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Design on Odd-even Steps Third Order Approach Interpolation Algorithm for Logarithmic Curve

Jiang Xinhua [a], Chen Xingwu [a,b]

[a] *School of Information Science and Engineering, Central South University, Changsha 410083 P.R.China (Tel: 0086-591-83968157; e-mail: xhjiang@ fjut.edu.cn).*
[b] *Institute of Control and Information Technology, Fujian University of Technology, Fuzhou 350014 P.R.China(e-mail: cxw@fjut.edu.cn)*

Abstract: Logarithmic curve interpolation is essential for precise machining conical cutters with constant rake and spiral angle in CNC machine. In this paper, a novel odd-even steps third order approach algorithm is presented to realize the logarithmic interpolation. In each interpolating cycle there are odd and even steps divided. At the odd step, the interpolating point moves in tangent direction for an instruction interpolation length. The even step is in succession to interpolate same length through the coordinate increments computed by using the Taylor Mean-value Theorem in third order approach algorithm for the interpolation point approaching the ideal curve closely. The third order approach interpolating formulae for calculating coordinate increments are deduced in the results with simplified arithmetic operations that can be quickly computed for real time application. Test results indicate the third order approach algorithm is featured with high accuracy, suitable to be executed by the 32-bit micro-processor in ARM9 embedded CNC system for multi-axis servo control to accurate grind the conical teeth.

Keywords: Logarithmic interpolation; Odd-even steps algorithm; CNC; Parametric interpolator

1. INTRODUCTION

Conventional CNC system generally supports linear and circular interpolation. For machining complex parametric curves, a CNC system has to segment the curves into many straight lines and/or circular arcs to make approximation to a desired accuracy. This leads to enormous data transmissions between CNC unit and servo systems besides the approach error, which will definitely cause the transmission error and increase the complexity in multi-axis simultaneous control. Furthermore, the feed rate will be fluctuated due to the interpolation switching of the segments, start/stop motions and directional changes in velocity (Yong et al., 2003).

Consequently, parametric interpolators are being developed to eliminate the weaknesses of the segmentation approximating in the basic line type (Chou et al., 1991). Zhao et al (2000) proposed the direct interpolator for the involute, the spiral, the cycloid, the sine and the cosine curves based on the pulse increment interpolation technology. Jeong et al (2006) presented a parametric interpolator by use of tabulated parameter and length data. You et al (2000) developed an adaptive interpolation algorithm for the parametric curve. Muller et al (2004) raised a spline interpolation algorithm for 5-axis machining. For free-form curves and surfaces, it is normal to describe them in general mathematic form of segmental NURBS (non-uniform rational B-spline) consisted of values of control points, their weighs and knot vectors. By manipulating these values a wide variety of complex shapes can be designed and machined through NURBS interpolation (Tsai et al., 2003; Liang et al., 2006; Lei et al., 2007). Among above algorithms, the pulse increment interpolation technology can not meet the needs in modern AC servo system for multi-axis control. Some algorithms are complicated with enormous arithmetic operations and time-consuming, which need supports from high performance of CPU and CAD/CAM. The main micro-processor in the embedded CNC kernel can hardly process these algorithms because of its compact but hardware-limited structure.

For machining a conical cutter with constant rake and spiral angle, the interpolation algorithm for logarithmic curve is needed with 4-axis simultaneous control. In this paper, a novel odd-even steps third order approach interpolation algorithm is presented to realize the logarithmic interpolation by the algorithm designing and formula deducing. The interpolation algorithm is featured with high accuracy and simplified computation. It can be used to precise grind the conical tooth of mills in micro-processor embedded CNC machine with multi-axis AC servo control in real time.

The paper is organized as follows: Section 2 indicates the mathematic model of logarithmic curve for machining conical cutters. Section 3 develops the odd-even steps third order approach interpolation algorithm. Section 4 makes the accuracy test. The last section concludes the paper.

