out of the domain $[-\varepsilon;+\varepsilon]$. This can directly be inferred from the contraposition of (5): $\theta_k \notin [-\varepsilon;+\varepsilon] \Rightarrow \neg SysOK$.

Moreover, in any case (including the faulty case), it also seems quite reasonable to fix some physical limits to θ_k in order to deal with scenarios involving a lack of input excitation. As it will be further discussed in the paper, if some a priori knowledge allows to bound the values of θ_k within a (possibly large) finite domain, i.e.,

$$\theta_k \in [-\varphi;+\varphi] \tag{6}$$

for some constant vector $\varphi \in \Re^p$, this information can be incorporated in the fault diagnosis algorithm though such information is not necessary in the design of the proposed residual generator. When available, $[-\varphi;+\varphi]$ represents a centred p-dimensional interval vector (or box). Notice that $[-\varphi;+\varphi] = D(\varphi)[-1;+1]^p$ where $D(\varphi)$ is a diagonal matrix, the diagonal of which is the vector φ.

The above specifications (G_k, ε, φ) are based on the entire value of the vector θ_k. If the components of the vector θ_k have physical meanings, then faults can be similarly specified for each component of θ_k. This kind of specifications may be useful for fault diagnosis, as illustrated in Section 5.

3. RESIDUAL GENERATION

3.1 Residual computation form

In (Guyader and Zhang, 2003) an adaptive observer for estimating x_k and θ_k satisfying (1), (2) and (3) has been proposed in the following form:

$$\Gamma_{k+1} = (A_k - K_k C_k)\Gamma_k + \Psi_k \tag{7}$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \mu_k \Gamma_k^T C_k^T (y_k - C_k \hat{x}_k) \tag{8}$$

$$\hat{x}_{k+1} = A_k \hat{x}_k + B_k u_k + \Psi_k \hat{\theta}_k + K_k (y_k - C_k \hat{x}_k) + \Gamma_{k+1}(\hat{\theta}_{k+1} - \hat{\theta}_k) \tag{9}$$

where $K_k \in \Re^{n \times m}$ is a bounded matrix sequence designed so that $\Phi_k = A_k - K_k C_k$ is exponentially stable. Typically K_k is chosen to be the Kalman gain matrix. Its computation normally requires A_k, C_k and also the covariance matrices of state and output noises. Since these covariance matrices are unknown in the present case, they are considered as tuning parameters. It is known that, if the matrix pair (A_k, C_k) is uniformly completely observable, then for any chosen positive definite covariance matrices, the Kalman gain is bounded and (A_k-K_kC_k) is exponentially stable (Jazwinski, 1970). $\Gamma_k \in \Re^{n \times p}$ can be viewed as a linearly filtered version of Ψ_k, the vector sequences $\hat{x}_k$ and $\hat{\theta}_k$ are respectively the state and parameter estimates, μ_k>0 is a scalar parameter adaptation gain. The vector sequence $\hat{\theta}_k$ computed in the above algorithm can be considered as a residual for the detection of the fault influence $f_k = \Psi_k\theta_k$. This residual generator (7)-(9) is similar to that of (Zhang et al., 2001; Zhang et al., 2002). However, a saturation projection operator is used in the latter to ensure the boundedness of the residual, whereas in this paper the residual is naturally bounded.

3.2 Residual analysis form

Equations (7)-(9) define the computation form of the residual corresponding to an algorithm for computing the residual $\hat{\theta}_k$ from input-output signals u_k, y_k. In the following, let us establish the analysis form of the residual which describes the relationship between the monitored fault and the residual, without forgetting the influence of the modelling errors.

Theorem 1. The residual $\hat{\theta}_k$ is driven by θ_k, w_k, v_k, e_k through the following equations:

$$\eta_{k+1} = (A_k - K_k C_k)\eta_k + E_k w_k - K_k F_k v_k - \Gamma_{k+1} e_k \qquad (10)$$

$$\tilde{\theta}_{k+1} = (I - \mu_k \Gamma_k^T C_k^T C_k \Gamma_k)\tilde{\theta}_k - \mu_k \Gamma_k^T C_k^T C_k \eta_k - \mu_k \Gamma_k^T C_k^T F_k v_k + e_k \qquad (11)$$

$$\hat{\theta}_k = \theta_k - \tilde{\theta}_k \qquad (12)$$

where A_k, C_k, E_k are as in the system (1)-(2), K_k, μ_k, Γ_k as in the residual generator (7)-(9) and $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$, $\eta_k = x_k - \hat{x}_k - \Gamma_k \tilde{\theta}_k$. The proof of this result is given in (Guyader and Zhang, 2003).

