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Coordinated Control of Multiple Mobile Agents with Connectivity Preserving ⋆

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Abstract: The underlying topology of the network remaining connected frequently enough during the evolution is a basic assumption seen in many previous works on coordinated control in a network of multi-agent systems to guarantee the stability of the coordinated motion. However, for a given set of initial conditions, this assumption is very difficult to verify. In particular, connectivity of the initial network can not guarantee connectivity of the network during the evolution. In this paper, we propose a coordinated control protocol, which combines the roles of motion control and connectivity control. This protocol can enable the group to achieve velocity alignment and a desired group shape while preserving connectivity of the network during the evolution only if the initial network is connected. Moreover, we investigate the coordinated control with a virtual leader.

1. INTRODUCTION

Recently, multi-agent distributed coordination problems have attracted much attention among researchers studying biology, physics, computer science and control engineering Vicsek et al. [1995], Reynolds [1987], Olfati-Saber [2006]. This is partly due to broad applications of multi-agent systems in many areas including cooperative control of mobile robots, unmanned air vehicles (UAVs), and so on. Over the years, many variants of distributed coordination control protocols have been proposed, which can be roughly divided into three classes: i) protocols for single integrator dynamics, which including first-order consensus protocols Jadbabaie et al. [2003], Olfati-Saber and Murray [2004], Ren and Beard [2005] and swarming algorithms Gazi and Passino [2003, 2004]; ii) protocols for double integrator dynamics, which including second-order consensus protocols Ren and Atlkins [2007], Ren [2007], Lee and Spong [2007] and flocking algorithms Olfati-Saber [2006], Tanner et al. [2007]; iii) protocols for high-order integrator dynamics, which including high-order consensus protocols Ren et al. [2007].

Subjected to limited sensing and communication capabilities of agents, the interaction topology among agents may change over time. A basic assumption made in stability analysis of collective dynamics for most previous works is that the underlying topology remaining connected frequently enough during the evolution. However, this assumption is very difficult to verify in practice. In fact, connectivity of the initial network can not guarantee connectivity of the network over time. This motives the following question: can we design a distributed control algorithm so that it enables the group to achieve the desired coordinated motion while preserving connectivity?

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Distributed connectivity control of mobile network provides an effective method to preserve connectivity Spanos and Murray [2004], Ji and Egerstedt [2006], Zavlanos and Pappas [2005]. In Spanos and Murray [2004], a measure of local connectivity of a network is introduced to achieve global connectivity. In Ji and Egerstedt [2006], the network stay connected by adding appropriate weights to the edges in the network. Potential method is proposed for maintain connectivity of dynamic mobile network Zavlanos and Pappas [2005]. However, most of these protocols were designed for single integrator dynamics. In Zavlanos et al. [2007], flocking algorithm combined with network connectivity is proposed for double integrator dynamics to achieve velocity alignment.

In this paper, we investigate the second-order consensus problem. Previous algorithms Ren and Atlkins [2007], Ren [2007], Lee and Spong [2007] rely on the connectivity assumption of the network during the evolution, which can not be guaranteed even if the initial network is connected. We propose a coordinated control protocol for double integrator dynamics, which combines the roles of motion control and connectivity control. This protocol can enable the group to achieve velocity alignment and a desired group shape while preserving connectivity of the network during the evolution only if the initial network is connected. In addition, we investigate the coordinated control with a virtual leader and show that all agents can asymptotically attain the desired velocity when only one agent in the team has access to the virtual leader.

2. PROBLEM STATEMENT

We consider N agents moving in an $n-$dimensional Euclidean space. The motion of each agent is described by a double integrator of the form

$$\begin{aligned} \dot{q}_i &= p_i \\ \dot{p}_i &= u_i,\ i = 1, \cdots, N \end{aligned} \tag{1}$$