2. LOGARITHMIC MODEL FOR CONICAL MILLS

To grind a conical mills' tooth with constant rake and spiral angle as illustrated in Fig. 1, a CNC tool grinding machine is

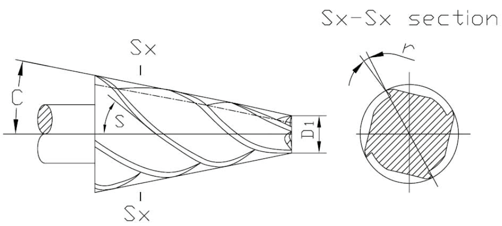

Fig. 1. Drawing of a conical mills with constant rake and spiral angle.

required with three linear axes X, Y, Z and one rotational axis A that can be controlled for feed simultaneously (Chen,2003). As per the result, the mathematic and speed relationships for the four-coordinate are

$$Y = X \times \tan C \tag{1}$$

$$Z = X \times \tan C \times \mathrm{Sin} r \tag{2}$$

$$\frac{dA}{dt} = \frac{dX}{dt} \times 2\tan S / (D_1 + 2X \times \tan C) \tag{3}$$

$$\frac{dY}{dt} = \frac{dX}{dt} \times \tan C$$

$$\frac{dZ}{dt} = \frac{dX}{dt} \times \tan C \times \mathrm{Sin} r$$

Where, X, Y, Z and A are relative coordinate values for feed simultaneously. $\frac{dX}{dt}$, $\frac{dY}{dt}$, $\frac{dZ}{dt}$ and $\frac{dA}{dt}$ are their velocity. C is the conical angle, r is the rake, S is the spiral angle and D_1 is the end diameter of the cutter.

By taking integral of (3), we have

$$A = \int \frac{dA}{dt} dt = \int \left(\frac{dX}{dt} \times 2\tan S/(D_1 + 2X \times \tan C)\right) dt$$

The initial condition is $A = 0$ when $X = 0$, so

$$A = \frac{\tan S}{\tan C} \ln\left(1 + \frac{2\tan C}{D_1} X\right)$$

Now we make two constants of a and b as

$$a = \frac{\tan S}{\tan C}, \; b = \frac{2\tan C}{D_1}, \text{ thus}$$

$$A = a \times \ln(1 + bX) \tag{4}$$

From (4), the kinematical relationship between A and X is of the logarithmic curve. So, to design a reasonable logarithmic interpolation algorithm is key solution to realize the precise machining for the conical cutters.

3. THE ODD-EVEN STEPS INTERPOLATION ALGORITHM

3.1 Idea of the Algorithm

To interpolate the logarithmic curve in general form of (4), we designed the interpolating cycle with odd and even step. At the odd step, the point interpolates for a short instruction length in tangent direction of the curve. Surely, the point will deviate from the ideal curve after the odd step. But it is acceptable as long as the interpolated length is short enough to assure the deviation is within the accuracy range. In succession an even step is executed in which the interpolating direction is corrected and the point is approached toward the ideal path for same instruction length. In this even step, the Taylor Mean-value Theorem is employed to make calculation for the coordinate increments in third order expansion. The odd and even steps are executed alternately once in every interpolating cycle, to make the interpolation from start to end point to realize the logarithmic interpolation.

3.2 Design on the Interpolation Points

The logarithmic curve of (4) is depicted in Fig. 2. The start point is named as P_0 with coordinate (X_0, A_0). From P_0 a short line $\overline{P_0P_1}$ is made in tangent direction with instruction length of Δl that is ended at P_1. At P_1 a circle is made with radius of Δl, which intersects the logarithmic curve at another point P_2. The coordinate values are $P_1(X_1, A_1)$ and $P_2(X_2, A_2)$.

The coordinate increments between P_0 and P_1 are

$$\Delta X_{10} = X_1 - X_0, \; \Delta A_{10} = A_1 - A_0$$

and between P_1 and P_2 they are

$$\Delta X_{21} = X_2 - X_1, \; \Delta A_{21} = A_2 - A_1$$

so we have

$$\Delta X_{21}^2 + \Delta A_{21}^2 = \Delta l^2 \tag{5}$$

$$X_2 - X_0 = \Delta X_{21} + \Delta X_{10} \tag{6}$$

$$A_2 - A_0 = \Delta A_{21} + \Delta A_{10} \tag{7}$$

The odd step interpolation is planned along line $\overline{P_0P_1}$ and the even step interpolation is along line $\overline{P_1P_2}$ as developed in following subsections.

