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Operability Analysis for Process Systems with Recycle and Bypass Streams

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Abstract: Recycle and bypass streams exist in chemical plants in various configurations. They may lead to plantwide operability problems due to the strong interaction between process units. Based on the concept of dissipative systems, this paper provides a new approach to plantwide operability analysis for processes with by-pass and recycle streams, particularly on plantwide stability, stabilizability, and effects of disturbance. By using the topology of process interconnections, this analysis approach is developed from a network perspective and can be used for large scale process systems.

Keywords: Process control; Nonlinear systems; Plantwide operability analysis; Dissipativity; Process network

1. INTRODUCTION

Process operability determines whether a process can be effectively controlled using a feedback control system, e.g., the stabilizability and the extent of disturbance rejection a control system can achieve. It is an inherent property of the process, independent to the choice of controllers implemented. Ignoring operability considerations during process design may lead to costly retro-fitting and re-design if the process is found to be difficult to control during commissioning (Perkins [2001]).

A number of operability analysis tools have been developed in the literature, ranging from linear operability indicators such as process singular values (Morari [1983]), relative gain array (RGA) (Bristol [1966], Skogestad and Morari [1987]) and the condition numbers (Bahri et al. [1997]) to the nonlinear methods such as nonlinearity analysis (Guay et al. [1995]) and steady-state operability index (Vinson and Georgakis [2000]). More recently, the concept of passive systems was used to develop methods for analyzing the steady-state region of attraction Rojas et al. [2006] and nonlinear dynamic operability (Rojas et al. [2007a,b]). The above approaches are useful in analyzing the operability of stand-alone processes. However, in modern chemical plants which consist a large number of process units, it is often the coupling between process units that has the major effect on plantwide controllability.

In this paper, a new approach is developed to study how a process network (i.e., interconnections of process units) influences the controllability of the plantwide system, in particular, the effects of process recycle and by-pass streams on the entire process network dynamics and operability. Materials recycle and heat integration are now two major components in chemical plants. Recycle and bypass streams exist in chemical plants in various configurations. They may lead to plantwide operability problems due to the strong interaction between process units. Current methods of analysis involve mainly approaches that are based on heuristics and simulations (Kiss et al. [2007], Baldea et al. [2006], Baldea and Daoutidis [2007]). Based on the implication of process dissipativity on its operability, this work extends the results in Rojas et al. [2007b] and Moylan and Hill [1978] to provide a network perspective of plantwide process operability analysis. A distinctive feature of this approach is that process units are explicitly modeled with two sets of input/output variables: one set for the physical material flow (interconnecting flow) between the process units in the network and the other set include the rest of inputs and outputs (including both information flow and physical flow of the control system and disturbance). This allows the topology of the material flow (modeled using an interconnecting matrix), such as by-pass and recycle streams, to be used in plantwide operability analysis. Based on the (Q, S, R)-dissipativity of each process unit and the network topology, the stability, stabilizability and disturbance effects of recycle and by-pass streams can be analyzed. The network approach inherently scalable and can be applied to large process systems since it is only based on the model of each process unit rather than that of the entire plantwide system.

2. SYSTEM REPRESENTATION

The operability analysis approach presented in this paper involves a view of processes from network perspective. As shown in Figure 1, every single process unit (unit i) in the plantwide system is modeled to have two classes of inputs: (1) the input of physical material and energy flow $\tilde{u}_i$ between process units (2) inputs of other physical flow and the information flow, including the manipulated variables $\hat{u}_i$ and disturbances d_i. All the output variable of

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the process unit is denoted y_i, including the physical outlet flow from the unit and the independent measured process variables which cannot be derived from the physical outlet flow. This presents the interconnection of physical flow among the process units and information flow between the process units and the controller and enables their effects in a large network to be analyzed. Information flow is usually obtained from measuring device on a physical stream which is later used as process output variable from the point of view of the controller.

