Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Delay-dependent Decentralized Stabilization of Multi-channel Singular Linear Continuous-time Systems with Multiple Point Delays

Wei-Hua GUI *, Zhao-Hui JIANG, Chun-Hua YANG***, Yong-Fang XIE******

School of Information Science and Engineering, Central South University, Changsha, 410083, China
(Tel: 86-731-8836876; e-mail: *gwh@mail.csu.edu.cn, **jzh0903@ mail.csu.edu.cn,
***ychh@mail.csu.edu.cn, ****yfxie@mail.csu.edu.cn).

Abstract: The delay-dependent decentralized stabilization problem of multi-channel singular time-delay time-invariant systems subject to multiple internal and external incommensurate constant point delays is discussed based on the descriptor integral-inequality approach. The descriptor integral-inequality Lemma is firstly established. Based on the new Lemma and the Lyapunov-Krasovskii functional approach, delay-dependent decentralized stabilization sufficient conditions are obtained. An LMI based algorithm to design the decentralized state feedback controls that stabilize the multi-channel singular time delays systems is provided. Finally, some numerical examples are presented to illustrate the effectiveness and the availability for the design.

1. INTRODUCTION

Singular systems provide a more natural description of dynamical systems than the standard state-space systems due to the fact that the singular systems can preserve the structure of physical systems and can include non-dynamic constraints and impulsive elements (George *et al*., 1981). On the other hand, Singular systems have many important applications in, for example, circuit systems (Newcomb and Dziurla, 1989), robotics (Mills and Goldenberg, 1989), and aircraft modeling (Stevens and Lewis, 1991). From these viewpoints, considerable attention has been devoted to the analysis and synthesis of linear singular systems in the references Stefanovski (2006), Xu *et al*., (2002), Yu *et al*., (2003), Ishihara *et al*., (2003), Wang *et al*., (2006), Gui *et al*., (2006).

Delay systems have attracted the attention of many researchers over the past decades since time delays are one of the main causes for instability and poor performance of many control systems and frequently encountered in many industrial processes such as the steel industry, oil industry etc (Richard, 2003). Delays may be classified as point delays or distributed delays according to their nature and as internals (i.e. in the state) and externals (i.e. in the input or out put) according to the signals they influence. Point delays may be commensurate if each delay is an integer multiple of a base delay or, more generally, incommensurate if they are arbitrary real numbers (De la Sen, 2007). The presence of internal delays leads to a large complexity in the resulting system dynamic since the whole dynamical systems becomes infinite-dimensional. In addition, the fact increases the difficulty in the study of basic properties, like for instance controllability, observability, stability and stabilization, compared to the delay-free case since the transfer functions consist of transcendent numerator and denominator quasi-polynomials (Zheng and Frank, 2001). By those reasons, the stability of multiple internal and external incommensurate constant point delay systems becomes of increased difficulty related to the delay-free case (Lee *et al*., 2004). Singular delay systems, which are those where the delays influence the system's behavioral such as regular, impulsive, asymptotically stable and so on, present even a higher analysis and design difficulty. It should be pointed out that the stability problem for singular time delay systems is much more complicated than that for regular systems because it requires to consider not only asymptotically stable , but also regularity and impulse free at the same time (Fridman and Shaked, 2002), and the latter two need not be considered in regular systems. Very recently, a great effort has been devoted to the investigation of the behavior of time-delay singular systems in he references Kim , (2005), Boukas *et al*., (2003), Jiang *et al*., (2007).

During the last decades, the decentralized control and analysis of singular systems have been studied extensively, such as Wen *et al*., (2003), Wo *et al*., (2007), Lewis *et al*., (1991), Liu *et al*., (1995), Xie *et al*., (2006), due to the fact that many real-world systems can be modeled as multi-channel or large-scale singular systems, such as electric power systems, economic systems and so on. It is well known that the stabilization of multi-channel or large-scale singular systems can become very complicated owing to the high dimensionality and interconnection of the system model. In practices, due to the information transmission between subsystems, time delays naturally exist in multi-channel or large-scale singular systems, so, in recent years, some researchers have considered the stability problem of various multi-channel or large-scale singular systems with time delays (Guan *et al*., (1995)). However, to the best of our knowledge, there are no results on the problems of delay-dependent stabilization of multi-channel singular linear continuous-time systems with multiple internal and external incommensurate constant point delays.

The goal of this paper is to deal with the delay-dependent stabilization of singular linear continuous-time systems with multiple internal and external incommensurate constant point delays. A new method called the descriptor integral-inequality method that can be used to study the delay-dependent stabilization issue of singular linear continuous-time systems with time-varying state and input delays is proposed. Based on the Lyapunov-Krasovskii functional approach, new delay-dependent stabilization sufficient conditions are developed. All the sufficient conditions can be easily solved by the linear matrix inequality (LMI) Matlab toolbox.

This paper is organized as follows. In section 2, the regular independent of time delays problem is discussed and the delay-dependent decentralized stabilization problem is defined for the multi-channel singular time-delay time-invariant systems with multiple internal and external incommensurate constant point delays. Then a new lemma called the descriptor integral-inequality Lemma that can be used to study the delay-dependent decentralized stabilization issue for singular time-delay systems is proposed. In section 3, by employing the descriptor integral-inequality Lemma, the sufficient conditions of delay-dependent decentralized stabilization are completely characterized based on nonlinear matrix inequality (NLMI), which are associated with the time delays and cannot be solved directly. From the viewpoint of LMI, the design method of state feedback control law is summarized. In section 4, the delay-dependent decentralized stabilization problems are illustrated by numerical examples.

Notation: Throughout the paper, I and 0 denote the appropriately dimensioned identity matrix and zero matrix. $\mathrm{diag}\{\cdots\}$ is a block-diagonal matrix. The symmetric terms in a symmetric matrix are denoted by $*$, e.g., $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix}$.