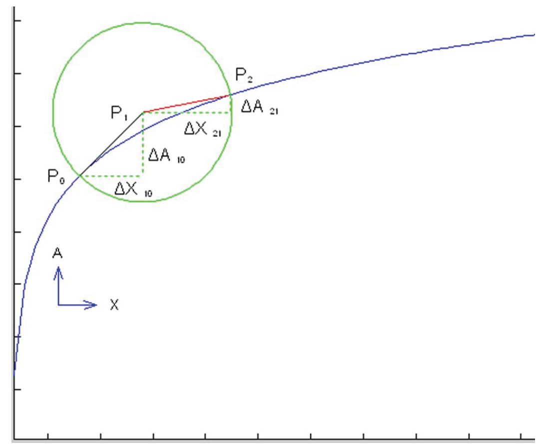

Fig. 2. The interpolation points for the logarithmic curve $A = a \times \ln(1+bX)$.

3.3 Tangent Interpolation at the Odd Step

The odd step interpolation is made along line $\overline{P_0P_1}$. As illustrated in Fig. 2 we have

$\Delta X_{10} = \Delta l \times \cos\theta$, $\Delta A_{10} = \Delta l \times \sin\theta$

where, θ is the crossing angle between line $\overline{P_0P_1}$ and X axis.

The slope of the line $\overline{P_0P_1}$ is

$K = \tan\theta = A' = \dfrac{dA}{dX} = (a\times\ln(1+bX))' = ab/(1+bX)$, so

$$\Delta X_{10} = \Delta l \times \cos\theta = \Delta l \times \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$\Delta X_{10} = \frac{(1+bX_0)\Delta l}{\sqrt{(1+bX_0)^2+(ab)^2}} \tag{8}$$

and

$$\Delta A_{10} = \Delta l \times \sin\theta = \Delta l \times \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

$$\Delta A_{10} = \frac{ab\Delta l}{\sqrt{(1+bX_0)^2+(ab)^2}} \tag{9}$$

Based on (8), the coordinate increments for Y and Z axes between point P_0 and P_1 can be computed according to (1) and (2) as under

$$\Delta Y_{10} = \Delta X_{10} \times \tan C \tag{10}$$

$$\Delta Z_{10} = \Delta X_{10} \times \tan C \times \mathrm{Sin} r \tag{11}$$

Consequently the coordinate increments for the four axes X, A, Y and Z computed from (8), (9), (10) and (11) are named as the coordinate increments in odd interpolation step.

Before interpolation the instruction interpolation length Δl must be decided according to the error allowance, and the interpolating cycle T_1 can be determined by the feed-rate v (from the formula of $\Delta l = vT_1$), the system dynamic feature and the related factors. Surely the Δl should be set in a small length to keep the interpolation accuracy. The instruction interpolating increments for the four axes can be accordingly determined based on the short length Δl.

During the interpolating cycle T_1 at the odd step, the four axes of X, Y, Z and A feed simultaneously for increment of ΔX_{10}, ΔY_{10}, ΔZ_{10}, ΔA_{10} respectively. The short line $\overline{P_0P_1}$ is therefore interpolated. When interpolation arrives at point P_1 the machining point deviates apart from the ideal logarithmic curve gradually. It is acceptable in case the deviation is within the allowance error. But if it is not, the instruction interpolating length Δl needs to be shortened until it meets the error requirements.

Following the odd step, the even step starts from point P_1 to approach the ideal logarithmic curve along the line $\overline{P_1P_2}$ at same speed, which is designed in following subsection.

3.4 Third Order Approach Algorithm at the Even Step

In Fig. 2, among P_0, P_1 and P_2 we have following equation derived from the Taylor Mean-value Theorem

$$A_2 = A_0 + A_0' (X_2-X_0) + \frac{A_0^{(2)}}{2!}(X_2-X_0)^2 + \frac{A_0^{(3)}}{3!}(X_2-X_0)^3 + \ldots + \frac{A_0^{(n)}}{n!}(X_2-X_0)^n + \frac{A_\xi^{(n+1)}}{(n+1)!}(X_2-X_0)^{n+1} \tag{12}$$

where, $\xi \in [X_0, X_2]$, $A_\xi^{(n+1)}$ is the $(n+1)$ orders of derivative of function A at the point of $X = \xi$.