A network of process units is presented in Figure 2. The outputs of a system that are used in interconnection as the physical flow inputs to the other systems are represented as $\tilde{y}$. The relationship between $\tilde{y}$ and the system outputs y is given by:

$$\widetilde{y} = F_I\, y, \tag{1}$$

where F_I is an appropriate constant matrix with only one none-zero element of "1" in each row. It is used to form the interconnecting (physical) outputs of process units, $\tilde{y}$, as a subset of all outputs of the process units in the network, y. The measured outputs to be controlled are denoted by $\hat{y}$. These outputs are used as feedback signal sent to the controller and thus regarded as information flow of the network. The relationship between measured outputs $\hat{y}$ and overall network outputs y is given by:

$$\hat{y} = F_c\, y, \tag{2}$$

where F_c is a constant matrix which selects the outputs that are measured by the sensors as a subset of the overall outputs of the network of systems y. For those measured variables that relate to the physical output variables, then F_c also converts the physical output flow (e.g., extensive variables) into measured variables (e.g., intensive variables). These measured outputs are then used as the input signals to the controller. The controller is assumed to be a negative output feedback loop such that:

$$\begin{aligned} u_c &= -\hat{y} = -F_c\, y \\ \hat{u} &= y_c. \end{aligned} \tag{3}$$

For a plantwide process system, the interconnection between the physical flow (inputs and outputs) between different process units can be represented as:

$$\begin{aligned} \widetilde{u} &= -H\widetilde{y} \\ &= -HF_I\, y \\ &= -\widetilde{H}y, \end{aligned} \tag{4}$$

where H, called the interconnection matrix, relates the physical outputs of the process units to the physical inputs of other units in the network. Therefore H represents the topology of the process network.

3. OPERABILITY ANALYSIS

Dissipativity is an input-output property of nonlinear process systems and has been shown to relate to the achievable

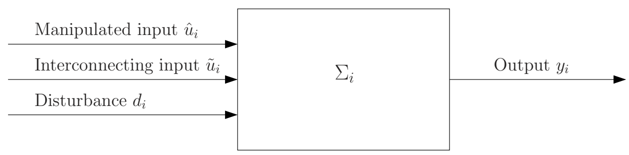

Fig. 1. Single System: Inputs and Outputs

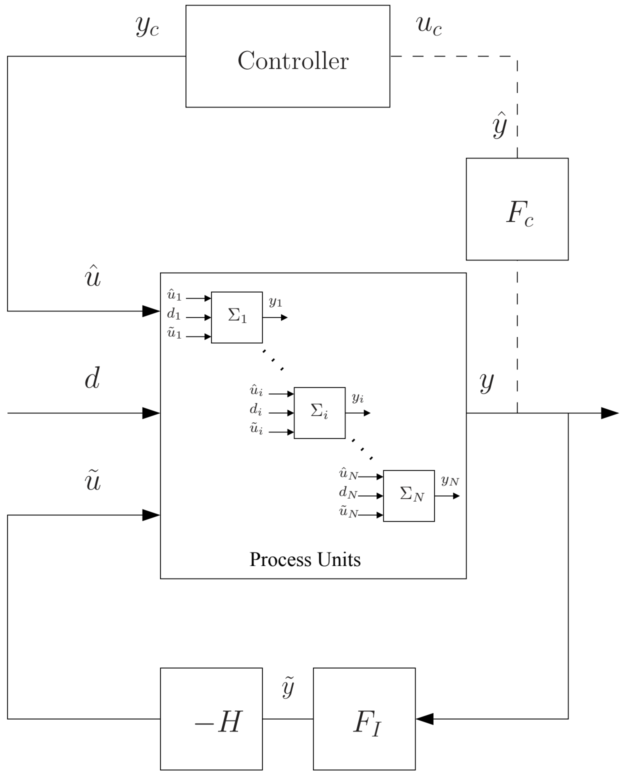

Fig. 2. Network of Systems with interconnected flow (continuous line) and information flow (dashed line)

dynamic performance of nonlinear processes (Rojas et al. [2007b]). In this paper, based on the topology of the material flow and dissipativity of process units, an approach for studying the effects of recycle and by-pass streams on the open-loop plantwide process stability, disturbance impacts and process stabilizability (via feedback controllers) is developed.

