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Region Coverage for Planar Sensor Network via Sensing Sectors ⋆

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Abstract: In this paper, asymptotical k-coverage of a planar sensor network with a sensing sector assigned to each node/sensor is considered. Sufficient conditions of the node density for the complete k-covering with probability approaching 1 are proposed when nodes deployment forms a planar Poisson point process and the sensing area of each node only covers a sector of a certain angle. Both fixed and varying sector direction cases are investigated, and related comparisons are also shown.

1. INTRODUCTION

In recent years, there has been an increasing research interest for agent-based problems. Although different models for flocks of birds, robotic networks, and groups of particles (Reynolds [1987], Cortes et al. [2005], Hong et al. [2007]) have been established to characterize the coordination behaviors, the distributed local information plays a key role in the studies of these multi-agent systems in order to achieve a collective objective. In fact, to serve a global aim, all the related information of the whole agent groups needs to be fully exchanged with local rules or obtained by coverage.

There are several agent-based coverage problems such as communication coverage (related to the network connectivity) and sensing coverage in various sensor networks. In fact, connectivity-coverage was introduced by Erdös and Rényi in as early as 1940s using random graph theory (Bollobas [2001]). Since then this probabilistic idea has been employed to investigate asymptotical connectivity for models with many kinds of nodes deployment (Meester et al. [1996], Gupta et al. [1998]). On the other hand, sensing coverage problem has been widely studied, too. Probabilistic methods have also been applied to the vacancy problem in coverage processes since it was brought forward in the 1960s (Hall [1988], Zhang et al. [2004], Kumar et al. [2004]). Different formulations, such as k-coverage (Zhang et al. [2004], Kumar et al. [2004]) and θ-coverage (Xue et al. [2006]), were proposed for different coverage problems. In addition, coverage problems have been studied in various ways such as differential inclusions (Cortes et al. [2005]). In most results of coverage, the local sensing region of each agent was supposed to be a disk with a certain radius.

However, in practice, there are other geometric sensing forms. In fact, if we consider the coverage in a plane, the sensing region of some agents, including visual field of a bird (Reynolds [1987]) and a robotic camera (Arkin [1998]), becomes a sector with a certain angle.

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In this paper, we will formulate and study a coverage problem with local sensing area as a sector in order to extend the result given on disks. Note that a sector will turn to a disk when the central angle of the sector equals 2π. Therefore, the sector-coverage problem is a generalization of a disk-coverage problem. In comparison with a disk, a sector is sensitive to its direction, which implies that the sector-coverage problem is more challenging. Here we will consider the k-coverage with fixed direction or randomly selected direction to investigate threshold problems for the sensing coverage.

This paper is organized as follows. In Section 2, we set up a sector coverage model with node deployment forming a planar Poisson process and then in Section 3 we present a sufficient condition for the nodes intensity to maintain k-covering with probability approaching 1. Following that, we report similar results when we consider two varying direction cases in Section 4. Finally, we give concluding remarks in Section 5.

2. PROBLEM FORMULATION

In this section, we will introduce some basic concepts related to coverage and then formulate our problem.

There have been various coverage models. One of the ideas is to consider the coverage as a stochastic process. A coverage process can be described as $\mathcal{C} = \{\xi_i + S_i : i = 1, 2, \cdots\}$ (Hall [1988]), where $X = \{\xi_1, \xi_2, \cdots\}$ is a countable collection of points in a Euclidean space, and $S = \{S_1, S_2, \cdots\}$ is a collection of nonempty sets. With appointing a point process on X and the distribution on S, we get a coverage model. Point ξ_i in X is usually considered as a sensing node and $\xi_i + S_i$ is usually considered as its sensing region in practice.

At first, we introduce k-coverage, which has been widely studied in coverage problems (referring to Zhang et al. [2004], Kumar et al. [2004]).

Definition 1. Coverage number of a point p is the number of nodes, whose sensing regions cover p. Region R is said to be k-covered if the coverage number of each point is at least k.

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Obviously, the region R is covered if it is k-covered, where $k \geq 1$.

Note that a sufficient condition to guarantee k-covering was given when each local sensing set is taken as a disk (Wang et al. [2003]). To study the case with sensing sectors, we introduce the following concepts.

Suppose n nodes are placed in a convex region R in the plane and each node has an open and bounded sensing region. In practice, the area of each sensing region is usually far smaller than that of R, which is therefore assumed to be true in the following.