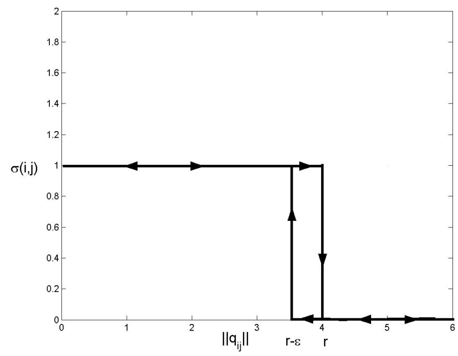

Fig. 1. Indicator function $\sigma(i, j)$

where $q_i \in \mathrm{R}^n$ is the position vector of agent i, $p_i \in \mathrm{R}^n$ is its velocity vector and $u_i \in \mathrm{R}^n$ is the (force) control input acting on agent i. Suppose that each agent has the same influencing/sensing radius r. Let $\varepsilon \in (0, r)$ be a given constant. We call $G(t) = (V, E(t))$ a dynamic undirected graph consisting of a set of vertices $V = \{1, 2, \cdots, N\}$ indexed by the set of agents and a time varying set of links $E(t) = \{(i, j) | i, j \in V\}$ such that, i) initial links are generated by $E(0) = \{(i, j) | \|q_i(0) - q_j(0)\| < r, i, j \in V\}$, ii) if $(i, j) \notin E(t^-)$ and $\|q_i(t) - q_j(t)\| < r - \varepsilon$ then, (i, j) is a new link to be added to $E(t)$, iii) if $\|q_i(t) - q_j(t)\| \geq r$ then, $(i, j) \notin E(t)$. The symmetric indicator function $\sigma(i, j) = \sigma(j, i) \in \{0, 1\}$ determines whether or not there is a link between agent i and agent j, what is defined as (Fig. 1)

$$\sigma(i,j)[t^+] =$$
$$\begin{cases} 0, \; if(\sigma(i,j)[t^-] = 0 \cap r - \varepsilon \leq \|q_{ij}\| < r) \cup \|q_{ij}\| \geq r \\ 1, \; if(\sigma(i,j)[t^-] = 1 \cap r - \varepsilon \leq \|q_{ij}\| < r) \cup \|q_{ij}\| < r - \varepsilon \end{cases}$$

Therefore, there is a hysteresis in addition of new links in $G(t)$, which was firstly proposed in Ji and Egerstedt [2006]. This hysteresis is crucial in preserving connectedness of the dynamical interaction network.

Our objective is to make all agents move with a common velocity while keeping a desired group shape under the assumption that initial network is connected. Specifically, we want $p_i(t) - p_j(t) \to 0$ and $q_i(t) - q_j(t) \to k_{ij}$ for all $i, j \in V$, where $k_{ij} \in \mathrm{R}^n$ is an offset between the agent i and agent j to make a certain desired group shape. Here, we assume that this offset k_{ij} is constant and also compatible in the sense that $k_{im} + k_{mj} = k_{ij}$ for any $i, j, m \in V$. We choose the control law u_i for agent i to be

$$u_i = \alpha_i + \beta_i \tag{2}$$

where gradient-based term α_i is designed to enforce a desired group shape, and also to guarantee connectivity preserving of the dynamic interaction network. In this paper, for convenience and without loss of generality, we assume that the desired group shape is given by $q_1(t) = q_2(t) = \cdots = q_N(t)$, which is the same as that of Ren and Atlkins [2007], Ren [2007], Lee and Spong [2007]. Consensus term β_i is used to regulate the velocity of each agent to a common value. Furthermore, in the situation where there is a virtual leader, the coordinated control law should be designed to enable all agents asymptotic tracking the virtual leader. In this case, we modify the control law u_i to

$$u_i = \alpha_i + \beta_i + \gamma_i \tag{3}$$

where γ_i is the navigational feedback term. In this paper, the virtual leader is a desired common constant velocity.

3. FUNDAMENTAL COORDINATED CONTROL ALGORITHM

3.1 Main Result

We present explicit control input in Eq. (2), which is described as:

$$u_i = \underbrace{- \sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_i - q_j\|)}_{\alpha_i} \underbrace{- \sum_{j \in N_i(t)} a_{ij}(t)(p_i - p_j)}_{\beta_i} \tag{4}$$

where $N_i(t)$ is the neighborhood region of agent i at time t defined as

$$N_i(t) = \{ j | \sigma(i,j)[t] = 1, j \neq i, j = 1, \cdots, N\} \tag{5}$$

The nonnegative potential $\psi(\|q_i - q_j\|)$ is a function of distance $\|q_i - q_j\|$ between agent i and agent j, which is differentiable for $\|q_i - q_j\| \in [0, r)$, such that, 1) $\psi(\|q_i - q_j\|) \to \infty$ as $\|q_i - q_j\| \to r$, 2) $\frac{\partial \psi(\|q_i - q_j\|)}{\partial \|q_{ij}\|} \geq 0$ for $\|q_i - q_j\| \in [0, r)$ and $\lim_{\|q_i - q_j\| \to 0} \left(\frac{\partial \psi(\|q_i - q_j\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_i - q_j\|} \right)$ is finite. One example of such a potential function is

