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A New Perspective on Anti-Windup Design based on Experimental Results

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Abstract: Whereas stable linear systems with inputs have a unique steady-state solution, which is independent on the initial conditions and only depends on the input signals, nonlinear systems in general do not possess this property. Due to the lack of this property it is often hard to evaluate the exact behavior of the system, and thus to evaluate the 'exact' performance. In this article we present the results of three experiments performed on a simple nonlinear system, i.e. an anti-windup system consisting of a PI controlled integrator plant with input saturation and linear anti-windup controller. These results, which are validated by simulation and substantiated by theoretical results, clearly show the problems that prevent such an exact performance evaluation of the system. Based on these experimental results we come up with another definition of performance-based anti-windup design, based on the notion of convergent systems.

Keywords: Anti-Windup Design, Experimental Results, Nonlinear System Behavior, Performance, Convergent Systems

1. INTRODUCTION

Stable linear systems with inputs have the property that every solution of such a system, independent of the initial condition, converges after some transient time to a *unique* steady-state solution (not necessarily constant), which is independent on the initial conditions and only depends on the input signal(s). Nonlinear systems with inputs on the other hand, do in general not possess this property: several steady-state solutions may exist, and it depends on the initial conditions or system disturbances to which of these steady-state solutions the system converges. Due to this nonlinear behavior, it is often hard to predict the 'exact' performance of nonlinear systems. Although various $\mathcal{L}_p$ gains are often used as a performance measure for nonlinear systems, these measures generally do not describe the exact performance of the system: they provide a bound on the worst-case time-average behavior of the system.

In order to further investigate this nonlinear behavior, we focus in this article on an anti-windup system, since this is a relatively simple nonlinear system with inputs, and has many practical applications. The type of anti-windup system that we consider here is a PI controlled integrator plant with input saturation and linear anti-windup control. The presence of input saturation in the otherwise linear closed-loop system can cause a severe performance degradation, and may even have disastrous consequences such as the fighter crashes in April 1992 (see Dornheim [1992]) and August 1993 (see Shifrin [1993]). This performance degradation is caused by the so-called 'controller windup', and may be counteracted by adding an anti-windup controller to the system. In the past decades, the study on anti-windup controllers received a lot of attention and has resulted in many proposals for both linear and nonlinear anti-windup design, see e.g. Kothare et al. [1994], Teel and Kapoor [1997], Fromion and Scorletti [2000], Grimm et al. [2003], Galeani et al. [2004], Galeani and Teel [2006]. Most of these anti-windup designs, however, are based on minimizing an (incremental) $\mathcal{L}_2$ gain, and do not pursue the exact performance of the system.

In this paper we show the results of a series of experiments which we performed on the mentioned anti-windup system. These results, which we validate by simulation and substantiate by theoretical results, clearly indicate some of the problems in the nonlinear behavior of this system that prevent an exact performance evaluation. Based on these experiments we come up with another definition of anti-windup design, which is based on the notion of convergent systems, see Demidovich [1961], Yakubovich [1964], Pliss [1966], Demidovich [1967]. This definition follows the point of view originated in Fromion and Scorletti [2000] and later in van den Berg et al. [2006]. Using this definition for anti-windup design, a unique steady-state solution of the anti-windup is guaranteed and therefore the exact performance of the system can be found and optimized.

The outline of this paper is as follows. Section 2 describes the experimental setup and discusses the simulation and experimental results that have been obtained. The obtained results are theoretically substantiated in Section 3 and based on these results a new perspective on anti-windup design is given. Section 4 concludes the paper.

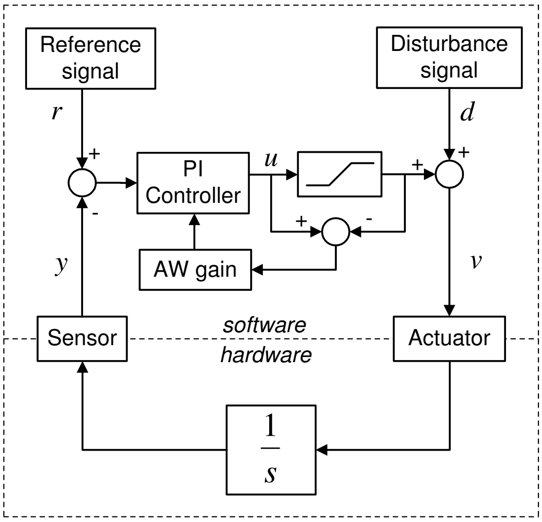

Fig. 1. Schematic representation of the experimental setup

2. EXPERIMENTS

In this section, first the experimental setup is described that we used during our experiments. Then, the experiments that we performed are described and the corresponding results are presented. Besides the experimental results, also numerical results are presented, which were obtained by computer simulations. The section is concluded with a discussion on the obtained results.