2. PROBLEM STATEMENT

Consider the following multi-channel singular system with $q'+1$ local control station and multiple internal and external incommensurate constant point delays described by

$$\begin{cases} E\dot{x}(t) = \sum_{j=0}^{q} A_j x(t-h_j) + \sum_{j=0}^{q'} B_j u_j(t-h'_j), \\ x(t) = \phi(t), \quad t \in [-\max\{h, h'\}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ represents the state and $u_j(t) \in \mathcal{R}^{m_j}$ is the local control input at the jth local station, and $A_j\,(j=0,1,\cdots,q)$, $B_j\,(j=0,1,\cdots,q')$ are real matrices of compatible orders with the dimensions of those vectors, and $h_j,(j=1,\cdots,q)$, $h'_j,(j=1,\cdots,q')$ are the q internal and q' external point delays respectively. The zero delays $h_0 = h'_0 = 0$ corresponding to the delay-free dynamics and current delay-free input are added for notational simplification convenience. $\phi(t)$ is a compatible vector valued continuous different initial function. The maximum delays h and h' are defined as $h = \max_{1 \le j \le q}(h_j)$ and $h' = \max_{1 \le j \le q'}(h'_j)$. The singular matrix $E \in \mathcal{R}^{n \times n}$ with $\mathrm{rank}(E) = r < n$ gives the singular character to the system (1) compared to the standard system $(E = I_n)$.

The unforced singular delay system of (1) can be written as

$$E\dot{x}(t) = A_0 x(t) + \sum_{j=1}^{q} A_j x(t-h_j) \tag{2}$$

The following definition is useful to discuss a wide class of the so-called singular regular systems:

Definition 1. The system (2) is said to be regular if there exists a constant $s \in \mathbb{C}$ such that $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \ne 0$.

The definition 1 is not easy to test because the $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \ne 0$ depends on the internal point delays $h_j\,(j=1,\cdots,q)$. An alternative characterization of regularity is now given. First, the generic rank (g.r.) in $\mathbb{C}$ of a complex matrix $Q(s)$ is defined as $\mathrm{g.r.}(Q(s)) = \max_{s\in\mathbb{C}}(\mathrm{rank}[Q(s)])$. The subsequent result formulates equivalent condition for regularity of the system (2) to that given explicitly in Definition 1.

Theorem 1. The system (2) is said to be regular independent of the delays $h_j\,(j=1,\cdots,q)$ if the $\mathrm{rank}[E, \bar{A}_j] = n$ where $\bar{A}_j = [A_0, \bar{A}_{j1}]$ with $\bar{A}_{j1} = [A_1, A_2, \cdots, A_q]$.

Proof From definition 1, the direct calculation yields

$$sE - \sum_{j=0}^{q} A_j e^{-h_j s} = [E, \bar{A}_j]\left[sI_n, -I_n, -e^{-h_1 s} I_n, \cdots, -e^{-h_q s} I_n\right]^{\mathrm{T}} \tag{3}$$

Thus, from (3) it is clear that the $\exists s \in \mathbb{C}$: $\left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \ne 0$ is equivalent to $\mathrm{rank}[E, \bar{A}_j] = n$. Since $\mathrm{rank}\left[sI_n, -I_n, -e^{-h_1 s} I_n, \cdots, -e^{-h_q s} I_n\right] = n, \forall s \in \mathbb{C}$ then $\mathrm{g.r.}_{s\in\mathbb{C}}\left(sE - \sum_{j=0}^{q} A_j e^{-h_j s}\right) = \mathrm{rank}[E, \bar{A}_j]$. So, $\mathrm{rank}[E, \bar{A}_j] = n \Rightarrow \mathrm{g.r.}_{s\in\mathbb{C}}\left(sE - \sum_{j=0}^{q} A_j e^{-h_j s}\right) = n \Rightarrow \left| sE - \sum_{j=0}^{q} A_j e^{-h_j s} \right| \ne 0$ for $s \in \mathbb{C}$ and Theorem 1 has been fully proved.

It is well known that a singular system has a more complicated structure and contains not only finite dynamical modes (exponential modes), but also infinite frequency modes, including infinite nondynamical and dynamical modes that may generate undesired impulse behaviors, they should be eliminated. In order to guarantee the system (2) is regular and impulse-free, the following lemmas are given:

Lemma 1 (Xu *et al.*, 2002). Suppose the pair (E, A_0) is said to be regular and impulse free, then the solution to (2) exists and is impulse free and unique on $[0, \infty)$.

Lemma 2 (Masubuchi *et al.*, 1997). The pair (E, A_0) is said to be regular, impulse free and stable if and only if there exist a matrix P such that $EP^{\mathrm{T}} = PE^{\mathrm{T}} \ge 0$ and $P^{\mathrm{T}} A_0 + A_0^{\mathrm{T}} P < 0$.

The state feedback controllers

$$u_j(t) = K_j x(t), j = 0, 1, \cdots, q' \tag{4}$$

are employed to stabilize (1). The goal of this paper is to develop a new delay-dependent stabilization method that provides the gain, K_j, of the controllers such that the resulting closed-loop system

$$E\dot{x}(t)=\sum_{j=0}^{q}A_j x\left(t-h_j\right)+\sum_{j=0}^{q'}\bar{B}_j x\left(t-h_j'\right) \tag{5}$$

is asymptotically stable, where $\bar{B}_j=B_jK_j$. For this purpose, the following lemmas are first introduced.