In (12), the items with order of more than fourth are truncated. So the third order approach equation becomes

$$A_2 = A_0 + A_0' (X_2-X_0) + \frac{A_0^{(2)}}{2!}(X_2-X_0)^2 + \frac{A_0^{(3)}}{3!}(X_2-X_0)^3$$

By substituting (A_2-A_0) and (X_2-X_0) in (6) and (7) into the above equation, yields

$$\Delta A_{21} + \Delta A_{10} = A_0' (\Delta X_{21}+\Delta X_{10}) + \frac{A_0^{(2)}}{2!}(\Delta X_{21}+\Delta X_{10})^2 + \frac{A_0^{(3)}}{3!}(\Delta X_{21}+\Delta X_{10})^3 \tag{13}$$

Now taking second and third derivative of (4), yields

$A^{(2)} = -ab^2/(1+bX)^2 = -\tan^2\theta/a$

$A^{(3)} = 2ab^3/(1+bX)^3 = 2\tan^3\theta/a^2$

Substitute values of A_0', $A^{(2)}$ and $A^{(3)}$ into (13), yields

$$\Delta A_{21} + \Delta l \times \sin\theta = \tan\theta \times \Delta X_{21} + \tan\theta \times \Delta l \times \cos\theta - \frac{\tan^2\theta}{2a} \times (\Delta X_{21}+\Delta X_{10})^2 + \frac{\tan^3\theta}{3a^2}(\Delta X_{21}+\Delta X_{10})^3$$

and simplifying

$$\Delta A_{21} = \tan\theta \times \Delta X_{21} - \frac{\tan^2\theta}{2a}(\Delta X_{21}+\Delta X_{10})^2 + \frac{\tan^3\theta}{3a^2}(\Delta X_{21}+\Delta X_{10})^3$$

Put ΔA_{21} from the above equation into (5), yields

$$[\tan\theta \times \Delta X_{21} - \frac{\tan^2\theta}{2a}(\Delta X_{21}+\Delta X_{10})^2 + \frac{\tan^3\theta}{3a^2}(\Delta X_{21}+\Delta X_{10})^3]^2 + \Delta X_{21}^2 = \Delta l^2$$

. And it can be simplified into

$$\frac{\Delta X_{21}-\Delta X_{10}}{\sin^2\theta} - \frac{2\tan\theta}{a}(\Delta X_{21}+\Delta X_{10})[\frac{1}{2} - \frac{\tan\theta}{3a}(\Delta X_{21}+\Delta X_{10})]\Delta X_{21} + \frac{\tan^2\theta}{a^2}(\Delta X_{21}+\Delta X_{10})^3[\frac{1}{2} - \frac{\tan\theta}{3a}(\Delta X_{21}+\Delta X_{10})]^2 = 0$$

To assure the continuity of the interpolation, the variation between consecutive coordinate increments should be as small as possible for smooth interpolating. So there is less difference between ΔX_{21} and ΔX_{10}. Due to the fact that the coordinate increment is essentially small in the CNC interpolation, the higher order of the coordinate increments is much smaller in value. Therefore value ΔX_{10} can be used to approach the mean value of ΔX_{21} and ΔX_{10}, i.e., (ΔX_{21} + ΔX_{10})/2, in the higher order items of the above equation. So

$$\frac{\Delta X_{21}-\Delta X_{10}}{\sin^2\theta} - \frac{4\tan\theta}{a}\Delta X_{10}\left(\frac{1}{2} - \frac{2\tan\theta}{3a}\Delta X_{10}\right)\Delta X_{21} + \frac{8\tan^2\theta}{a^2}\Delta X_{10}^3\left(\frac{1}{2} - \frac{2\tan\theta}{3a}\Delta X_{10}\right)^2 = 0,$$ and simplifying into

$$[a^2 - 4a\sin^2\theta\Delta A_{10}(\frac{1}{2} - \frac{2}{3a}\Delta A_{10})]\Delta X_{21} = a^2\Delta X_{10} - 8\sin^2\theta\Delta A_{10}^2\Delta X_{10}(\frac{1}{4} - \frac{2}{3a}\Delta A_{10} + \frac{4}{9a^2}\Delta A_{10}^2)$$

The value of the four orders of the ΔA_{10} at last item of above equation is very small and can be ignored. Thus

$$[a^2 - 4a\sin^2\theta\Delta A_{10}(\frac{1}{2} - \frac{2}{3a}\Delta A_{10})]\Delta X_{21} = a^2\Delta X_{10} - 8\sin^2\theta\Delta A_{10}^2\Delta X_{10}(\frac{1}{4} - \frac{2}{3a}\Delta A_{10})$$

Finally

$$\Delta X_{21} = \frac{3a^3 - 2\Delta A_{10}^2\sin^2\theta(3a-8\Delta A_{10})}{3a^3 - 2a\Delta A_{10}\sin^2\theta(3a-4\Delta A_{10})}\Delta X_{10} \tag{14}$$