2.1 Experimental Setup

In order to investigate some typical nonlinear behavior, we considered a relatively simple nonlinear system, which is shown schematically in Fig. 1. This system consists of a PI controlled integrator plant with input saturation and a static anti-windup (AW) gain. As indicated in Fig. 1, the hardware of this setup (see Fig. 2) consists of an actuator (brushless DC motor) an integrator plant and a sensor (incremental encoder, 8192 counts/revolution). The hardware is connected (sample rate: 1kHz) to a computer with a Matlab Simulink model (Real Time Workshop), which contains the software elements described in Fig. 1; both the reference and disturbance signal, and the controller parameters are defined in this model. The actuator is driven by a velocity controller (not shown in Fig. 1), which receives its reference value v from the Simulink model. The actuator rotates a mass at the given speed, and the rotation angle of the mass is measured by the incremental encoder and fed back to the Simulink model. This transition from angular velocity to rotation angle forms the integrator plant.

2.2 Experiments and Results

In this subsection three experiments are described that we performed on the setup described in Section 2.1. The results of these experiments are presented and compared to simulation results that we obtained from a Simulink model. This simulation model has the same structure as in Fig. 1, except for the hardware components which are replaced by a numerical integrator block.

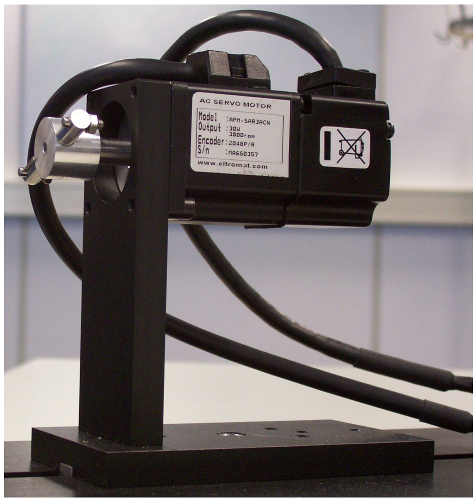

Fig. 2. Photo of the hardware construction

For all the experiments we used the following parameters for the PI controller: $K_P = 10$ and $K_I = 20$. These parameters have been chosen in such a way that the system without saturation has a satisfactory performance. Furthermore, the saturation function is defined as $\mathrm{sat(x)} = \mathrm{sign(x)} \min\{1, |x|\}$. Finally, the settling time of the actuator's velocity controller is negligible, so that we may assume that the motor follows the reference velocity v exactly.

Experiment 1: dependency on initial conditions Consider the system in Fig. 1 with reference signal $r = \sin(t)$, disturbance signal $d = 0$, and $K_A = 0$ (i.e. no anti-windup). The experiment and simulation are performed for two different initial conditions, i.e. the initial rotation angle of the mass is set to respectively $y(0) = 3$ and $y(0) = 4$ revolutions, while the initial value of the integrator in the PI controller is set to $I_{\mathrm{PI}}(0) = 0$ in both cases. The resulting rotation angle y as a function of time is shown for both instantiations in Fig. 3.