3.1 Dissipative Systems

The concept of dissipative systems was introduced by Willems [1972a,b] as an extension to the concept of passivity. The energy stored in a dissipative system is never greater than the amount supplied to it by the surroundings. In other words, dissipative systems can only dissipate but not generate energy.

Definition 1. (Dissipative Systems, Willems [1972a,b]). A dynamical system Σ is called dissipative if:

$$\phi(x(\tau)) - \phi(x_0) \leq \int_0^\tau w(u(t), y(t))\, dt, \tag{5}$$

where $\phi(x) : X \to \mathbb{R}^+$ is a nonnegative function of the system's states called the *storage function* and $w(u, y)$ is a real valued function of the system's inputs and outputs called the *supply rate*. The inequality in (5) formalizes the property of dissipative systems, which states that the increase in stored energy is never greater than the amount of energy supplied by the environment.

In this article, the considered supply rate follows the form of:

$$w(u, y) = y^T Qy + 2y^T Su + u^T Ru, \tag{6}$$

where $Q \in \mathbb{R}^{p\times p}$, $S \in \mathbb{R}^{p\times m}$, $R \in \mathbb{R}^{m\times m}$ are constant matrices with Q, and R symmetric. These type of systems are called (Q, S, R)-dissipative to emphasize the specific structure of the associated supply rate in (6). A comprehensive analytical framework for (Q, S, R)-dissipative systems was developed during the initial study by Hill and Moylan [1976, 1977, 1980]. An important feature of (Q, S, R)-dissipative systems is that the dissipation inequality in (5) describes the way the systems absorb energy, carrying additional information on both the (nonlinear counterpart of the) phase and gain of the process systems.

Assuming that the i-th process unit in the network is (Q_i, S_i, R_i)-dissipative with a differentiable storage function $\phi_i(x_i)$, inputs $[\hat{u}_i^T\ \widetilde{u}_i^T\ d_i^T]^T$, and outputs y_i as shown in Figure 1, i.e.:

$$\frac{d\phi_i(x_i)}{dt} \le y_i^T Q_i y_i + 2y_i^T S_i \begin{bmatrix}\hat{u}_i\\ \widetilde{u}_i\\ d_i\end{bmatrix} + \begin{bmatrix}\hat{u}_i^T & \widetilde{u}_i^T & d_i^T\end{bmatrix} R_i \begin{bmatrix}\hat{u}_i\\ \widetilde{u}_i\\ d_i\end{bmatrix}, \tag{7}$$

It is then possible to consider the overall storage function of the entire network of N-process units (subsystems) given by:

$$\phi(x) = \sum_{i=1}^{N} \phi_i(x_i). \tag{8}$$

Thus, by summing the dissipativity inequality of every process unit in the network, the dissipativity of the overall plant can be determined.

$$\frac{d\phi(x)}{dt} \le y^T \mathbf{Q}\, y + 2y^T \mathbf{S} \begin{bmatrix}\hat{u}\\ \widetilde{u}\\ d\end{bmatrix} + \begin{bmatrix}\hat{u}^T & \widetilde{u}^T & d^T\end{bmatrix} \mathbf{R} \begin{bmatrix}\hat{u}\\ \widetilde{u}\\ d\end{bmatrix}. \tag{9}$$

Matrices $\mathbf{S}$ and $\mathbf{R}$ can be split into submatrices corresponding to the subsets of the inputs ($\hat{u}$, $\tilde{u}$ and d):

$$\mathbf{S} = [\mathbf{S}_1\ \mathbf{S}_2\ \mathbf{S}_3], \quad \mathbf{R} = \begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_4 & \mathbf{R}_5\\ \mathbf{R}_4^T & \mathbf{R}_2 & \mathbf{R}_6\\ \mathbf{R}_5^T & \mathbf{R}_6^T & \mathbf{R}_3\end{bmatrix}. \tag{10}$$