If $\hat{\theta}_k$ is seen as an estimate of θ_k, then $\tilde{\theta}_k$ is the estimation error. Clearly, $\tilde{\theta}_k$ depends on w_k, v_k, e_k which are unknown but bounded with known bounds. It will be shown that $\tilde{\theta}_k$ is bounded under some assumptions.

Lemma 1. If $\Phi_k = A_k - K_kC_k$ is exponentially stable and Ψ_k is bounded, then Γ_k generated by (7) is also bounded.

This classical result is known as the bounded input -bounded state (BIBS) stability. See, for example, (Freeman, 1965, page 168).

Theorem 2. Assume that:

A1. $A_k, C_k, \Psi_k, K_k, \mu_k, w_k, v_k$ e_k are all bounded and $\Phi_k = A_k - K_kC_k$ is exponentially stable,

A2. $\mu_k > 0$ is small enough so that

$$\| \sqrt{\mu_k} C_k \Gamma_k \| \leq 1 \qquad (13)$$

where ||•|| denotes the spectral norm (the largest singular value) of a matrix,

A3. there exists a constant $\alpha > 0$ and an integer $L > 0$ such that, for all $k \geq L$-1,

$$J_L(k) = \tfrac{1}{L} \sum_{i=k-L+1}^{k} \mu_i \Gamma_i^T C_i^T C_i \Gamma_i \geq \alpha I \qquad (14)$$

Then η_k and $\tilde{\theta}_k$ governed by equations (10)-(11) are bounded. A proof can be found in (Guyader and Zhang, 2003).

Inequality (14) is a persistent excitation condition: the regression matrix Ψ_k must be persistently exciting so that Γ_k, obtained by linearly filtering Ψ_k through (7), satisfies inequality (14). $J_L(k)$ can be interpreted as an input excitation criterion based on the data over a temporal window having a size of L samples.

4. FAULT DIAGNOSIS

4.1 Principle of the proposed fault diagnosis scheme

For the purpose of fault diagnosis, it is important to compute the uncertainty envelope of $\tilde{\theta}_k$ following the uncertainty bounds of w_k, v_k, e_k, since it then provides the error bounds of the fault parameter estimate $\hat{\theta}_k$. Noticing that (12) can obviously be rewritten as:

$$\theta_k = \hat{\theta}_k + \tilde{\theta}_k \qquad (15)$$

the equations (10), (11) and (15) can be viewed as a linear time varying (LTV) discrete time state space system with η_k, $\tilde{\theta}_k$ as states (concatenated in the vector z_k), with w_k, v_k, e_k (all concatenated in $\underline{w}_k$) and $\hat{\theta}_k$ as inputs, and with θ_k as output:

$$z_{k+1} = \underline{A}_k z_k + \underline{E}_k \underline{w}_k \qquad (16)$$

$$\theta_k = \underline{C}_k z_k + \hat{\theta}_k \qquad (17)$$

where:

$$\underline{A}_k = \begin{bmatrix} A_k - K_k C_k & 0 \\ -\mu_k \Gamma_k^T C_k^T C_k & (I - \mu_k \Gamma_k^T C_k^T C_k \Gamma_k) \end{bmatrix} \qquad (18)$$

$$\underline{E}_k = \begin{bmatrix} E_k & -K_k F_k & -\Gamma_{k+1} G_k \\ 0 & -\mu_k \Gamma_k^T C_k^T F_k & G_k \end{bmatrix}, \; \underline{C}_k = \begin{bmatrix} 0 & I \end{bmatrix} \qquad (19)$$

