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Time-Varying Compensation for Mid-Frequency Repeatable Runout in Hard Disk Drives via a Linear Feedback Scheme

Chin Kwan Thum [*,**] **Chunling Du** [*] **Ben M. Chen** [**]
Eng Hong Ong [*] **Kim Piew Tan** [*]

[*] *A*STAR Data Storage Institute, Singapore 117608.*
E-mail: DU_Chunling@dsi.a-star.edu.sg
[**] *Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576.*

Abstract: Conventional add-on feedback filters for mid-frequency (mid-f) repeatable runout (RRO) compensation in a hard disk drive (HDD) servo system either have a long filter transient or constitute a large sensitivity hump and poor stability margins. This paper presents a novel linear time-varying (LTV) group filtering scheme for the compensation of a few mid-f RRO harmonics. While having a short filter transient that ensures a fast disturbance attenuation, the proposed group filter does not cause any substantial unnecessary sensitivity function gain amplification at the steady state. Implementation results show the effectiveness of the proposed scheme used to compensate two mid-f RRO harmonics simultaneously.

1. INTRODUCTION

Inside hard disk drives (HDDs), the eccentricity of the servo tracks introduces written-in repeatable runout (RRO), which is periodic and synchronized with the rotation of the disk platters. They are typically of known frequencies (generally integer multiples of the frequency of the spindle operating speed), but of uncertain phases and magnitudes. Another main source of RRO comes from the spindle motor. As RRO inside a HDD servo system can greatly degrade the HDD servo performance and thus can constitute to a lower achievable track density, the effect of RRO must be attenuated. Further, with the continuous demand for a faster data transfer rate, future HDD servo system shall not only attenuate the effect of RRO substantially, but also shortly.

As the effect of RRO can be readily shown as [Jia et al (2005)],

$$PES_{RRO}(t) = d_{RRO}(t) * s(t), \qquad (1)$$

where $PES_{RRO}(t)$ represents the repeatable runout that appears in the position error signal (PES), $s(t)$ is the impulse response of the sensitivity transfer function, $S(s)$, and $d_{RRO}(t)$ stands for the repeatable disturbances. It is more difficult to compensate the impact of the mid-frequency (mid-f) RRO that is located closely to servo bandwidth on HDD servo performance than the low-frequency (low-f) RRO as the sensitivity gain, $|S(s)|$, at the lower frequency can be significantly and easily reduced by using any classical linear control techniques. Hence, in this paper, we focus on the development a novel RRO attenuation scheme that is capable of compensating the effect of mid-f RRO substantially and quickly.

Over these years, several control feedback methods [Wu et al (2006);Kempf et al (1993)] that are based on the internal-model-principle (IMP) [Francis et al (1976)] have been proposed to compensate the effect of RRO. From the frequency domain point-of-view, these methods typically incorporate add-on RRO compensator to deliberately insert lowly damped open loop poles at the RRO frequencies so as to create deep gain notches in $S(s)$ at the RRO frequencies. And to ensure a fast error attenuation convergence, these compensator gains have to be large such that the resultant closed loop transient characteristics of the $S(s)$ are not dominated by the lowly damped poles of the filters. Consequently, if any of those gain notches are placed at the mid-f region, i.e. close to the servo bandwidth, the width of those gain notches are usually too wide such that, under the constraints imposed by the Bode's integral theorem [Bode (1945)] and Bode's gain-phase relationship [Bode (1945)], they consitute a huge sensitivity hump that will lead to the amplification of disturbances at higher frequencies as well as poor stability margins at the steady state.

Here, we propose a new LTV add-on RRO compensator made of a group filter to compensate for the effect of mid-f RROs. By varying the dampings and the gains of the group filter appropriately with time, our proposed RRO compensator can have a short transient that enusres a fast error convergence and a mild distortion to the basic servo performance and stability achieved by the main feedback loop so as to avoid unnecessary disturbance amplification at higher frequencies and degradation of the stability margins at the steady state.

