

# Stabilization for Networked Control Systems with Nonlinear Perturbation

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**Abstract:** This paper discusses the stabilization controller designs for a class of networked control systems with nonlinear perturbation. Under consideration of both network-induced delay and the data packet dropout in the transmission, a state feedback controller and a static output feedback controller are constructed, respectively. In addition, the effectiveness of our approach is demonstrated by two numerical examples.

# 1. INTRODUCTION

Feedback control systems wherein the control loops are closed through a real-time network are called networked control systems (NCS). Recently, extensive research has been devoted to NCS due to it's great advantages, including simpler installation, higher reliability, ease of system diagnosis and maintenance and increased system agility. See Hu [2003], Azimi-Sadjadi [2003], Liu [2003]. At the same time, some new analytical challenges are raised because the insertion of the communication network in the feedback control loop makes the analysis and design of an NCS complex: conventional control theory must be revalued before it can be applied to NCS. In an NCS, several important issues needed to be treated, which include the network-induced delay, data dropout and multiplepackets transmission of plant output. The delay occurs while exchanging data among the devices connected to the shared medium, it may be constant, time varying, or even random.

A basic problem in an NCS is its stability and stabilization. Recently, much research work has been done on the stability for networked control systems, see Zhang [2001]. Kim [2003], Nilsson [1998], Branicky [2002]. By means of a hybrid systems technique, stability for NCS with constant delays less than the sampling period reduces to examine the Schur-ness of the corresponding matrix in Zhang [2001]. The try-once-discard (TOD) control network protocol is introduced and a criterion for global exponential stabilization presented for an NCS in Walsh [2002]. In Montestruque [2004], an NCS is addressed in which transmission times are varying within a time interval or driven by a stochastic process with identically independently distributed and Markov-chain driven transmission times. A new method for stabilization analysis for an NCS is proposed. However, it can only be used to treat the systems with sensor-to-controller delay case. Considering

time varying delay, output feedback controller based LMI is proposed to guarantee the system stability and reduce the state error in Jung [2004]. In Yue [2004], state feedback controller is designed under consideration of both the networked-induced delay and data packed dropout, and the feedback gain can be derived by solving a set of LMIs. In Liu [2003], the problem of state feedback control of networked systems with an uncertain plant is considered and a necessary and sufficient condition for the robust exponential stability of the closed loop system are derived. However, most of previous work is based on discrete-time model, and the information on the intersample behavior may be lost when discretizing the continuous-time plant, as pointed in Yue [2004]. In addition, the plants mentioned above are linear and nonlinear plants are seldom addressed in the literature.

The objective of this paper is to discuss the state and static output feedback controller designs for a class of nonlinear networked control systems. Some sufficient conditions for stabilization for the systems is presented by means of LMI. In addition, both state feedback controller and static output feedback controller are constructed provided the corresponding LMI is feasible.

Compared with the existing results in the literature, this paper discusses the more general class of systems than those in Yue [2004], Mu [2004]. In addition, our proposed method is in the continuous-time domain and the intersample behavior is considered. Moreover, a state feedback controller design in Yue [2004] is a special case of this paper. Finally, as shown by the numerical examples, our results are less conservative than those in Zhang [2001], Yue [2004].

**Notation.**  $\mathbf{R}^n$  denotes the *n*-dimension Euclidean space,  $\mathbf{R}^{m \times n}$  is the set of  $m \times n$  real matrix, *I* is identity matrix of appropriate dimensions, *W'* denotes transpose of matrix *W*.  $||x|| = \sqrt{x'x}$ , where  $x = (x_1 \ x_2 \ \cdots \ x_n)' \in \mathbf{R}^n$ . Throughout this note, for symmetric matrices *X* and *Y*,  $X \ge Y$  (respectively, X > Y): X - Y is positive

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semi-definite (respectively, positive definite),  $X \leq Y$  (respectively, X < Y): X - Y is negative semi-definite (respectively, negative definite). If  $\begin{pmatrix} A & * \\ B & C \end{pmatrix}$  is a real symmetric matrix, then \* denotes the entries implied by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. MODELLING OF NETWORKED CONTROL SYSTEMS

Consider an NCS described by the following continuous nonlinear system.

