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Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Control of Discrete-Time Markovian Jump Systems via Static Output-Feedback Controllers ⋆

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Abstract: This paper investigates the static output-feedback mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem of discrete-time Markovian jump systems from a novel perspective. Unlike traditional methods, the closed-loop system is represented as an augmented form, in which input and gain-output matrices are decoupled. By virtue of the augmented representation, new characterizations on stochastic stability and $\mathcal{H}_2/\mathcal{H}_\infty$ performance of the closed-loop system are established in terms of matrix inequalities. Based on these, a sufficient condition with redundant matrices for the existence of the mode-dependent controller is proposed, and an iteration algorithm is given to solve the condition. An extension to the mode-independent case is provided as well.

Keywords: Iteration, linear matrix inequality (LMI), Markovian jump systems, mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control, static output-feedback.

1. INTRODUCTION

Discrete-time Markovian jump linear systems (DMJLSs), modeled by a set of discrete-time linear systems with transitions among the models determined by a Markov chain taking values in a finite set, have appealed to a lot of researchers in the control community. This is due to their widespread applications to model various practical processes that experience abrupt changes in their structure and parameters, possibly caused by phenomena such as component failures or repairs, sudden environmental disturbances, changing subsystem interconnections. Stability of DMJLSs has been investigated thoroughly in Costa and Fragoso [1993], and the equivalence of different second moment stability has been established in Ji et al. [1991]. The linear quadratic optimal control problem for DMJLSs has been studied in Chizeck et al. [1986] and Costa and Fragoso [1995], and the filtering problem has been considered in Costa and Marques [2000]. Some results on the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems are available in Costa and Marques [1998], Seiler and Sengupta [2003] and references therein. As for robust stability analysis, we refer readers to de Souza [2006] and references therein. More details on DMJLSs can be found in Costa et al. [2005].

In the literature mentioned above, it is often assumed that the system state is completely accessible to the controller. However, in practice, this assumption may not be always true, and only partial information through the measured output is available. Therefore, it is necessary to consider the more practical case that the system state is partially accessible, i.e., the output-feedback case. Although Costa et al. [1997] proposed a non-convex cutting-plane algorithm based on the output structural constraint approach by Geromel et al. [1993] to solve the static output-feedback $\mathcal{H}_2$ control problem of DMJLSs, it is not easy to apply due to its nonlinearity and complexity. Apart from this work, there are very few results on output-feedback control of DMJLSs. This motivates us to seek an effective and easy-to-use approach for output-feedback control of DMJLSs.

In this paper, we investigate the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem of DMJLSs via static output-feedback controllers from a new point of view. The closed-loop system is represent as an augmented form with algebraic constraints. By virtue of the augmented representation, new characterizations on stochastic stability and $\mathcal{H}_2/\mathcal{H}_\infty$ performance of the closed-loop system are established in terms of matrix inequalities. Two advantages of our characterizations lie in the decoupling of the input matrix and the gain-output matrix, which enables us to parameterize the controller matrix by free matrix, and the separation of the Lyapunov matrix and the system matrix, which avoids imposing any constraint on the Lyapunov matrix when the controller matrix is parameterized. Based on these, a sufficient condition with redundant matrix variables for the existence of the mode-dependent controller is proposed, and an iterative algorithm is given to solve the condition. An extension to the mode-independent case is provided as well. When Markovian jumps disappear, the obtained results are also applicable to the usual deterministic discrete-time linear systems.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive-semidefinite

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(respectively, positive-definite). I is the identity matrix with appropriate dimension, and the superscript "T" represents the transpose. $|\cdot|$ denotes the Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm for matrices. $\boldsymbol{E}\{\cdot\}$ stands for the mathematical expectation with respect to some probability measure. l_2 refers to the space of mean square summable infinite vector sequences with norm $|f|_2 = \sqrt{\boldsymbol{E}\left\{\sum_{k=0}^{\infty} |f(k)|^2\right\}}$. The symbol $\#$ is used to denote a matrix which can be inferred by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. PRELIMINARIES

Consider the following class of discrete-time stochastic systems, denoted $\mathcal{T}$:

$$(\mathcal{T}): \begin{cases} x(k+1) = A(r(k))x(k) + B(r(k))u(k) \\ \qquad\qquad + B_w(r(k))w(k) \\ z(k) = C(r(k))x(k) + D(r(k))u(k) \\ y(k) = C_y(r(k))x(k) \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^l$, $y(k) \in \mathbb{R}^m$, $z(k) \in \mathbb{R}^p$, $w(k) \in \mathbb{R}^q$ are the system state, control input, measured output, regulated output to be controlled and exogenous noise process, respectively, and $A(r(k)) \in \mathbb{R}^{n\times n}$, $B(r(k)) \in \mathbb{R}^{n\times l}$, $B_w(r(k)) \in \mathbb{R}^{n\times q}$, $C(r(k)) \in \mathbb{R}^{p\times n}$, $D(r(k)) \in \mathbb{R}^{p\times l}$, $C_y(r(k)) \in \mathbb{R}^{m\times n}$ are the system matrices of the stochastic jumping process $\{r(k), k > 0\}$; the parameter $r(k)$ represents a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, s\}$ with transition probabilities

$$\Pr\{r(k+1) = j \,|r(k) = i\} = \pi_{ij}, \quad (2)$$

where $\pi_{ij} \geq 0$, and for any $i \in \mathcal{S}$, $\sum_{j=1}^{s} \pi_{ij} = 1$. The processes $w(k)$ and $r(k)$ are mutually independent. To simplify the notation, $M(r(k))$ and $M_N(r(k))$ will be denoted by $M_{r(k)}$ and $M_{Nr(k)}$, respectively, and, for a set of matrices M_i, $i \in \mathcal{S}$, $\hat{M}_i$ denotes $\sum_{j=1}^{s} \pi_{ij} M_j$.

Definition 1.