The interconnection between the inputs and outputs of each process unit in the entire network can then be used to simplify the (Q, S, R)-dissipativity of the process network. By replacing (4) in (9), the network then obeys the following new inequality:

$$\begin{aligned}\frac{d\phi(x)}{dt} \le\, & y^T(\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H})y +\\ & 2y^T\begin{bmatrix}\mathbf{S}_1 - \widetilde{H}^T\mathbf{R}_4^T & \mathbf{S}_3 - \widetilde{H}^T\mathbf{R}_6\end{bmatrix}\begin{bmatrix}\hat{u}\\ d\end{bmatrix} +\\ & \begin{bmatrix}\hat{u}^T & d^T\end{bmatrix}\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_5\\ \mathbf{R}_5 & \mathbf{R}_3\end{bmatrix}\begin{bmatrix}\hat{u}\\ d\end{bmatrix}.\end{aligned} \tag{11}$$

In a special case of an open-loop system when the controller is not considered, the input vector $\hat{u}$ is empty. Hence, the dissipativity of the system can simply be written as:

$$\begin{aligned}\frac{d\phi(x)}{dt} \le\, & y^T(\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H})y +\\ & 2y^T\left[\mathbf{S}_3 - \widetilde{H}^T\mathbf{R}_6\right]d + d^T\mathbf{R}_3 d.\end{aligned} \tag{12}$$

The dissipativity of the overall process can be used to determine the stability and the disturbance effects on the open-loop plantwide process. Furthermore, it can infer the stabilizability of the entire process system.

3.2 Plantwide Stability

The stability of a (Q, S, R)-dissipative plantwide process systems can be inferred directly from the negative definiteness of matrix Q (Hill and Moylan [1976]). The formal result is given in the following theorem:

Theorem 2. (Hill and Moylan [1976]). Let Σ be (Q, S, R)-dissipative. Assume Σ is zero-state detectable (i.e., if $u(t) = 0$ and $y(t) = 0$ then $\lim_{t\to\infty} x(t) = 0$). The equilibrium $x = 0$ of the free system $\dot{x} = f(x)$ is Lyapunov stable if Q is negative semi-definite ($Q \le 0$) and asymptotically stable if the matrix Q is negative definite ($Q < 0$).

This result can be used immediately for a network of dissipative systems. According to (11), if

$$\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H} < 0 \tag{13}$$

then the open-loop process network (without the controller) has an asymptotically stable equilibrium $x = 0$.

3.3 Disturbance Effect on Output

The open-loop disturbance effects can be determined based the $\mathcal{L}_2$ gain of the plantwide process from disturbance to process outputs. The $\mathcal{L}_2$ gain of a nonlinear system can be estimated from its (Q, S, R)-dissipativity:

Theorem 3. (Moylan and Hill [1978]). Consider a nonlinear (Q, S, R)-dissipative system Σ with $Q < 0$. Assume $x_0 = 0$. Then Σ is finite gain $\mathcal{L}_2$ stable i.e.,

$$\|y_\tau\|_{\mathcal{L}_2} \le \gamma\ \|u_\tau\|_{\mathcal{L}_2}, \tag{14}$$

where the subscript τ denotes truncation, e.g.:

$$y_\tau(t) \triangleq \begin{cases} y(t), & 0 \le t \le \tau\\ 0, & t > \tau\end{cases} \tag{15}$$

The $\mathcal{L}_2$ gain γ of Σ is bounded by:

$$\gamma\ \le\ \|\widehat{Q}^{-\frac{1}{2}}\|\big(\alpha + \|\widehat{Q}^{-\frac{1}{2}}S\|\big), \tag{16}$$

where $\|\cdot\|$ is the (induced) matrix 2-norm (that is $\|A\| = \bar{\sigma}(A)$), $\widehat{Q} \triangleq -Q$ and $\alpha > 0$ is a finite scalar such that:

$$R + S^T\widehat{Q}^{-1}S - \alpha^2 I \le 0. \tag{17}$$

Here we need to quantify the disturbance effect on some process outputs. Consider specific process outputs of interest obtained from:

$$y_v = Vy, \tag{18}$$

where V is a constant matrix for selecting a subset of outputs of interest, y_v, from the entire output vector y. Since

$$y^TV^TVy - y_v^Ty_v = 0, \tag{19}$$

Inequality (12) can be rewritten as follows:

$$\begin{aligned}\frac{d\phi(x)}{dt} &\le y^T(\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H} + V^TV)y\\ &+ 2y^T(\mathbf{S}_3 - \widetilde{H}^T\mathbf{R}_6)d + d^T\mathbf{R}_3 d - y_v^Ty_v.\end{aligned} \tag{20}$$

Define:

$$\widehat{Q} \triangleq -(\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H} + V^TV). \tag{21}$$

Assume $\widehat{Q} > 0$. According to Theorem 2, this implies that the open-loop system is stable. Completing squares in (20) results in:

$$\frac{d\phi(x)}{dt} \le -\left\|\widehat{Q}^{\frac{1}{2}}\, y - \widehat{S}\, d\right\|^2 - y_v^Ty_v + d^T(\mathbf{R}_3 + \widehat{S}^T\widehat{S})d, \tag{22}$$

where

$$\widehat{S} \triangleq \widehat{Q}^{-\frac{1}{2}}(\mathbf{S}_3 - \widetilde{H}^T \mathbf{R}_6). \tag{23}$$

Thus the open-loop system with input d and output y_v satisfies the following inequality:

$$\frac{d\phi(x)}{dt} \le -y_v^T y_v + d^T(\mathbf{R}_3 + \widehat{S}^T \widehat{S})d. \tag{24}$$

According to Theorem 3, the open-loop system above has a finite $\mathcal{L}_2$ gain γ which is bounded by $\alpha > 0$, which in turn, satisfies the following condition:

$$\mathbf{R}_3 + \widehat{S}^T \widehat{S} - \alpha^2 I \le 0. \tag{25}$$

By means of the Schur complement in Boyd et al. [1994], the following linear matrix inequality (LMI) is constructed:

$$\begin{bmatrix} \mathbf{R}_3 - \alpha^2 I & \mathbf{S}_3^T - \mathbf{R}_6^T \widetilde{H} \\ (\mathbf{S}_3^T - \mathbf{R}_6^T \widetilde{H})^T & \mathbf{Q} - \mathbf{S}_2 \widetilde{H} - \widetilde{H}^T \mathbf{S}_2^T + \widetilde{H}^T \mathbf{R}_2 \widetilde{H} + V^T V \end{bmatrix} \le 0. \tag{26}$$

For a stable system *i.e.* $\widehat{Q} > 0$, the gain from the disturbance to the output is always bounded. This implies that the above LMI (or equivalently, Inequality (25)) can always be solved if the system is stable.

Treat α^2 as a single decision variable (and positive). The upper bound of the $\mathcal{L}_2$ gain of the open-loop system from the disturbance d to the process outputs of interest y_v can then be obtained by solving the following optimization problem:

Problem 4.

$$\min_{\alpha^2} \quad \alpha^2$$

subject to Inequality (26).

For chemical processes with recycle and and bypass streams, the above method can be used to quantify the effects of disturbances on the process outputs with different recycle or bypass ratios.

3.4 Stabilizability of Plantwide Processes

Chemical processes that involve chemical reaction are often unstable (e.g., exothermal reactions). It is necessary to employ controllers to stabilize these types of processes. Using the concept of Q, S, R-dissipativity, it is possible to determine whether the process can be stabilized by using a feedback controller (i.e., the stabilizability analysis).