$$\dot{x}(t) = Ax(t) + Bu(t) + Hf(t, x(t), u(t)),$$
  
 $y(t) = Cx(t),$ 
(1)

where  $x(t) \in \mathbf{R}^n$  is the state vector,  $u(t) \in \mathbf{R}^m$  is the input,  $y(t) \in \mathbf{R}^q$  is the output of the system; A, B, C and H are constant matrices with appropriate dimensions; f = f(t, x, u) is a vector-valued nonlinear function which is regarded as a nonlinear perturbation and satisfies the following quadratic inequality for all (t, x, u).

$$f'(t, x, u)f(t, x, u) \le {\binom{x}{u}}' {\binom{M'}{N'}} (M \quad N) {\binom{x}{u}}, \quad (2)$$

where  ${\cal M}$  and  ${\cal N}$  are constant matrices with appropriate dimensions.

In this paper, we consider the case where the sensor is clock-driven, the controller and actuator are event driven, and the data are transmitted with a single packet. Under consideration of both network-induced delay and the data packet dropout in the transmission, the NCS can be modeled as

$$\dot{x}(t) = Ax(t) + Bu(i_kh) + Hf(t, x(t), u(i_kh)),$$

$$y(t) = Cx(t),$$

$$t \in [i_kh + \tau_k, i_{k+1}h + \tau_{k+1}), k = 1, 2, \cdots,$$
(3)

where h is the sampling period,  $i_k(k = 1, 2, \cdots)$  are integers and  $\{i_1, i_2, \cdots\} \subset \{1, 2, \cdots\}, \tau_k$  is networked induced delay. Suppose  $t_0$  is the instant when the first control signal reaches the plant and there exists a constant  $\eta$  such that

$$(i_{k+1} - i_k)h + \tau_{k+1} \le \eta, \ k = 1, 2, \cdots.$$
 (4)

Remark 2.1. Since  $\{i_1, i_2, \cdots\}$  is a subset of  $\{1, 2, \cdots\}$ , thus the effect of data packet dropout is considered. Specially, if  $\{i_1, i_2, \cdots\} = \{1, 2, \cdots\}$ , it means that no data packet dropout occurs in the transmission.

Remark 2.2. If  $i_k = k$ , then (4) implies  $h + \tau_{k+1} \leq \eta$ , it can conclude from this relation that faster sampling can allow for a larger networked-induced delay. Moreover, for the given sampling period h and constant  $\eta$ , we can also determine the maximum allowable size of the delay.

## 3. STATE FEEDBACK CONTROLLER DESIGN

In this section, we assume that the sensor has the full state vector available and consider the following form of linear state feedback controller

$$u(t) = Kx(t), \tag{5}$$

where K is the constant matrix gain to be determined.

Then the system (3) can be rewritten as

$$\dot{x}(t) = (A + BK)x(t) - BKe(t) + Hg(t, x(t), e(t)),$$
  

$$t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \quad k = 1, 2, \cdots,$$
(6)  

$$x(t) = \phi(t), \quad t \in [t_0 - \eta, t_0],$$

where  $e(t)=x(t)-x(i_kh)$  ,  $\phi(t)$  is a continuous initial function of the system.  $g(t,x(t),e(t))=f(t,x(t),Kx(i_kh))$  satisfies

$$g'(t, x, e)g(t, x, e) \leq {\binom{x}{e}}' {\binom{M' + K'N'}{-K'N'}} \times (M + NK - NK) {\binom{x}{e}},$$
(7)

Definition 3.1. The system (6) is said to be exponentially asymptotically stable if there exist constants  $\alpha > 0$  and  $\beta > 0$  such that  $||x(t)|| \leq \alpha sup_{t_0-\eta \leq s \leq t_0} ||\phi(s)|| e^{-\beta t}$ ,  $t \geq t_0$ .

The purpose of this section is to find a controller in the form of (5) such that the closed-loop system (6) is exponentially asymptotically stable.

The following theorem presents a way to construct state feedback controller law (5), in which sufficient condition is presented by means of LMI.

Theorem 1. The system (6) is exponentially asymptotically stable if there exist positive matrices X > 0, S > 0 and a real matrix Y such that the following LMI holds

$$\begin{pmatrix} \Omega_1 & * & * & * & * \\ -Y'B' & -2\gamma X + \gamma S & * & * & * \\ H' & 0 & -I & * & * \\ AX + BY & -BY & H & -\gamma S & * \\ MX + NY & -NY & 0 & 0 & -I \end{pmatrix} < 0, (8)$$

where  $\Omega_1 = AX + XA' + BY + Y'B', \gamma = \eta^{-1}$ . Furthermore, the controller parameter K is given by  $K = YX^{-1}$ .