Put ΔX_{21} into (5), yields

$$\Delta A_{21} = \Delta A_{10} \times \frac{\sqrt{[3a^3\text{-}2a\Delta A_{10}(3a\text{-}4\Delta A_{10})]^2\text{-}64a^2\Delta A_{10}^4\cos^2\theta - 4\Delta A_{10}^4\sin^2\theta(3a-8\Delta A_{10})^2\cos^2\theta}}{3a^3 - 2a\Delta A_{10}\sin^2\theta(3a-4\Delta A_{10})}$$

The value at the items of higher than four orders within the radical sign at the numerator of above equation is small and ignored for simplified calculation. Thus

$$\Delta A_{21} = \frac{3a^2\text{-}2\Delta A_{10}(3a\text{-}4\Delta A_{10})}{3a^2 - 2\Delta A_{10}\sin^2\theta(3a-4\Delta A_{10})}\Delta A_{10} \tag{15}$$

In (14) and (15), due to ΔX_{10} and ΔA_{10} have been figured out in the odd step, the ΔX_{21} and ΔA_{21} can be quickly computed by putting into the related constants.

Through (1) and (2), ΔY_{21} and ΔZ_{21} can be computed accordingly as under.

$$\Delta Y_{21} = \Delta X_{21}\times\tan C \tag{16}$$

$$\Delta Z_{21} = \Delta X_{21}\times\tan C\times\mathrm{Sin}r \tag{17}$$

The calculation equations of (14), (15), (16) and (17) constitute the calculation formulae for the four coordinate increments in the even step interpolation. Combined with calculation formulae of (8), (9), (10), (11) in odd step interpolation, these formulae constitute the complete odd-even steps third order approach interpolation algorithm.

After the first odd-even steps interpolation cycle, the interpolating point arrives at point P_2 beginning from the start point P_0. It needs the next odd-even interpolation cycle for the point to interpolate from P_2 to P_3, and then to P_4. And then go on and on until the end point. In Fig. 2, there is not illustrated for points of P_3 and P_4, and the corresponding subsequent points. But it is same for the interpolation as of the points of P_1 and P_2. From P_2 to P_3 the odd step interpolation is executed and from P_3 to P_4 it is the even step interpolation. Beginning from the start point, the coordinate increments are computed by the related formula in recursion way. The interpolation cycle is run in turn until the interpolating point arrives at end point

From the above calculation formulae it is clear that there is not complicated function operation, but with only the basic operations of addition, subtraction multiplication and division. So the instruction increments of the four-axis can be worked out quickly in the four-axis CNC tool grinding machine developed by authors based on the ARM9 embedded MCU (micro controller unit) with 203 MIPS of processing speed. The logarithmic interpolation is quickly realized in real time.

4. INTERPOLATION ACCURACY TEST

The formulae for coordinate increments in the odd-even steps third order approach algorithm are listed as under

$$\Delta X_{10} = \frac{(1+bX_0)\Delta l}{\sqrt{(1+bX_0)^2+(ab)^2}}$$

$$\Delta A_{10} = \frac{ab\Delta l}{\sqrt{(1+bX_0)^2+(ab)^2}}$$

$$\Delta Y_{10} = \Delta X_{10}\times\tan C$$

$$\Delta Z_{10} = \Delta X_{10}\times\tan C\times\mathrm{Sin}r$$

$$\Delta X_{21} = \frac{3a^3 - 2\Delta A_{10}^2\sin^2\theta(3a-8\Delta A_{10})}{3a^3 - 2a\Delta A_{10}\sin^2\theta(3a-4\Delta A_{10})}\Delta X_{10}$$

$$\Delta A_{21} = \frac{3a^2\text{-}2\Delta A_{10}(3a\text{-}4\Delta A_{10})}{3a^2 - 2\Delta A_{10}\sin^2\theta(3a-4\Delta A_{10})}\Delta A_{10}$$

$$\Delta Y_{21} = \Delta X_{21}\times\tan C$$

$$\Delta Z_{21} = \Delta X_{21}\times\tan C\times\mathrm{Sin}r$$

In the formulae the related constants can be computed by preprocessing. Beginning from the start $P_0(0,\ 0)$, the subsequent points could be working out in recursion way.