(1) System $\mathcal{T}$ is said to be stochastically stable if, when $w(k) \equiv 0$, $u(k) \equiv 0$, there exists a scalar $\tilde{M}(x_0, r_0) > 0$, for $x(0) = x_0$ and $r(0) = r_0$, such that

$$\lim_{\nu\to\infty} \boldsymbol{E}\left\{ \sum_{k=0}^{\nu} |x(k)|^2 \middle| x_0, r_0 \right\} \leq \tilde{M}(x_0, r_0).$$

(2) Assume that system $\mathcal{T}$ is stochastically stable. The $\mathcal{H}_\infty$ norm of system $\mathcal{T}$ with $x(0) \equiv 0$ and $u(k) \equiv 0$, denoted as $\|\mathcal{T}\|_\infty$, is defined as $\|\mathcal{T}\|_\infty \triangleq \sup_{r_0\in\mathcal{S}} \sup_{0\neq w\in l_2} \frac{|z|_2}{|w|_2}$.

(3) The $\mathcal{H}_2$ norm of system $\mathcal{T}$ with $x(0) \equiv 0$ and $u(k) \equiv 0$, denoted as $\|\mathcal{T}\|_2$, is defined as $\|\mathcal{T}\|_2^2 \triangleq \sum_{i=1}^{q} \sum_{j=1}^{s} |z_{i,j}|_2^2$, where $z_{i,j}$ represents the output sequence generated by (1), i.e., $(z(0), z(1), \ldots)$, when
   (a) the input sequence is $w = (w(0), w(1), \ldots)$, where $w(0) = e_i$, the unit vector formed by one at the ith position and zero elsewhere, and $w(k) = 0$, for $k > 0$.
   (b) $r(0) = r(1) = j$.

The static output-feedback controller under consideration is of the form

$$u(k) = K_{r(k)} y(k). \quad (3)$$

When static mode-dependent controller (3) is applied to (1), the closed-loop system becomes

$$(\mathcal{T}_{cl}): \begin{cases} x(k+1) = A_{clr(k)}x(k) + B_{wr(k)}w(t), \\ z(k) = C_{clr(k)}x(t), \end{cases} \quad (4)$$

where

$$A_{clr(k)} = A_{r(k)} + B_{r(k)}K_{r(k)}C_{yr(k)},$$
$$C_{clr(k)} = C_{r(k)} + D_{r(k)}K_{r(k)}C_{yr(k)}.$$

Our goal is to design a controller in (3) such that system $\mathcal{T}_{cl}$ is stochastically stable and satisfies

$$\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty, \quad \|\mathcal{T}_{cl}\|_2 < \gamma_2,$$

where $\gamma_\infty > 0$ and $\gamma_2 > 0$ are prescribed scalars. Since $K_{r(k)}$ is embedded in the middle of two matrices, it is hard to parameterize it by matrix variables. Hence, our fundamental idea is to extract $K_{r(k)}$ from the middle of two matrices. To this end, we view the input $u(k)$ as a state component and choose $\left[\, x^T(k) \; u^T(k) \,\right]^T$ as a new state variable. Then the closed-loop system can be re-written as the following augmented form:

$$\begin{cases} \mathbf{E}\xi(k+1) = \mathbf{A}_{r(k)}\xi(k) + \mathbf{B}_{wr(k)}w(k), \\ z(k) = \mathbf{C}_{r(k)}\xi(k), \end{cases}$$

where

$$\xi(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}, \qquad \mathbf{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$
$$\mathbf{A}_{r(k)} = \begin{bmatrix} A_{r(k)} & B_{r(k)} \\ K_{r(k)}C_{yr(k)} & -I \end{bmatrix}, \quad \mathbf{B}_{r(k)} = \begin{bmatrix} B_{wr(k)} \\ 0 \end{bmatrix},$$
$$\mathbf{C}_{r(k)} = \left[\, C_{r(k)} \; D_{r(k)} \,\right].$$

An advantage of this augmented representation lies in the separation of $B_{r(k)}$ and $K_{r(k)}C_{yr(k)}$, which enables us to parameterize $K_{r(k)}$ by matrix variables. It is noted that if we choose $\left[\, x^T(k) \; y^T(k) \,\right]^T$ as a new state variable, we may also obtain a similar augmented representation, which we call *dual augmented representation.* In this paper, we do not intend to present any results on dual augmented representation, due to the page length consideration, and further discussion on this issue will appear in our future work. In addition, many dynamic output-feedback synthesis problems can be reformulated as a static output-feedback control design involving augmented plants. Therefore, the approach presented in this paper is applicable to the dynamic output-feedback case as well.

We end this section by giving several lemmas, which will be useful in the sequel.

Lemma 1. (Seiler and Sengupta [2003]). Assuming system $\mathcal{T}_{cl}$ is weakly controllable [1], it is stochastically stable with $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$ if and only if there exist matrices $P_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\begin{bmatrix} A_{cli} & B_{wi} \\ C_{cli} & 0 \end{bmatrix}^T \begin{bmatrix} \hat{P}_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{cli} & B_{wi} \\ C_{cli} & 0 \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma_\infty^2 I \end{bmatrix} < 0.$$

[1] System $\mathcal{T}_{cl}$ is said to be weakly controllable with respect to $w(k)$ if for every initial state/mode, (x_0, r_0), and any final state/mode, (x_f, r_f), there exists a finite time t_c and an input $w(k)$ such that $\mathbf{Pr}[x(t_c) = x_f, r(t_c) = r_f] > 0$.

Lemma 2. (Costa et al. [1997], Costa and Marques [1998]). Assuming that system $\mathcal{T}_{cl}$ is stochastically stable, if $P_i > 0$ is the unique solution of the following equations:

$$A_{cli}^T \hat{P}_i A_{cli} - P_i + C_{cli}^T C_{cli} = 0, \quad i \in \mathcal{S},$$

then $\|\mathcal{T}_{cl}\|_2^2 = \sum_{i=1}^s \text{trace}\left(B_{wi}^T P_i B_{wi}\right)$.

Lemma 3. If system $\mathcal{T}_{cl}$ is stochastically stable, then, for any $Q_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, there exists a unique solution $P_i \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, such that $P_i - A_{cli}^T \hat{P}_i A_{cli} = Q_i$. Moreover, if $Q_{1i} \geq Q_{2i} \geq 0$ (> 0, respectively) and $P_{1i} - A_{cli}^T \hat{P}_{1i} A_{cli} = Q_{1i}$, $P_{2i} - A_{cli}^T \hat{P}_{2i} A_{cli} = Q_{2i}$, then $P_{1i} \geq P_{2i} \geq 0$ (> 0, respectively).