**Proof.** Let  $P = X^{-1}, Q = S^{-1}$ , and multiplying both sides of LMI (8) with  $diag\{X^{-1}, X^{-1}, I, I, I\}$ , and noticing  $K = YX^{-1}$ , (8) is equivalent to

$$\begin{pmatrix} \Omega_2 & * & * & * & * \\ -K'B'P & \Omega_3 & * & * & * \\ H'P & 0 & -I & * & * \\ A+BK & -BK & H & -\gamma Q^{-1} & * \\ M+NK & -NK & 0 & 0 & -I \end{pmatrix} < 0, \quad (9)$$

where  $\Omega_2 = PA + A'P + PBK + K'B'P$  and  $\Omega_3 = -2\eta^{-1}P + \eta^{-1}PQ^{-1}P$ .

By means of Schur Complement Lemma, LMI (9) is equivalent to the following matrix inequality

$$\begin{pmatrix} PA_0 + A'_0P + M'_0M_0 & * & * & * \\ -K'B'P + N'_0M_0 & N'_0N_0 & * & * \\ H'P & 0 & -I & * \\ A_0 & -BK & H & -\gamma Q^{-1} \end{pmatrix} < 0,(10)$$

where  $A_0 = A + BK$ ,  $M_0 = M + NK$ ,  $N_0 = -NK$ . By Schur Complements, LMI (10) is equivalent to  $\Gamma < 0$ , where  $\Gamma$  is defined as

$$\Gamma = \begin{pmatrix} \Gamma_{11} & * & * \\ \Gamma_{21} & \Gamma_{22} & * \\ H'P + \eta H'QA_0 & -\eta H'QBK & \eta H'QH - I \end{pmatrix}, (11)$$

where

$$\Gamma_{11} = PA_0 + A'_0 P + \eta A'_0 Q A_0 + M'_0 M_0,$$
  

$$\Gamma_{21} = -K' B' P - \eta B K' B' Q A_0 + N'_0 M_0,$$
(12)

$$\Gamma_{22} = -2\eta^{-1}P + \eta^{-1}PQ^{-1}P + \eta K'B'QBK + N'_0N_0.$$

In order to obtain the stability for system (6), we construct the following Lyapunov functional candidate

$$V(t) = x'(t)Px(t) + \int_{t-\eta}^{t} \int_{s}^{t} \dot{x}'(v)Q\dot{x}(v)dvds.$$
 (13)

Then for  $t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1})$ , we can get the derivation of V(t) along with system (6)

$$\dot{V}(t) = 2x'(t)P\dot{x}(t) + \eta \dot{x}'(t)Q\dot{x}(t) - \int_{t-\eta}^{t} \dot{x}'(s)Q\dot{x}(s)ds.$$
 (14)

From (4), it can be seen that, when  $t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$ 

$$e'(t)Qe(t) = \int_{i_{k}h}^{t} \dot{x}'(s)dsQ\int_{i_{k}h}^{t} \dot{x}(s)ds$$
  
$$= \|\int_{i_{k}h}^{t} Q^{\frac{1}{2}}\dot{x}(s)ds\|^{2}$$
  
$$\leq (\int_{i_{k}h}^{t} \|Q^{\frac{1}{2}}\dot{x}(s)\|ds)^{2}$$
  
$$\leq (t - i_{k}h)\int_{i_{k}h}^{t} \|Q^{\frac{1}{2}}\dot{x}(s)\|^{2}ds$$
  
$$\leq \eta \int_{t-\eta}^{t} \dot{x}'(s)Q\dot{x}(s)ds$$
  
(15)

Combining (14) and (15), we have

$$\dot{V}(t) \leq 2x'(t)P\dot{x}(t) + \eta\dot{x}'(t)Q\dot{x}(t) - \eta^{-1}e'(t)Qe(t) 
\leq 2x'(t)P\dot{x}(t) + \eta\dot{x}'(t)Q\dot{x}(t) 
+ \eta^{-1}e'(t)[-2P + PQ^{-1}P]e(t) 
-g'(t,x(t),e(t))g(t,x(t),e(t)) 
+ {x(t) \choose e(t)}' {M_0 \choose N_0} {M_0 \choose N_0}' {x(t) \choose e(t)} 
= {x(t) \\ e(t) \\ g(t,x,e(t))}' \Gamma {x(t) \\ e(t) \\ g(t,x,e(t))}.$$
(16)

Then

$$\dot{V}(t) \le -\lambda \|x(t)\|^2 - \lambda \|e(t)\|^2,$$

where  $\lambda = \lambda_{min}(-\Gamma)$ .