Proposition 5. An unstable plantwide process Σ which is dissipative according to (11) can be stabilized by a (Q_c, S_c, R_c)-dissipative controller Σ_c (in negative feedback) if:

$$\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} < 0, \tag{27}$$

where:

$$\begin{aligned} \mathbf{Q}_{11} &= (\mathbf{Q} - \mathbf{S}_2 \widetilde{H} - \widetilde{H}^T \mathbf{S}_2^T + \widetilde{H}^T \mathbf{R}_2 \widetilde{H} + F_c^T R_c F_c), \\ \mathbf{Q}_{12} &= (\mathbf{S}_1 - F_c^T S_c^T - \widetilde{H}^T \mathbf{R}_4^T), \\ \mathbf{Q}_{21} &= \mathbf{Q}_{12}^T, \\ \mathbf{Q}_{22} &= (\mathbf{R}_1 + Q_c). \end{aligned} \tag{28}$$

Proof. The addition of the control feedback loop to the system results in a network of two systems. Assume Σ_c is (Q_c, S_c, R_c)-dissipative:

$$\frac{d\phi_c(x_c)}{dt} \le y_c^T Q_c y_c + 2y_c^T S_c u_c + u_c^T R_c u_c. \tag{29}$$

Using feedback relationship in (3), the network of dissipative systems with dissipative controller can be represented by the following inequality:

$$\begin{aligned} \frac{d\hat{\phi}(\hat{x})}{dt} \le & y^T(\mathbf{Q} - \mathbf{S}_2 \widetilde{H} - \widetilde{H}^T \mathbf{S}_2^T + \widetilde{H}^T \mathbf{R}_2 \widetilde{H})y + \\ & 2y^T \begin{bmatrix} \mathbf{S}_1 - \widetilde{H}^T \mathbf{R}_4^T & \mathbf{S}_3 - \widetilde{H}^T \mathbf{R}_6 \end{bmatrix} \begin{bmatrix} \hat{u} \\ d \end{bmatrix} + \\ & \begin{bmatrix} \hat{u}^T & d^T \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_5 \\ \mathbf{R}_5 & \mathbf{R}_3 \end{bmatrix} \begin{bmatrix} \hat{u} \\ d \end{bmatrix} + \\ & y_c^T Q_c y_c + 2y_c^T S_c u_c + u_c^T R_c u_c, \end{aligned} \tag{30}$$

where $\hat{x} = \begin{bmatrix} x^T & x_c^T \end{bmatrix}^T$, and $\hat{\phi}(\hat{x}) = \phi(x) + \phi_c(x_c)$. Replacing the relationship (3) in the above expression, the new inequality is obtained:

$$\begin{aligned} \frac{d\hat{\phi}(\hat{x})}{dt} \le & \begin{bmatrix} y^T & y_c^T \end{bmatrix} \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} \begin{bmatrix} y \\ y_c \end{bmatrix} \\ & + 2 \begin{bmatrix} y^T & y_c^T \end{bmatrix} \begin{bmatrix} \mathbf{S}_3 - \widetilde{H}^T \mathbf{R}_6 \\ \mathbf{R}_5 \end{bmatrix} d \; + \; d^T \mathbf{R}_3 \, d. \end{aligned} \tag{31}$$

where $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{21}$ and $\mathbf{Q}_{22}$ are given in (28). The unstable system Σ can be stabilized if there exists a set of (Q_c, S_c, R_c) such that the modified Q in (31) is negative definite as shown in Theorem 2. $\Diamond\Diamond\Diamond$

4. RECYCLE AND BYPASS

Recycle and bypass streams are common configurations used in chemical processes. Reactant in chemical reactor is often recycled back to reduce production cost, while the bypass configuration is commonly used in the control of heat exchanger networks. It is well-known that recycle streams can cause stability issues in chemical processes. Snowball effect is one of the major problem encountered in process with recycle streams (Kiss et al. [2007]). In most cases, the recycle streams also contain some inert materials. This then requires the addition of purge streams in order to eliminate the accumulation of the inert materials in the process network. With this configuration of recycle and purge streams, processes become more complex and exhibit multiple time-scale dynamic behaviors (Baldea et al. [2006], Baldea and Daoutidis [2007]). Unlike recycle streams, bypass streams do not cause stability issues. Both recycle and bypass may have significant impact on plantwide operability. In this section, the methods discussed in Section 3 are used as analytical tools to assess operability of chemical processes with recycle and by-pass streams.