Lemma 3 is an analogue in the real number field of Proposition 6 in Costa and Fragoso [1993]. Its proof can be conducted in a similar way, and thus omitted here.

3. NEW CHARACTERIZATIONS ON STOCHASTIC STABILITY AND $\mathcal{H}_2/\mathcal{H}_\infty$ PERFORMANCE

3.1 Stochastic Stability and $\mathcal{H}_\infty$ Performance (Bounded Real Lemma)

On the basis of the proposed augmented system representation, we establish a new bounded real lemma for the closed-loop system in the following theorem.

Theorem 1. Assuming that system $\mathcal{T}_{cl}$ is weakly controllable, it is stochastically stable with $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$, if and only if there exist $P_{1i} = P_{1i}^T$, $P_{4i} = P_{4i}^T$, P_{2i}, $Q_{4i} = Q_{4i}^T$, and scalars $\alpha_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\Omega_{\infty i} = \mathcal{A}_i^T \hat{\mathcal{P}}_i \mathcal{A}_i - \mathcal{E}_\infty \mathcal{P}_i \mathcal{E}_\infty + \mathcal{Q}_i \mathcal{L}_i + \mathcal{L}_i^T \mathcal{Q}_i^T < 0, \quad (5)$$

where

$$\mathcal{A}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_{wi} \\ \mathbf{C}_i & 0 \end{bmatrix},\ \mathcal{L}_i = \begin{bmatrix} \mathbf{A}_i & 0 \end{bmatrix},\ \mathcal{E}_\infty = \begin{bmatrix} \mathbf{E} & 0 \\ 0 & \gamma_\infty I \end{bmatrix},$$

$$\mathcal{P}_i = \begin{bmatrix} P_{1i} & P_{2i}^T & 0 \\ P_{2i} & P_{4i} & 0 \\ 0 & 0 & I \end{bmatrix} > 0,\ \mathcal{Q}_i = \begin{bmatrix} 0 & -\alpha_i C_{yi}^T K_i^T Q_{4i} \\ 0 & \alpha_i Q_{4i} \\ 0 & 0 \end{bmatrix}.$$

Proof: (Sufficiency) Define two nonsingular transformation matrices as follows:

$$T_{1i} = \begin{bmatrix} I & 0 & 0 \\ K_i C_{yi} & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix}.$$

Pre- and post-multiplying (5) by $T_2^T T_{1i}^T$ and its transpose yields that

$$\begin{aligned} &T_2^T T_{1i}^T \Omega_{\infty i} T_{1i} T_2 \\ &= \left[\begin{matrix} A_{cli}^T \hat{P}_{1i} A_{cl_i} - P_{1i} + C_{cli}^T C_{cli} & A_{cli}^T \hat{P}_{1i} B_{wi} \\ B_{wi}^T \hat{P}_{1i} A_{cli} & -\gamma_\infty^2 I + B_{wi}^T \hat{P}_{1i} B_{wi} \\ B_i^T \hat{P}_{1i} A_{cli} - \hat{P}_{2i} A_{cli} + D_i^T C_{cli} & B_i^T \hat{P}_{1i} B_{wi} - \hat{P}_{2i} B_{wi} \end{matrix}\right. \\ &\qquad \left.\begin{matrix} A_{cli}^T \hat{P}_{1i} B_i - A_{cli}^T \hat{P}_{2i}^T + C_{cli}^T D_i \\ B_{wi}^T \hat{P}_{1i} B_i - B_{wi}^T \hat{P}_{2i}^T \\ \begin{pmatrix} B_i^T \hat{P}_{1i} B_i - B_i^T \hat{P}_{2i}^T - \hat{P}_{2i} B_i \\ + \hat{P}_{4i} + D_i^T D_i - 2\alpha_i Q_{4i} \end{pmatrix} \end{matrix}\right] \\ &< 0, \end{aligned} \quad (6)$$

which implies that

$$\begin{aligned} \Psi_i &= \begin{bmatrix} A_{cli}^T \hat{P}_{1i} A_{cl_i} - P_{1i} + C_{cli}^T C_{cli} & A_{cli}^T \hat{P}_{1i} B_{wi} \\ B_{wi}^T \hat{P}_{1i} A_{cli} & -\gamma_\infty^2 I + B_{wi}^T \hat{P}_{1i} B_{wi} \end{bmatrix} \\ &= \begin{bmatrix} A_{cli} & B_{wi} \\ C_{cli} & 0 \end{bmatrix}^T \begin{bmatrix} \hat{P}_{1i} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{cli} & B_{wi} \\ C_{cli} & 0 \end{bmatrix} \\ &\quad - \begin{bmatrix} P_{1i} & 0 \\ 0 & \gamma_\infty^2 I \end{bmatrix} \\ &< 0. \end{aligned}$$

Therefore, according to Lemma 1, system $\mathcal{T}_{cl}$ is stochastically stable with $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$.

(Necessity) If system $\mathcal{T}_{cl}$ is stochastically stable with $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$, then according to Lemma 1, there exist matrices $P_{1i} > 0$ such that

$$\Psi_i < 0.$$

Now set P_{4i} to be any positive definite matrices, $P_{2i} = 0$, $Q_{4i} = \hat{P}_{4i}$, and $\alpha_i > 0$ to be sufficiently large scalars such that

$$-\digamma_i^T \Psi_i^{-1} \digamma_i + B_i^T \hat{P}_{1i} B_i + D_i^T D_i - (2\alpha_i - 1)\hat{P}_{4i} < 0, \quad (7)$$

where

$$\digamma_i = \begin{bmatrix} A_{cli}^T \hat{P}_{1i} B_i + C_{cli}^T D_i \\ B_{wi}^T \hat{P}_{1i} B_i \end{bmatrix}$$

Then, directly manipulating together with (6), (7), and Schur complement equivalence yields that

$$\Omega_{\infty i} = T_{1i}^{-T} T_2^{-T} \begin{bmatrix} \Psi_i & \digamma_i \\ \digamma_i^T & \begin{pmatrix} B_i^T \hat{P}_{1i} B_i + D_i^T D_i \\ -(2\alpha_i - 1)\hat{P}_{4i} \end{pmatrix} \end{bmatrix} T_2^{-1} T_{1i}^{-1} < 0.$$

This completes the proof. □

Remark 1. When the assumption of weak controllability is not satisfied, the necessity of Theorem 1 may be lost, but the sufficiency still holds. For the subsequent synthesis, we only need to use the sufficiency of Theorem 1, since the controller matrices are unknown before they are computed.

