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H_∞ Output Feedback Control for Singular Systems with Time-Varying Delay *

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Abstract: This paper is devoted to the problem of H_∞ output feedback control for linear singular systems with state time-varying delay. The purpose is to design a non-singular dynamic output feedback controller to ensure that the closed-loop system is regular, impulse free and stable with a prescribed H_∞ performance level. Using the linear matrix inequality (LMI) approach, a new delay-dependent stability criterion is obtained by introducing a free-weighting matrix when estimating the upper bound of the derivative of Lyapunov Functional. And the corresponding stability controller design algorithm is proposed. Finally, a numerical example is included to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Singular systems, known as descriptor systems, generalized systems or semi-state space systems, can describe physical systems more comprehensively and naturally than regular ones, and have been widely applied in the fields of electric systems, economic systems, robotic systems and space navigation systems. The robust stability problem of singular system is much more complicated in contrast to that of regular ones since regularity and absence of impulses are necessary to be considered simultaneously. In many practical systems, time-delay arises frequently and can severely degrade closed-loop system performance and in some cases drive the system to instability Dugard [1998]. Therefore, a great deal of attention has been paid to singular systems with time-delay.

Recently, many important results on singular systems with time-delays have been reported in literature. For example, in Xu et al. [2002], a delay-independent sufficient and necessary condition of robust stability for uncertain singular system with time-delay was given in terms of LMI, and based on this condition, a state feedback controller was presented. Kim [2005] considered the H_∞ state feedback control problem of singular time-delay system and proposed the corresponding stability controller. It is well known that the study for output feedback control is more complicated than that for state feedback control. Vladimir et al. [2006] investigated the static output feedback control for singular system with multiple delays. Jun'e et al [2005] presented the sufficient conditions for the existence of a dynamic output feedback controller, in which the conditions are delay-independent, and then an improved method of dynamic output feedback control was proposed based on the delay-dependent conditions in Zhu et al. [2005]. However, these above output feedback controllers are all singular. In this paper, we consider the H_∞ dynamic output feedback control problem of singular systems with time-varying delay. Using the free-weighting matrix technique, a new delay-dependent stability criterion is given in terms of LMIs. And the corresponding feedback controller is designed to guarantee that the closed-loop system is regular, impulse free and stable with a given H_∞ performance level.

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Notations: The matrix $X > 0$ means that X is symmetry and positive definite. I_r is an identity matrix with the dimension r. The symbol $*$ denotes the symmetry part of a symmetry matrix, and $diag\{\cdots\}$ denotes a block diagonal matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the linear singular system with time-varying delay described by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Bu(t) + B_\omega \omega(t) \\ z(t) &= Cx(t) + Du(t) \\ y(t) &= C_1 x(t) \\ x(t) &= \phi(t), \forall t \in [-d, 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $\omega(t) \in \mathbb{R}^p$ is the non-zero disturbance input which belongs to $\mathbb{L}_2[0,\infty]$, $z(t) \in \mathbb{R}^q$ is the controlled output, $y(t) \in \mathbb{R}^l$ is the measured output, $\phi(t)$ is a continuous initial function, and $d(t)$ is the time-varying delay satisfying

$$0 < d(t) \le d, \dot{d}(t) \le \mu < 1. \tag{2}$$

where τ and μ are known constant scalars. The matrix $E \in \mathbb{R}^{n\times n}$ is singular with $rankE = r < n$, and the system matrices $A, A_d, B, B_\omega, C, D$ and C_1 are known real matrices of appropriate dimensions. Here, we assume that C is a row full rank matrix.

Since $rankE = r < n$, there exist nonsingular matrices $P, Q \in \mathbb{R}^{n\times n}$, such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Then, in this paper, without loss of generality, we write

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, A_d = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}$$
$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, B_\omega = \begin{bmatrix} B_{\omega 1} \\ B_{\omega 2} \end{bmatrix}$$
$$C = [\, C_1 \;\; C_2 \,], C_1 = [\, C_{11} \;\; C_{12} \,].$$

Define a matrix $E_0 = \begin{bmatrix} I_r & 0_{r\times(n-r)} \end{bmatrix}$, and we have $E_0 E = E_0$, $E_0^T E_0 = E$.