To use the proposed techniques of operability analysis in this paper, every system needs to be transformed into the representation discussed in Section 2. The inputs to a single system needs to be distinguished into three subsets, namely the disturbances d, the manipulated inputs $\hat{u}$ and the interconnecting inputs $\tilde{u}$ as shown in Figure 1. After each system has been transformed into this representation, a network approach as shown in Figure 2 can then be used describe the interconnection of every single systems as well as with the controller. Following these steps, the operability analysis can then be performed on the network of chemical processes.

The (Q, S, R)-dissipativity of a system can be established via various methods based on nonlinear control theory.

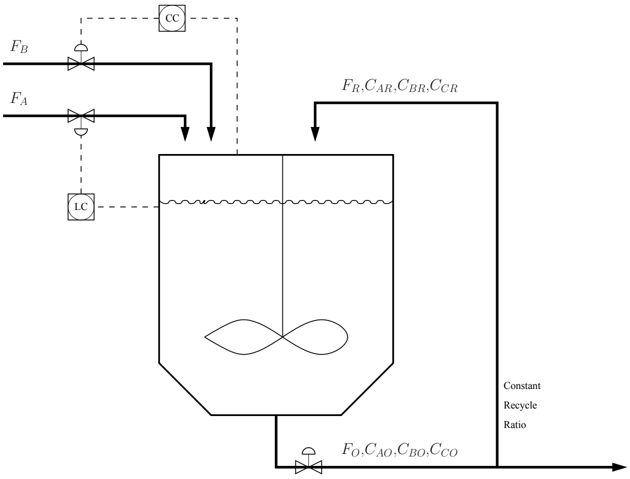

Fig. 3. CSTR with recycle stream

For chemical process it is possible to determine their dissipativity based on the process knowledge, e.g., based on process thermodynamics and conservation of mass (Alonso and Ydstie [1996], Ydstie and Alonso [1997], and Farschman et al. [1998]).

In order to give better understanding of the approach proposed in this paper, an illustrative example on a chemical process with recycle stream is given as follows:

Example 6. Consider an endothermic CSTR with three input streams, the inlet flows of component A (F_A) and component B (F_B) and recycle flow (F_R) (as shown in Figure 3). The reaction involved in the reactor is given by:

$$A + B \rightarrow C \tag{32}$$

A controller is implemented to control the liquid level in the reactor by manipulating the flowrate F_A. Another controller is employed to control the concentration of product C in the reactor by manipulating flowrate F_B. In this example, it is assume temperature is not considered for simplicity. However, the it will be discussed in the later part of the paper.

To analyze the stability of the open-loop system, the controller is ignored. The inlet flows F_A and F_B are then considered as disturbances to the open-loop system. The process is transformed into an appropriate representation as shown in Figure 4. Since there are no manipulated inputs in the open-loop system, there exist only two subsets of inputs, the disturbances d and the interconnecting input $\tilde{u}$. Observe that the disturbances are the external inlet flows, and the interconnecting input is the inlet flow from the recycle stream. The outputs of the process consists of several variables, the liquid level h, outlet flow rate F_O, and other variables, such as temperatures, and pressures which are not considered at this stage. This representation of the process allows the use of network analysis framework proposed in this paper. Figure 5 describes the network representation of the process. It is now possible to do open-loop stability analysis, and effect of changes in the external inlet flow (d) to the outputs of the process. In this case, it is assumed that the recycle ratio is constant at K_R for simplicity. Observe that outlet flow rate and the concentration of the components are the outputs interconnecting to the inputs back into the reactor via recycle. It is obvious