Lemma 3(Jiang *et al*., 2007). Let $x(t)\in\mathcal{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following descriptor integral-inequality holds for any matrices E, M_1, M_2, Y and $X=X^{\mathrm{T}}>0$, and a scalar $h\geq 0$:

$$-\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}XE\dot{x}(s)\mathrm{d}s\leq\xi^{\mathrm{T}}(t)\Upsilon\xi(t)+h\xi^{\mathrm{T}}(t)Y^{\mathrm{T}}X^{-1}Y\xi(t) \tag{6}$$

where

$$\xi(t):=\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix},\ \Upsilon:=\begin{bmatrix}M_1^{\mathrm{T}}E+E^{\mathrm{T}}M_1 & -M_1^{\mathrm{T}}E+E^{\mathrm{T}}M_2\\ * & -M_2^{\mathrm{T}}E-E^{\mathrm{T}}M_2\end{bmatrix},\ Y:=\begin{bmatrix}M_1 & M_2\end{bmatrix}.$$

3. MAIN RESULTS

This section addresses the sufficient conditions of delay-dependent stabilization obtained by the descriptor integral-inequality method. The following theorem is obtained for system (5).

Theorem 2. For given scalars $h>0$ and $h'>0$, if there exist symmetric and positive definite matrices $X=X^{\mathrm{T}}>0$, $Y_j=Y_j^{\mathrm{T}}>0,\ (j=1,\cdots,q)$, $Y_j'=Y_j'^{\mathrm{T}}>0,(j=1,\cdots,q')$ $R=R^{\mathrm{T}}>0$, $R'=R'^{\mathrm{T}}>0$, any matrices M_{1j},M_{2j} $(j=1,\cdots,q)$ and M_{1j}',M_{2j}' $(j=1,\cdots,q')$ such that:

$$EX^{\mathrm{T}}=XE^{\mathrm{T}}\geq 0 \tag{7}$$

$$\Psi=\begin{bmatrix}\Xi & qh\mathrm{T}_0^{\mathrm{T}} & q'h'\mathrm{T}_0^{\mathrm{T}} & h\mathrm{H}_1^{\mathrm{T}} & \cdots & h\mathrm{H}_q^{\mathrm{T}} & h'\mathrm{H}_1'^{\mathrm{T}} & \cdots & h'\mathrm{H}_{q'}'^{\mathrm{T}}\\ * & -qhR^{-1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0\\ * & * & -q'h'R'^{-1} & 0 & \cdots & 0 & 0 & \cdots & 0\\ * & * & * & -hR & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & * & \cdots & -hR & 0 & \cdots & 0\\ * & * & * & * & \cdots & * & -h'R' & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & * & \cdots & * & * & \cdots & -h'R'\end{bmatrix}<0 \tag{8}$$

where

$$\Xi=\begin{bmatrix}(1,1) & \Pi_1 & \cdots & \Pi_q & \Pi_1' & \cdots & \Pi_{q'}'\\ * & -\amalg_1 & \cdots & 0 & 0 & \cdots & 0\\ * & * & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & -\amalg_q & 0 & \cdots & 0\\ * & * & * & * & -\amalg_1' & \cdots & 0\\ * & * & * & * & * & \ddots & \vdots\\ * & * & * & * & * & * & -\amalg_{q'}'\end{bmatrix},$$

$$(1,1)=X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}}X+\sum_{j=1}^{q}Y_j+\sum_{j=1}^{q'}Y_j'$$
$$+\left(\sum_{j=1}^{q}M_{1j}^{\mathrm{T}}+\sum_{j=1}^{q'}M_{1j}'^{\mathrm{T}}\right)E+E^{\mathrm{T}}\left(\sum_{j=1}^{q}M_{1j}+\sum_{j=1}^{q'}M_{1j}'\right),$$

$\Pi_j=XA_j-M_{1j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}$, $\amalg_j=Y_j+M_{2j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}$,

$\Pi_j'=X\bar{B}_j-M_{1j}'^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}'$, $\amalg_j'=Y_j'+M_{2j}'^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j}'$,

$$\mathrm{H}_1=\begin{bmatrix}M_{11} & M_{21} & \cdots & 0 & 0 & \cdots & 0\end{bmatrix},\mathrm{H}_q=\begin{bmatrix}M_{1q} & 0 & \cdots & M_{2q} & 0 & \cdots & 0\end{bmatrix},$$
$$\mathrm{H}_1'=\begin{bmatrix}M_{11}' & 0 & \cdots & 0 & M_{21}' & \cdots & 0\end{bmatrix},\mathrm{H}_q'=\begin{bmatrix}M_{1q}' & 0 & \cdots & 0 & 0 & \cdots & M_{2q}'\end{bmatrix},$$
$$\mathrm{T}_0=\begin{bmatrix}A_0+\bar{B}_0 & A_1 & \cdots & A_q & \bar{B}_1 & \cdots & \bar{B}_{q'}\end{bmatrix}.$$

Then the closed-loop system (5) is regular independent of the delays and impulse free and asymptotically stable.

Proof. Suppose (7)-(8) hold for $X=X^{\mathrm{T}}>0$, $Y_j=Y_j^{\mathrm{T}}>0$, $Y_j'=Y_j'^{\mathrm{T}}>0$, $R=R^{\mathrm{T}}>0$, $R'=R'^{\mathrm{T}}>0$, M_{1j},M_{2j}, M_{1j}',M_{2j}', then from (8) it is easy to see that

$$X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}}X<0 \tag{9}$$

By Theorem 1 and Lemma 2, it follows from (7) and (9) that the system (5) is regular independent of the delays and the pair $\left(E,A_0+\sum_{j=1}^{q}\bar{B}_{j1}\right)$ is regular and impulse free. Next, we shall examine the stability of the singular delay system (5). To this end, we choose a Lyapunov-Krasovskii functional candidate as:

$$V(t)=V_1(t)+V_2(t) \tag{10}$$

with

$$V_1(t)=x^T(t)XEx(t)+\sum_{j=1}^{q}\int_{t-h_j}^{t}x^T(s)Y_jx(s)\mathrm{d}s+\sum_{j=1}^{q'}\int_{t-h_j'}^{t}x^T(s)Y_j'x(s)\mathrm{d}s,$$

$$V_2(t)=\sum_{j=1}^{q}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+\sum_{j=1}^{q'}\int_{-h'}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

where $X=X^{\mathrm{T}}>0$, $Y_j=Y_j^{\mathrm{T}}>0$, $Y_j'=Y_j'^{\mathrm{T}}>0$, $R=R^{\mathrm{T}}>0$ and $R'=R'^{\mathrm{T}}>0$ are to be determined. Then, the time derivative of $V(t)$ along the trajectory (5) satisfies