In the following theorem, we provide an equivalent characterization of the bounded real lemma, which will play a key role in the subsequent controller synthesis.

Theorem 2. (5) holds if and only if there exist $P_{1i} = P_{1i}^T$, $P_{4i} = P_{4i}^T$, P_{2i}, $Q_{4i} = Q_{4i}^T$, $H_{\nu i}$, $G_{\nu i}$, ($\nu = 1, 2, \ldots, 6$), and scalars $\alpha_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\begin{bmatrix} \begin{pmatrix} \mathcal{H}_i \mathcal{A}_i + \mathcal{A}_i^T \mathcal{H}_i^T - \mathcal{E}_\infty \mathcal{P}_i \mathcal{E}_\infty \\ + \mathcal{Q}_i \mathcal{L}_i + \mathcal{L}_i^T \mathcal{Q}_i^T \end{pmatrix} & \mathcal{A}_i^T \mathcal{G}_i^T - \mathcal{H}_i \\ \mathcal{G}_i \mathcal{A}_i - \mathcal{H}_i^T & \hat{\mathcal{P}}_i - \mathcal{G}_i - \mathcal{G}_i^T \end{bmatrix} < 0, \quad (8)$$

where $\mathcal{A}_i$, $\mathcal{P}_i$, $\mathcal{Q}_i$, $\mathcal{L}_i$, and $\mathcal{E}_\infty$ are defined as in Theorem 1, and

$$\mathcal{H}_i = \begin{bmatrix} H_{1i} & 0 & H_{2i} \\ H_{3i} & 0 & H_{4i} \\ H_{5i} & 0 & H_{6i} \end{bmatrix},\ \mathcal{G}_i = \begin{bmatrix} G_{1i} & 0 & G_{2i} \\ G_{3i} & Q_{4i} & G_{4i} \\ G_{5i} & 0 & G_{6i} \end{bmatrix}.$$

Proof: (Sufficiency) By pre- and post-multiplying (8) by $\begin{bmatrix} I & \mathcal{A}_i^T \end{bmatrix}$ and its transpose, we obtain (5) immediately.

(Necessity) If there exist $P_{1i} > 0$, $P_{2i} = 0$, $P_{4i} > 0$, $Q_{4i} = \hat{P}_{4i}$, and sufficiently large $\alpha_i > 0$ such that (5) holds, then, by simple manipulating and Schur complement equivalence, we can obtain that (8) holds with

$H_{\nu i} = 0$, $(\nu = 1, 2, \ldots, 6)$, $G_{2i} = G_{3i} = G_{4i} = G_{5i} = 0$, $G_{1i} = \hat{P}_{1i}$, $G_{6i} = I$. □

Remark 2. The major merit of the equivalent characterization is the separation of the Lyapunov matrices P_{1i} and the system matrices, which avoids imposing any constraint on the Lyapunov matrices when K_i is parameterized. In addition, by following the idea proposed in de Oliveira et al. [1999], redundant matrices $H_{\nu i}$ and $G_{\nu i}$ are introduced to reduce the conservatism and to improve the solvability of the iterative calculation to be presented later.

Remark 3. It should be pointed out that, without loss of generality, the matrices P_{4i} and Q_{4i} in Theorems 1 and 2 can be set to be mode-independent, i.e., $P_{41} = P_{42} = \ldots = P_{4s}$ and $Q_{41} = Q_{42} = \ldots = Q_{4s}$, and the corresponding conditions are still necessary and sufficient. In view of this feature, it is easy to design a mode-independent controller for the case that the jump variable $r(k)$ is not available without imposing any restriction on the Lyapunov matrices P_{1i}, which may cause excessive conservatism.

3.2 $\mathcal{H}_2$ Performance

Likewise, we first propose a new condition for $\mathcal{H}_2$ performance, and then give an equivalent characterization.

Theorem 3. System $\mathcal{T}_{cl}$ is stochastically stable with $\|\mathcal{T}_{cl}\|_2 < \gamma_2$ if and only if there exist $X_{1i} = X_{1i}^T$, $X_{4i} = X_{4i}^T$, X_{2i}, $W_{4i} = W_{4i}^T$, $\Lambda_i = \Lambda_i^T$, and scalars $\beta_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\sum_{i=1}^{s} \operatorname{trace}(\Lambda_i) < \gamma_2^2, \tag{9}$$

$$\begin{aligned}\Omega_{2i} = \mathcal{I}_{up}^T \mathcal{A}_i^T \hat{\mathcal{X}}_i \mathcal{A}_i \mathcal{I}_{up} + \mathcal{I}_{dn}^T \mathcal{A}_i^T \mathcal{X}_i \mathcal{A}_i \mathcal{I}_{dn} \\ -\mathcal{E}_{2i} \mathcal{X}_i \mathcal{E}_{2i} + \mathcal{W}_i \mathcal{L}_i + \mathcal{L}_i^T \mathcal{W}_i^T < 0,\end{aligned} \tag{10}$$

where $\mathcal{A}_i$ and $\mathcal{L}_i$ are defined as in Theorem 1, and

$$\mathcal{X}_i = \begin{bmatrix} X_{1i} & X_{2i}^T & 0 \\ X_{2i} & X_{4i} & 0 \\ 0 & 0 & I \end{bmatrix} > 0,\ \mathcal{W}_i = \begin{bmatrix} 0 & -\beta_i C_{yi}^T K_i^T W_{4i} \\ 0 & \beta_i W_{4i} \\ 0 & 0 \end{bmatrix},$$

$$\mathcal{I}_{up} = \begin{bmatrix} \mathbf{I}_{n+l} & 0 \\ 0 & 0 \end{bmatrix},\ \mathcal{I}_{dn} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I}_q \end{bmatrix},\ \mathcal{E}_{2i} = \begin{bmatrix} \mathbf{E} & 0 \\ 0 & \Lambda_i^{1/2} \end{bmatrix}.$$