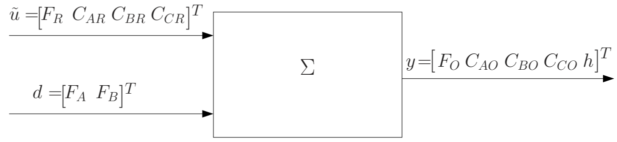

Fig. 4. System Representation of Reactor with Recycle

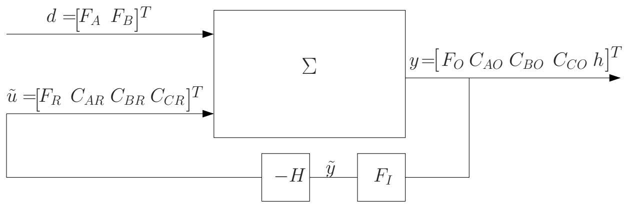

Fig. 5. Network Representation of Reactor with Recycle

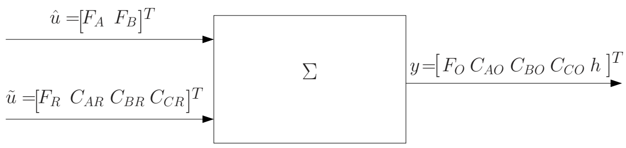

Fig. 6. System Representation of Reactor with Recycle (with controller)

from Figure 5 that the interconnection matrix H has the first diagonal entry of $-K_R$. The stability of the open-loop system can then be simply assessed by evaluating the negative definiteness of $(\mathbf{Q} - \mathbf{S}_2\widetilde{H} - \widetilde{H}^T\mathbf{S}_2^T + \widetilde{H}^T\mathbf{R}_2\widetilde{H})$ in (12). It has been shown that the recycle ratio K_R affects the interconnection matrix H. This implies that the selection of recycle ratio K_R has direct effect on the stability of the open-loop system. This technique of analysis can be used to assess the stability of the process during early stage of process design due to the fact that the recycle ratio K_R is considered explicitly. Different recycle ratios can then be tried to optimize the plant objectives while stability condition can be ensured.

If the stability of the process can be established via its dissipativity, it is also interesting to quantify the effect of disturbances (in this case, variation in external inlet flows A and B) to the outputs. By solving Problem 4, the $\mathcal{L}_2$ gain of the outputs from the disturbance can be quantified.

If the control loop is considered in the analysis, then it is possible to assess the stabilizability of an unstable process. In this case, the process must be transformed into a new representation as shown in Figure 6. The external inlet flow F_A and F_B have now become the manipulated input and are used to control the liquid level and the concentration of C in the reactor. The network representation can then be used to interconnect the process and the recycle stream with the controller. This configuration is shown in Figure 7. From the network representation, it is then possible to analyze the stabilizability of the process with a controller. The appropriate controller can be obtained via Proposition 5.

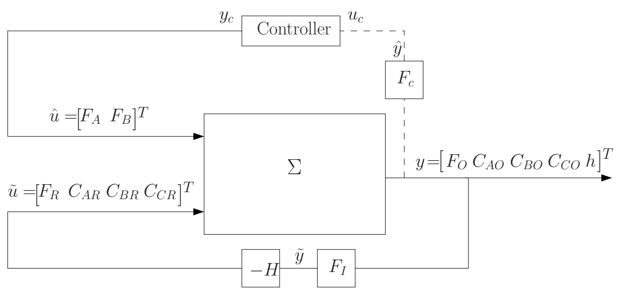

Fig. 7. Network Representation of Reactor with Recycle (with controller)

If the considered reactor is exothermic and includes a cooling water jacket, and temperature controller is necessary. In this case, the network representation can be expanded to allow proposed operability analysis.

Here we illustrated the proposed analysis approach using an example of a system with a recycle stream. The same method can be applied to processes with bypass streams (e.g., heat exchanger networks) in a similar manner.

5. CONCLUSION

Based on dissipativity and process network topology, an approach to plantwide operability analysis has been developed in this paper. Unlike many existing methods that are only based on heuristics and simulations, the proposed approach is an analytical method for plantwide stability, stabilizability and disturbance effects. The primary advantage of this technique lies in its scalability. It can be used for complex process systems with a large number of process units.

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