Now, denote the unforced singular system of (1) by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) \\ x(t) &= \phi(t), \forall t \in [-d, 0]. \end{aligned} \tag{3}$$

Definition 1. Xu et al. [2002] The pair (E,A) is said to be regular if there exists a scalar $s \in \mathscr{C}$ such that $\det(sE-A) \neq 0$, and the pair (E,A) is said to be impulse free if $\deg(\det(sE-A)) = rank(E)$. Further, if the pair (E,A) is regular and impulse free, the system (3) is said to be regular and impulse free.

The objective of this paper is to design a non-singular full order dynamic output feedback controller:

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + B_f y(t) \\ u(t) &= C_f x_f(t) + D_f y(t) \end{aligned} \tag{4}$$

such that the closed-loop system constructed by (1) and (4) is regular, impulse free and stable with a prescribed H_∞ performance level γ. That is, under the zero-valued initial state condition, the H_∞ norm of the closed-loop transfer function $T_{z\omega}(s)$ from external disturbance $\omega(t)$ to controlled output $z(t)$ satisfies

$$\|T_{z\omega}(s)\|_{H_\infty} = \sup_{0\neq\omega\in L_2[0,+\infty)} \frac{\|z\|_2}{\|\omega\|_2} < \gamma$$

where γ is a given constant and,

$$\|z(t)\|_2 = (\int_0^\infty z^{\mathrm{T}}(s)z(s)ds)^{1/2}$$
$$\|\omega(t)\|_2 = (\int_0^\infty \omega^{\mathrm{T}}(s)\omega(s)ds)^{1/2}.$$

where, $x_f \in \mathbb{R}^n$ is the controller state, the matrices A_f, B_f, C_f and D_f are controller gain to be determined.

Before ending this section, we introduce the following lemmas that will be used in the development of our main results.

Lemma 1. [Jia (2007)] (Schur Complement Formula) Let $S_{11} = S_{11}^{\mathrm{T}}$, S_{12} and $S_{22} = S_{22}^{\mathrm{T}}$ be matrices of appropriate dimensions. Then

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix} < 0$$

if and only if

$$S_{11} < 0, S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{11} < 0$$

or equivalently

$$S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0.$$

Lemma 2. [Xu (2002)] For the singular system $E\dot{x}(t) = Ax(t)$, the pair (E,A) is regular, impulse free if and only if there exists a matrix P such that

$$EP^{\mathrm{T}} = PE^{\mathrm{T}} \geq 0$$
$$AP^{\mathrm{T}} + PA^{\mathrm{T}} < 0.$$

3. MAIN RESULTS

In this section, we present the main results on stability and H_∞ dynamic output feedback control for system (1). Firstly, consider the unforced singular system (3), we have the following conclusion.

Theorem 1. Given positive scalars d and μ, the singular system (3) is regular, impulse free and sable for the time-varying delay $d(t)$ satisfying (2), if there exist matrices $R_i > 0 (i = 1,2)$, $Q > 0$, P and $N_i (i = 1,2,3)$, such that

$$E^T P = P^T E \geq 0 \tag{5}$$

$$\begin{bmatrix} \Omega & P^T A_1 & 0 & dA^T E_0^T & (d+1)N_1 & E_0^T \\ * & -(1-\mu)R_1 & 0 & dA_1^T E_0^T & (d+1)N_2 & 0 \\ * & * & -R_2 & 0 & (d+1)N_3 & -E_0^T \\ * & * & * & -dQ & 0 & 0 \\ * & * & * & * & -(d+1)Q & 0 \\ * & * & * & * & * & -Q \end{bmatrix} < 0 \tag{6}$$

where, $\Omega = A^T P + P^T A + R_1 + R_2$.

Proof: If (6) holds, then we can get that $\Omega < 0$ with $R_i > 0, i = (1,2)$. Hence,

$$A^T P + P^T A < 0 \tag{7}$$

By lemma 2, it follows from (5) and (7) that the pair (E,A) is regular and impulse free. From Definition 1, system (3) is regular and impulse free. Next, we will show the stability of the system (3).

For any free-weighting matrix $\tilde{N}$ of appropriate dimension, the following equation holds

$$2\xi^T(t)\tilde{N}(x_1(t) - x_1(t-d) - \int_{t-d}^{t} \dot{x}_1(s)ds) = 0$$

where, $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-d) \end{bmatrix}^T$.