Definition 2. Sensing boundary of node v is the boundary of its sensing region. *Coverage boundary* is the boundary of the coverage region R. A point $p \in R$ is called an *interior intersection point* if it is an intersection point between sensing boundaries. A point p is called an *boundary intersection point* if it is an intersection point between sensing boundary and coverage boundary.

Then we have a lemma, which is useful to estimate the k-covering probability in following theorems.

Lemma 3. Assume that the isolated interior and boundary intersection points are finite. Then region R is k-covered if:

(1) there exist both interior and boundary intersection points.

(2) all intersection points are at least k-covered.

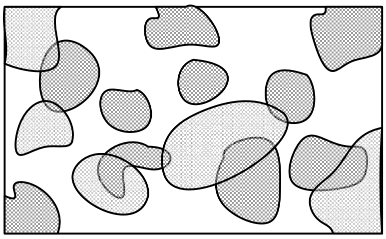

Figure 1. Coverage and sensing boundaries divide up *R* into disjoint open fractions

Proof: With the assumption of this lemma, region R is separated by sensing boundaries and coverage boundary into disjoint fractions (Fig. 2). Since each sensing region is open, it cannot happen that any sensing region covers the fraction boundary without covering its interior. On the other hand, if a sensing region covers one interior point of certain fraction, it will definitely cover the whole interior of that fraction. Thus the interior points of each fraction share the same coverage number and this coverage number is no less than that of the fraction boundary. At the same time, interior and boundary intersection points lie in the fraction boundaries, and their coverage number is locally minimal along each fraction boundary since they are intersection of boundaries. Therefore, if all intersection points are at least k-covered, then the whole coverage region will be k-covered. Then the conclusion follows immediately. □

Up to now, disks are used as the local coverage sets in many coverage models (Hall [1988], Kumar et al. [2004], Zhang et al. [2004], Meguerdichian et al. [2001a,b, 2003]). Here, we propose a novel coverage formulation of sensing sectors.

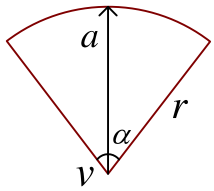

Figure 2. Four elements to determine the location of a sector

The location of a sector $(v, r, \boldsymbol{a}, \alpha)$ in 2-dimensional Euclidean space is totally determined by its vertex v, radius r, axis direction $\boldsymbol{a}$, and central angle α (see Fig. 1). In our formulation, the sensing area of a sensor node v is supposed to be the interior (hence, an open set) of the sector $(v, r, \boldsymbol{a}, \alpha)$, in which v is just located at the vertex. In what following, when there is no confusion, we will call the sensing area of node v its sensing sector. We say a point p is covered by node v if p is in the sensing sector of v. Thus, we consider the coverage problem as follows. The region to be monitored or covered is a square region R with side length ℓ; the deployment of sensor nodes (whose locations are determined by the points belonging to set X) forms a planar Poisson point process with density λ for convenience; and the sensing area of each node is a sector with the same area (that is, S is a set of sectors). Note that n nodes placed in R randomly, each with an identical uniform distribution that is essentially a Poisson process with density n/ℓ^2, as pointed out in Hall [1988] (page 39).

Obviously, if $\alpha = 2\pi$, then the sensing sector turns into a disk in the conventional coverage models. Without loss of generality, we take $r = \frac{1}{\sqrt{\pi}}$ in this paper, which makes the sector area simply equal 1 when $\alpha = 2\pi$.

Denote $\mathcal{P}_k$ as the probability that R is k-covered. Then R is said to be asymptotically k-covered if $\mathcal{P}_k \to 1$ as $\ell \to +\infty$. Note that sectors are directional (described by $\boldsymbol{a}$), different from disks, which are special forms of sectors. Once we fix the sensing sectors' direction (that is, $\boldsymbol{a}$) and central angle, each node has a fixed sensing area. In the following sections, we will give sufficient conditions to guarantee asymptotical k-covering in two different cases: fixed directions and variable directions.

3. SECTOR COVERAGE WITH FIXED DIRECTION

As we know, different from sensing disk, sensing sector depends on the direction of the central axis. In this section, we will discuss the simplest case when all the sensing sectors to cover region R share the same axis direction, or in other words, there is a common sector-axis direction for all the nodes.

It is known that the coverage problem does not rely on the selection of coordinate systems. However, to simplify the expression of problem formulation, we first fix any coordinate system in the plain with corresponding axes x and y. Then, we assume the common sector-axis direction to be the same as that of axis-x for convenience.