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)$$

where

$$\begin{aligned}\dot{V}_1(t)&=2x^{\mathrm{T}}(t)XE\dot{x}(t)+x^{\mathrm{T}}(t)\left(\sum_{j=1}^{q}Y_j+\sum_{j=1}^{q'}Y_j'\right)x(t)\\ &\quad-\sum_{j=1}^{q}x^{\mathrm{T}}\left(t-h_j\right)Y_jx\left(t-h_j\right)-\sum_{j=1}^{q'}x^{\mathrm{T}}\left(t-h_j'\right)Y_j'x\left(t-h_j'\right)\\ &=\eta^{\mathrm{T}}(t)\Xi_0\eta(t)\end{aligned} \tag{11}$$

with

$$\eta(t)=\begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h_1) & \cdots & x^{\mathrm{T}}\left(t-h_q\right) & x^{\mathrm{T}}\left(t-h_1'\right) & \cdots & x^{\mathrm{T}}\left(t-h_q'\right)\end{bmatrix}^{\mathrm{T}},$$

$$\Xi_0=\begin{bmatrix}(1,1)_0 & XA_1 & \cdots & XA_q & X\bar{B}_1 & \cdots & X\bar{B}_{q'}\\ * & -Y_1 & \cdots & 0 & 0 & \cdots & 0\\ * & * & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & -Y_q & 0 & \cdots & 0\\ * & * & * & * & -Y_1' & \cdots & 0\\ * & * & * & * & * & \ddots & \vdots\\ * & * & * & * & * & * & -Y_{q'}'\end{bmatrix},$$

$$(1,1)_0=X\left(A_0+\bar{B}_0\right)+\left(A_0+\bar{B}_0\right)^{\mathrm{T}}X+\sum_{j=1}^{q}Y_j+\sum_{j=1}^{q'}Y_j',$$

and

$$\begin{aligned}\dot{V}_2(t)&=\eta^{\mathrm{T}}(t)\left(\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0+\mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0\right)\eta(t)-\sum_{j=1}^{q}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s\\ &\quad-\sum_{j=1}^{q'}\int_{t-h'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s\end{aligned} \tag{12}$$

with

$$\mathrm{T}_0=\begin{bmatrix}A_0+\bar{B}_0 & A_1 & \cdots & A_q & \bar{B}_1 & \cdots & \bar{B}_{q'}\end{bmatrix}$$

Applying the Lemma 2, It is clear that the following is true

$$-\sum_{j=1}^{q}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s-\sum_{j=1}^{q'}\int_{t-h'}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s$$
$$\le-\sum_{j=1}^{q}\int_{t-h_j}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\mathrm{d}s-\sum_{j=1}^{q'}\int_{t-h'_j}^{t}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R'E\dot{x}(s)\mathrm{d}s \quad (13)$$
$$\le\eta^{\mathrm{T}}(t)\left(\Xi_1+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}'_j\right)\eta(t)$$

where

$$\Xi_1=\begin{bmatrix}(1,1)_1 & \mathrm{T}_1 & \cdots & \mathrm{T}_q & \mathrm{T}'_1 & \cdots & \mathrm{T}'_{q'}\\ * & \Gamma_1 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \Gamma_q & 0 & \cdots & 0\\ * & * & \cdots & * & \Gamma'_1 & \vdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & * & * & \cdots & \Gamma'_{q'}\end{bmatrix},\quad \begin{aligned}&\mathrm{T}_j=-M_{1j}^{\mathrm{T}}E+E^{\mathrm{T}}M_{2j},\\ &\Gamma_j=-M_{2j}^{\mathrm{T}}E-E^{\mathrm{T}}M_{2j},\ j=1,\cdots,q\\ &\mathrm{T}'_j=-M_{1j}'^{\mathrm{T}}E+E^{\mathrm{T}}M'_{2j},\\ &\Gamma'_j=-M_{2j}'^{\mathrm{T}}E-E^{\mathrm{T}}M'_{2j},\ j=1,\cdots,q'\end{aligned}$$

$$(1,1)_1=\left(\sum_{j=1}^{q}M_{1j}^{\mathrm{T}}+\sum_{j=1}^{q'}M_{1j}'^{\mathrm{T}}\right)E+E^{\mathrm{T}}\left(\sum_{j=1}^{q}M_{1j}+\sum_{j=1}^{q'}M'_{1j}\right),,.$$

$$\mathrm{H}_1=\begin{bmatrix}M_{11} & M_{21} & \cdots & 0 & 0 & \cdots & 0\end{bmatrix},\mathrm{H}_q=\begin{bmatrix}M_{1q} & 0 & \cdots & M_{2q} & 0 & \cdots & 0\end{bmatrix},$$
$$\mathrm{H}'_1=\begin{bmatrix}M'_{11} & 0 & \cdots & 0 & M'_{21} & \cdots & 0\end{bmatrix},\mathrm{H}'_q=\begin{bmatrix}M'_{1q} & 0 & \cdots & 0 & 0 & \cdots & M'_{2q}\end{bmatrix}.$$

Substituting (13) into (12) gives us

$$\dot{V}_2(t)\le\eta^{\mathrm{T}}(t)\left(\Xi_1+\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0+\mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}'_j\right)\eta(t) \quad (14)$$