Define the following Lyapunov functional candidate by

$$\begin{aligned} V(t) &= x^T(t)E^T Px(t) + \int_{t-d(t)}^{t} x^T(s)R_1 x(s)ds \\ &+ \int_{t-d}^{t} x^T(s)R_2 x(s)ds + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E_0^T Q^{-1} E_0 \dot{x}(s)d\theta ds \end{aligned} \tag{8}$$

Calculating the time derivative of $V(t)$ along the solution of the system (3), one yields

$$\begin{aligned} \dot{V}(t) &\leq 2x^T(t)E^T P\dot{x}(t) + x^T(t)(R_1+R_2)x(t) - (1-\mu) \\ &x^T(t-d(t))R_1 x(t-d(t)) - x^T(t-d)R_2 x(t-d) \\ &+ d\dot{x}^T(t)E_0^T Q^{-1} E_0 \dot{x}(t) - \int_{t-d}^{t} \dot{x}_1^T(s)Q^{-1}\dot{x}_1(s)ds \end{aligned} \tag{9}$$

$$
\begin{aligned}
&= \xi^T(t)\Phi\xi(t) + d\xi^T(t)\hat{A}^T E_0^T Q^{-1} E_0 \hat{A}\xi(t) \\
&\quad + 2\xi^T(t)\tilde{N}(x_1(t) - x_1(t-d) - \int_{t-d}^{t} \dot{x}_1(s)ds) \\
&\quad - \int_{t-d}^{t} \dot{x}_1^T(s) Q^{-1} \dot{x}_1(s) ds \\
&= \xi^T(t)(\Phi + d\hat{A}^T E_0^T Q^{-1} E_0 \hat{A} + 2\tilde{N}\hat{E})\xi(t) \\
&\quad - 2\xi^T(t)\tilde{N}\int_{t-d}^{t} \dot{x}_1(s)ds - \int_{t-d}^{t} \dot{x}_1^T(s) Q^{-1} \dot{x}_1(s) ds
\end{aligned}
$$

where $\hat{A} = [\, A \;\; A_d \;\; 0 \,], \hat{E} = [\, E_0 \;\; 0 \;\; -E_0 \,]$.

For any vectors of appropriate dimensions x, y and any positive definite matrix X, we have $-2x^T y \leq x^T X^{-1} x + y^T X y$, hence

$$
\begin{aligned}
&-2\xi^T(t)\tilde{N}\int_{t-d}^{t} \dot{x}_1(s)ds \\
&\leq d\xi^T(t)\tilde{N}Q\tilde{N}^T\xi(t) + \int_{t-d}^{t} \dot{x}_1^T(s) Q^{-1} \dot{x}_1(s) ds
\end{aligned}
\tag{10}
$$

Substituting (10) into (9), one gets

$$
\dot{V}(t) \leq \xi^T(t)(\Phi + d\hat{A}^T E_0^T Q^{-1} E_0 \hat{A} + 2\tilde{N}\hat{E} + d\tilde{N}Q\tilde{N}^T)\xi(t)
$$

where

$$
\Phi = \begin{bmatrix} A^T P + P^T A + R_1 + R_2 & P^T A_d & 0 \\ A_d^T P & -(1-\mu)R_1 & 0 \\ 0 & 0 & -R_2 \end{bmatrix} \tag{11}
$$

Further, by Lemma 1, we can get that $\dot{V}(t) < 0$ is equivalent to

$$
\begin{bmatrix} \Phi & d\hat{A}^T E_0^T & (d+1)\tilde{N}Q & E^{1T} \\ E_0\hat{A} & -dQ & 0 & 0 \\ dQ\tilde{N}^T & 0 & -(d+1)Q & 0 \\ E^1 & 0 & 0 & -Q \end{bmatrix} < 0 \tag{12}
$$

Let

$$
N = \tilde{N}Q = \begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T \tag{13}
$$

The inequality (12) can be rewritten as (6). Thus, the stability of the system (3) is proved. It completes the proof.