$$\psi(\|q_{ij}\|) = \begin{cases} \dfrac{\|q_{ij}\|^{m_1}}{(r - \|q_{ij}\|)^{m_2}}, & \|q_{ij}\| < r \\ +\infty \qquad , & \|q_{ij}\| = r \\ m_3 \qquad , & \|q_{ij}\| > r \end{cases} \tag{6}$$

where $q_{ij} = q_i - q_j$, integers $m_1 \geq 2$ and $m_2 \geq 1$, and constant $m_3 \geq 0$. Note that the potential function in Ren and Atlkins [2007], Ren [2007], Lee and Spong [2007] can be written as

$$V(\|q_i - q_j\|) = \frac{1}{2} a_{ij}(t)(\|q_i - q_j\|)^2 \tag{7}$$

The main difference between potential functions (6) and (7) is that the potential (6) tends to infinite when the distance between agent i and agent j tends to r. This property can guarantee no initial edges to be lost. The adjacent matrix $A(t)$ of the graph $G(t)$ is defined as $a_{ij}(t) = a_{ji}(t) > 0$ if $(i, j) \in E(t)$ and $a_{ij}(t) = 0$ otherwise. The corresponding Laplacian is $L(t) = \Delta(A(t)) - A(t)$, where the degree matrix $\Delta(A(t))$ is a diagonal matrix with $i-$th diagonal element equals to $\sum_{j=1, j \neq i}^{N} a_{ij}(t)$ Godsil and Royle [2001]. The corresponding $n-$dimensional graph Laplacian is defined as $\widehat{L}(t) = L(t) \otimes I_n$, where I_n is the identity matrix of order n and $\otimes$ stands for the Kronecker product. This multi-demensional Laplacian satisfies the following sum of squares property Olfati-Saber [2006]:

$$z^T \widehat{L}(t) z = \frac{1}{2} \sum_{(i,j) \in E} a_{ij}(t) \|z_j - z_i\|^2 \tag{8}$$

where $z = col(z_1, z_2, \cdots, z_N) \in \mathrm{R}^{Nn}$ and $z_i \in \mathrm{R}^n$ for all i.

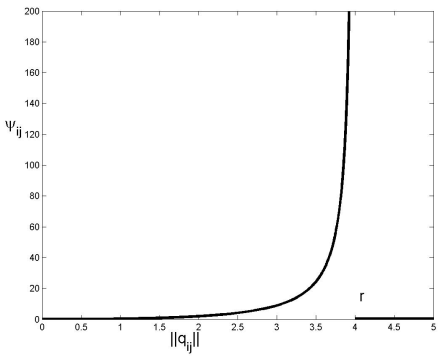

Fig. 2. Example of inter-agent artificial potential function $\psi(\|q_{ij}\|)$ ($m_1 = 2$, $m_2 = 1$ and $m_3 = 0$)

Denote the position and velocity of the center of mass (COM) of all agents in the group as

$$\bar{q} = \frac{\sum_{i=1}^{N} q_i}{N}, \quad \bar{p} = \frac{\sum_{i=1}^{N} p_i}{N}$$

The sum of artificial potential energy and the total kinetic energy is defined as follows:

$$Q = \frac{1}{2}\sum_{i=1}^{N}\Big(\sum_{j\in N_i(t)} \psi(q_{ij}) + p_i^T p_i\Big) \tag{9}$$

Clearly, Q is a positive semi-definite function.

Theorem 1 Consider a system of N mobile agents with dynamics (1), each steered by protocol (4). Suppose that the initial network $G(0)$ is connected and the initial energy Q_0 is finite. Then the following hold:

i) $G(t)$ will be connected for all $t \geq 0$;
ii) All agents asymptotically converge at the same position and move with the same velocity;
iii) The velocity of the COM $\bar{p}(t)$ will be invariant for all $t \geq 0$.