Denote the distance between a node v and a point p in region R as $d(v,p)$ and the argument of vector $\boldsymbol{vp}$ (the vector from point v to point p) as $\arg \boldsymbol{vp}$ (noting that the valued field of function $\arg \boldsymbol{x}$ is $(-\pi, \pi]$). Hence with fixed direction the sensing region of node v is the set

$$S_v = \{p \mid d(v,p) < \frac{1}{\sqrt{\pi}}, \arg \boldsymbol{vp} \in (-\frac{\alpha}{2}, \frac{\alpha}{2})\}.$$

Then we estimate the node density for asymptotical k-covering.

Theorem 4. Suppose $0 < \alpha \leq 2\pi$ and

$$\lambda = \frac{2\pi}{\alpha}(\log \ell^2 + (k+1)\log\log \ell^2) + c(\ell). \tag{1}$$

If $c(\ell) \to +\infty$, then $\mathcal{P}_k \to 1$ as $\ell \to +\infty$.

Proof: Set $\hat{\mathcal{P}}_k := 1 - \mathcal{P}_k$, and then $\hat{\mathcal{P}}_k$ can be rewritten in three terms:

$$\hat{\mathcal{P}}_k \triangleq P_1 + P_2 + P_3,$$

where

$$\begin{aligned}
P_1 \equiv & \mathcal{P}(\textit{no sensing sector vertex in } R) = \exp(-\lambda\ell^2) \to 0,\\
P_2 \equiv & \mathcal{P}(\textit{at least one sector vertex in } R\\
& \quad \textit{but no intersections each other}\\
& \quad \textit{or with the boundary of } R)\\
& \leq \mathcal{P}(\textit{at least one sector vertex in } R) \times \mathcal{P}(\textit{a}\\
& \quad \textit{given sensing sector with no intersects})\\
& \leq (1 - \exp(-\lambda\ell^2)) \times \exp(-4\lambda) \to 0,\\
P_3 \equiv & \mathcal{P}(R \textit{ is not completely } k-\textit{covered, at least one}\\
& \quad \textit{sector vertex in } R\textit{, there are intersections}\\
& \quad \textit{each other or with the boundary of } R).
\end{aligned}$$

Therefore, to show $\hat{\mathcal{P}}_k \to 0$, we only need to prove $P_3 \to 0$ as $\ell \to +\infty$.

According to Lemma 3, if R is not completely k-covered and there exist interior and boundary intersection points, then there should exist at least one un-k-covered intersection point. Denote $\mathcal{M}_k$ as the number of un-k-covered intersection points, and then we have,

$$P_3 \leq \mathcal{P}(\mathcal{M}_k \geq 1) \leq \mathbb{E}(\mathcal{M}_k). \tag{2}$$

Take $\mathcal{T}_1$ and $\mathcal{T}_2$ as the number of interior and boundary intersection points, respectively. Then we will give the upper bound of $\mathbb{E}(\mathcal{T}_1)$ *and* $\mathbb{E}(\mathcal{T}_2)$.

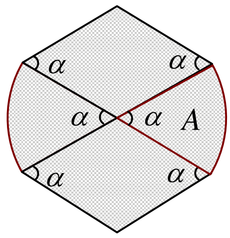

Figure 3. Only sensing sectors whose nodes fall into the shadowed area will intersect the given sensing sector *A*

Let us see that the expected number of sensing nodes in region R is $\lambda\ell^2$. When $0 < \alpha \leq \pi$, see Fig. 3 for that only sensing sectors whose nodes fall into the shadowed area, which is exactly composed of two diamonds and two sectors, would intersect the given sector A. Since the area of this shadowed area is $\frac{\alpha + 2\sin\alpha}{\pi}$ and two intersection points generated once intersection happened, the expected number of interior intersection points created by a single sensing sector would be $\frac{2\lambda(\alpha+2\sin\alpha)}{\pi}$. Note that each interior intersection point is counted twice in this case. Hence,

$$\mathbb{E}(\mathcal{T}_1) = \frac{\lambda^2\ell^2(\alpha + 2\sin\alpha)}{\pi}. \tag{3}$$

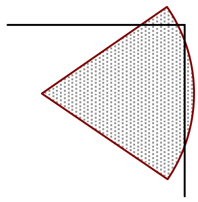

Figure 4. At most four boundary intersection points generated for nodes near the coverage boundary