Proof: (Sufficiency) By pre- and post-multiplying (10) by $T_2^T T_{1i}^T$ and its transpose, we obtain that

$$\begin{aligned} & T_2^T T_{1i}^T \Omega_{2i} T_{1i} T_2 \\ &= \left[\begin{matrix} A_{cli}^T \hat{X}_{1i} A_{cl_i} - X_{1i} + C_{cli}^T C_{cli} & 0 \\ 0 & B_{wi}^T X_{1i} B_{wi} - \Lambda_i \\ B_i^T \hat{X}_{1i} A_{cli} - \hat{X}_{2i} A_{cli} + D_i^T C_{cli} & 0 \end{matrix}\right. \\ &\qquad \left.\begin{matrix} A_{cli}^T \hat{X}_{1i} B_i - A_{cli}^T \hat{X}_{2i}^T + C_{cli}^T D_i \\ 0 \\ \begin{pmatrix} B_i^T \hat{X}_{1i} B_i - B_i^T \hat{X}_{2i}^T - \hat{X}_{2i} B_i \\ + \hat{X}_{4i} + D_i^T D_i - 2\beta_i W_{4i} \end{pmatrix} \end{matrix}\right] \\ &< 0, \end{aligned}$$

which implies that

$$A_{cli}^T \hat{X}_{1i} A_{cl_i} - X_{1i} + C_{cli}^T C_{cli} < 0, \tag{11}$$

$$B_{wi}^T X_{1i} B_{wi} < \Lambda_i. \tag{12}$$

Hence, the stochastic stability of system $\mathcal{T}_{cl}$ follows from (11) immediately. On one hand, it follows from (11) that there must exist some $F_i \geq 0$ such that

$$X_{1i} - A_{cli}^T \hat{X}_{1i} A_{cl_i} = C_{cli}^T C_{cli} + F_i^T F_i. \tag{13}$$

On the other hand, by Lemma 2,

$$\|\mathcal{T}_{cl}\|_2^2 = \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T S_i B_{wi}\right),$$

where

$$S_i - A_{cli}^T \hat{S}_i A_{cl_i} = C_{cli}^T C_{cli}. \tag{14}$$

Therefore, from (13), (14) and Lemma 3, we obtain that $X_{1i} \geq S_i > 0$, and thus

$$\begin{aligned} \|\mathcal{T}_{cl}\|_2^2 &= \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T S_i B_{wi}\right) \leq \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T X_{1i} B_{wi}\right) \\ &< \sum_{i=1}^{s} \operatorname{trace}(\Lambda_i) < \gamma_2^2, \end{aligned}$$

where (12) is used.

(Necessity) If system $\mathcal{T}_{cl}$ is stochastically stable with $\|\mathcal{T}_{cl}\|_2 < \gamma_2$, then there exist $Z_{1i} > 0$ and $S_i > 0$ such that

$$A_{cli}^T \hat{Z}_{1i} A_{cl_i} - Z_{1i} < 0 \tag{15}$$

$$\|\mathcal{T}_{cl}\|_2^2 = \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T S_i B_{wi}\right) < \gamma_2^2, \tag{16}$$

where S_i satisfying (14). Now, define

$$\begin{aligned} X_{1i} &= S_i + \varepsilon Z_{1i}, \\ \Lambda_i &= B_{wi}^T X_{1i} B_{wi} + \delta I, \end{aligned}$$

where $\varepsilon > 0$ and $\delta > 0$ are sufficiently small numbers such that

$$\begin{aligned} & \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T S_i B_{wi}\right) + \varepsilon \sum_{i=1}^{s} \operatorname{trace}\left(B_{wi}^T Z_{1i} B_{wi}\right) \\ & + \sum_{i=1}^{s} \operatorname{trace}(\delta I) \\ & < \gamma_2^2. \end{aligned}$$

Then, it follows that

$$B_{wi}^T X_{1i} B_{wi} < \Lambda_i,\ \sum_{i=1}^{s} \operatorname{trace}(\Lambda_i) < \gamma_2^2. \tag{17}$$

Meanwhile, from (14) and (15), we have that

$$\begin{aligned} & A_{cli}^T \hat{X}_{1i} A_{cl_i} - X_{1i} + C_{cli}^T C_{cli} \\ &= \varepsilon \left(A_{cli}^T \hat{Z}_{1i} A_{cl_i} - Z_{1i}\right) \\ &< 0. \end{aligned} \tag{18}$$

Combining (17)–(18), and following the same line as used in the proof of Theorem 1, we obtain that (9) and (10) hold. This completes the proof. □

Theorem 4. (9) and (10) hold if and only if there exist $X_{1i} = X_{1i}^T$, $X_{4i} = X_{4i}^T$, X_{2i}, $W_{4i} = W_{4i}^T$, $U_{\nu i}$, $V_{\nu i}$, $(\nu = 1, 2, \ldots, 6)$, and scalars $\beta_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\sum_{i=1}^{s} \text{trace}(\Lambda_i) < \gamma_2^2, \tag{19}$$

$$\begin{bmatrix} \begin{pmatrix} \mathcal{V}_i \mathcal{A}_i \mathcal{I}_{up} + \mathcal{I}_{up}^T \mathcal{A}_i^T \mathcal{V}_i^T \\ +\mathcal{I}_{dn}^T \mathcal{A}_i^T \mathcal{X}_i \mathcal{A}_i \mathcal{I}_{dn} \\ -\mathcal{E}_{2i} \mathcal{X}_i \mathcal{E}_{2i} + \mathcal{W}_i \mathcal{L}_i + \mathcal{L}_i^T \mathcal{W}_i^T \end{pmatrix} & \mathcal{I}_{up}^T \mathcal{A}_i^T \mathcal{U}_i^T - \mathcal{V}_i \\ \mathcal{U}_i \mathcal{A}_i \mathcal{I}_{up} - \mathcal{V}_i^T & \hat{\mathcal{X}}_i - \mathcal{U}_i - \mathcal{U}_i^T \end{bmatrix} < 0, \tag{20}$$

where $\mathcal{A}_i$, $\mathcal{X}_i$, $\mathcal{W}_i$, $\mathcal{L}_i$, and $\mathcal{E}_{2_i}$ are defined as in Theorem 3, and

$$\mathcal{V}_i = \begin{bmatrix} V_{1i} & 0 & V_{2i} \\ V_{3i} & 0 & V_{4i} \\ V_{5i} & 0 & V_{6i} \end{bmatrix}, \ \mathcal{U}_i = \begin{bmatrix} U_{1i} & 0 & U_{2i} \\ U_{3i} & W_{4i} & U_{4i} \\ U_{5i} & 0 & U_{6i} \end{bmatrix}.$$

The proof can be conducted by following the same line as used in the proof of Theorem 2, and thus omitted here for brevity.