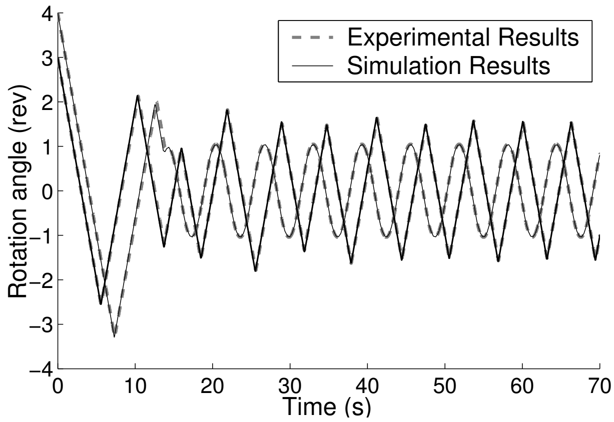

Fig. 3. Experiment 1, results for $K_A = 0$

We observe that the experimental and simulation results match very well, from which we can conclude that the hardware components can indeed be modeled accurately by a numerical integrator. Furthermore, we observe that there are two periodic steady-state solutions, and we can conclude that it depends on the initial conditions to which of these solutions the system converges. One can also see that one of the steady-state solutions follows the reference signal $r = \sin(t)$ quite well, while the other steady-state solution has a larger amplitude and is out of phase. This undesired solution is the result of a 'bang-bang'-control signal: the control signal u changes from large positive values to large negative values (and vice versa) and therefore is saturated most of the time, which results in a successive linear increase and decrease of the steady-state solution.

If, on the other hand, we slightly change the system, that is we apply an anti-windup gain $K_A = 0.5$, and repeat the experiment and simulation, then we obtain the results visualized in Fig. 4.

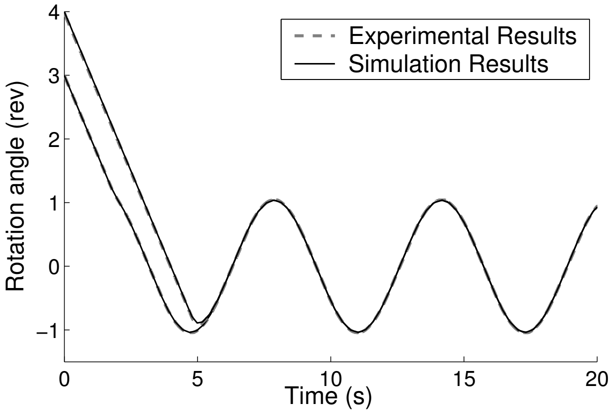

Fig. 4. Experiment 1, results for $K_A = 0.5$

Again the experimental and simulation results match very well, but now it seems that the undesired steady-state solution is no longer present, whereas the other steady-state solution has not changed. However, to be 100% certain that there do not exist other steady-state solutions anymore, we would have to evaluate the solution for all possible (i.e. an infinite amount of) initial conditions, which is practically not realizable by experiment or simulation. In Section 3 we will reconsider this problem. In the following experiments, we focus further on the situation with $K_A = 0$, where multiple steady-state solutions coexist.

Experiment 2: effect of disturbance Consider again the system in Fig. 1 with reference signal $r = \sin(t)$ and $K_A = 0$. If we now run the experiment and simulation for one initial condition ($I_{\mathrm{PI}}(0) = 0$ and $y(0) = 0$), and apply a disturbance that is described by

$$d(t) = \begin{cases} 1 \text{ if } & t \in [21, 21.4] \vee [73, 75] \\ 0 \text{ otherwise} \end{cases}$$

then we obtain the result in Fig. 5.

This result shows that even a relatively small disturbance can cause the system to switch from one steady-state solution to the other. The disturbance at $t = 21$ causes the system to move from the desirable steady-state solution to the undesirable steady-state solution, whereas the slightly larger disturbance at $t = 73$ forces the system back to the initial steady-state solution. Again the experimental and simulation results match very well.