On the other hand, only the sensing sectors of the nodes, whose distance away from the coverage boundary is less than $r = \frac{1}{\sqrt{\pi}}$, could possibly intersect the coverage boundary. Furthermore, when $0 < \alpha \leq \pi$, at most 4 isolated boundary intersection points would be generated for a given sector (the case when 4 boundary intersection points are generated can be found in Fig. 4). Thus,

$$\mathbb{E}(\mathcal{T}_2) \leq \frac{16\lambda\ell}{\sqrt{\pi}}. \tag{4}$$

When $\pi < \alpha \leq 2\pi$, with analysis similar to the above process, we also get the upper bound of the expected number of $\mathcal{T}_1$ and $\mathcal{T}_2$:

$$\mathbb{E}(\mathcal{T}_1) \leq \frac{1}{2} \times 4 \times 4\lambda \times \lambda\ell^2 = 8\lambda^2\ell^2, \tag{5}$$

and

$$\mathbb{E}(\mathcal{T}_2) \leq \frac{1}{2} \times 6 \times \frac{8\lambda\ell}{\sqrt{\pi}} = \frac{24\lambda\ell}{\sqrt{\pi}}. \tag{6}$$

Since we assume the deployment of sensor nodes form a planar Poisson point process with density λ, the probability that a given point is covered by at most $k-1$ sensor sectors is clearly

$$e^{-\frac{\alpha\lambda}{2\pi}}\sum_{i=o}^{k-1}\frac{(\frac{\alpha\lambda}{2\pi})^i}{i!}.$$

Thus,

$$\begin{aligned}\mathbb{E}(\mathcal{M}_k) &= (\mathbb{E}(\mathcal{T}_1)+\mathbb{E}(\mathcal{T}_2))\times(e^{-\frac{\alpha\lambda}{2\pi}}\sum_{i=o}^{k-1}\frac{(\frac{\alpha\lambda}{2\pi})^i}{i!})\\ &\le (8\lambda^2\ell^2+\frac{24\lambda\ell}{\sqrt{\pi}})\times(e^{-\frac{\alpha\lambda}{2\pi}}\sum_{i=o}^{k-1}\frac{(\frac{\alpha\lambda}{2\pi})^i}{i!})\\ &= \ell^2 e^{-\frac{\alpha\lambda}{2\pi}}(\frac{\alpha\lambda}{2\pi})^{k+1}\frac{32\pi^2}{\alpha^2(k-1)!}(1+o(1)). \quad (7)\end{aligned}$$

According to (2) and (7), along with (1),

$$\begin{aligned}P_3 \le \mathbb{E}(\mathcal{M}_k) &\le \frac{[\log\ell^2+(k+1)\log\log\ell^2+\frac{\alpha}{2\pi}c(\ell)]^{k+1}}{(\log\ell^2)^{k+1}e^{\frac{\alpha}{2\pi}c(\ell)}}\\ &\times\frac{32\pi^2}{\alpha^2(k-1)!}(1+o(1)). \quad (8)\end{aligned}$$

Since $c(\ell)\to+\infty$ as $\ell\to+\infty$, the right hand side of (8) converges to 0. Therefore, $P_3\to 0$ as $\ell\to+\infty$, which implies the conclusion. □

When $\alpha=2\pi$, sensing sectors become sensing disks, and the sufficient intensity to guarantee the asymptotically complete k-covering becomes $\lambda=\log\ell^2+(k+1)loglog\ell^2+c(\ell)$ with $c(\ell)\to+\infty$ as $\ell\to+\infty$, which is exactly the same as Theorem 1 in Zhang et al. [2004]. In other words, our result extended that obtained in Zhang et al. [2004], and we found that the sector-coverage is not less effective than the disk-coverage as we might think.

4. SECTOR COVERAGE WITH VARIABLE DIRECTIONS

In the preceding section, we considered the asymptotical k-covering with fixed sector directions, which may be applied to the robots that march or line up in the same direction. Here, we will change the directions for different sensor nodes, which may be applied to the robots with more freedom to move their direction. In what follows, we consider two direction-varying cases.

First, we consider N-*direction policy* for the sector-coverage. For any fixed integer N, we first define a set

$$\Lambda_N=\{\boldsymbol{a}\,|\,\boldsymbol{a}=(\cos\frac{2m\pi}{N},\ \sin\frac{2m\pi}{N}),\ m=0,...,N-1\},$$

which gives N directions in a plain. N-direction policy is carried out in the following way, after the position of a node is located according to a point process, the section direction will chosen randomly from Λ_N with equal probability (that is, $1/N$).