2. DESIGN OF LTV GROUP FILTER

Fig. 1 shows the open loop block diagram representation of a typical HDD servo system pre-compensated with the main servo compensator, $C(s)$, for basic servo performance and stability. $P(s)$ represents the transfer function of

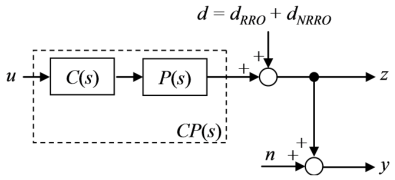

Fig. 1. Open loop block diagram of a typical HDD servo loop precompensated with a main servo compensator.

the Voice Coil Motor (VCM) actuator plant. Denote $CP(s) = C(s)P(s)$. z is the PES whereas y represents the measured PES. d_{NRRO} represents the equivalent summed effect of all non-repeatable runout (NRRO) disturbances. d_{RRO} denotes the equivalent summed effect of all RRO disturbances. n is measurement noise.

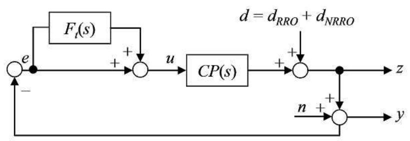

Fig. 2. Block diagram of the proposed $F_t(s)$ added onto the main feedback loop via parallel realization.

2.1 Group Filter Structure: Parallel Realization

The proposed LTV group filter, $F_t(s)$, is added onto the main feedback loop via parallel realization, as shown in Fig. 2. [Zheng et al (2006)] shows that by adopting such a filter structure, the control design for the main servo compensator, $C(s)$ (*basic servo performance and stability*), and the proposed $F_t(s)$ (*mid-f RRO attenuation*), can be decoupled. The final $S(s)$ is given by

$$S(s) = S_0(s)S_F(s) \tag{2}$$

where

$$S_0(s) = \frac{1}{1+CP(s)};\ S_F(s) = \frac{1}{1+T_0(s)F_t(s)};$$
$$T_0(s) = \frac{CP(s)}{1+CP(s)}.$$

Motivated by [Zheng et al (2006)], in order to preserve the good stability margins of the main feedback loop with $C(s)$, we propose the following group filter structure for $F_t(s)$:

$$F_t(s) = \sum_{i=1}^{n} F_t^i(s), \tag{3}$$

$$F_t^i(s) = K_i(t)\frac{s\left[w_i\cos(\varphi_i) - \sin(\varphi_i)s\right]}{s^2 + 2\zeta_i(t)w_i s + w_i^2}, \tag{4}$$

where w_i is the i-th mid-f RRO frequency, φ_i is the phase angle determined by

$$\varphi_i = \arg[T_0(jw_i)] \in (-\pi, \pi], \tag{5}$$

$\zeta_i(t)$ and $K_i(t)$ represent, respectively, the time-varying damping ratio and the time-varying positive filter gain of the i-th subfilter, $F_t^i(s)$. For $\zeta_i(t)$:

i) $\zeta_i(t) \in \Re$;
ii) $0 \le \zeta_i^{\min} \le \zeta_i(t) \le \zeta_i^{\max} \le 1,\ \forall t$;
iii) $0 \le |\zeta_i(t_1) - \zeta_i(t_2)| \le M_i^{\zeta}(t_1 - t_2), \forall t_1, t_2$ for some $M_i^{\zeta} > 0$ and $t_1 < t_2$.

And for $K_i(t)$:

i) $K_i(t) \in \Re$;
ii) $0 \le K_i^{\min} \le K_i(t) \le K_i^{\max},\ \forall t$;
iii) $0 \le |K_i(t_1) - K_i(t_2)| \le M_i^{K}(t_1 - t_2), \forall t_1, t_2$ for some $M_i^{K} > 0$ and $t_1 < t_2$.

2.2 Closed Loop Stability

We have the following theorem about the global uniform exponentially stability (GUES) of the resultant LTV closed loop system, which is derived from the small-gain theorem [Liberzon (2003)].

Theorem 1. Consider the given pre-compensated plant, $CP(s)$ of Fig. 1 and the proposed LTV RRO compensator group filter of (3). Let there exists a constant LTI group filter, $G(s)$, which is of similar structure as $F_t(s)$ such that the transfer function $T(s)$, which is given by

$$T(s) = -\frac{[1+G(s)]\,CP(s)}{1+[1+G(s)]\,CP(s)}, \tag{6}$$

is asymptotically stable. Let $\Delta(j\omega)$ be a multiplicative perturbation defined by