3.2 Proof of the Theorem

A. Proof of part i)

Assume that $G(t)$ switches at time $t_k (k = 1, 2, \cdots)$. In each time-interval $[t_{k-1}, t_k)$, $G(t)$ is a fixed graph. The time derivative of Q in $[t_{k-1}, t_k)$ is

$$\dot{Q} = -p^T[L(t) \otimes I_n]p \tag{10}$$

Because L is a positive semi-definite matrice Horn and Johson [1987], $\dot{Q}(t) \leq 0$ in $[t_{k-1}, t_k)$, which implies that

$$Q(t) \leq Q(t_{k-1}) \qquad for \quad t \in [t_{k-1}, t_k) \tag{11}$$

In particular, $Q(t) \leq Q_0$ for $[t_0, t_1)$. Since the definition of potential function $\lim_{\|q_{ij}(t)\|\to r} \psi(\|q_{ij}(t)\|) = \infty$, no edge-distance will tend to r for $t \in [t_0, t_1)$, which implies that no edge will be lost at time t_1. Hence, new edges must be added in the interaction network at switching time t_1. Note that the hysteresis ensures that if finite links are added to $G(t)$, then the associated potentials are bounded. Thus, $Q(t_1)$ is bounded. Similar to the aforementioned analysis, we can get that no edge-distance will tend to r for $t \in [t_{k-1}, t_k)$, which implies that no edge will be lost at time t_k, and $Q(t_k)$ is bounded. Since $G(0)$ is connected and no edges in $E(0)$ will be lost, $G(t)$ will be connected for all $t \geq 0$.

B. Proof of part ii) and iii)

Assume there are $m_k (0 < m_k \leq (\frac{N(N-1)}{2} - (N-1)) \triangleq M)$ new links that are added to the interaction network at time t_k. From (9) and (11), we have $Q(t_k) \leq Q_0 + (m_1 + \ldots + m_k)\psi(\|r - \varepsilon\|)$. Since there are at most M new links that may be added to $G(t)$, we have $k \leq M$ and $Q(t) \leq Q_0 + M\psi(\|r - \varepsilon\|) \triangleq Q_{\max}$ for all $t \geq 0$. Hence, the set

$$\Omega = \left\{ \widehat{q} \in D_g, p \in \mathrm{R}^{Nn} \middle| Q(\widehat{q}, p) \leq Q_{\max} \right\} \tag{12}$$

is positively invariant, where $\widehat{q} = col(q_{11}, \cdots, q_{1N}, \cdots, q_{N1}, \cdots, q_{NN})$, $D_g = \left\{ \widehat{q} \in \mathrm{R}^{N^2 n} \middle| \|q_{ij}\| \in [0, r), \forall (i,j) \in E(t) \right\}$ and $p = col(p_1, p_2, \cdots, p_N)$. Since $G(t)$ is connected for all $t \geq 0$, $\|q_{ij}\| < (N-1)r$ for any i and j. Since $Q(t) \leq Q_{\max}$, we have $p_i^T p_i \leq 2Q_{\max}$, and then $\|p_i\| \leq \sqrt{2Q_{\max}}$. Therefore, the set Ω is compact. It follows from LaSalle's invariant principle Khalil [2002] that if the initial conditions of the system lie in Ω, its trajectories will converge to the largest invariant set inside the region $S = \{ \widehat{q} \in D_g, p \in \mathrm{R}^{Nn} \big| \dot{Q} = 0\}$. From (10), we have

$$\dot{Q} = -p^T[L(t) \otimes I_n]p = -p^T(L(t) \otimes I_n)p = 0 \tag{13}$$

which implies that $p_1 = \cdots = p_N$.

It follows from the control protocol (4) and the symmetry of $\psi(\|q_{ij}\|)$ and $A(t)$ that

$$\bar{u} = \dot{\bar{p}} = \frac{\sum_{i=1}^{N} u_i}{N} = 0 \tag{14}$$

which implies that the velocity of the COM will be invariant for all $t \geq 0$.