Then we have the following result.

Theorem 5. With the N-direction policy, a sufficient condition for the node intensity to maintain the asymptotical k-covering for $0<\alpha\le\frac{2\pi}{N}$ and λ given in (1) is still $c(\ell)\to+\infty$ as $\ell\to+\infty$.

Proof: Given a point q, we define the valid area $\mathcal{D}_q$ of q as the set each point of which could cover q possibly under the N-direction policy if a sensor node is placed at it. Then we have,

$$\begin{aligned}\mathcal{D}_q=\{v\mid d(q,v)<\frac{1}{\sqrt{\pi}},\ \ \arg\boldsymbol{qv}\in(\pi-\frac{\alpha}{2},\pi]\bigcup\\ (-\pi+\frac{\alpha}{2},-\pi)\bigcup_{m=1}^{N-1}(\frac{2m\pi}{N}-\frac{\alpha}{2}-\pi,\frac{2m\pi}{N}+\frac{\alpha}{2}-\pi)\}.\end{aligned}$$

See Figs. 5 and 6 with $N=3$ for illustration. In Fig. 5, the shadowed area is composed of three possible sensing sectors when a node is located at q, while Fig. 6 shows $\mathcal{D}_q$. Clearly, $\mathcal{D}_q$ is just the same as the region of possible sensing sectors after a π-angle turning around q.

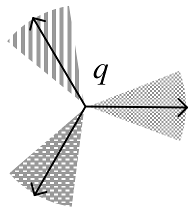

Figure 5. Three possible sensing sectors for nodes located at point *q* when *N*=3

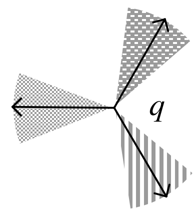

Figure 6. The valid area of point *q* when *N*=3

Note that the probability for nodes falling into $\mathcal{D}_q$ to cover q is exactly $\frac{1}{N}$. Therefore, the probability that a given point q is covered by at most $k-1$ sectors nodes is

$$\begin{aligned}&\mathcal{P}\{q \text{ is covered by at most } k-1 \text{ nodes}\}\\ &=\mathcal{P}\{\text{there are at most } k-1 \text{ nodes in } \mathcal{D}_q\}\\ &\quad+\sum_{i=k}^{\infty}\mathcal{P}\{\text{there are } i \text{ nodes in } \mathcal{D}_q\}\\ &\quad\times\mathcal{P}\{\text{at most } k\text{-1 nodes cover } q\,|\,i \text{ nodes located in } \mathcal{D}_q\}\\ &=e^{-\frac{N\alpha\lambda}{2\pi}}\sum_{i=0}^{k-1}\frac{(\frac{N\alpha\lambda}{2\pi})^i}{i!}+e^{-\frac{N\alpha\lambda}{2\pi}}\sum_{i=k}^{\infty}\frac{(\frac{N\alpha\lambda}{2\pi})^i}{i!}\sum_{j=0}^{k-1}b(j;i,\frac{1}{N}).\end{aligned}$$

where $b(j;i,\frac{1}{N})$ is the probability that a random variable equals j if it is binomial distributed with parameters i and $\frac{1}{N}$.

It is not hard to see that

$$\begin{aligned}
&\sum_{i=k}^{+\infty} e^{-N\theta}\frac{(N\theta)^i}{i!}\sum_{j=0}^{k-1} b(j;i,\frac{1}{N})\\
=&\sum_{i=k}^{2k-2} e^{-N\theta}\frac{(N\theta)^i}{i!}\sum_{j=0}^{k-1} b(j;i,\frac{1}{N})\\
&+\sum_{i=2k-1}^{+\infty} e^{-N\theta}\frac{(N\theta)^i}{i!}\sum_{j=0}^{k-1} b(j;i,\frac{1}{N})\\
=&\,e^{-\theta}\theta^{k-1}o(1)\\
&+\sum_{i=2k-1}^{+\infty} e^{-N\theta}(N\theta)^i\sum_{j=0}^{k-1}\frac{1}{j!(i-j)!}(\frac{1}{N})^j(\frac{N-1}{N})^{i-j}\\
<&\,e^{-\theta}\theta^{k-1}o(1)+\sum_{i=2k-1}^{+\infty} e^{-N\theta}(N\theta)^i\sum_{j=0}^{k-1}\frac{1}{j!(i-j)!}(\frac{N-1}{N})^i.
\end{aligned} \tag{9}$$