$$\Delta(j\omega) = \max_t |\Delta_F(j\omega)|, \quad \forall t, \tag{7}$$

where

$$\Delta_F(s) = \frac{F_t(s) - G(s)}{1+G(s)}, \quad s = j\omega, \tag{8}$$

and there exists a bounding function $W_U(s)$ such that $|\Delta(j\omega)| \le |W_U(j\omega)|$ and $W_U \in RH_\infty$. Ignoring plant uncertainty, the resultant LTV closed loop system is GUES stable if

$$\|TW_U\|_\infty < 1. \tag{9}$$

Proof. Ignoring d and n, which do not affect closed loop stability, the block diagram in Fig. 2 can be represented by the block diagram in Fig. 3.(A), which is equivalent to the block diagram in Fig. 3.(B) if and only if $\Delta_F(s)$ is given by (8). From Fig. 3.(B), ignoring $\Delta_F(s)$, since the nominal complementary transfer function is given by $T(s)$ of (6), following the result of small gain theorem, we have the complete proof of the GUES stability of the resultant LTV closed loop system.

3. SELECTION OF DESIGN PARAMETERS

Designing of the LTV group filter is done by means of appropriate designing of its sub-filters, $F_t^i(s)$. Prior to the design of $F_t^i(t)$, we assume that the main servo compensator, $C(s)$, has been properly designed to achieve basic servo stability and performance.

Here, we are interested in the speed of attenuation for any mid-f RRO with a center frequency of w_i, RRO_i.

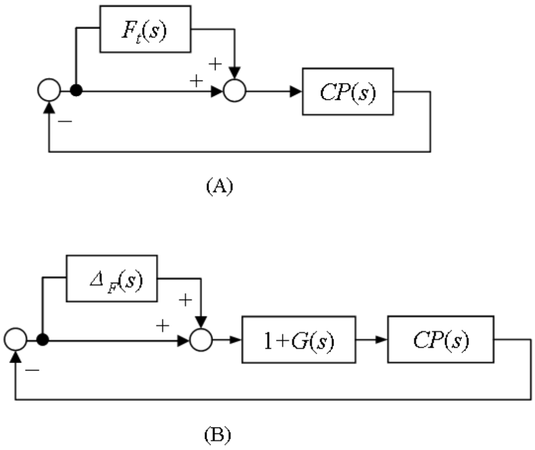

Fig. 3. Different representations of the block diagram in Fig. 2.

Generally, the transient time of any harmonic, t_s, inside a LTI closed loop system is dominantly determined by the pair of closed loop poles that is located closest to the harmonic frequency, which is given by

$$t_s \approx \frac{4.6}{\zeta_p w_p}, \tag{10}$$

where ζ_p and w_p stands for the damping ratio and center frequency of the closest pair of closed loop poles, respectively. In our case, without loss of generality, $w_p = w_i$. And using the well-known Q-factor rule [Antoniou (1993)] that identifies the strong association between the -3dB bandwidth, Δw, of any gain notch in the frequency domain that centers at w_p and the damping ratio ζ_p of the pair of poles centering at w_p, which is given by

$$\Delta w = \zeta_p w_p, \tag{11}$$

we can easily approximate the required -3 dB bandwidth of the resultant $|S(s)|$ notch at w_i for any desired value of ζ_p, which in turn, decides the transient time or the speed of RRO attenuation for RRO_i. Since the -3 dB bandwidth of the required $|S(s)|$ notch at w_i can be approximated by the 3 dB bandwidth of $F_t^i(s)$, that is dominantly controlled by the value of $K_i(t)$, it becomes apparent that by adjusting $K_i(t)$, we can decide the effective closed loop damping ratio, i.e., ζ_p, and so, the speed of RRO attenuation at any time t.

Our control strategy is as follows. Initially, when the effect of RRO_i is large, $K_i(t)$ shall be large, which consequently ensures a wide sensitivity gain notch at w_i and also a large ζ_p, and thus, a high speed RRO_i attenuation. As the effect of RRO_i is reduced exponentially with time, to avoid distortion to the basic servo performance and good stability margins achieved by $C(s)$, during the steady state, the -3 dB bandwidth of the resultant sensitivity gain notch at w_i should be significantly reduced, meaning a small $K_i(t)$ as $t \to \infty$.