Defining a new function as

$$W(t) = e^{\varepsilon t} V(t),$$

then using the similar analysis method developed in Mao [1998], there exist a sufficiently small constant  $\varepsilon > 0$  and a constant  $\alpha > 0$  such that

$$V(t) \le \alpha \ \sup_{t_0 - \eta \le s \le t_0} \|\phi(s)\| e^{-\varepsilon t}, \quad t \ge t_0.$$
(17)

Noticing that

$$V(t) \ge \lambda_{min}(P) \|x(t)\|^2, \quad t \ge t_0,$$
 (18)

then we have

$$\|x(t)\| \le \mu \ \sup_{t_0 - \eta \le s \le t_0} \|\phi(s)\| e^{-\frac{\varepsilon}{2}t}, \quad t \ge t_0,$$
(19)

where  $\mu = (\lambda_{\min}^{-1}(P)\alpha)^{\frac{1}{2}}$ .

By Definition 3.1, system (6) is exponentially asymptotically stable, this completes the proof.

Remark 3.1. (3) is a generalized nonlinear networked control system, the following linear system is its spacial case.  $\dot{x}(t) = (A + BK)x(t) - BKe(t),$ 

$$t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \quad k = 1, 2, \cdots,$$
 (20)

$$x(t) = \phi(t), \ t \in [t_0 - \eta, t_0].$$

*Remark 3.2.* Theorem 1 can be regarded as an extension of Theorem 1 in Yue [2004], where the system (20) is considered.

In Theorem 1, assume H = 0, M = 0 and N = 0, then the following result presents a way to construct state feedback controller law (5) such that system (20) is exponentially asymptotically stable.

Corollary 2. The system (20) is exponentially asymptotically stable if there exist positive matrices X > 0, S > 0 and a real matrix Y such that the following LMI holds

$$\begin{pmatrix} \Omega_4 & * & * \\ -Y'B' & -2\eta^{-1}X + \eta^{-1}S & * \\ AX + BY & -BY & -\eta^{-1}S \end{pmatrix} < 0, \quad (21)$$

where  $\Omega_4 = AX + BY + XA' + Y'B'$ . Furthermore, the controller parameter K is given by  $K = YX^{-1}$ .

## 4. STATIC OUTPUT FEEDBACK CONTROLLER DESIGN

In this section, we consider a static output feedback controller

$$u(t) = Gy(t), \tag{22}$$

where G is the constant matrix gain to be determined. In this case, the closed-loop system is given by

$$\dot{x}(t) = (A + BGC)x(t) - BGCe(t) + Hh(t, x(t), e(t)),$$

$$t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}), \quad k = 1, 2, \cdots,$$
(23)

where  $h(t, x(t), e(t)) = f((t, x(t), GCx(i_kh))).$ 

The purpose of this section is to find a controller in the form of (22) such that the closed-loop system (23)

is exponentially asymptotically stable. Without loss of generality, we assume that  $C \in \mathbf{R}^{q \times n}$  is full-row rank, then the singular value decomposition of C is

$$C = U(C_0 \ 0) V', \tag{24}$$

where  $U \in \mathbf{R}^{q \times q}$  and  $V \in \mathbf{R}^{n \times n}$  are unitary matrices and  $C_0 \in \mathbf{R}^{q \times q}$  is a diagonal matrix with positive diagonal elements in decreasing order.

Similarly to the case of state feedback, we have the following result.