As the fault parameter estimate $\hat{\theta}_k$ is computed from (7)-(9), a finite domain bounding the fault parameter θ_k can be computed from (17) if the bounds of z_k can be derived from the assumed modeling uncertainty bounds. This task can be achieved through (16) where $\underline{w}_k$ is the collection of the assumed modeling uncertainties: $w_k \in [-1;+1]^n$, $v_k \in [-1;+1]^m$ and $e_k \in [-1;+1]^p$. It follows that $\underline{w}_k$ is bounded by a unit hypercube: $\underline{w}_k \in [-1;+1]^{n+m+p}$. Moreover, it is a reasonable assumption to consider that z_0 belongs to a finite domain: $z_0 \in [z_0]$. As a result, computing a tight outer approximation of the set of possible values for the fault parameter θ_k consists in computing a tight domain enclosing all the possible outputs of the LTV system (16)-(17) when $z_0 \in [z_0]$ and $\underline{w}_k \in [-1;+1]^{n+m+p}$ (i.e. when the initial state set and the inputs are bounded). The computation of a set $[\theta_k]$ such that $\theta_k \in [\theta_k]$ allows to achieve the three steps of a fault diagnosis scheme in a unified way:

Fault detection. Following (5), a fault is guaranteed to have occurred when $[\theta_k] \cap [-\varepsilon;+\varepsilon] = \emptyset$.

Fault isolation. The i^{th} fault (i=1…p) is guaranteed to have occurred when $[\theta_k]_i \cap [-\varepsilon_i;+\varepsilon_i] = \varnothing$. The subscript i refers to the projection along the i^{th} axis (which is very easy compute when $[\theta_k]$ is either a box or a zonotope: see Remark 2).

Fault identification. The set $[\theta_k]$ encloses the set of possible values for the fault parameter vector θ_k. Its centre, $\hat{\theta}_k$, and its shape $[\tilde{\theta}_k]$ respectively provide an estimation of the faults and a characterization of the uncertainty around the estimate. $[\theta_k]$ is thus a fault identification result.

4.2. Computation of a tight enclosure of the output of a LTV system with bounded initial state and bounded inputs

The aim of this paragraph is to outline how the set $[\theta_k]$ can be efficiently computed using zonotopes (Kühn, 1998). A detailed description of the corresponding algorithm can be found in (Combastel and Zhang, 2006). Some definitions are first recalled. The name of a variable v in brackets, $[v]$, will denote a domain of possible values for v: $v \in [v]$. Such domains will be described by zonotopes which are a special class of convex polytopes. More precisely, a p-zonotope in $\Re^n$ with centre $c \in \Re^n$ and with shape matrix $R \in \Re^{n \times p}$ is the linear image of a p-dimensional unit hypercube in $\Re^n$:

$$c + \mathbf{Z}(R) = c + R.[-1;+1]^p \subset \Re^n \tag{20}$$

The (Minkowski) sum of two zonotopes is a zonotope the matrix shape of which can be computed by a matrix concatenation: $\mathbf{Z}(R_1) + \mathbf{Z}(R_2) = \mathbf{Z}([R_1\ R_2])$. The linear image of a zonotope is a zonotope obtained from a matrix product: $L\mathbf{Z}(R) = \mathbf{Z}(LR)$. The smallest aligned box enclosing a zonotope $\mathbf{Z}(R)$ (also called interval hull) is $\mathbf{Z}(box(R))$ where $box(R)$ is a diagonal matrix, the i^{th} diagonal element of which is the 1-norm of the i^{th} line of R. Using the zonotope properties and assuming $z_0 \in [z_0] = c_{z,0} + \mathbf{Z}(R_{z,0})$, an algorithm that can be used to compute $[\theta_k] = c_{\theta,k} + \mathbf{Z}(R_{\theta,k})$ from (16)-(17) is (for $k \geq 0$) :

$$c_{z,k+1} = \underline{A}_k c_{z,k}, \qquad R_{z,k+1} = [\underline{A}_k R_{z,k} \quad \underline{E}_k] \tag{21}$$

$$R_{z,k} = red(R_{z,k}) \tag{22}$$

$$c_{\theta,k} = \hat{\theta}_k + \underline{C}_k c_{z,k}, \quad R_{\theta,k} = \underline{C}_k R_{z,k} \tag{23}$$

$red(.)$ is a reduction operator used to limit to a fixed value the increasing number of columns of $R_{z,k}$ involved by (21). It mainly consists in sorting the columns of $R_{z,k}$ on decreasing Euclidean norm and to enclose the influence of the smaller ones into a box in order to limit the zonotope complexity: see (Combastel, 2005) for details.