Let the required extra amount of attenuation at w_i provided by the i-th sub-filter during steady state be given by $\left|S^i_{f(desired)}(jw_i)\right|$. Then from (2), the steady state value of $\left|F_t^i(jw_i)\right|$, $\left|F^i(jw_i)\right|$, can be designed to be given by

$$\left|F^i(jw_i)\right| = \left[\left|S^i_{f(desired)}(jw_i)\right|^{-1} - 1\right] |T_0(jw_i)|^{-1}. \tag{12}$$

During the transient state of the RRO_i attenuation, as the value $K_i(t)$ is varying, $\zeta_i(t)$ should be varying accordingly to ensure that the value of $\left|F_t^i(jw_i)\right|$, which is important to magnitude of RRO_i attenuation, is approporiate. By letting $s = jw_i$, from (4), the relationship between $K_i(t)$, $\zeta_i(t)$ and $\left|F_t^i(jw_i)\right|$ is derived and given by

$$\zeta_i(t) = 0.5\left|F^i(jw_i)\right|^{-1} K_i(t). \tag{13}$$

Thus, assuming $\left|F_t^i(jw_i)\right|$, $\forall t$, is a constant, using (13), the initial value and final value of $\zeta_i(t)$ and $K_i(t)$ can be designed based on the following 2 relationships:

$$\zeta_i^0 = 0.5\left|F^i(jw_i)\right|^{-1} K_i^0; \tag{14}$$

$$\zeta_i^\infty = 0.5\left|F^i(jw_i)\right|^{-1} K_i^\infty. \tag{15}$$

Remark 1: When $K_i(t)$ reduces too rapidly so as to quickly reduce the effective width of the gain notch, the speed of RRO_i attenuation also deteroriates quickly naturally. And it is noted that in order to avoid a rapid deteroriation in the speed of RRO_i attenuation as $K_i(t)$ reduces exponentially, it helps by either slowing the reduction rate of $K_i(t)$ or designing the effective $\left|S_f^i(jw_i)\right|$ to be appropriately lower than $\left|S^i_{f(desired)}(jw_i)\right|$ during the transient state. As a result, in order to achieve excellent attenuation performance during the transient state, $\zeta_i(t)$ and $K_i(t)$ should be designed such that

$$\zeta_i^{\min} \le \zeta_i(t) \le 0.5\left|F^i(jw_i)\right|^{-1} K_i(t) \le \zeta_i^{\max} \quad \forall t. \tag{16}$$

In this paper, our choice of $\zeta_i(t)$ and $K_i(t)$ are respectively given by

$$K_i(t) = K_i^\infty + (K_i^0 - K_i^\infty)\exp(-\beta_i t); \tag{17}$$

$$\zeta_i(t) = \zeta_i^\infty + (\zeta_i^0 - \zeta_i^\infty)\exp(-\gamma_i t) - \zeta_i^\infty \exp(-\lambda_i |t - \sigma_i|), \tag{18}$$

where β_i, γ_i, λ_i and σ_i are positive scalar tuning parameters that determine how fast $K_i(t)$ and $\zeta_i(t)$ approaches their steady state values, which in turns decides how fast the the notch widths of the group filter shrink while maintaining an excellent attenuation performance.

4. APPLICATION TO A HDD SERVO SYSTEM

4.1 Identification of the VCM Actuator Plant Model

Through experiments, the frequency response of the actual VCM of a 3.5" HDD is obtained and is shown in Fig. 4. The transfer function of the VCM actuator is given by

$$P(s) = \frac{2.18 \times 10^8}{s^2 + 1131s + 3.948 \times 10^5} \prod_{i=1}^{5} P_i^{\mathrm{rm}}(s) \tag{19}$$

with

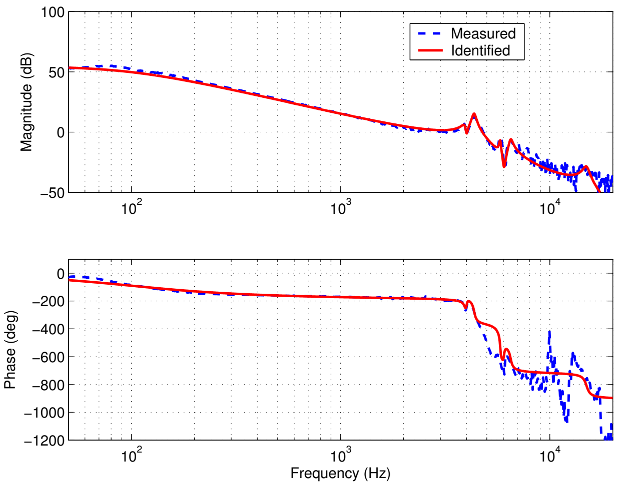

Fig. 4. Frequency response of the VCM actuator (LDV range 2 μm/V).