Note that, when $i \geq 2k-1$, we have

$$\frac{1}{j!(i-j)!} < \frac{1}{(k-1)!(i-k+1)!},$$

for $0 \leq j \leq k-2$. Therefore,

$$\begin{aligned}
&\sum_{i=2k-1}^{+\infty} e^{-N\theta}(N\theta)^i\sum_{j=0}^{k-1}\frac{1}{j!(i-j)!}(\frac{N-1}{N})^i\\
<&\sum_{i=2k-1}^{+\infty} e^{-N\theta}(\frac{N-1}{N})^i(N\theta)^i\frac{k}{(k-1)!(i-k+1)!}\\
=&\frac{k}{(k-1)!}e^{-N\theta}((N-1)\theta)^{k-1}\sum_{i=2k-1}^{+\infty}\frac{((N-1)\theta)^{i-k+1}}{(i-k+1)!}\\
=&\frac{k}{(k-1)!}(N-1)^{k-1}e^{-N\theta}\theta^{k-1}(e^{(N-1)\theta}\\
&-\sum_{i=k-1}^{2k-2}\frac{((N-1)\theta)^{i-k+1}}{(i-k+1)!}).\\
=&\frac{k}{(k-1)!}(N-1)^{k-1}e^{-\theta}\theta^{k-1}(1+o(1)).
\end{aligned} \tag{10}$$

With taking $\theta = \frac{\alpha}{2\pi}$ in (9) and (10), we have

$$\begin{aligned}
&\mathcal{P}\{q \text{ is covered by at most } k\text{-1 nodes}\}\\
&< \frac{k}{(k-1)!}(N-1)^{k-1}e^{-\frac{\alpha\lambda}{2\pi}}(\frac{\alpha\lambda}{2\pi})^{k-1}(1+o(1)).
\end{aligned}$$

Note that under our axis direction deciding strategy, the upper bound of the expected number of the intersection points could also be controlled by $\bar{\theta}\lambda^2\ell^2$ for some sufficiently large constant $\bar{\theta}$. Thus, the conclusion follows by similar analysis used in the proof of Theorem 4. □

In fact, based on the above analysis, we can re-define the set

$$\begin{aligned}
\Lambda_N = \{\boldsymbol{a}\,|\,\boldsymbol{a} = (\cos(\frac{2m\pi}{N}+\bar{\theta}),\ \sin(\frac{2m\pi}{N}+\bar{\theta}))^T;\\
m = 0,...,N-1\}
\end{aligned}$$

for some fixed angle $\bar{\theta}$ and still make Theorem 5 hold.

Next, we consider *U-direction policy*: the sector direction for each node is selected randomly with a uniform distribution in the plane.

Here is the result for U-direction policy.

Theorem 6. With the U-direction policy, a sufficient condition for the node intensity to maintain the asymptotical k-covering for $0 < \alpha \leq 2\pi$ and λ given in (1) is still $c(\ell) \to +\infty$ as $\ell \to +\infty$.

Proof: With this policy, it is obvious to see that the valid area $\mathcal{D}_q$ for a given point q is exactly the disk centered at q with radius $\frac{1}{\sqrt{\pi}}$, denoted as $\mathcal{D}_q = \{v\,|\,d(q,v) < \frac{1}{\sqrt{\pi}}\}$. Furthermore, the probability of node v covering a given point p on condition that $v \in \mathcal{D}_q$ is $\frac{\alpha}{2\pi}$. Therefore, the probability that a given point q is covered by at most $k-1$ nodes is

$$\begin{aligned}
&\mathcal{P}\{q \text{ is covered by at most } k-1 \text{ nodes}\}\\
=&\,\mathcal{P}\{\text{there are at most } k-1 \text{ nodes in } \mathcal{D}_q\}\\
&+\sum_{i=k}^{\infty}\mathcal{P}\{\text{there are } i \text{ nodes in } \mathcal{D}_q\}\\
&\times\mathcal{P}\{\text{at most } k-1 \text{ nodes cover } q\,|\,i \text{ nodes located}\\
&\text{in } \mathcal{D}_q\}\\
=&\,e^{-\lambda}\sum_{i=0}^{k-1}\frac{\lambda^i}{i!}+e^{-\lambda}\sum_{i=k}^{\infty}\frac{\lambda^i}{i!}\sum_{j=0}^{k-1}b(j;i,\frac{\alpha}{2\pi}).
\end{aligned}$$