4. CONTROLLER SYNTHESIS

We are now in a position to establish a sufficient condition for the existence of desired mode-dependent controllers.

Theorem 5. If there exist $P_{1i} = P_{1i}^T$, $P_{4i} = P_{4i}^T$, P_{2i}, $H_{\nu i}$, $G_{\nu i}$, $X_{1i} = X_{1i}^T$, $X_{4i} = X_{4i}^T$, X_{2i}, $U_{\nu i}$, $V_{\nu i}$, $(\nu = 1, 2, \ldots, 6)$, M_i, L_i, $Q_{4i} = Q_{4i}^T$, $\Lambda_i = \Lambda_i^T$, and scalars $\alpha_i > 0$, $\beta_i > 0$ such that, for each $i \in \mathcal{S}$,

$$\begin{bmatrix} P_{1i} & P_{2i}^T \\ P_{2i} & P_{4i} \end{bmatrix} > 0, \ \begin{bmatrix} X_{1i} & X_{2i}^T \\ X_{2i} & X_{4i} \end{bmatrix} > 0, \tag{21}$$

$$\sum_{i=1}^{s} \text{trace}(\Lambda_i) < \gamma_2^2, \tag{22}$$

$$\Phi_{\infty i}(\alpha_i, M_i) = \begin{bmatrix} \hat{\Phi}_{11i} & \# & \# & \# & \# & \# \\ \hat{\Phi}_{21i} & \hat{\Phi}_{22i} & \# & \# & \# & \# \\ \hat{\Phi}_{31i} & \hat{\Phi}_{32i} & \hat{\Phi}_{33i} & \# & \# & \# \\ \hat{\Phi}_{41i} & \hat{\Phi}_{42i} & \hat{\Phi}_{43i} & \hat{\Phi}_{44i} & \# & \# \\ \hat{\Phi}_{51i} & \hat{\Phi}_{52i} & \hat{\Phi}_{53i} & \hat{\Phi}_{54i} & \hat{\Phi}_{55i} & \# \\ \hat{\Phi}_{61i} & \hat{\Phi}_{62i} & \hat{\Phi}_{63i} & \hat{\Phi}_{64i} & \hat{\Phi}_{65i} & \hat{\Phi}_{66i} \end{bmatrix} < 0 \tag{23}$$

$$\Phi_{2i}(\beta_i, M_i) = \begin{bmatrix} \check{\Phi}_{11i} & \# & \# & \# & \# & \# \\ \check{\Phi}_{21i} & \check{\Phi}_{22i} & \# & \# & \# & \# \\ \check{\Phi}_{31i} & \check{\Phi}_{32i} & \check{\Phi}_{33i} & \# & \# & \# \\ \check{\Phi}_{41i} & \check{\Phi}_{42i} & \check{\Phi}_{43i} & \check{\Phi}_{44i} & \# & \# \\ \check{\Phi}_{51i} & \check{\Phi}_{52i} & 0 & \check{\Phi}_{54i} & \check{\Phi}_{55i} & \# \\ \check{\Phi}_{61i} & \check{\Phi}_{62i} & \check{\Phi}_{63i} & \check{\Phi}_{64i} & \check{\Phi}_{65i} & \check{\Phi}_{66i} \end{bmatrix} < 0 \tag{24}$$

where

$$\begin{aligned}
\hat{\Phi}_{11i} &= H_{1i}A_i + H_{2i}C_i + A_i^T H_{1i}^T + C_i^T H_{2i}^T - P_{1i} \\
&\quad 2\alpha_i M_i^T Q_{4i} M_i - 2\alpha_i C_{yi}^T L_i^T M_i - 2\alpha_i M_i^T L_i C_{yi}, \\
\check{\Phi}_{11i} &= V_{1i}A_i + V_{2i}C_i + A_i^T V_{1i}^T + C_i^T V_{2i}^T - X_{1i} \\
&\quad 2\beta_i M_i^T Q_{4i} M_i - 2\beta_i C_{yi}^T L_i^T M_i - 2\beta_i M_i^T L_i C_{yi}, \\
\hat{\Phi}_{21i} &= H_{3i}A_i + H_{4i}C_i + B_i^T H_{1i}^T + D_i^T H_{2i}^T + 2\alpha_i L_i C_{yi}, \\
\check{\Phi}_{21i} &= V_{3i}A_i + V_{4i}C_i + B_i^T V_{1i}^T + D_i^T V_{2i}^T + 2\beta_i L_i C_{yi}, \\
\hat{\Phi}_{22i} &= H_{3i}B_i + H_{4i}D_i + B_i^T H_{3i}^T + D_i^T H_{4i}^T - 2\alpha_i Q_{4i}, \\
\check{\Phi}_{22i} &= V_{3i}B_i + V_{4i}D_i + B_i^T V_{3i}^T + D_i^T V_{4i}^T - 2\beta_i Q_{4i}, \\
\hat{\Phi}_{31i} &= H_{5i}A_i + H_{6i}C_i + B_{wi}^T H_{1i}^T, \check{\Phi}_{31i} = V_{5i}A_i + V_{6i}C_i, \\
\hat{\Phi}_{32i} &= H_{5i}B_i + H_{6i}D_i + B_{wi}^T H_{3i}^T, \check{\Phi}_{32i} = V_{5i}B_i + V_{6i}D_i, \\
\hat{\Phi}_{33i} &= H_{5i}B_{wi} + B_{wi}^T H_{5i}^T - \gamma_\infty I, \check{\Phi}_{33i} = B_{wi}^T X_{1i} B_{wi} - \Lambda_i, \\
\hat{\Phi}_{41i} &= G_{1i}A_i + G_{2i}C_i - H_{1i}^T, \check{\Phi}_{41i} = U_{1i}A_i + U_{2i}C_i - V_{1i}^T, \\
\hat{\Phi}_{42i} &= G_{1i}B_i + G_{2i}D_i - H_{2i}^T, \check{\Phi}_{42i} = U_{1i}B_i + U_{2i}D_i - V_{2i}^T, \\
\hat{\Phi}_{43i} &= G_{1i}B_{wi} - H_{3i}^T, \check{\Phi}_{43i} = -V_{3i}^T, \\
\hat{\Phi}_{44i} &= \hat{P}_{1i} - G_{1i} - G_{1i}^T, \check{\Phi}_{44i} = \hat{X}_{1i} - U_{1i} - U_{1i}^T, \\
\hat{\Phi}_{51i} &= G_{3i}A_i + G_{4i}C_i + L_i C_{yi}, \\
\check{\Phi}_{51i} &= U_{3i}A_i + U_{4i}C_i + L_i C_{yi}, \\
\hat{\Phi}_{52i} &= G_{3i}B_i + G_{4i}D_i - Q_{4i}, \check{\Phi}_{52i} = U_{3i}B_i + U_{4i}D_i - Q_{4i}, \\
\hat{\Phi}_{53i} &= G_{3i}B_{wi}, \hat{\Phi}_{54i} = \hat{P}_{2i} - G_{3i}, \check{\Phi}_{54i} = \hat{X}_{2i} - U_{3i}, \\
\hat{\Phi}_{55i} &= \hat{P}_{4i} - 2Q_{4i}, \check{\Phi}_{55i} = \hat{X}_{4i} - 2Q_{4i}, \\
\hat{\Phi}_{61i} &= G_{5i}A_i + G_{6i}C_i - H_{4i}^T, \check{\Phi}_{61i} = U_{5i}A_i + U_{6i}C_i - V_{4i}^T, \\
\hat{\Phi}_{62i} &= G_{5i}B_i + G_{6i}D_i - H_{5i}^T, \check{\Phi}_{62i} = U_{5i}B_i + U_{6i}D_i - V_{5i}^T, \\
\hat{\Phi}_{63i} &= G_{5i}B_{wi} - H_{6i}^T, \check{\Phi}_{63i} = -V_{6i}^T, \\
\hat{\Phi}_{64i} &= -G_{5i} - G_{2i}^T, \check{\Phi}_{64i} = -U_{5i} - U_{2i}^T, \\
\hat{\Phi}_{65i} &= -G_{4i}^T, \check{\Phi}_{65i} = -U_{4i}^T, \\
\hat{\Phi}_{66i} &= I - G_{6i} - G_{6i}^T, \check{\Phi}_{66i} = I - U_{6i} - U_{6i}^T,
\end{aligned}$$