$$P_1^{\mathrm{rm}}(s) = \frac{0.9752s^2 + 490.2s + 6.16 \times 10^8}{s^2 + 992.7s + 6.16 \times 10^8};$$
$$P_2^{\mathrm{rm}}(s) = \frac{0.5625s^2 - 1640s + 7.47 \times 10^8}{s^2 + 1093s + 7.47 \times 10^8};$$
$$P_3^{\mathrm{rm}}(s) = \frac{0.9191s^2 + 698.7s + 1.328 \times 10^9}{s^2 + 583.1s + 1.328 \times 10^9};$$
$$P_4^{\mathrm{rm}}(s) = \frac{0.02641s^2 - 1327s + 1.668 \times 10^9}{s^2 + 1634s + 1.668 \times 10^9};$$
$$P_5^{\mathrm{rm}}(s) = \frac{8.883 \times 10^9}{s^2 + 5655s + 8.883 \times 10^9}.$$

4.2 Main Servo Compensator Design

$C(s)$ is designed using the classical control techniques and it is given as follows

$$C(s) = C_{PI-Lead}(s) C_{nf}(s) \tag{20}$$

with

$$C_{PI-Lead}(s) = 6.59 \frac{s + 3302}{s} \frac{s + 6.283}{s + 2.466 \times 10^5};$$
$$C_{nf}(s) = \frac{s^2 + 1913s + 7.470 \times 10^8}{s^2 + 2.733 \times 10^4 s + 7.470 \times 10^8}$$

where $C_{PI-Lead}$ is a PI-Lead controller and C_{nf} is a notch filter designed to gain-stabilize the first two resonance modes.

4.3 Design of the LTV Group Filter for Mid-f RRO Compensation

To evaluate the effectiveness of the proposed technique, we first assume that we have to attenuate 2 mid-f RRO that center at 700 Hz and 2 kHz respectively. And the design specifications are detailed in Table 1. Based on the specifications given above, obviously,

$$w_1 = 2\pi 700 \text{ rad/s}, \quad w_2 = 2\pi 2000 \text{ rad/s}. \tag{21}$$

With the given desired settling time, using (10) and (11), the desired initial Δw of the $F_t^1(s)$ and $F_t^2(s)$, are supposed

Table 1. Design Specifications

Specifications		
	700 Hz	2 kHz
1. Transient settling time (ms)	≤ 4	≤ 2
2. Extra attenuation (dB)	≥ 15	≥ 15

to be greater than 184 Hz and 367 Hz, respectively. As a result,$K_1^0 = 0.5$ and $K_2^0 = 0.3$. To ensure a low sensitivity hump as well as good stability margins during the steady state, K_1^∞ and K_2^∞ are designed to be 0.15 and 0.06, respectively, such that $K_i^\infty << K_i^0, \ \forall i$. Consequently, to ensure an attenuation of greater than 15 dB at the steady state,

$$\left|F^1(jw_1)\right| = \left|F^2(jw_2)\right| = 15\text{dB}. \tag{22}$$

So naturally, following (14) and (15), ζ_1^0, ζ_1^∞, ζ_2^0, ζ_2^∞ are respectively given by 0.0445, 0.0133, 0.0267 and 0.0053. Subsequently, the final proposed group filter for RRO compensation is given by (3) with $n = 2$ and

$$w_1 = 2\pi 700 \text{ rad/s}; \ \varphi_1 = -37.0 \text{ deg};$$
$$K_1(t) = 0.15 + 0.35 \exp(-\beta_1 t); \tag{23}$$
$$\zeta_1(t) = 0.0133 + 0.0311 \exp(-\gamma_1 t) \tag{24}$$
$$-0.0133 \exp(-\lambda_1 |t - \sigma_1|),$$

$$w_2 = 2\pi 2000 \text{ rad/s}; \ \varphi_2 = -109.4 \text{ deg};$$
$$K_2(t) = 0.06 + 0.24 \exp(-\beta_2 t); \tag{25}$$
$$\zeta_2(t) = 0.0053 + 0.0213 \exp(-\gamma_2 t) \tag{26}$$
$$-0.0053 \exp(-\lambda_2 |t - \sigma_2|),$$

and

$$\beta_1 = \beta_2 = 231, \ \gamma_1 = \gamma_2 = 400,$$
$$\lambda_1 = \lambda_2 = 105, \ \sigma_1 = \sigma_2 = 0.005.$$