In the above analysis, we have obtained a delay-dependent sufficient condition that can guarantee the unforced singular system (3) is regular, impulse free and stable. In the following, based on Theorem 1, consider the closed-loop system constructed by (1) and (4):

$$
\begin{aligned}
E_c \dot{x}_c(t) &= A_c x_c(t) + A_{cd} x_c(t - d(t)) + B_{c\omega}\omega(t) \\
z(t) &= C_c x_c(t)
\end{aligned}
\tag{14}
$$

where

$$
E_c = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_c = \begin{bmatrix} A_f & B_f C_{11} & B_f C_{12} \\ B_1 C_f & A_{11} & A_{12} \\ B_2 C_f & A_{21} & A_{22} \end{bmatrix}
$$

$$
A_{cd} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{d11} & A_{d12} \\ 0 & A_{d21} & A_{d22} \end{bmatrix}, B_{c\omega} = \begin{bmatrix} 0 \\ B_{\omega 1} \\ B_{\omega 2} \end{bmatrix}
$$

$$
C_c = [\, DC_f \;\; C_1 \;\; C_2 \,],\; x_c(t) = \begin{bmatrix} x_f^T(t) & x_1^T(t) & x_2^T(t) \end{bmatrix}^T
$$

We give a sufficient condition such that the system (14) is regular, impulse free and stable with a given H_∞ performance γ in the following Theorem 2.

Theorem 2. Given positive scalars d, γ and μ, the closed-loop system (14) is regular, impulse free and sable while satisfying the prescribed H_∞ performance level $\gamma^{1/2}$ for any time-varying delay $d(t)$ satisfying (2), if there exist matrices $P_1 > 0$, $R_1 > 0$, $R_2 > 0$, $Q > 0$, P_2, P_3, M_i $(i = 1,2,3)$ and N_j $(j = 1,2,3,4)$ such that

$$
E_c^T P = P^T E_c \geq 0 \tag{15}
$$

$$
\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix} < 0 \tag{16}
$$

where

$$
\Psi_{11} = \begin{bmatrix}
\Gamma & M_2 + M_3^T B^T + P_2^T A^T + R_1^{12} + R_2^{12} & 0 & 0 & 0 & 0 & 0 \\
* & AP_3 + P_3^T A^T + R_1^{22} + R_2^{22} & 0 & A_d P_3 & 0 & 0 & B_\omega \\
* & * & \bar{\mu}R_1^{11} & \bar{\mu}R_1^{12} & 0 & 0 & 0 \\
* & * & \bar{\mu}R_1^{22} & 0 & 0 & 0 & \\
* & * & * & * & -R_2^{11} & -R_2^{12} & 0 \\
* & * & * & * & * & -R_2^{22} & 0 \\
* & * & * & * & * & * & -\gamma I
\end{bmatrix}
$$

$$
\Psi_{12} = \begin{bmatrix}
dM_1^T & (d+1)N_{11} & P_1^T & M_3^T D^T + P_2^T C^T \\
dM_2^T & (d+1)N_{12} & 0 & P_3^T C^T \\
0 & (d+1)N_{21} & 0 & 0 \\
0 & (d+1)N_{22} & 0 & 0 \\
0 & (d+1)N_{31} & -P_1^T & 0 \\
0 & (d+1)N_{32} & 0 & 0 \\
0 & (d+1)N_4 & 0 & 0
\end{bmatrix}
$$

$$
\Psi_{22} = diag\{-dQ, -(d+1)Q, -Q, -I\}
$$

$$
\Gamma = M_1 + M_1^T + R_1^{11} + R_2^{11},\; M_1 = A_f P_1 + B_f C P_2
$$

$$
M_2 = B_f C_1 P_3,\; M_3 = C_f P_1,\; \bar{\mu} = -(1-\mu) < 0
$$

$$
N_1 = \begin{bmatrix} N_{11} \\ N_{12} \end{bmatrix}, N_2 = \begin{bmatrix} N_{21} \\ N_{22} \end{bmatrix}, N_3 = \begin{bmatrix} N_{31} \\ N_{32} \end{bmatrix}
$$

$$
R_1 = \begin{bmatrix} R_1^{11} & R_1^{12} \\ * & R_1^{22} \end{bmatrix}, R_2 = \begin{bmatrix} R_2^{11} & R_2^{12} \\ * & R_2^{22} \end{bmatrix}.
$$

and the corresponding controller can be taken as:

$$
\begin{aligned}
A_f &= M_1 P_1^{-1} + M_2 P_3^{-1} P_2 P_1^{-1} \\
B_f &= M_2 P_3^{-1} C_{1R}^{(-1)} \\
C_f &= M_3 P_1^{-1}
\end{aligned}
$$

where, $C_{1R}^{(-1)}$ is the right inverse matrix of C_1, i.e., $C_1 C_{1R}^{(-1)} = I$.