then, a mode-dependent control law

$$u(k) = Q_{4i}^{-1} L_i y(k) \tag{25}$$

exists, and makes the closed-loop system stochastically stable with $\|\mathcal{T}_{cl}\|_2 < \gamma_2$ and $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$.

Proof: It follows from Theorems 2 and 4 that a desired control law exists if (8), (19), and (20) hold. For the purpose of parameterization, Q_{4i} in Theorem 2 and W_{4i} in Theorem 4 can be set to be equal without loss of generality, i.e., $Q_{4i} = W_{4i}$, for $i \in \mathcal{S}$. By expanding (8) and (20), and noting that

$$\begin{aligned}
-2C_{yi}^T K_i^T Q_{4i} K_i C_{yi} &\leq -2\left(C_{yi}^T K_i^T Q_{4i}^T\right) M_i \\
&\quad -2M_i^T \left(Q_{4i} K_i C_{yi}\right) + 2M_i^T Q_{4i} M_i,
\end{aligned}$$

we obtain that (8), (19) and (20) hold if (22)–(24) hold, where the parameterization $L_i = Q_{4i} K_i$ is used. □

When α_i, β_i, and M_i are fixed, (23) and (24) become strict LMIs, which could be verified easily by conventional LMI solver. According to the proof of Theorems 1 and 3, the larger the α_i and β_i, the higher the reduction in conservatism of (23) and (24). If (21)–(24) do not hold for sufficiently large $\alpha_i > 0$ and $\beta_i > 0$, it is plausible to conclude that a desired controller does not exist. As a matter of fact, when $M_i = Q_{4i}^{-1} L_i C_{yi}$, the left sides of (23) and (24) are monotonic decreasing matrix functions with respect to α_i and β_i, respectively, i.e., for $\alpha_i^{(1)} > \alpha_i^{(2)}$ and $\beta_i^{(1)} > \beta_i^{(2)}$,

$$\begin{aligned}
\Phi_{\infty i}\left(\alpha_i^{(1)}, Q_{4i}^{-1} L_i C_{yi}\right) &\leq \Phi_{\infty i}\left(\alpha_i^{(2)}, Q_{4i}^{-1} L_i C_{yi}\right), \\
\Phi_{2i}\left(\beta_i^{(1)}, Q_{4i}^{-1} L_i C_{yi}\right) &\leq \Phi_{2i}\left(\beta_i^{(2)}, Q_{4i}^{-1} L_i C_{yi}\right).
\end{aligned}$$

Hence, we can set α_i and β_i to be large values. The remaining problem is how to select M_i. It can be seen from the proof of Theorem 3 that the left sides of (23) and (24), $\Phi_{\infty i}(\alpha_i, M_i)$ and $\Phi_{2i}(\alpha_i, M_i)$, achieve their minima only if $M_i = Q_{4i}^{-1} L_i C_{yi}$. Therefore, we adopt a simple iterative algorithm to solve the condition of Theorem 5.

Algorithm:

(1) Set $\nu = 1$ and α_i, β_i to be sufficiently large values (for example, $\alpha_i = \beta_i = 10^4$, for each $i \in \mathcal{S}$). Select initial values $M_i^{(\nu)}$, $i = 1, 2, \ldots, s$, such that system $\mathcal{T}$ with $u(k) = M_i^{(\nu)} x(k)$ (denoted as $\mathcal{T}_{sf}$) is stochastically stable with $\|\mathcal{T}_{sf}\|_2 < \gamma_2$ or $\|\mathcal{T}_{sf}\|_\infty < \gamma_\infty$.

(2) For fixed α_i, β_i, and $M_i^{(\nu)}$, solve the following convex optimization problem with respect to $L_i^{(\nu)}$, $Q_{4i}^{(\nu)}$, $P_{1i}^{(\nu)} > 0$, $P_{4i}^{(\nu)} > 0$, $P_{2i}^{(\nu)}$, $X_{1i}^{(\nu)} > 0$, $X_{4i}^{(\nu)} > 0$, $X_{2i}^{(\nu)}$, $H_{\tau i}^{(\nu)}$, $V_{\tau i}^{(\nu)}$, $G_{\tau i}^{(\nu)}$, $U_{\tau i}^{(\nu)}$, $(\tau = 1, 2, \ldots, 6)$.