4.4 Stability Analysis

Here, we select $G(s)$ to be given by

$$G(s) = F_t(s)|_{t=0.005}. \tag{27}$$

Thus, $G(s)$ is the LTI group filter of the same structure as $F_t(s)$, whose parameters are given by (23)-(26) when time $t = 5$ ms. From the nyquist plot of $[1 + G(s)] CP(s)$ as shown in Fig. 5, it is obvious that $T(s)$ of (6) is an asymptotically stable transfer function. Using MATLAB simulation, the multiplicative perturbation

$$\left|\frac{F_t(j\omega) - G(j\omega)}{1 + G(j\omega)}\right|, \quad \forall t \tag{28}$$

caused by the time-varying control parameters of $F_t(s)$ is computed and shown in Fig 6. And the bounding function $W_U(s)$ is found to be

$$W_U(s) =$$
$$\frac{0.23617\left(s^2 + 4283s\right)}{s^2 + 825.7s + 1.666 \times 10^7} \times \frac{s^2 + 276.1s + 1.934 \times 10^7}{s^2 + 321.4s + 1.934e007}$$
$$\times \frac{s^2 + 2105s + 9.56 \times 10^7}{s^2 + 295.6s + 1.423 \times 10^8} \times \frac{s^2 + 371.5s + 1.583 \times 10^8}{s^2 + 589.7s + 1.579 \times 10^8}.$$

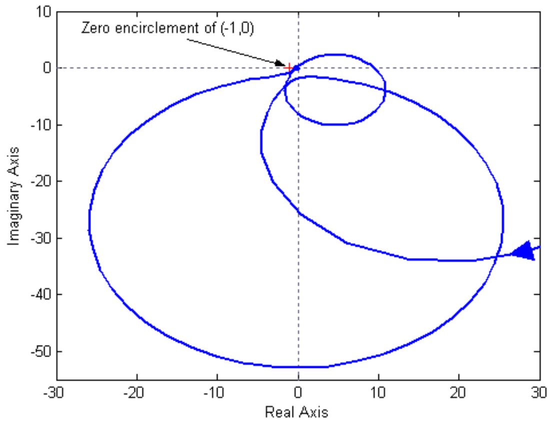

Fig. 5. Nyquist plot of $[1 + G(s)]\, CP(s)$.

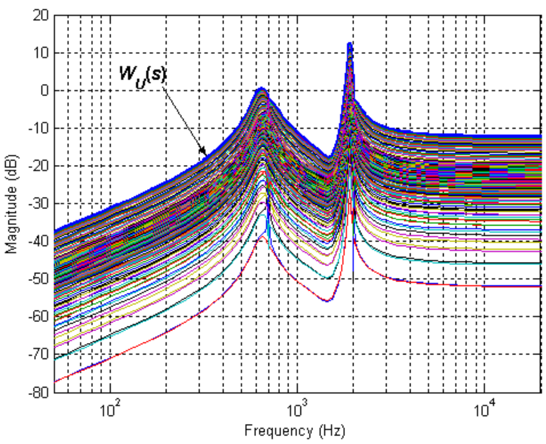

Fig. 6. Multiplicative perturbation of (28) and $W_U(s)$.

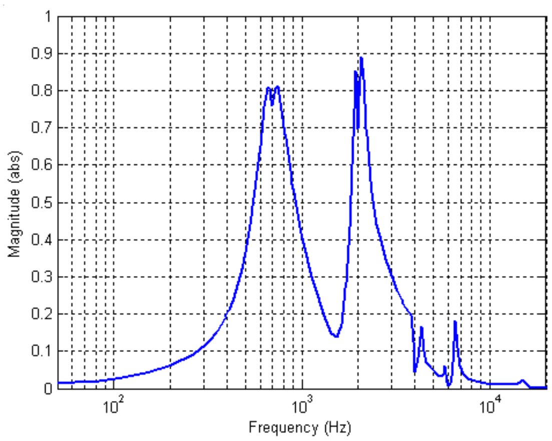

Fig. 7. Frequency response of $|TW_U|$.