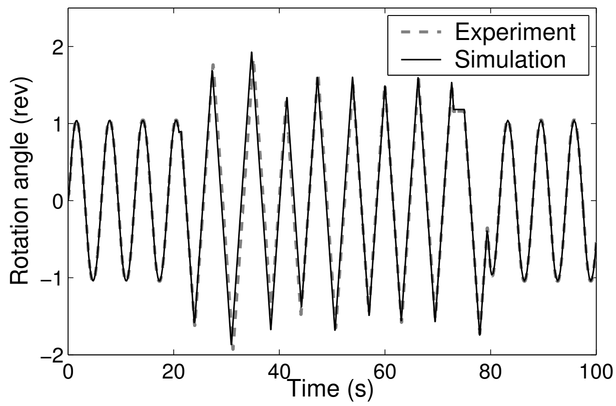

Fig. 5. Experiment 2, effect of disturbance

Experiment 3: 'hysteresis' The previous experiment showed that the system can switch between the different steady-state solutions as a result of a (possibly external) disturbance. Experiment 3 elaborates on this result. For this experiment we used again $K_A = 0$, disturbance $d = 0$ and initial condition $\{I_{\mathrm{PI}}(0) = 0, y(0) = 0\}$. We applied $r = b(t)\sin(t)$ with $b(0) = 0.1$ and slightly increased b every 200π seconds, so that after each increase of b steady-state is reached again. Once the amplitude was large enough, we slightly decreased b every 200π seconds until $b = 0.1$ again. Fig. 6 displays the resulting amplitude of the controller output u as a function of the amplitude b of the reference signal, once steady-stated is reached.

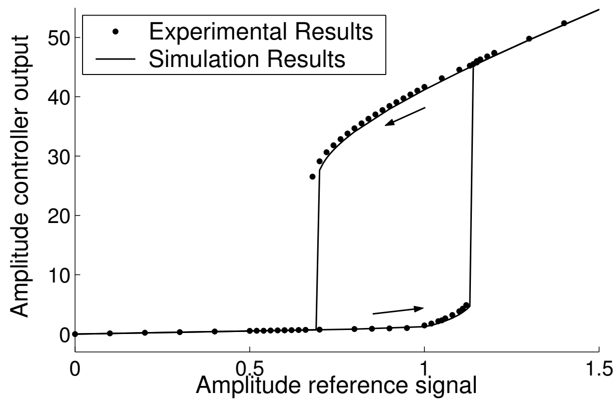

Fig. 6. Experiment 3, 'hysteresis'

We observe in Fig. 6 that for a small amplitude b there is only one amplitude for the controller output u, which suggests that there is only one steady-state solution. If b has a value between approximately 0.69 and 1.13, then u can have two different amplitudes, and thus there are (at least) two steady-state solutions. Finally, if b is larger then approximately 1.13, then there is only one steady-state solution left. We note again that we can not be 100% sure that there are no other steady-state solutions, based on the experiments and simulation alone. See Section 3 for further comments on this issue.

Another observation that we can make on Fig. 6 is that by slowly increasing b (starting from $b = 0.1$) the amplitude of the controller output u also slowly increases but does not switch to the other steady-state solution, until it makes a large jump at $b = 1.13$ and then slowly increases further. Then, if b is decreased again, the amplitude of u only slowly decreases and does not switch back to the other steady-state solution until $b = 0.69$. This gives Fig. 6 a hysteresis-like shape. During the experiment it was also noted that the closer the increasing amplitude b gets to 1.13 the smaller the required effort to switch from the desirable solution to the undesirable solution becomes (and vice versa for decreasing $b \to 0.69$). This suggests that the region of attraction of the desirable steady-state solution (small amplitude) decreases with increasing b, while the region of attraction of the undesired steady-state solution (large amplitude) decreases with decreasing b. In Experiment 2 we used $b = 1$, which is closer to 1.13 than to 0.69. This would imply a larger domain of attraction for the undesirable steady-state solution. Therefore, it makes sense that we required a larger disturbance to move from the undesirable steady-state solution to the desirable steady-state solution than vice versa.

2.3 Discussion

The three experiments described in Section 2.2 demonstrate some typical properties of nonlinear systems with inputs. Experiment 1 shows that multiple steady-state solutions may coexist that have a totally different performance, and that it depends on the initial conditions which of the steady-state solutions is followed by the system. Experiment 2 shows that once a certain (e.g. desired) steady-state solution is reached even a small disturbance can force the system to another (e.g. less desired) steady-state solution. Finally, Experiment 3 suggests that under certain circumstances the system can switch to another steady-state solution due to a very small disturbance, while the required effort to switch back to the initial steady-state solution may be huge.

Due to the properties described above it is hard to evaluate the performance of such a nonlinear system. Currently the performance of such systems is often estimated by an $\mathcal{L}_2$ gain. A disadvantage hereof is that this gain is an upper bound which only depends on the 'worst' steady-state solution in terms of $\mathcal{L}_2$ norms. The $\mathcal{L}_2$ gain does not indicate that other (desirable or undesirable) steady-state solutions may also exist. And although some anti-windup designs (e.g. Teel and Kapoor [1997]) guarantee that the steady-state solution for their system is unique (since it equals the solution of the corresponding *linear* system), this result only holds for external signals that eventually drive the system to the region of linear dynamics (i.e. within saturation bounds). For other external signals, these papers do not provide a proof that a unique steady-state solution exists, and hence the $\mathcal{L}_2$ gain can only be interpreted as an upper bound on the 'worst' performance.