In steady state, $p_1 = \cdots = p_N = \bar{p}$, and then $\dot{p}_i = \dot{\bar{p}} = 0, \forall i \in V$. From (4),

$$u_i = -\sum_{j\in N_i(t)} \frac{\partial\psi(\|q_i - q_j\|)}{\partial\|q_{ij}\|} \cdot \frac{1}{\|q_i - q_j\|}(q_i - q_j) = 0 \tag{15}$$

We can rewrite Eq. (15) in matrix form as $-(\hat{L}(t) \otimes I_n)q \to 0$, where $\hat{l}_{ii} = -\sum_{j=1, j\neq i}^{N} (\frac{\partial\psi(\|q_i - q_j\|)}{\partial\|q_{ij}\|} \cdot \frac{1}{\|q_i - q_j\|})$, $\hat{l}_{ij} = \frac{\partial\psi(\|q_i - q_j\|)}{\partial\|q_{ij}\|} \cdot \frac{1}{\|q_i - q_j\|}$ and $q^T = [q_1^T, \cdots, q_N^T]$. From definition, we know $\frac{\partial\psi(\|q_i - q_j\|)}{\partial\|q_{ij}\|} \cdot \frac{1}{\|q_i - q_j\|} \geq 0$ is uniformly bounded for $\|q_{ij}\| \in [0, \psi^{-1}(Q_{\max}))$. Since the dynamic network is connected all the time, from the property of the Laplacian matrix $\hat{L}(t) \otimes I_n$ Godsil and Royle [2001], q converges asymptotically to span$\{1\}$, which implies that $q_1 = \cdots = q_N$.

4. COORDINATED CONTROL ALGORITHM WITH A VIRTUAL LEADER

In some situation, the regulation of agents has certain purposes, such as achieving the desired common velocity, or arriving at a desired destination. In this section, we investigate the coordination control algorithm with a virtual leader.

4.1 Main Result

We present explicit control input in Eq. (3), which is described as:

$$u_i = -\underbrace{\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_i - q_j\|)}_{\alpha_i} - \underbrace{\sum_{j\in N_i(t)} a_{ij}(t)(p_i - p_j)}_{\beta_i} - \underbrace{c_1(p_i - p_\gamma)}_{\gamma_i},\ c_1 > 0 \tag{16}$$

where p_γ is the desired constant velocity (virtual leader).

We define the sum of artificial potential energy and the total relative kinetic energy between all agents and the virtual leader as following:

$$U = \frac{1}{2}\sum_{i=1}^{N}\Big(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + (p_i - p_\gamma)^T(p_i - p_\gamma)\Big) \tag{17}$$

Clearly, U is a positive semi-definite function.

Theorem 2 Consider a system of N mobile agents with dynamics (1), each steered by protocol (16). Suppose that the initial network $G(0)$ is connected and the initial energy U_0 is finite. Then the following hold:

i) $G(t)$ will be connected for all $t \geq 0$.
ii) All agents asymptotically converge to the same position and move with the desired velocity p_γ.
iii) If the initial velocity of the COM $\bar{p}(0)$ is equal to the desired velocity p_γ, then $\bar{p}(t)$ will be equal to the desired velocity p_γ for all $t \geq 0$; otherwise $\bar{p}(t)$ will exponentially converge to the desired velocity with a time constant of c_1s.

4.2 Proof of the Theorem

We first prove part *i*) of Theorem 2.

Denote the position difference vector and the velocity difference vector between agent i and virtual leader as $\tilde{q}_i = q_i - p_\gamma t$ and $\tilde{p}_i = p_i - p_\gamma$, respectively. We have

$$\begin{aligned}\dot{\tilde{q}}_i &= \tilde{p}_i \\ \dot{\tilde{p}}_i &= u_i,\ i = 1, ..., N\end{aligned} \tag{18}$$

By the definition of $\psi(\|q_{ij}\|)$, potential function can be rewritten as

$$\psi(\|q_{ij}\|) = \psi(\|\tilde{q}_{ij}\|) \tag{19}$$

Thus the control input (16) for agent i can be rewritten as

$$u_i = -\sum_{j\in N_i(t)} \nabla_{\tilde{q}_i}\psi(\|\tilde{q}_{ij}\|) - \sum_{j\in N_i(t)} a_{ij}(t)(\tilde{p}_i - \tilde{p}_j) - c_1\tilde{p}_i \tag{20}$$

and the positive semi-definite energy function (17) can be rewritten as

$$U = \frac{1}{2}\sum_{i=1}^{N}(\psi(\|\tilde{q}_{ij}\|) + \tilde{p}_i^T\tilde{p}_i) \tag{21}$$