The frequency response of $|TW_U|$ is shown in Fig. 7. It is obvious that $\|TW_U\|_\infty < 1$. Hence, following the result of *theorem 1*, the resultant LTV closed loop system is GUES stable.

4.5 Implementation Results

All controllers are discretized using the bilinear rule, except for the notch filter, $C_{nf}(s)$, which is discretized using the matched pole-zero rule, at a sampling frequency of 40 kHz and implemented using a dSpace DSP. Experiments are conducted on a dissected 3.5” HDD that is placed on a vibration free platform and its R/W head displacement is measured using a scanning laser doppler vibrometer (LDV).

For performance comparison, a conventional LTI group filter, $F_{LTI1}(s)$, is designed based on the same specifications. $F_{LTI1}(s)$ shares the same filter structure with the proposed $F_t(s)$, but its filter gains and dampings are constant throughout time instead. It is given by

$$F_{LTI1}(s) = F_t(s)|_{t=0} \tag{29}$$

On the other hand, another LTI group filter, $F_{LTI2}(s)$, is designed to share the similar steady state frequency domain properties with $F_t(s)$, and given by

$$F_{LTI2}(s) = F_t(s)|_{t=\infty} \tag{30}$$

During the experiments, 700 Hz and 2 kHz sinosudal disturbances with amplitude of 0.4 μm (200 mV) and 0.1 μm (50 mV), respectively, are separately injected into the closed loop. The transient responses of RRO attenuation using respective schemes are shown in Fig. 8-11. Please note that for Fig. 8-11, Ch 1 is the LDV measurement at 2μm/V and Ch 2 is the VCM driver input. The transient performance of the proposed scheme as shown in Fig. 8 and 9, are very similar to those when using the conventional $F_{LTI1}(z)$. On the other hand, the measured transient responses of RRO attenuation are significantly more oscillatory and took longer times to settle down (133 % longer for 700 Hz and 300 % longer for 2 kHz), when using the conventional $F_{LTI2}(z)$, as shown in Fig. 10 and 11. The measured $|S(s)|$ and bode plots of the Open loop transfer function with respective methods are shown in Fig. 12 and 13. As displayed in Fig. 12 and 13, comparing with the conventional $F_{LTI1}(z)$, the proposed $F_t(z)$ constitutes to a significantly smaller increase of the sensitivity hump and lesser degradation of the good stability margins achieved by $C(s)$.

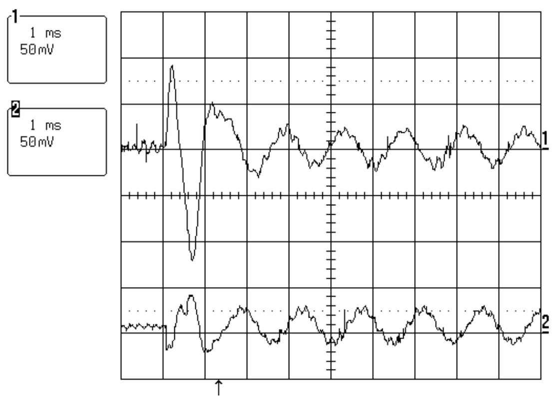

Fig. 8. Proposed LTV method, $F_t(z)$. 700 Hz. $t_s \approx 3$ ms.

5. CONCLUSION

In this paper, a novel LTV group filtering method for a few mid-f RRO compensation has been proposed. By varying the filter gains and dampings exponentially with time, the proposed LTV method shows that it can achieve good

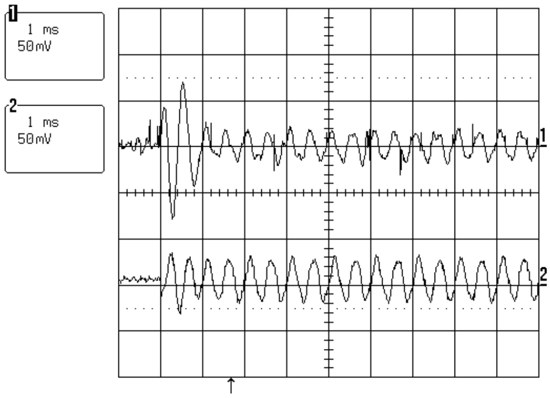

Fig. 9. Proposed LTV method, $F_t(z)$. 2 kHz. $t_s \approx 1.5$ ms.