Combining (11)-(14) yields

$$V(t)\le\eta^{\mathrm{T}}(t)\left(\Xi+\mathrm{T}_0^{\mathrm{T}}qhR\mathrm{T}_0+\mathrm{T}_0^{\mathrm{T}}q'h'R'\mathrm{T}_0+\sum_{j=1}^{q}\mathrm{H}_j^{\mathrm{T}}hR^{-1}\mathrm{H}_j+\sum_{j=1}^{q'}\mathrm{H}_j'^{\mathrm{T}}h'R'^{-1}\mathrm{H}'_j\right)\eta(t) \quad (15)$$

where

$$\Xi=\Xi_0+\Xi_1$$

From (15), it is clear that $\Psi<0$ guarantees $\dot{V}(t)<0$ by the Schur complement (Boyd et al., 1994). According to the Lyapunov-Krasovskii functional theorem (Gu et al., 2003), the closed-loop system (5) is asymptotically stable. This completes the proof of Theorem 2.

As $K_j\ (j=0,1,\cdots,q')$ are design matrixes, Ψ is nonlinear in the design parameters K_j and X, the nonlinearities also come from R and R^{-1}, R' and R'^{-1}. Thus in this case (8) cannot be solved directly by LMI toolbox. In order to obtain a controller gain, K_j, from the nonlinear matrix inequality (8), the following theorem is given.

Theorem 3. For given numbers $h>0$, $h'>0$, λ_j, $\mu_j\ne0$, $j=1,\cdots,q$ and λ'_j, $\mu'_j\ne0$, $j=1,\cdots,q'$, if there exist symmetric and positive matrices $\bar{Y}_j$, $j=1,\cdots,q$, $\bar{Y}'_j$, $j=0,1,\cdots,q'$, $\bar{R}$, $\bar{R}'$ and any matrices $\bar{K}_j$, $j=0,1,\cdots,q'$ such that the following LMI holds

$$\bar{Y}'_0E^{\mathrm{T}}=E\bar{Y}'_0\ge0 \quad (16)$$

$$\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15}\\ * & \Sigma_{22} & 0 & 0 & 0\\ * & * & \Sigma_{33} & 0 & 0\\ * & * & * & \Sigma_{44} & 0\\ * & * & * & * & \Sigma_{55}\end{bmatrix}<0 \quad (17)$$

Where

$$\Sigma_{11}=\begin{bmatrix}\Omega_1 & qh\Omega_2^{\mathrm{T}} & q'h'\Omega_2^{\mathrm{T}}\\ * & -qh\bar{R} & 0\\ * & * & -q'h'\bar{R}'\end{bmatrix},\ \Sigma_{12}=\begin{bmatrix}h\mathrm{N}_1^{\mathrm{T}} & \cdots & h\mathrm{N}_q^{\mathrm{T}}\\ 0 & \cdots & 0\\ 0 & \cdots & 0\end{bmatrix},$$

$$\Sigma_{13}=\begin{bmatrix}h'\mathrm{N}_1'^{\mathrm{T}} & \cdots & h'\mathrm{N}_{q'}'^{\mathrm{T}}\\ 0 & \cdots & 0\\ 0 & \cdots & 0\end{bmatrix},\ \Sigma_{14}=\Sigma_{15}=\begin{bmatrix}\bar{Y}'_0 & \cdots & \bar{Y}'_0\\ 0 & \cdots & 0\\ 0 & \cdots & 0\end{bmatrix},$$

$$\Sigma_{22}=\mathrm{diag}\left\{-h\bar{R},\cdots,-h\bar{R}\right\},\ \Sigma_{33}=\mathrm{diag}\left\{-h'\bar{R}',\cdots,-h'\bar{R}'\right\},$$
$$\Sigma_{44}=\mathrm{diag}\left\{-\bar{Y}_1,\cdots,-\bar{Y}_q\right\},\ \Sigma_{55}=\mathrm{diag}\left\{-\bar{Y}'_1,\cdots,-\bar{Y}'_q\right\},$$

$$\Omega_1=\begin{bmatrix}\Omega_{11} & \mho_{11} & \cdots & \mho_{1q} & \mho'_{11} & \cdots & \mho'_{1q'}\\ * & -\mho_{21} & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & -\mho_{2q} & 0 & \cdots & 0\\ * & * & \cdots & * & -\mho'_{21} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & * & * & \cdots & -\mho'_{2q'}\end{bmatrix}$$

$$\mho_{1j}=\mu_j^{-1}EA_j\bar{Y}_j+\left(\bar{Y}'_0+\lambda_j\mu_j^{-1}\bar{Y}_j\right)E^{\mathrm{T}}+\lambda_j\mu_j^{-2}\bar{Y}_j,\ j=1,\cdots,q$$
$$\mho_{2j}=\mu_j^{-1}\left(E\bar{Y}_j+\bar{Y}_jE^{\mathrm{T}}\right)+\mu_j^{-2}\bar{Y}_j,\ j=1,\cdots,q$$
$$\mho'_{1j}=\mu_j'^{-1}EB_j\bar{K}_j+\left(\bar{Y}'_0+\lambda'_j\mu_j'^{-1}\bar{Y}'_j\right)E^{\mathrm{T}}+\lambda'_j\mu_j'^{-2}\bar{Y}'_j,\ j=1,\cdots,q'$$
$$\mho'_{2j}=\mu_j'^{-1}\left(E\bar{Y}'_j+\bar{Y}'_jE^{\mathrm{T}}\right)+\mu_j'^{-2}\bar{Y}'_j,\ j=1,\cdots,q'$$

$$\Omega_{11}=E\left(A_0\bar{Y}'_0+B_0\bar{K}_0\right)-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}EA_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}EB_j\bar{K}_j-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}\bar{Y}_j$$
$$+\left(E\left(A_0\bar{Y}'_0+B_0\bar{K}_0\right)-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}EA_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}EB_j\bar{K}_j\right)^{\mathrm{T}}-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}\bar{Y}'_j$$