The time derivative of U in $[t_{k-1}, t_k)$ is

$$\dot{U} = -\tilde{p}^T[(L(t) + c_1 I_N)\otimes I_n]\tilde{p} \tag{22}$$

Because L is a positive semi-definite, $\dot{U}(t) \leq 0$ in $[t_{k-1}, t_k)$, which implies that

$$U(t) \leq U(t_{k-1}) \qquad for \quad t \in [t_{k-1}, t_k) \tag{23}$$

Similar to the proof of part *i*) of Theorem 1, we can get that no edge-distance will tend to r for $t \in [t_{k-1}, t_k)$, which implies that no edge will lost at time t_k, and $U(t_k)$ is bounded. Since $G(0)$ is connected and no edges in $E(0)$ will be lost, $G(t)$ will be connected for all $t \geq 0$.

We now prove part *ii*) of Theorem 2.

Similar to the proof of part *ii*) of Theorem 1, the set

$$\Omega = \left\{ \widehat{\tilde{q}} \in D_g, \tilde{p} \in \mathrm{R}^{Nn} \,\middle|\, U(\widehat{\tilde{q}}, \tilde{p}) \leq U_{\max} \right\} \tag{24}$$

is compact, where $\widehat{\tilde{q}} = col(\tilde{q}_{11}, \cdots, \tilde{q}_{1N}, \cdots, \tilde{q}_{N1}, \cdots, \tilde{q}_{NN})$, $D_g = \left\{ \widehat{\tilde{q}} \in \mathrm{R}^{N^2 n} \,\middle|\, \|\tilde{q}_{ij}\| \in [0, r), \forall (i,j) \in E(t) \right\}$, $\tilde{p} = col(\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_N)$ and $U_{\max} \triangleq U_0 + (\frac{N(N-1)}{2} - (N-1))\psi(\|r - \varepsilon\|)$. From LaSalle's invariant principle, we have

$$\dot{U} = -\tilde{p}^T(L(t)\otimes I_n)\tilde{p} - c_1\tilde{p}^T\tilde{p} = 0 \tag{25}$$

which implies that $\tilde{p}_1 = \cdots = \tilde{p}_N = 0$. This occurs only when $p_1 = \cdots = p_N = p_\gamma$. This also implies that $u_i = \dot{p}_i = \dot{p}_\gamma = 0$. From (16),

$$u_i = -\sum_{j\in N_i(t)} \frac{\partial\psi(\|q_i - q_j\|)}{\partial\|q_{ij}\|}\cdot\frac{1}{\|q_i - q_j\|}(q_i - q_j) = 0 \tag{26}$$

Similar to the proof of part *ii*) of Theorem 1, we have $q_1 = \cdots = q_N$.

Finally, we prove part *iii*) of Theorem 2.

It follows from the control protocol (16) and the symmetry of $\psi(\|q_{ij}\|)$ and $A(t)$ that

$$\dot{\bar{p}} = \frac{\sum_{i=1}^{N} u_i}{N} = -c_1\bar{p} + c_1 p_\gamma \tag{27}$$

By solving Eq. (27), we get

$$\bar{p} = p_\gamma + (\bar{p}(0) - p_\gamma)e^{-c_1 t} \tag{28}$$

where $\bar{p}(0)$ is initial velocity of COM. Thus, it follows that, if $\bar{p}(0) = p_\gamma$, then $\bar{p}(t) = p_\gamma$ for all $t \geq 0$; otherwise $\bar{p}(t)$ will exponentially converge to the desired velocity with a time constant of c_1s.

4.3 Extensions and discussion

In the coordinated control algorithm (16), it is assumed that each agent is an informed agent who has information about the virtual leader. However, in some nature examples, few individuals have the pertinent information, such as knowledge of the location of a food source, or of a migration route Couzin et al. [2005]. In this subsection, we assume that only one agent is the informed agent which is given information about the virtual leader. The control input for agent i is designed as

$$u_i = -\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_i - q_j\|) - \sum_{j\in N_i(t)} a_{ij}(t)(p_i - p_j) - h_i c_1(p_i - p_\gamma) \tag{29}$$

If agent i is the informed agent, then $h_i = 1$; otherwise, $h_i = 0$. Without loss of generality, we can assume that the first agent is informed agent, that is, $h_i = 1$ for $i = 1$ and $h_i = 0$ for others. Similar to the proof of Theorem 2, we can prove the follows:

Theorem 3 Consider a system of N mobile agents with dynamics (1), each steered by protocol (29). Suppose that the initial network $G(0)$ is connected and the initial energy U_0 is finite. Then the following hold:

i) $G(t)$ will be connected for all $t \geq 0$.
ii) All agents asymptotically converge to the same position and move with the desired velocity p_γ.