For test purpose, a typical specification of the conical mill is appointed as

End diameter of the mill D_1 = 10mm
Spiral angle S = 30°, i.e. $\pi/6$ (rad)
Conical angle C = 5°, i.e. $\pi/36$ (rad)
Rake of the conical cutter r = 8°, i.e. $\pi/22.5$ (rad)

The logarithmic path is interpolated from X = 0 to 30mm. The A interpolating value is listed and compared to its theoretical value respectively based on X increment. The values for other axes are out of the table for simplification because their relations to X axis are simple proportional.

The odd-even steps third order approach interpolation program is designed and executed in the embedded CNC system with kernel of ARM9. The instruction length Δl is set at 0.2mm. The interpolating and theoretical value of A is displayed with respect to X interpolating value, and recorded in table 1. By limited of the table length, only first and last 10 points amongst the 152 interpolating points are listed.

In table 1, column 1 shows the interpolating point's numbers. Column 2 shows X interpolating value. Column 3 shows A interpolating value. Column 4 is the theoretical value for A calculated from the formula of $A = a\times\ln(1+bX)$), which is named as PA. Column 5 is the error between PA and the A interpolating value, which is named as $ErPA$.

The maximum error for the $ErPA$ amongst the 152 interpolating points is found as

$MaxErPA$ = 3.9785e-005

The max deviation of A between its interpolating and theoretical value is within $3.9785\times10^{-5}\times180^{o}/\pi = 0.00228^{o}$. So it is very small and can be ignored compared to the spiral angle's deviation value of 0~3° for the ordinary milling cutters which is specified according to the American National Standard No.ANSI B94.19 – 1985.

From the table 1 we see the maximum error amongst all the odd points is

$MaxErPAo$ = 3.9785e-005

The maximum error amongst all the even points is

$MaxErPAe$ = 3.5000e-007

It is about 114 times smaller than the error at the odd points. Consequently we see the accuracy from the third approach interpolation is much higher than that from the tangent line interpolation.

By simulation the interpolating trace is illustrated in Fig. 3. The paths are enlarged for easy visualizing, of which the start part is depicted in Fig. 4 and the end part in Fig. 5. From the charts it is clear the interpolating points are much close to the ideal path, also the curve is fitted evenly in the whole path

To valuate the accuracy affected by the instruction interpolation length Δl, we change Δl from 0.2mm to 0.5mm and repeat the same interpolation process. Thus, only 62 points are needed to interpolate the corresponding length of X = 30.8667mm. But the maximum error is enlarged to be $MaxErPA$ = 2.4780e-004, that is about 6 times bigger than the error when Δl=0.2mm. The related data at details for this test are not listed here due to the limitation of the paper's length. So enlarging the instruction interpolation length will decrease the interpolation accuracy, but can increase the interpolating efficiency. The tradeoff should be made between them in the practical applications.

Table 1. Test data from the odd-even steps third order approach algorithm.

Nos.	Interpolating value of X	Interpolating value of A	Theoretical value of A (PA)	Error value $ErPA(\times10^{-4})$
0	0	0	0	0
1	0.1987	0.0229	0.0229	-0.3979
2	0.3974	0.0457	0.0457	-0.0001
3	0.5961	0.0685	0.0685	-0.3926
4	0.7948	0.0911	0.0911	-0.0002
5	0.9935	0.1138	0.1137	-0.3874
6	1.1922	0.1363	0.1363	-0.0003
7	1.3910	0.1587	0.1587	-0.3823
8	1.5897	0.1811	0.1811	-0.0004
9	1.7885	0.2034	0.2033	-0.3773
…	…	…	…	…
143	28.4736	2.6679	2.6679	-0.1829
144	28.6730	2.6833	2.6833	-0.0034
145	28.8724	2.6986	2.6986	-0.1813
146	29.0718	2.7138	2.7138	-0.0035
147	29.2713	2.7291	2.7291	-0.1797
148	29.4707	2.7443	2.7443	-0.0035
149	29.6701	2.7595	2.7595	-0.1781
150	29.8695	2.7746	2.7746	-0.0035
151	30.0689	2.7897	2.7897	-0.1765
152	30.2684	2.8048	2.8048	-0.0035
MaxErPA				0.39785
MaxErPAo				0.39785
MaxErPAe				0.00350

Remark: effective digits listed for 4 bits after the point.
MaxErPA: Max error for interpolating value of coordinate A.
MaxErPAo: Max error of interpolating value A at odd steps.
MaxErPAe: Max error of interpolating value A at even steps.