$$\Omega_2=\begin{bmatrix}\Omega_{21} & \mu_1^{-1}A_1\bar{Y}_1 & \cdots & \mu_q^{-1}A_q\bar{Y}_q & \mu_1'^{-1}B_1\bar{K}_1 & \cdots & \mu_{q'}'^{-1}B_{q'}\bar{K}_{q'}\end{bmatrix},$$
$$\Omega_{21}=\left(A_0\bar{Y}'_0+B_0\bar{K}_0\right)-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}A_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}B_j\bar{K}_j,$$

$$\mathrm{N}_1=\begin{bmatrix}0 & \bar{R} & \cdots & 0 & 0 & \cdots & 0\end{bmatrix},\mathrm{N}_q=\begin{bmatrix}0 & 0 & \cdots & \bar{R} & 0 & \cdots & 0\end{bmatrix},$$
$$\mathrm{N}'_1=\begin{bmatrix}0 & 0 & \cdots & 0 & \bar{R}' & \cdots & 0\end{bmatrix},\mathrm{N}'_{q'}=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & \cdots & \bar{R}'\end{bmatrix}.$$

then the decentralized controller (4) with $K_j=\bar{K}_j\bar{Y}_j'^{-1}$, $j=0,1,\cdots,q'$ stabilises system (1) and the closed-loop system (5) is regular independent of the delays and impulse free.

Proof. To cast the problem of designing a stabilising controller (4) into the LMI formulation, it is assumed that $\bar{Y}'_0=X^{-1}$. Under this condition, the (16) is equivalent to (7). Due to $EX^{\mathrm{T}}=XE^{\mathrm{T}}\ge0$ and define the following matrices

$$W=\begin{bmatrix}X & 0\\ W_1 & W_2\end{bmatrix},\ \bar{A}=\begin{bmatrix}\left(A_0+\sum_{j=1}^{q}\bar{B}_{j1}\right) & \bar{A}_{11}\\ \bar{A}_{12} & \bar{A}_{13}\end{bmatrix},$$

$$\mathrm{U}=\mathrm{diag}\left\{\sum_{j=1}^{q}\left(Y_j+Y'_j\right),-Y_1,\cdots,-Y_q,-Y'_1,\cdots,-Y'_q\right\},$$

$$W_1=\begin{bmatrix}M_{11}^{\mathrm{T}} & \cdots & M_{1q}^{\mathrm{T}} & M_{11}'^{\mathrm{T}} & \cdots & M_{1q}'^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$W_2=\mathrm{diag}\left\{M_{21},\cdots,M_{2q},M'_{21},\cdots,M'_{2q}\right\},$$

$$\bar{A}_{11}=\begin{bmatrix}A_1 & \cdots & A_q & \bar{B}_{12} & \cdots & \bar{B}_{q2}\end{bmatrix},\bar{A}_{12}=\begin{bmatrix}E^{\mathrm{T}} & \cdots & E^{\mathrm{T}} & E^{\mathrm{T}} & \cdots & E^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$\bar{A}_{13}=\mathrm{diag}\left\{-E,\cdots,-E,-E,\cdots,-E\right\},$$

then

$$\Xi=W^{\mathrm{T}}\bar{A}+\bar{A}^{\mathrm{T}}W+\mathrm{U},$$
$$\mathrm{H}_1=\begin{bmatrix}0 & I & \cdots & 0 & 0 & \cdots & 0\end{bmatrix}W,\ \mathrm{H}_q=\begin{bmatrix}0 & 0 & \cdots & I & 0 & \cdots & 0\end{bmatrix}W,$$
$$\mathrm{H}'_1=\begin{bmatrix}0 & 0 & \cdots & 0 & I & \cdots & 0\end{bmatrix}W,\ \mathrm{H}'_{q'}=\begin{bmatrix}0 & 0 & \cdots & 0 & 0 & \cdots & I\end{bmatrix}W,$$

when $M_{1j}=\lambda_j X$, $M_{2j}=\mu_j Y_j$, $M'_{1j}=\lambda'_j X$, $M'_{2j}=\mu'_j Y'_j$, $\mu_j \neq 0$, and $\mu'_j \neq 0$, it is obvious that W is invertible, and

$$W^{-1}=\begin{bmatrix}\bar{X} & 0\\ \bar{W}_1 & \bar{W}_2\end{bmatrix},$$

$$\bar{W}_1=\left[-\lambda_1\mu_1^{-1}\bar{Y}_1 \ \cdots \ -\lambda_q\mu_q^{-1}\bar{Y}_q \ -\lambda'_1\mu_1'^{-1}\bar{Y}'_1 \ \cdots \ -\lambda'_q\mu_q'^{-1}\bar{Y}'_q\right]^{\mathrm{T}},$$

$$\bar{W}_2=\mathrm{diag}\left\{\mu_1^{-1}\bar{Y}_1,\cdots,\mu_q^{-1}\bar{Y}_q,\mu_1'^{-1}\bar{Y}'_1,\cdots,\mu_q'^{-1}\bar{Y}'_q\right\},$$