Minimize $\gamma^{(\nu)}$ subject to, for each $i \in \mathcal{S}$, (21), (22) and

$$\Phi_{\infty i}\left(\alpha_i, M_i^{(\nu)}\right) < \gamma^{(\nu)} I,$$

$$\Phi_{2i}\left(\alpha_i, M_i^{(\nu)}\right) < \gamma^{(\nu)} I.$$

If a $\gamma_{ltz}^{(\nu)} \leq 0$ is found during solving the convex optimization problem, then the system is output-feedback stabilizable, and a controller law can be obtained as (25). **STOP**.

(3) Denote $\gamma_*^{(\nu)}$ as the optimal value of $\gamma^{(\nu)}$. If

$$\left|\gamma_*^{(\nu)} - \gamma_*^{(\nu-1)}\right| \leq \delta,$$

where δ is a prescribed tolerance, then go to next step, else update $M_i^{(\nu+1)}$ as

$$M_i^{(\nu+1)} = \left(Q_{4i}^{(\nu)}\right)^{-1} L_i^{(\nu)} C_{yi},$$

and set $\nu = \nu + 1$, then go to Step 2.

(4) A desired control law may not exist. **STOP**.

Remark 4. The convergence of the iteration is not guaranteed. However, it can be shown easily that the sequence $\gamma_*^{(\nu)}$ is monotonic decreasing with respect to ν, i.e., $\gamma_*^{(\nu)} \leq \gamma_*^{(\nu-1)}$. If $\gamma_*^{(\nu)}$ does not converge to a positive number, then, after a sufficiently large number of iterations, $\gamma_*^{(\nu)}$ will always be negative, which means that the system is output-feedback stabilizable. Therefore, the case that the iteration is non-convergent is trivial.

Remark 5. Initial values $M_i^{(1)}$ are $\mathcal{H}_2$ or $\mathcal{H}_\infty$ state-feedback controller matrices, which can be found by existing approaches Ji et al. [1991], Costa et al. [1997]. It should be pointed out that the optimum of the converged value $\gamma_*^{(\infty)}$ is affected by the initial values $M_i^{(1)}$, α_i, and β_i, and the optimization of $M_i^{(1)}$, α_i, and β_i will be investigated in the future.

By setting P_{4i}, X_{4i}, and Q_{4i} to be mode-independent, as stated in Remark 3, we give a sufficient condition for the existence of mode-independent controllers.

Theorem 6. If there exist $P_{1i} = P_{1i}^T$, P_4, P_{2i}, $H_{\nu i}$, $G_{\nu i}$, $X_{1i} = X_{1i}^T$, X_4, X_{2i}, $U_{\nu i}$, $V_{\nu i}$, $(\nu = 1, 2, \ldots, 6)$, M_i, L, Q_4, and scalars $\alpha_i > 0$, $\beta_i > 0$ such that, for each $i \in \mathcal{S}$, (21)–(24) hold, then a mode-independent control law $u(k) = Q_4^{-1} L y(k)$ exists, and makes the closed-loop system stochastically stable with $\|\mathcal{T}_{cl}\|_2 < \gamma_2$ and $\|\mathcal{T}_{cl}\|_\infty < \gamma_\infty$.

5. CONCLUSION

The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem of discrete-time Markovian jump systems via static output-feedback controllers has been solved by employing an augmented system representation. New characterizations on stochastic stability and $\mathcal{H}_2/\mathcal{H}_\infty$ performance of the closed-loop system are established in terms of the new representation and the matrix inequality technique. Based on these new results, a sufficient condition with redundant matrix variables for the existence of the mode-dependent controller is proposed, and an iterative algorithm is given to solve the condition. An extension to the mode-independent case is provided as well.

REFERENCES

H. J. Chizeck, A. S. Willsky, and D. Castanon. Discrete-time Markovian-jump linear quadratic optimal-control. *Int J. Control*, 43(1):213–231, January 1986.

O. L. V. Costa and M. D. Fragoso. Stability results for discrete-time linear-systems with Markovian jumping parameters. *Journal of Mathematical Analysis and Applications*, 179(1):154–178, October 1993.

O. L. V. Costa and M. D. Fragoso. Discrete-time LQ-optimal control problems for infinite Markov jump parameter systems. *IEEE Trans. Automatic Control*, 40 (12):2076–2088, December 1995.

O. L. V. Costa and R. P. Marques. Robust $\mathcal{H}_2$-control for discrete-time Markovian jump linear systems. *Int J. Control*, 73(1):11–21, January 2000.

O. L. V. Costa and R. P. Marques. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$-control of discrete-time Markovian jump linear systems. *IEEE Trans. Automatic Control*, 43(1):95–100, January 1998.

O. L. V. Costa, J. B. R. Do Val, and J. C. Geromel. A convex programming approach to $\mathcal{H}_2$ control of discrete-time Markovian jump linear systems. *Int J. Control*, 66 (4):557–579, March 1997.

O. L. V. Costa, M. D. Fragoso, and R. P. Marques. *Discrete-Time Markov Jump Linear Systems*. Springer, London, 2005.

M. C. de Oliveira, J. Bernussou, and J. C. Geromel. A new discrete-time robust stability condition. *Syst. Control Lett.*, 37(4):261–265, July 1999.

C. E. de Souza. Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems. *IEEE Trans. Automatic Control*, 51(5):836–841, May 2006.

J. C. Geromel, P. L. D. Peres, and S. R. Souza. Convex analysis of output feedback structural constraints. In *Proceedings of the 32th Conference on Decision and Control*, volume 2, pages 1363–1364, December 1993.

Y. Ji, H. J. Chizeck, X. Feng, and K. A. Loparo. Stability and control of discrete-time jump linear-systems. *Control-Theory and Advanced Technology*, 7(2):247–270, June 1991.

P. Seiler and R. Sengupta. A bounded real lemma for jump systems. *IEEE Trans. Automatic Control*, 48(9): 1651–1654, September 2003.