In case the nonlinear system has a unique steady-state solution, the problems described above do not exist and performance can be evaluated in an exact way. As the results in Fig. 4 and Fig. 6 already suggested, nonlinear systems can have a unique steady-state solution under certain conditions. In Section 3 we discuss two theories that guarantee the existence and uniqueness of a (not necessarily constant) steady-state solution for the presented nonlinear system under certain conditions. Based on these theorems we present a new perspective on performance-based anti-windup design.

3. UNIQUE STEADY-STATE SOLUTION AND PERFORMANCE OF ANTI-WINDUP SYSTEMS

The system that has been investigated during the experiments presented in Section 2 can be described using the following state-space notation:

$$\begin{aligned} \dot{x} &= Ax - B\mathrm{sat}(u) + Fw \\ u &= Cx + Dw \\ y &= Hx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^2$ represents the state, $w = [d, r]^T \in \mathbb{R}^2$ are the system inputs, $u \in \mathbb{R}$ is the controller output, $y \in \mathbb{R}$ is the system output, and

$$A = \begin{bmatrix} 0 & 0 \\ -(1 - K_A K_P) & -K_I K_A \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -K_A \end{bmatrix},$$

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 1 - K_A K_P \end{bmatrix}, \quad C = [-K_P \ K_I],$$

$$D = [0 \ K_P], \quad H = [1 \ 0],$$

with $K_P = 10$ and $K_I = 20$.

In Section 2 we focussed mainly on the system with $K_A = 0$, i.e. the system without anti-windup control, and we observed that this system has undesirable properties.

In the past decades, many anti-windup designs have been proposed in literature to improve the performance of this nonlinear system. Most of them are based on (incremental) $\mathcal{L}_2$-gain approaches, and do not guarantee a unique steady-state solution for the closed-loop system. In this section, we discuss the theory that explains the behavior exhibited during the experiments, and focus on an anti-windup controller design that rules out the undesirable properties mentioned above by guaranteeing a unique steady-state solution for the closed loop system. For such closed-loop systems with a unique steady-state solution, a numerically efficient approach to evaluate the performance exists, which is based on harmonic linearization.

Note that for systems with an (eventually) *constant* reference signal and $K_P > 0$, $K_I > 0$, global asymptotic stability of the system can be proven in a standard way using for example absolute stability theory.

3.1 Theoretical substantiation of the experimental results

The results as presented in Section 2 can be substantiated and explained using two theoretic results: the notion of convergent systems (see e.g. Pavlov et al. [2004] and references therein) and the theory on harmonic balance equations for harmonically forced Lur'e systems as described in van den Berg et al. [2007]. We first give a definition of convergent systems.

Definition 1: Uniformly convergent systems
Consider system (1) and assume input w belongs to the class $\mathbb{PC}_m$ of piecewise continuous inputs $w(t) : \mathbb{R} \to \mathbb{R}^m$ which are bounded for all $t \in \mathbb{R}$. Then, system (1) is said to be *uniformly convergent* for a class of inputs $\mathcal{W} \subset \mathbb{PC}_m$ if for every input $w(t) \in \mathcal{W}$ there is a solution $\bar{x}(t) = x(t, t_0, \bar{x}_0)$ satisfying the following conditions:

(1) $\bar{x}(t)$ is defined and bounded for all $t \in (-\infty, +\infty)$,
(2) $\bar{x}(t)$ is globally uniformly asymptotically stable for every input $w(t) \in \mathcal{W}$.

This implies that if system (1) is uniformly convergent, then it has only *one* steady-state solution, i.e. $\bar{x}(t)$. Now consider the following theorem (van den Berg et al. [2006]).

Theorem 1. If $K_A K_P > 1$ then system (1) is uniformly convergent for all $w(\cdot) \in \mathcal{W}$, where $\mathcal{W}$ is defined as the class of inputs $w = [d, r]^T \in \mathbb{PC}_2$ for which $|d(t)| < 1$, $\forall t$ and r is a harmonic signal with bounded amplitude and finite frequency.