Define $T=\mathrm{diag}\left\{W^{-1},I,I,R^{-1},\cdots,R^{-1},R'^{-1},\cdots,R'^{-1}\right\}$ and set $\bar{Y}_j=Y_j^{-1}$ $j=1,\cdots,q$, $\bar{Y}'_j=Y_j'^{-1}$, $j=1,\cdots,q'$, $\bar{R}=R^{-1}$, $\bar{R}'=R'^{-1}$, $\bar{K}_j=K_j\bar{Y}'_j$ $j=0,1,\cdots,q'$, then

$$T^{\mathrm{T}}\Psi T=\begin{bmatrix}(1,1)_2 & qh\Omega_2^{\mathrm{T}} & q'h'\Omega_2^{\mathrm{T}} & h\mathrm{N}_1^{\mathrm{T}} & \cdots & h\mathrm{N}_q^{\mathrm{T}} & h'\mathrm{N}_1'^{\mathrm{T}} & \cdots & h'\mathrm{N}_{q'}'^{\mathrm{T}}\\ * & -qh\bar{R} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0\\ * & * & -q'h'\bar{R}' & 0 & \cdots & 0 & 0 & \cdots & 0\\ * & * & * & -h\bar{R} & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & * & \cdots & -h\bar{R} & 0 & \cdots & 0\\ * & * & * & * & \cdots & * & -h'\bar{R}' & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & * & \cdots & * & * & \cdots & -h'\bar{R}'\end{bmatrix}<0 \tag{18}$$

where

$$(1,1)_2=\begin{bmatrix}(1,1)_3 & \mho_{11} & \cdots & \mho_{1q} & \mho'_{11} & \cdots & \mho'_{1q'}\\ * & -\mho_{21} & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & -\mho_{2q} & 0 & \cdots & 0\\ * & * & \cdots & * & -\mho'_{21} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & * & * & \cdots & -\mho'_{2q'}\end{bmatrix}$$

$$\begin{aligned}(1,1)_3 &= E\left(A_0\bar{Y}'_0+B_0\bar{K}_0\right)-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}EA_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}EB_j\bar{K}_j-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}\bar{Y}_j\\ &-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}\bar{Y}'_j+\left(E\left(A_0\bar{Y}'_0+B_0\bar{K}_0\right)-\sum_{j=1}^{q}\lambda_j\mu_j^{-1}EA_j\bar{Y}_j-\sum_{j=1}^{q'}\lambda'_j\mu_j'^{-1}EB_j\bar{K}_j\right)^{\mathrm{T}}\\ &+\bar{Y}'_0\left(\sum_{j=1}^{q}Y_j+\sum_{j=1}^{q'}Y'_j\right)\bar{Y}'_0\end{aligned}$$

and $\mho_{1j}$, $\mho_{2j}$, $\mho'_{1j}$, $\mho'_{2j}$, N_1 , N_q , N'_1 , N'_q , Ω_2 are defined in Theorem 3.

Therefore, (18) follows from (17). From this derivation, the conclusion can be drawn that if there exist symmetric and positive matrices $\bar{Y}_j$, $j=1,\cdots,q$, $\bar{Y}'_j$, $j=0,1,\cdots,q'$, $\bar{R}$, $\bar{R}'$ and any matrices $\bar{K}_j$, $j=0,1,\cdots,q'$ satisfying (16) and (17), then the following symmetric and positive matrices X , Y_j , $j=1,\cdots,q$, Y'_j , $j=1,\cdots,q'$, R , R' , and any matrices M_{1j},M_{2j} $j=1,\cdots,q$ and M'_{1j},M'_{2j} $j=1,\cdots,q'$ satisfy (7) and (8). So, the resulting closed-loop systems (5) is regular independent of the delays , impulse free and asymptotically stable, and the desired controller is defined by (4) with $K_j=\bar{K}_j\bar{Y}_j'^{-1}$, $j=0,1,\cdots,q'$. This completes the proof of Theorem 3.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to demonstrate the validity of the results described above.

Example. Consider a multi-channel singular time-delay system (1), with the following parameters:

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, A_0=\begin{bmatrix}0.2 & 0.1 & -0.5\\ 0.25 & 0 & -0.2\\ 0 & 1 & 0\end{bmatrix}, A_1=\begin{bmatrix}-0.4 & 0.2 & -0.6\\ 0 & -0.5 & 0\\ 0 & 0 & -2\end{bmatrix},$$

$$B_0=\begin{bmatrix}1 & 0 & 0\end{bmatrix}^{\mathrm{T}}, B_1=\begin{bmatrix}0 & 1 & 0\end{bmatrix}^{\mathrm{T}}, B_2=\begin{bmatrix}0 & 0 & 1\end{bmatrix}^{\mathrm{T}}.$$

In this example, we assume that the max point time delays are $h=4.46$ and $h'=0.2$, and λ_j , μ_j λ'_j , μ'_j are chosen as $\lambda_1=-1.8953$, $\lambda'_1=-1.4451$, $\lambda'_2=-0.3451$, $\mu_1=14.7388$, $\mu'_1=0.3654$, $\mu'_2=4.3654$ and using Matlab LMI Control Toolbox to solve the feasible problems (22), (23) and (24), a decentralized stabilizing state feedback control law can be obtained as:

$$u_0(t)=\begin{bmatrix}-0.4687 & -0.1629 & 0.4769\end{bmatrix}x(t),$$

$$u_1(t)=\begin{bmatrix}-0.5624 & -0.4637 & 2.7508\end{bmatrix}x(t),$$

$$u_2(t)=\begin{bmatrix}-0.0035 & -0.0476 & 0.0039\end{bmatrix}x(t),$$

The purpose is to design a decentralized state feedback control law such that the closed-loop system is regular independent of the delays, impulse free and asymptotically stable. To this end, we can validate easily by computing $\mathrm{rank}\left[E,A_0,A_1,B_0K_0,B_1K_1,B_2K_2\right]$, so, from the Theorem 1, we can obtain that $\mathrm{rank}\left[E,A_0,A_1,B_0K_0,B_1K_1,B_2K_2\right]=3$, i.e. the system (5) is regular independent of the delays h_j , $j=1,\cdots,q$. Hence, according to Theorem 1 and Theorem 3, controller (4) with gain K_j given in the preceding can make the closed-loop systems (5) regular independent of the delays, impulse free and asymptotically stable.