5. SIMULATION STUDY

5.1 Coordinated control without a leader

Simulations were performed on 10 agents moving in a 2-dimensional space under the influence of the control protocol (4). Initial positions and initial velocities of the 10 agents are chosen randomly from the boxes $[0, 8] \times [0, 8]$ and $[0, 4] \times [0, 4]$, respectively, and the initial positions are chosen to satisfy the condition that the initial interaction network is connected. Under the same initial positions and velocities of the 10 agents and $r = 4$, the simulations were performed with $\varepsilon = 0.05$ and 1.0, respectively. Fig. 3 (a) shows the initial states of the agents, the solid lines represent the neighboring relations, and the solid lines with arrows represent the direction of velocities.; Fig. 3 (b) depicts the motion trajectories of all agents from $t = 0$ to 5 seconds with difference constant ε; Fig. 3 (c) and (d) show the convergence of positions with difference constant ε for $x-$axis and $y-$axis, respectively, from which we can see that all agents eventually achieve the same position; Fig. 3 (e) and (f) show the convergence of velocities with difference constant ε for $x-$axis and $y-$axis, respectively. We find that the smaller the value of the constant ε, the larger the maximum overshoot and the smaller the settling time of the velocity. Therefore, there exists a fundamental trade-off between maximum overshoot and settling time for choosing the parameter ε. Large overshoot is due to the fact that smaller ε corresponds to larger potential force. On the other hand, smaller ε leads to earlier adding edges.

5.2 Coordinated control with a virtual leader

Simulations for the protocol (16) and the protocol (29) were performed with 50 agents moving in a 2-dimensional space. In both simulations, $r = 4$, and $\varepsilon = 0.5$, and initial positions and velocities of the 50 agents were chosen randomly from the boxes $[0, 15] \times [0, 15]$ and $[0, 4] \times [0, 4]$, respectively. The initial positions were chosen to satisfy the condition that the initial interaction network is connected. The desired velocity was chosen as $p_\gamma = [3, 3]$. Fig. 4 (a) shows the initial states of the agents; Fig. 4 (b) depicts the motion trajectories of all agents from $t = 0$ to 5 seconds; Fig. 4 (c) and (d) show the convergence of positions for $x-$axis and $y-$axis, respectively; Fig. 4 (e) and (f) show the convergence of velocities for $x-$axis and $y-$axis, respectively.

In Fig. 5, the informed agent is chosen randomly from the group and mark with a solid circle; Fig. 5 (a) shows

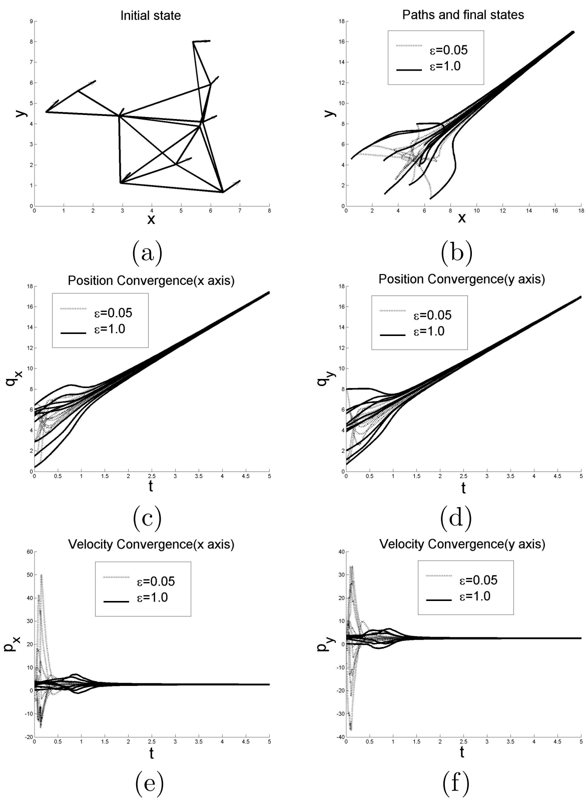

Fig. 3. Coordinated control of 10 agents applying algorithm (4)