With Theorem 1 we can explain the results from Experiment 1. The system for which the results are presented in Fig. 4 has $K_A = 0.5$, $d = 0$, and $r = \sin(t)$. Therefore, all conditions of Theorem 1 are met and thus this system is uniformly convergent, which implies that the system has only one steady-state solution and all other solutions converge to this solution, as can be seen in Fig. 4. Although Theorem only gives sufficient (and not necessary) conditions for uniform convergency, there is no theoretical proof that the system in Experiment 1 with $K_A = 0$ is *not* convergent, but Fig. 3 clearly shows multiple steady-state solutions, which implies that the system is not convergent. More research is required to investigate how conservative the condition $K_A K_P > 1$ is.

To substantiate the results of Experiments 2 and 3, we need the following theory on harmonic linearization.
Harmonic linearization (see e.g. Khalil [2002]) is often used to study periodic solutions of a nonlinear system. To do so, the nonlinear system is approximated by a linear system; in our case system (1) is approximated by:

$$\begin{aligned} \dot{\xi} &= A\xi - BK\zeta + Fw \\ \zeta &= C\xi + Dw \end{aligned} \tag{2}$$

where the gain K is to be determined. If we suppose that $d(t) = 0$ and the reference input is given by

$$r(t) = b \sin \omega t, \tag{3}$$

then the steady-state output $\bar{\zeta}$ can be described by

$$\bar{\zeta}(t) = a \sin(\omega t + \psi), \tag{4}$$

for some amplitude $a > 0$ and phase ψ. Gain K can now be described as a function of the output amplitude a (see e.g. Khalil [2002]):

$$K(a) = \begin{cases} 1, & a \leq 1 \\ \dfrac{2}{\pi}\left(\sin^{-1}\left(\dfrac{1}{a}\right) + \dfrac{1}{a}\sqrt{1 - \dfrac{1}{a^2}}\right), & a > 1 \end{cases}$$

Amplitude a in turn is a function of the system parameters and amplitude b and frequency ω of the input signal, i.e. $a = a(b, \omega)$. This relationship between a and (b, ω) for system (1) with input (3) is given by the following harmonic balance equation (van den Berg et al. [2007]):

$$|1 + K(a)C(i\omega I_n - A)^{-1}B|^2 a^2 = |C(i\omega I_n - A)^{-1}F + D|^2 b^2. \tag{5}$$

As one can see, the right-hand side of (5) is completely known for given values of b and ω, while on the left-hand side of (5) amplitude a is the only unknown parameter. If we now consider the left-hand side of (5):

$$\pi(a) = |a + K(a)aG(i\omega)|^2$$

and fill in the frequency that was used during the experiments in Section 2, i.e. $\omega = 1$, then we can plot $\pi(a)$ as a function of a for the case that $K_A = 0$, see Fig. 7.

Observe that if amplitude b is small (and thus $\pi(a)$ is small), then there is only one solution for amplitude a. For

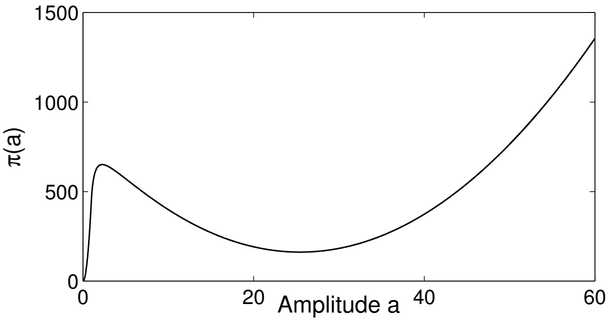

Fig. 7. $\pi(a)$ versus a for $\omega = 1$ and $K_A = 0$

larger values of b (and thus larger values of $\pi(a)$) there are multiple solutions for amplitude a, and if b increases even further there is again only one solution for amplitude a. Although the fact that there is one solution (resp. multiple solutions) for a of the harmonic balance equation does not prove that the nonlinear system (1) has a unique steady-state solution (resp. multiple steady-state solutions), it does make it plausible and it supports the findings of Experiment 3. In Experiment 2 we used amplitude $b = 1$, which corresponds to $\pi(a) = 500$. Fig. (7) shows that there are three solutions for a and thus system (1) probably has multiple steady-state solutions. Due to a large enough disturbance the solution can move into the domain of attraction of another steady-state solution, which explains the results of Experiment 2.