Proof: From Theorem 1, the closed-loop system (14) is regular, impulse free and stable if there exist the matrices P and $N_i, i = 1,2,3$, the positive definite matrices $R_i, i = 1,2$ and Q satisfying (15) and the following inequality

$$\begin{bmatrix} \Phi & dM^T & dN & S^T & N \\ dM & -dQ & 0 & 0 & 0 \\ dN^T & 0 & -dQ & 0 & 0 \\ S & 0 & 0 & -Q & 0 \\ N^T & 0 & 0 & 0 & -Q \end{bmatrix} < 0 \qquad (17)$$

where, $M = E_0 [\, A_c \ A_{cd} \ 0 \,]$, $S = [\, E_0 \ 0 \ -E_0 \,]$, N is defined in (13) while Φ is of the form similar to (11), where A and A_d are replaced by A_c and A_{cd}.

Consider the following performance index

$$J = \int_0^T (z^T(s)z(s) - \gamma\omega^T(s)\omega(s))ds.$$

Then, under the zero initial state condition, i.e., $\phi(t) = 0, \forall t \in [-d, 0]$, for the Lyapunov functional (8) and any nonzero $\omega(t) \in \mathscr{L}_2[0, \infty), t \geq 0$, we can obtain

$$\begin{aligned} J &= \int_0^T (z^T(s)z(s) - \gamma\omega^T(s)\omega(s) + \dot{V}(s))ds - V(x(T)) \\ &\leq \int_0^T (z^T(s)z(s) - \gamma\omega^T(s)\omega(s) + \dot{V}(s))ds \end{aligned}$$

Then, using the similar method in Theorem 1, we have

$$J \leq \int_0^T \zeta^T(s)(\Psi + d\tilde{M}^T Q^{-1}\tilde{M} + 2\tilde{N}\tilde{S} + d\tilde{N}Q\tilde{N}^T)\zeta(s)ds$$

where, $\zeta(t) = \left[\, x_c^T(t) \ x_c^T(t-d(t)) \ x_c^T(t-d) \ \omega^T(t) \,\right]^T$ and $\tilde{M} = E_0 [\, A_c \ A_{cd} \ 0 \ B_\omega \,]$, $\tilde{S} = [\, E_0 \ 0 \ -E_0 \ 0 \,]$.

Let $\tilde{N}Q = N$. By Lemma 1, $J < 0$ is equivalent to

$$\begin{bmatrix} \Psi & dM^T & dN & S^T & N & C_c^T \\ dM & -dQ & 0 & 0 & 0 & 0 \\ dN^T & 0 & -dQ & 0 & 0 & 0 \\ S & 0 & 0 & -Q & 0 & 0 \\ N^T & 0 & 0 & 0 & -Q & 0 \\ C_c & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \qquad (18)$$

where

$$\Psi = \begin{bmatrix} P^T A_c + A_c^T P + R_1 + R_2 & P^T A_{cd} & 0 & P^T B_{c\omega} \\ A_{cd}^T P & -(1-\mu)R_1 & 0 & 0 \\ 0 & 0 & -R_2 & 0 \\ B_{c\omega}^T P & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

Pre- and post-multiplying $diag\{P^{-T}, P^{-T}, P^{-T}, I, \cdots, I\}$ and $diag\{P^{-1}, P^{-1}, P^{-1}, I, \cdots, I\}$ to (18), respectively. Then, let $P^{-T} = \bar{P}, \bar{P}R_1\bar{P}^T = \bar{R}_1, \bar{P}R_2\bar{P}^T = \bar{R}_2$ and write the matrix $\bar{P}$ as

$$\bar{P}^T = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \qquad (19)$$

where P_1 is a positive definite matrix, P_2 and P_3 are of the following form $\begin{bmatrix} 0 \\ \star \end{bmatrix}$ and $\begin{bmatrix} \star & 0 \\ \star & \star \end{bmatrix}$. $\star$ denotes a non-zero matrix of appropriate dimension. At the moment, (18) can be rewritten as