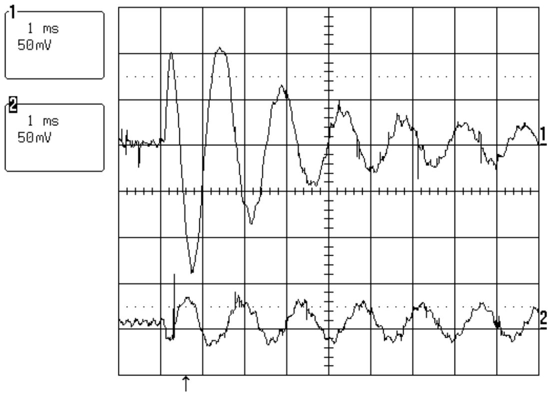

Fig. 10. Conventional LTI method, $F_{LTI2}(z)$. 700 Hz. $t_s \approx$ 7 ms.

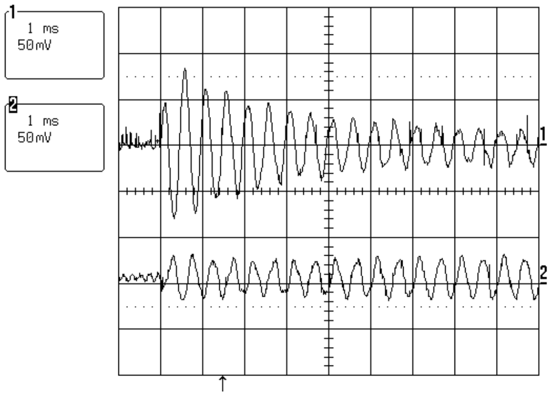

Fig. 11. Conventional LTI method, $F_{LTI2}(z)$. 2 kHz. $t_s \approx$ 6 ms.

transient disturbance attenuation performance without compromising the good steady state performance as well as stability robustness. Experimental results have verified the effectiveness of the proposed method in compensation for two mid-f RRO harmonics, and the comparison with the conventional LTI methods has also been made to demonstrate the benefits of the proposed LTV group filtering method.

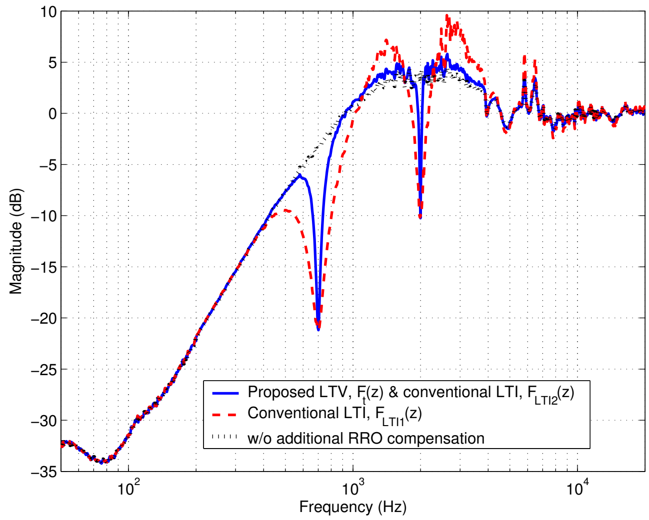

Fig. 12. Frequency responses of the respective sensitivity gain functions

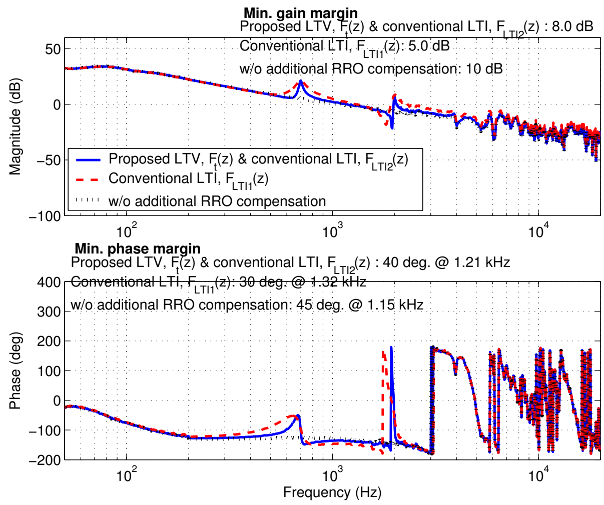

Fig. 13. Frequency responses of the respective Open loop transfer functions at steady state (LDV 2μm/V).

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