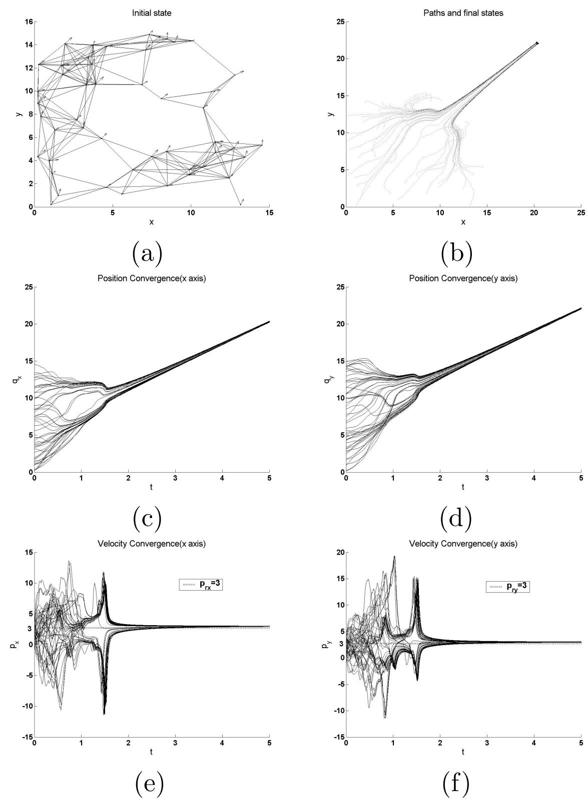

Fig. 4. Coordinated control of 50 agents applying algorithm (16)

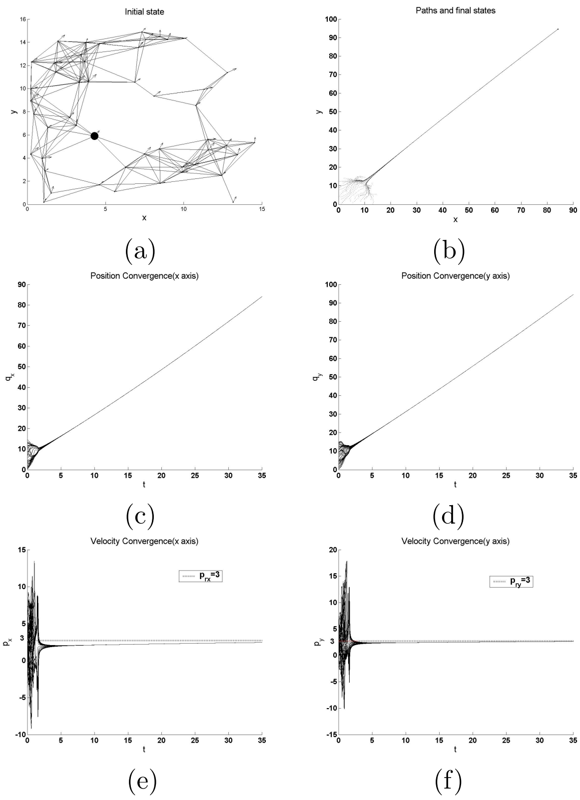

Fig. 5. Coordinated control of 50 agents applying algorithm (29)

the initial states of the agents; Fig. 5 (b) depicts the motion trajectories of all agents from $t = 0$ to 35 seconds; Fig. 5 (c) and (d) show the convergence of positions for $x-$axis and $y-$axis, respectively; Fig. 5 (e) and (f) show the convergence of velocities for $x-$axis and $y-$axis, respectively. We can clearly see that the group using control protocol (29) has a slower convergent time than that using control protocol (16).

6. CONCLUSION

In this paper, we present a class of coordinated control algorithms by a combination of roles of motion control and connectivity control. The goal of the control laws is to make the group to achieve velocity alignment and a desired group shape while preserving connectivity of the network during the evolution. We find that there is a trade-off between maximum overshoot and settling time of velocity convergence. Moreover, we investigate the coordinated control with a virtual leader and show that all agents can asymptotically attain the desired velocity even if only one agent in the team has access to the virtual leader. Other topics such as the effects of time delay and disturbance may warrant further studies.

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