Remark 1. if $[z_0]$ is centered ($c_{z,0} = 0$), then $\forall k \geq 0$, $c_{\theta,k} = \hat{\theta}_k$.

Remark 2. The projection $[\theta_k]_i$ of $[\theta_k]$ along the i^{th} reference frame axis is an interval that can be directly deduced from the interval hull of $[\theta_k]$: $[\theta_k]_i$ is an interval that can be defined by its centre, $(c_{\theta,k})_i$, the i^{th} (scalar) element of $c_{\theta,k}$, and by its half width, $box(R_{\theta,k})_{ii}$, the i^{th} diagonal element of $box(R_{\theta,k})$.

4.3 Dealing with non persistently exciting inputs

Theorem 2 proves that the estimation error $\tilde{\theta}_k$ is bounded under some assumptions. One of them (A3) states that the inputs should be persistently exciting. Such a condition does not rely on intrinsic properties of the system under study and may thus not always be fulfilled in practice. One idea to deal with non persistent excitation would consist in stopping the parameter adaptation (μ_k=0) when an excitation criterion based on $J_L(k)$ is not fulfilled for a predefined fixed L (14). Such a strategy could be implemented as follows:

$$\mu_k = \mu \ \text{ if } \ \forall s \in \{k\text{-}L\text{+}1;\ldots;k\},\ min(eig(J_L(s))) > \alpha_{min} \tag{24}$$
$$\mu_k = 0 \ \text{ otherwise}$$

where μ and α_{min}>0 are constants and $min(eig(.))$ computes the minimal eigenvalue of the argument. However, such an approach has not been chosen because it suffers from three drawbacks: Firstly, tuning L is not trivial. Secondly, the adaptation of all the parameters is locked when the input excitation criterion is not satisfied. It may happen that the excitation is not sufficient to adapt some parameters while it is still sufficient to adapt some others. This will be illustrated in section 5. Thirdly, locking the adaptation of the estimate $\hat{\theta}_k$ does not prevent the estimation error $\tilde{\theta}_k$ and the domain $[\tilde{\theta}_k]$ enclosing its possible values to diverge when the inputs are not persistently exciting. However, it can be noticed that the residual analysis form given by (16)-(17) is always valid and, consequently, the zonotope $[\theta_k]$ encloses all the possible values of θ_k no matter how the input excitation is. One solution to prevent the domain $[\theta_k]$ from diverging (at least in some directions) when the excitation is not sufficient consists in updating $[\theta_k]$ as an outer approximation of the intersection between $[\theta_k]$ and $[-\varphi;+\varphi]$ (6). This approach does not necessitate fixing any value for L. The domain $[-\varphi;+\varphi]$ ($=\mathbf{Z}(D(\varphi))$) represents some limits out of which θ_k has no physical meaning for the application under study. The relation $[\theta_k] = [\theta_k] \cap [-\varphi;+\varphi]$ ensures the boundedness of $[\theta_k]$ and thus makes the fault diagnosis scheme able to capture more precise information about θ_k when the input excitation comes back. To that purpose, the main difficulty consists in computing the outer approximation $\underline{\cap}$ of the intersection between two zonotopes which is not, in general, a zonotope. An algorithm allowing to compute $c_r+\mathbf{Z}(R_r) = c_1+\mathbf{Z}(R_1)\ \underline{\cap}\ c_2+\mathbf{Z}(R_2)$ (where $R_1 \in \Re^{n \times p1}$ and $R_2 \in \Re^{n \times p2}$) is:

$$M = [R_1 \ -R_2], \qquad b = (c_2\text{-}c_1) \tag{25}$$

$$M = U.S.V^T = \begin{bmatrix} U_1 & U_0 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_0^T \end{bmatrix} \tag{26}$$