5. CONCLUSIONS

In this paper we developed a novel interpolation algorithm for logarithmic curve by use of the third order Taylor Mean-Value Theorem. Through the algorithm design and formulae deduction the odd-even steps third order approach interpolation algorithm is presented that can precise interpolate the logarithmic path to accurate machine the conical cutters with constant rake and spiral angle. It is

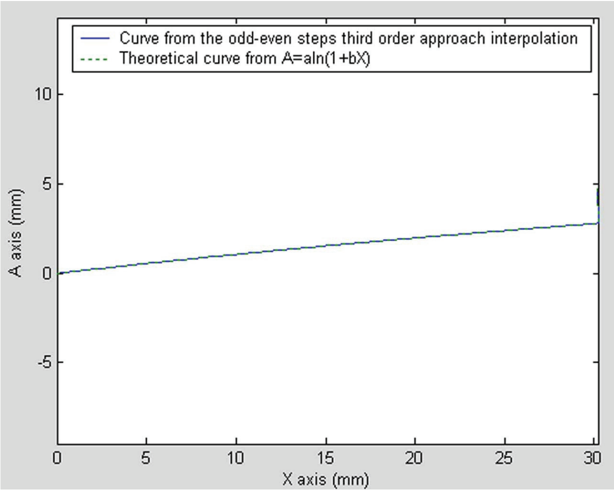

Fig. 3. Simulation charts from the odd-even third order approach interpolation and theoretical logarithmic curve.

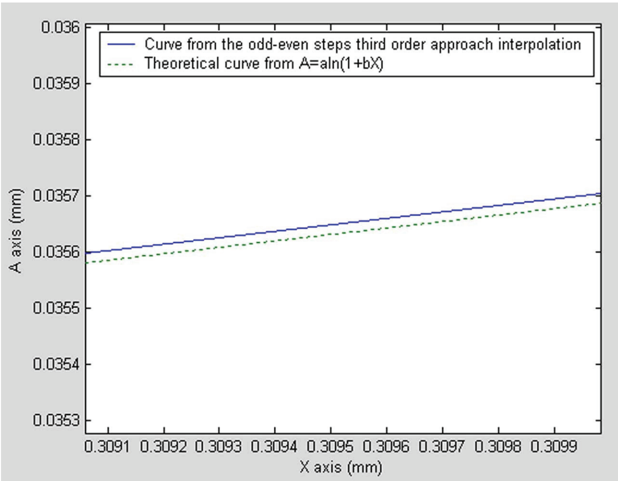

Fig. 4. Amplifying charts from the odd-even third order approach interpolation and theoretical logarithmic curve at the start part.

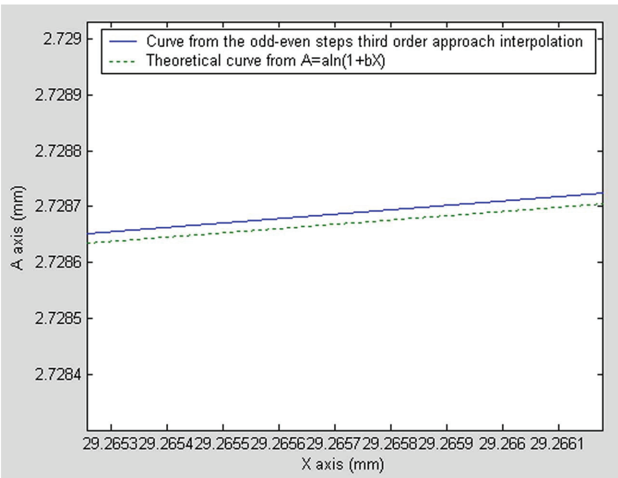

Fig. 5. Amplifying charts from the odd-even third order approach interpolation and theoretical logarithmic curve at the end part.

featured with high accuracy and can be directly executed by the micro-processor in the embedded system for multi-axis servo control in the mode of digital increments interpolation to meet modern CNC interpolation needs.

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Biographies:

Jiang Xinhua was born in 1956. He is a professor and specialized at fields of the industrial automation and computer application technology, the transportation information engineering and control.

Chen Xingwu was born in 1963. He is an associate professor and specialized at fields of the computer numeric control technology, and the transportation information engineering and control. He is the corresponding author of this paper.