Similarly to the proof of Theorem 5, we obtain:

$$\begin{aligned}
&e^{-\lambda}\sum_{i=0}^{k-1}\frac{\lambda^i}{i!}+e^{-\lambda}\sum_{i=k}^{\infty}\frac{\lambda^i}{i!}\sum_{j=0}^{k-1}b(j;i,\frac{\alpha}{2\pi})\\
=&\,e^{-\lambda}\sum_{i=0}^{k-1}\frac{\lambda^i}{i!}+e^{-\lambda}\sum_{i=k}^{\infty}\frac{\lambda^i}{i!}\sum_{j=0}^{k-1}\frac{i!}{j!(i-j)!}(\frac{2\pi-\alpha}{2\pi})^{i-j}(\frac{\alpha}{2\pi})^j\\
<&\begin{cases}
\frac{k}{(k-1)!}e^{-\frac{\alpha\lambda}{2\pi}}(\frac{2\pi-\alpha}{2\pi}\lambda)^{k-1}(1+o(1)), & 0<\alpha\leq\pi\\
\frac{k}{(k-1)!}e^{-\frac{\alpha\lambda}{2\pi}}(\frac{\alpha\lambda}{2\pi})^{k-1}(1+o(1)), & \pi<\alpha<2\pi\\
\frac{1}{(k-1)!}e^{-\lambda}\lambda^{k-1}(1+o(1)), & \alpha=2\pi
\end{cases}\\
<&\frac{k}{(k-1)!}(\frac{\pi+\mid\pi-\alpha\mid}{\alpha})^{k-1}e^{-\frac{\alpha\lambda}{2\pi}}(\frac{\alpha\lambda}{2\pi})^{k-1}(1+o(1)).
\end{aligned} \tag{11}$$

Again, the upper bound of the expected number of the intersection points can be controlled by $\hat{\theta}\lambda^2\ell^2$ for a sufficiently large $\hat{\theta}$. Repeating the estimation process leads to the conclusion. □

Note that, in (1), the coefficient $\frac{2\pi}{\alpha}$ is exactly the reciprocal of the sensing sector's area. In fact, if we take the sector radius as $\sqrt{\frac{2}{\alpha}}$ (equivalently, the area of the sensing sector becomes 1), similar conclusions in Theorems 4, 5, and 6 will also hold and density to maintain asymptotical k-covering will become $\log\ell^2 + (k+1)\log\log\ell^2 + \hat{c}(\ell)$ with $\hat{c}(\ell) \to +\infty$. On the other hand, take $\alpha = 2\pi$, and then Theorems 4, 5, and 6 take the same form of Theorem 1 in Zhang et al. [2004]. Therefore, we will see that the node density to maintain asymptotical k-covering for disk coverage will be

$$\log \ell^2 + (k+1)\log\log \ell^2 + \tilde{c}(\ell) \quad with \quad \tilde{c}(\ell) \to +\infty. \quad (12)$$

Therefore, when node deployment forms a planar Poisson point process with the same area of the sensing region (that is, 1 in the above discussion for convenience), node density guaranteeing asymptotical k-covering for a square region R will be in the form of (12), whatever sensing shape (either disk or sector) and axis direction policy (that is, either fixed direction policy, or N-direction policy, or U-direction policy) we choose. This observation implies that node density to make k-covering may not be sensitive to the sensing shape once the coverage process is fixed as a Poisson point process.

5. CONCLUSIONS

This paper considered the planar k-coverage problem using a set of sensing sectors. A sensor-network coverage model was established and sufficient conditions were given to secure the k-coverage with probability 1 asymptotically under three sector direction selection policies. The stochastic approach was used in the convergence analysis. All the results were obtained in a 2-dimensional space, but they could be easily extended somehow in the case of higher-dimensional spaces, where sectors become cones.

However, there are still many challenging problems in the study of static or dynamic coverage. For example, the sector-coverage shares essentially the same efficiency with the disk-coverage with a Poisson point process. It will be very interesting if we can tell what kind of point processes and the distributions for the node/sensor deployment can improve the coverage efficiency (for example, by reducing the node density in asymptotical k-covering).

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