Theorem 3. The system (23) is exponentially asymptotically stable if there exist positive matrices  $X_{11} > 0, X_{22} > 0, S > 0$  and a real matrix Z such that the following LMI holds

$$\begin{pmatrix} \Psi_{11} & * & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * & * \\ H' & 0 & -I & * & * \\ \Psi_{41} & -B\Psi_0 & H & -\eta^{-1}S & * \\ \Psi_{51} & -N\Psi_0 & 0 & 0 & -I \end{pmatrix} < 0, \qquad (25)$$

where

$$\begin{split} \Psi_{11} &= AX + XA' + B\Psi_0 + \Psi_0'B', \quad \Psi_{21} = -\Psi_0'B', \\ \Psi_{41} &= AX + B\Psi_0, \quad \Psi_{51} = MX + N\Psi_0, \\ \Psi_{22} &= -2\eta^{-1}X + \eta^{-1}S, \quad \Psi_0 = (Z \ 0 \ )V' \end{split}$$

and  $X = V diag(X_{11}, X_{22})V'$ . Furthermore, the controller gain G is given by

$$G = Z X_{11}^{-1} C_0^{-1} U'. (26)$$

**Proof.** Using (26) yields

$$GCX = GU(C_0 \quad 0) V'V diag\{X_{11}, X_{22}\}V' = \Psi_0.$$

Therefore, we can conclude that the condition (8) in Theorem 1 with  $Y = \Psi_0 = GCX$  is satisfied. This completes the proof of the Theorem 3.

## 5. NUMERICAL EXAMPLES

To illustrate the effectiveness of the design procedure, we give two numerical examples.

Example 5.1. Consider the linear system in Yue [2004]

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & 0.1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0.1 \end{pmatrix} u(t).$$
(27)

In Yue [2004], by applying Algorithm 1 with  $\rho_2 = 0.2$  and  $\rho_3 = 20$ , it has been found that the maximum allowable value of  $\eta_{max}$  is 402 and the corresponding state feedback gain is K = (-0.0025 - 0.0118). From Corollary 2, we have

$$X = \begin{pmatrix} 50.7818 & -0.8434 \\ -0.8434 & 0.1866 \end{pmatrix},$$
  

$$S = \begin{pmatrix} 32.7938 & -0.2797 \\ -0.2797 & 0.1063 \end{pmatrix},$$
  

$$Y = (0.1002 & -0.3401)$$

and the maximum allowable value of  $\eta_{max}$  is 802. In this case, the state feedback gain K can be computed as K = (-0.0306 - 1.9610). It is obvious that our result is less conservative than that of Yue [2004]. When we choose the initial conditions as x(0) = (-12), the sampling



Fig. 1. The state responses of the closed-loop system.



Fig. 2. The state responses of the closed-loop system.

period h = 0.2,  $i_k = 2k(k = 0, 1, 2, \dots)$ ,  $\tau_k = 0.2sin(k)$ , the simulation results of the state responses are given in Fig. 1.

Example 5.2. Considering the following system

$$\dot{x}(t) = \begin{pmatrix} 0 & 0.2 \\ 0 & 0.1 \end{pmatrix} x(t) + \begin{pmatrix} 0.1 & 0.2 \\ 1 & 0.4 \end{pmatrix} u(t) +H\sin(Mx(t) + Nu(t)),$$
(28)

where

$$H = (-0.5 \ 0.1)',$$
  

$$M = (0.2 \ -0.2),$$
  

$$N = (-0.1 \ 0.1).$$

Using Theorem 1, the following matrix solutions can be obtained

$$X = \begin{pmatrix} 7.4248 & 1.3077 \\ 1.3077 & 1.8948 \end{pmatrix},$$
  

$$S = \begin{pmatrix} 3.9913 & 0.4685 \\ 0.4685 & 1.1549 \end{pmatrix},$$
  

$$Y = \begin{pmatrix} 2.1916 & 0.1349 \\ -5.9820 & -1.2584 \end{pmatrix}$$

and the maximum allowable value of  $\eta_{max}$  is 804. In this case, the state feedback gain K can be obtained as follows.

$$K = \begin{pmatrix} 0.3217 & -0.1509 \\ -0.7840 & -0.1230 \end{pmatrix}$$

When we choose the same conditions as in Example 1, the simulation results of the state responses are given in Fig. 2.

### 6. CONCLUSIONS

This paper discusses the issues of state feedback and static feedback controller designs of for a class nonlinear NCS. Based on the Lyapunov-Krasovskii functional method, a state feedback and a static feedback controller are constructed in terms of linear matrix inequalities. It is shown that the approach presented in this paper is more effective than those in the literature. Dynamic output feedback approach for the given NCS is under investigation.

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