$$c_s = V_1 S_1^{-1} U_1^T b, \quad R_s = V_0 V_0^T, \quad P_1 = [I_{p_1} \quad 0_{p_1 \times p_2}] \tag{27}$$

$$c_r = c_1 + R_1 P_1 c_s, \qquad R_r = R_1 P_1 R_s \tag{28}$$

(26) is a singular value decomposition (SVD) of M. The practical implementation of $[\theta_k] = [\theta_k] \cap [-\varphi;+\varphi]$ is done with two refinements: Firstly, the zonotope $[\theta_k]$ is not directly intersected with the box $[-\varphi;+\varphi]$ but with the exact box intersection $c_{i,k}+\mathbf{Z}(R_{i,k})$ ($R_{i,k}$ is diagonal) between $[-\varphi;+\varphi]$ and the interval hull of $[\theta_k]$ (29). Secondly, the zonotope $[\theta_k] = c_{\theta,k} + \mathbf{Z}(R_{\theta,k})$ is first reduced using the red operator in order to simplify the computations related to the $\underline{\cap}$ operator (30):

$$c_{i,k} + \mathbf{Z}(R_{i,k}) = (c_{\theta,k} + \mathbf{Z}(box(R_{\theta,k}))) \cap [-\varphi;+\varphi] \quad (29)$$

$$[\theta_k] = (c_{\theta,k} + \mathbf{Z}(red(R_{\theta,k}))) \sqcap (c_{i,k} + \mathbf{Z}(R_{i,k})) \quad (30)$$

5. APPLICATION

The proposed fault detection and isolation scheme is illustrated with the simulation of a controlled satellite. The satellite nominal orbit is assumed to be circular with the radius normalized to 1. The nominal angular velocity of the satellite is 3.49×10^{-4} rad/s. The classic continuous time linearized satellite model (Brockett, 1970) is sampled with the period $T_s = 0.1$s, what results in a discrete time model like (1)-(3), with:

$$A_k = \begin{bmatrix} 1 & 0.1 & 0 & 3.49\times10^{-6} \\ 3.66\times10^{-8} & 1 & 0 & 6.98\times10^{-5} \\ -4.25\times10^{-14} & -3.49\times10^{-6} & 1 & 0.1 \\ -1.28\times10^{-12} & -6.98\times10^{-5} & 0 & 1 \end{bmatrix} \quad B_k = H.\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1.5 \end{bmatrix} \quad \Psi_k = H\begin{bmatrix} 0 & 0 \\ u_k^1 & 0 \\ 0 & 0 \\ 0 & u_k^2 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.1 & 0.005 & 0 & 1.16\times10^{-7} \\ 1.83\times10^{-9} & 0.1 & 0 & 3.49\times10^{-6} \\ -1.06\times10^{-15} & -1.16\times10^{-7} & 0.1 & 0.005 \\ -4.25\times10^{-14} & -3.49\times10^{-6} & 0 & 0.1 \end{bmatrix} \quad C_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E_k = 10^{-5}I_4,\ F_k = 10^{-2}I_2,\ G_k = 10^{-3}I_2$$

I_n is the n dimensional identity matrix. The components of the state vector $x_k \in \Re^4$ correspond to radial position, radial velocity, angular position and angular velocity, the components of the input vector $u_k = [u^1_k\ \ u^2_k]^T \in \Re^2$ are the radial and tangential thrusts, the output vector $y_k \in \Re^2$ correspond to distance and angle observations. The parameter vector $\theta_k \in \Re^2$ models reduced efficiencies of the radial and tangential thrusts. In the fault-free case, the admissible values of θ_k approximately represent 5% of the assumed range of possible values for θ_k: $\varepsilon = 0.05[1\ \ 1]^T$, $\varphi = [1\ \ 1]^T$. Moreover, the envelope $[\theta_k]$ is computed so as to be robust to variations of θ_k defined by G_k (4).