5. CONCLUSION

In this paper, the delay-dependent decentralized stabilization problem of large-scale singular time-delay time-invariant systems subject to multiple internal and external incommensurate constant point delays has been studied. The main contribution of this study is to obtain the control law, which can delay-dependent decentralized stabilises large-scale singular time-delay time-invariant systems, by the Lyapunov-Krasovskii functional approach combined with an descriptor integral- inequality. The numerical examples show that the proposed controller design method works very well.

6. ACKNOWLEDGEMENTS

This work was supported by the Key Program of Natural Science Foundation of China under Grant No. 60634020, Doctor Subject Foundation of China under Grant No. 20050533028, Natural Science Foundation of Hunan province under Grant No. 06JJ5145 and Science Development Foundation of Central South University under Grant No. 07SDF13.

REFERENCES

Verghese, C., Bernard C. and Thomas K. (1981). A generalized state-space for singular systems, *IEEE Trans. Automat. Contr.* 26, 811-831.

Newcomb, R. W. and B. Dziurla (1989). Some circuits and systems application of semistate theory, *Circuits Syst. Sig. Proc.* 8, 235-260.

Mills, J. K. and A. A. Goldenberg (1989). Force and position control of manipulators during constrained motion tasks, *IEEE Trans. Robot. Automat.* 38, 30-46.

Stevens, B. L. and Lewis, F. L (1991). *Aircraft Modeling, Dynamics, and Control*, NewYork: Wiley.

Stefanovski, J (2006). LQ control of descriptor systems by cancelling structure at infinity, *Int. J. Control vol.* 79, 224-238.

Xu, S. Y., P. V. Dooren, R. Stefan and J. Lam (2002). Robust stability and stabilization for singular systems with state delay and parameter uncertainty, *IEEE Trans. Automat. Contr.* 47, 1122-1128.

Yu, R. and D. H. Wang (2003). Structural properties and poles assignability of LTI singular systems under output feedback, *Automatica* 39(4), 685-692.

Ishihara, J. Y., M. H. Terra and R. M. Sales (2003). The full information and state feedback H_2 optimal controllers fro descriptor systems, *Automatica* 39(3), 391-402.

Wang, J., V. Sreeram and W. Liu (2006). An improved H_∞ suboptimal model reduction for singular systems, *Int. J. Control* vol. 79, 798-804.

Gui, W. H., Jiang, Z. H., and Xie, Y. F (2006). Decentralized robust H_∞ control of singular large-scale systems based on descriptor output feedback, *Systems Engineering and Electronics*, 28(5), 736-740.

Richard, J. P (2003). Time-delay systems: an overview of some recent advances and open problems, *Automatica* 39(10), 1667-1694.

Lee, Y. S., Y. S. Moon and W. H. Kwon, Other (2004). Delay-dependent robust H_∞ control for uncertain systems with a state-delay, *Automatica* 40(1), 65-72.

Zhang, X. M., M. Wu and J. H. She, Other (2005). Delay-dependent stabilization of linear systems with time-varying state and input delays, *Automatica* 41(8), 1405-1412.

De la Sen, M (2007). Quadratic stability and stabilization of switched dynamic systems with uncommensurate internal point delays, *Applied Mathematics and Computation*, 185(1), 508-526.

Zheng, F. and P. M. Frank (2001). Finite-dimensional variable structure control design for distributed delay systems, *Int. J. Control* vol. 74, 398-408.

Fridman, E. and U. Shaked (2002). H_∞ -control of linear state-delay descriptor systems: an LMI approach, *Linear Algebra and Its Applications*, vol. 351-352, 271-302.

Kim, J. H (2005). New design method on memoryless H_∞ controller for singular systems with delayed stated and control using LMI, *Journal of The Franklin Institute*, 42(3), 321-327.

Boukas, E. K. and Z. K. Liu (2003). Delay-dependent stability analysis of singular linear continuous-time system, *IEE Proc.-Control Theory Appl.*, 150(4), 325-330.

Jiang, Z. H., Gui, W. H. and Xie, Y. F., Other (2007). Delay-dependent Stabilization of Singular Linear Continuous-time Systems with Time-varying State and Input Delays, *the 6th IEEE International Conference on Control and Automation*, China, pp. 1862-1867.

Wen, X. C. and Y. Q. Liu (1995). Stabilization of large scale singular systems, *Advances in Modeling & Analysis C*, 5(1-3), 53-64.

Wo, S. L., Zou, Y. and Sheng, M., Other (2007). Robust control for discrete-time singular large-scale systems with parameter uncertainty, *Journal of the Franklin Institute*, 344(2), 97-106.

Lewis, F. L., G. Beauchamp and K. Ozcaldiran, Other (1991). Large-scale dynamical interconnections of stochastic singular systems, *Circuits Syst. Sig. Proc*. 10, 97-106.

Liu, Y. Q., Z. H. Guan and X. C. Wen (1995). Application of auxiliary simultaneous equations to the problem in the stabilization of singular and impulsive large scale systems, *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*. 42, 46-51.

Xie, Y. F., Jiang, Z. H., and Gui, W. H. Other (2006). Decentralized Robust Delay-dependent Stabilization for Singular Large Scale Systems Based on Descriptor Output Feedback, *Proceedings of the 6th World Congress on Control and Automation*, China, pp. 1191-1195.

Guan, Z. H., Y. Q. Liu and X. C. Wen (1995). Decentralized stabilization of singular and time-delay large-scale control systems with impulsive solutions, *IEEE Trans. Automat. Contr*. 40, 1437-1441.

Boyd, S., L. E. Ghaoui, and E. Feron, Other (1994). *Linear matrix inequalities in systems and control theory*, Philadelphia: SIAM.

Gu, K., L. Kharitonov, and J. Chen (2003). *Stability of time-delay systems*, Boston: Birkhauser.

Masubuchi, I., Y. Kamitane and A. Ohara, Other (1997). H_∞ control for descriptor systems: A matrix inequalities approach, *Automatic* 33(3), 669-676.