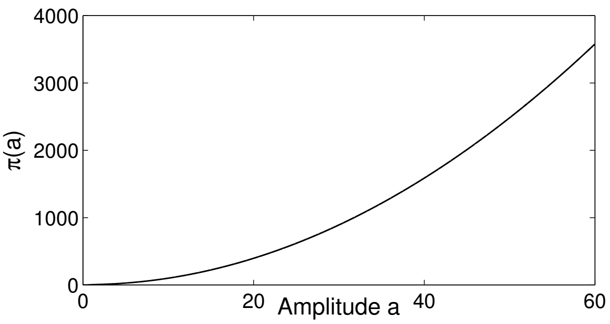

Fig. 8. $\pi(a)$ versus a for $\omega = 1$ and $K_A = 0.5 > 1/K_P$

Note that for $K_A > 1/K_P$ the function $\pi(a)$ becomes monotonically increasing in a for any value of ω, see Fig. 8, from which we can conclude that there is only one solution a for arbitrary ω and b. This is in accordance with Theorem 1, which states that this system with $K_A > 1/K_P$ is convergent.

3.2 Discussion: Performance-based anti-windup design

As stated before, most anti-windup design methods in literature are based on minimizing an (incremental) $\mathcal{L}_2$ gain. Although such an (incremental) $\mathcal{L}_2$ gain can provide valuable information on the worst-case time-average performance of a nonlinear system such as system (1), it does not give insight in the actual solution of the system, or the possibility that several steady-state solutions coexist. The rich dynamics, as indicated by the experiments in Section 2, is missed if a performance evaluation based on (incremental) $\mathcal{L}_2$ gain is used.

Based on the experimental results, we therefore propose a different definition for the performance-based design of anti-windup schemes, which consists of two steps:

(i) *Find conditions on the system parameters under which system* (1) *is (uniformly) convergent, i.e. it has a unique steady-state solution.*
Theorem 1 shows that for this case the required conditions are hardly restrictive.
(ii) *Within the range of parameter values for which the system is convergent, optimize the system's performance.*
Here, performance can be defined in terms of generalized frequency response functions (e.g. sensitivity function) as introduced by Pavlov et al. [2007], or any other desired (steady-state) performance measure.

Note that for convergent systems, the unique steady-state solution (and hence the exact performance) can be found using a *single* simulation run, due to the independency on initial conditions. For nonlinear systems in general this is not possible: an infinite amount of initial conditions should be evaluated in order to obtain a reliable analysis of the system's exact behavior.
Besides for performance *analysis*, simulation can also be used for *design*: a simulation-based optimization tool may be developed (in future work) to optimize in some sense the performance of the steady-state solution within the boundaries of the conditions for which the system is convergent. A disadvantage of the simulation-based performance analysis is that the simulations may be time-consuming under certain conditions. As a solution to this problem the authors presented in van den Berg et al. [2007] a numerically efficient method based on harmonic linearization to approximate the nonlinear system by a linear system and compute the error margins of the approximated solution.
Note that in this paper we focussed on steady-state performance. Transient performance, e.g. how fast the system converges to the steady-state solution, may be investigated as well using the notion of convergent systems, but this topic lies outside the scope of this paper.

The anti-windup system as presented in this article is only an example of the systems to which this performance-based design approach can be applied. The same approach can also be used for the performance-based design of various other nonlinear systems (e.g. more general anti-windup systems).

4. CONCLUSION

Nonlinear systems, such as the discussed anti-windup system, may exhibit multiple steady-state solutions, which causes trouble in defining the exact performance of such systems, i.e. performance based on the actual solution of the system, in contrast to performance which is based on an approximated bound such as (incremental) $\mathcal{L}_2$ gain.
In this paper we presented a new perspective on the anti-windup design: by first defining conditions which make the closed-loop system convergent, and then evaluating the performance of the unique steady-state solution, gives a better insight in the true behavior of the system. Simulation-based performance measures such as the generalized frequency response functions (see Pavlov et al. [2007]), or the numerically more efficient approach based on harmonic linearization (see van den Berg et al. [2007]) can be useful to optimize the performance of such systems.

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