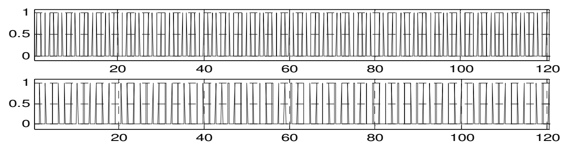

Fig. 1. Input signals u^1_k and u^2_k with respect to time.

The square impulse signals shown in Fig. 1 are used as exciting inputs. The initial values used in the simulation are $x_0 = [1, 0, 0, 3.49\times10^{-4}]^T$ for the "real" system, $\Gamma_0 = 0_{4\times2}$, $\hat{x}_0 = [0.1, 0, 0, 3.49\times10^{-5}]^T$, $\hat{\theta}_0 = [-0.8, 0.8]^T$ for the observer and $c_{z,0} = 0_{6\times1}$, $R_{z,0} = I_6$, d=40 (complexity parameter in $red()$) for the residual evaluator. The adaptive observer parameters are:

$$\mu_k = 4, \qquad K_k = \begin{bmatrix} 0.1412 & 4.93\times10^{-6} \\ 0.0932 & 5.26\times10^{-5} \\ -4.93\times10^{-6} & 0.1412 \\ -5.26\times10^{-5} & 0.0932 \end{bmatrix}$$

A first simulation aims at checking that the admissible variations of θ_k in the fault-free case are well enclosed in $[\theta_k]$. To that purpose, the two components of θ_k correspond to the sampling of a sinusoidal signal with amplitude = 0.047 and period = 30s. The amplitude and the maximum slope of this signal are consistent with the bounds for the admissible variations of θ_k specified by ε and G_k. For the sake of checking, the state and measurement uncertainty are temporarily assumed to be zero (E_k and F_k are zero matrices only for the simulation in Fig. 2).

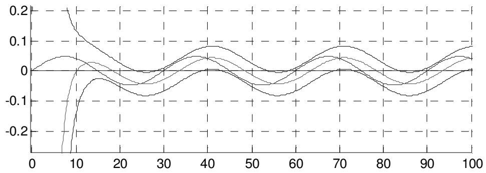

Fig. 2. Real value, estimated value and envelope of $\theta_{1,k}$ under variations consistent with the fault-free specifications.

Fig. 2 shows that the real value of θ_1 (starting from 0) is well enclosed by the computed envelope, the centre of which corresponds to the estimate of θ_1. The estimate follows θ_1 only with some delay due to the convergence of the adaptation toward the quickly varying sinusoid signal. The results for θ_2 (not reported here) are similar.

A fault scenario that has been studied is reported in Fig. 3. The efficiency of the radial thrust is decreased by θ_1=-0.25 between k=25 and k=75. θ_1=0 at other times, when the radial thrust is normal. The efficiency of the tangential thrust is decreased by θ_2=-0.25 between k=50 and k=75 (θ_2=0 elsewhere). The two faults appear and disappear through 10s ramps.

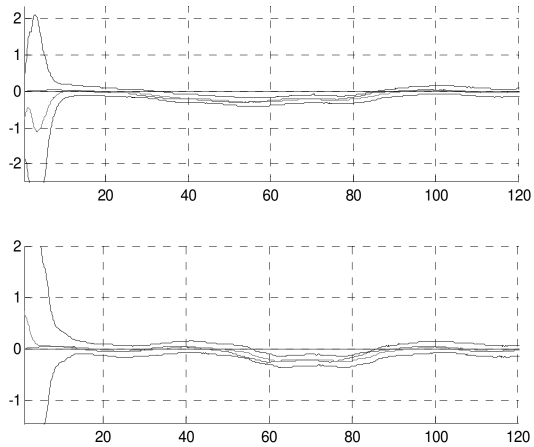

Fig. 3. Temporal evolution of the intervals bounding the fault parameters $\theta = [\theta_1\ \ \theta_2]^T$. Top: $[\theta_1]$. Bottom: $[\theta_2]$.

The envelopes reported on Fig. 3 take into account and characterize the large uncertainties induced by the imprecision of initial conditions and the transient related to the convergence of the observer. They also contribute to robustly detect and isolate the faults. The two grey plots in Fig. 3 correspond to the two elements of $\hat{\theta}_k$, providing an estimation of the faults (second plot within the envelopes). The fault isolation decision related to this multiple and intermittent faults scenario is reported on Fig. 4.