$$\left[\begin{array}{cccccccc} \Sigma & A_{cd}\bar{P}^T & 0 & B_{c\omega} & d\bar{P}A_c^T E_0^T & (d+1)\bar{P}\bar{N}_1 & \bar{P}E_0^T & \bar{P}C_c^T \\ \bar{P}A_{cd}^T & -(1-\mu)\bar{R}_1 & 0 & 0 & d\bar{P}A_{cd}^T E_0^T & (d+1)\bar{P}\bar{N}_2 & 0 & 0 \\ 0 & 0 & -\bar{R}_2 & 0 & 0 & (d+1)\bar{P}\bar{N}_3 & -\bar{P}E_0 & 0 \\ B_{c\omega}^T & 0 & 0 & -\gamma^2 I & dB_{c\omega}^T E_0^T & (d+1)\bar{N}_4 & 0 & 0 \\ dE_0A_c\bar{P}^T & dE_0A_{cd}\bar{P}^T & 0 & dE_0B_{c\omega} & -dQ & 0 & 0 & 0 \\ (d+1)\bar{N}_1^T\bar{P}^T & (d+1)\bar{N}_2^T\bar{P}^T & (d+1)\bar{N}_3^T\bar{P}^T & (d+1)\bar{N}_4 & 0 & -(d+1)Q & 0 & 0 \\ E_0\bar{P}^T & 0 & E_0\bar{P}^T & 0 & 0 & 0 & -Q & 0 \\ C_c\bar{P} & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{array}\right] < 0 \qquad (20)$$

$$\Sigma = A_c\bar{P}^T + \bar{P}A_c^T + \bar{R}_1 + \bar{R}_2$$

According to the form of $P, A_c, A_{cd}, B_{c\omega}, C_c$, we have

$$A_c\bar{P}^T = \begin{bmatrix} A_f P_1 + B_f C_1 P_2 & B_f C_1 P_3 \\ BC_f P_1 + AP_2 & AP_3 \end{bmatrix}$$

$$A_{cd}\bar{P}^T = \begin{bmatrix} 0 & 0 \\ 0 & A_d P_3 \end{bmatrix}, C\bar{P}^T = [\, DC_f P_1 + CP_2 \ \ CP_3 \,]$$

Set $M_1 = A_f P_1 + B_f C_1 P_2, M_2 = B_f C_1 P_3, M_3 = C_f P_1, \bar{\mu} = -(1-\mu) < 0$, then, (16) can be obtained.

Combining (12), (16), (17), (18) and 20, it is reached that

$J_t < 0 \Longleftrightarrow (18) \Longleftrightarrow (20) \Longleftrightarrow (16)$ which implies (12). Then, it is easy to see that (16) can guarantee the closed-loop system (14) is regular, impulse free and stable with the given H_∞ performance level γ. And the desired non-singular dynamic output feedback controller (3) can be obtained by solving the LMIs (15) and (16). Thus, it completes the proof.

4. NUMERICAL EXAMPLE

To show the effectiveness of the proposed method on the design of robust H_∞ controller for the singular systems with time-varying delays, a numerical example is presented in this section. Consider the linear singular time-delay system (1) with parameters:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}, A_d = \begin{bmatrix} -0.5 & 0 \\ 0.1 & -1 \end{bmatrix}$$

$$B_\omega = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$C = [\, 1 \ 0.5 \,], C_1 = [\, 0.2 \ 0.5 \,], D = -1.$$

The aim of this example is to design an output feedback controller such that, the closed-loop system is regular, impulse free and stable. Using LMI control toolbox to solve the LMIs (15) and (16), given the derivative upper bound of time-varying delay $\mu = 0.6$ and $\gamma = 0.25$, i.e. the H_∞ performance is 0.5, we get the maximal of the upper bound of the time-varying delay is $d = 11.75$.

Given $\gamma = 0.25$, $\mu = 0.6$. The closed-loop system is regular, impulse free and stable with H_∞ performance level 0.5 for the time-varying delay $0 < d(t) < 7.5$ with the following output feedback controller:

$$\dot{x}_f(t) = \begin{bmatrix} -0.7905 & -0.0313 \\ -0.0313 & -0.7905 \end{bmatrix} x_f(t) + \begin{bmatrix} -0.0098 \\ -0.0098 \end{bmatrix} y(t)$$

$$u(t) = \begin{bmatrix} -0.0928 & -0.0928 \end{bmatrix} x_f(t)$$

5. CONCLUSION

In this paper, the problem of H_∞ output feedback control for singular systems with time-varying delay has been investigated. By employing the free-weighting matrix and LMI approach, a new delay-dependent stability criterion that can ensure the unforced singular system is regular, impulse free and stable is established. And the desired non-singular dynamic output feedback controller is obtained by solving several LMIs. A numerical example has been shown to demonstrate the effectiveness of the proposed results.

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