The consequences of a loss of excitation on the first input are now studied on the same fault scenario. The results are reported on Fig. 5, Fig. 6 and Fig. 7.

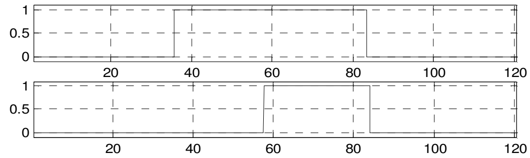

Fig. 4. Fault isolation. Top: fault θ_1. Bottom: fault θ_2.

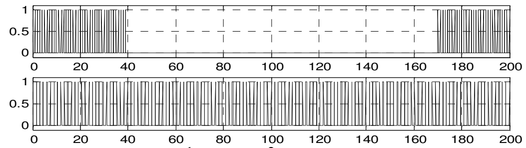

Fig. 5. Input signals u^1_k and u^2_k. No persistent excitation.

Fig. 6 shows that when u^1_k is not exciting the system ($t \in [40;170]$), the envelope $[\theta_1]$, that had previously converged after the transient due to the uncertain initial conditions, tends to diverge until it reaches the physical limits ([-1;+1] for θ_1). During this phase, the estimate of θ_1 does not follow the sinusoidal signal used to simulate the first fault. The accuracy of the estimation of θ_1 increases when u^1_k excites again the system. In the same time, the estimation of the parameter modelling the second fault does not suffer from the loss of excitation induced by u^1_k. Even if this point is rather specific to the application under study, it is also a consequence of not stopping the adaptation when the excitation criterion is not fulfilled ((24) is not implemented). The knowledge of $[\theta_1]$, even very imprecise at certain times, suffices to continue estimating θ_2 rather accurately and to follow the sinusoid signal used for its simulation.

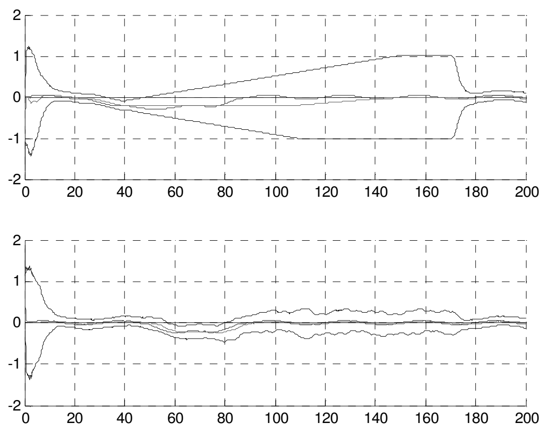

Fig. 6. Fault estimation. Top: $[\theta_1]$. Bottom: $[\theta_2]$.

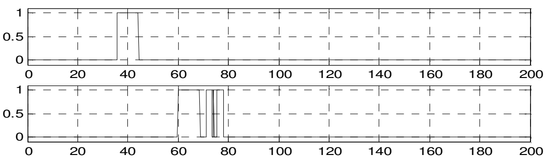

Fig. 7. Fault isolation. Top: fault θ_1. Bottom: fault θ_2.

It must be noticed that the direct link between the excitation of u^1_k (resp. u^2_k) and $[\theta_1]$ (resp. $[\theta_2]$) is due to the natural decoupling between tangential and radial values in the satellite model. However, the description of domains by zonotopes allows to deal with stronger dependencies that may appear in other applications. Finally, a scenario where both inputs are simultaneously null during some time has been studied and shows that both $[\theta_1]$ and $[\theta_2]$ first tend to reach their maximum range (physical limits) before converging toward a better accuracy once the input excitation is back.

6. CONCLUSION

In the proposed fault diagnosis scheme, a single adaptive observer combined with set-membership computations based on zonotopes performs the fault detection, fault isolation and fault identification tasks in an integrated way. The robustness properties are specified *a priori* by some uncertainty bounds. As shown by the satellite example, the case of multiple and intermittent faults is naturally handled, even when the inputs are not persistently exciting the system. The study of more complex applications will be the subject of future work.

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