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Gain-Scheduled Controller Synthesis Based on New LMIs for Dissipativity of Descriptor LPV Systems

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Abstract: This paper is concerned with synthesis of gain-scheduled controllers by representing LPV systems in the descriptor form. Based on a recent algebraic criterion characterizing dissipativity of descriptor systems, an improved synthesis method is proposed with removing limitations of descriptor representation in existing results. Specifications of exponential decay are also considered. A numerical example illustrates the procedure of the proposed synthesis method.

Keywords: Descriptor systems, gain-scheduled control, dissipativity, linear matrix inequalities (LMIs).

1. INTRODUCTION

Descriptor systems have been paid a great deal of attention as a general and natural representation of dynamical systems (See e.g., Lewis (1986)). The representation that can include algebraic constraints as well as differential equations is useful for such as analysis of time-delay systems (e.g., Fridman, & Shaked (2002)) and robustness analysis (e.g.,Cao, & Lin (2004); Chen (2004)). In particular, the descriptor form has advantage in representing linear parameter-varying (LPV) systems (Masubuchi, & Shimemura (1996)) in the sense that rational dependence on the scheduling parameters is reduced to affine dependence by employing the descriptor form. This leads to easily-handled parameter-dependent linear matrix inequality (LMI) problems, which merit has been exploited in gain-scheduled controller design (Chen, & Sugie (1998); Masubuchi et al. (2003, 2004); Polat et al. (2007)).

These previous papers aim to optimize L_2-gain of the control system based on LMI conditions of L_2-gain for descriptor systems. Including positive and bounded realness, dissipativity or integral quadratic constraints (IQCs) provides important criteria to evaluate performances and robustness of control systems. There are a considerable number of criteria for these specifications related to dissipativity generalized to descriptor systems (Freund, & Jarre (2004); Masubuchi et al. (1997); Rehm, & Allgöwer (2000, 2002); Rehm (2004); Takaba et al. (1994); Uezato, & Ikeda (1999); Wang et al. (1998); Zhang et al. (2002)). Recently, a new matrix inequality condition is proposed in Masubuchi (2005) that is necessary and sufficient for dissipativity of descriptor systems without restricting the choice of descriptor realization. Synthesis problems have been considered based on this result for linear time-invariant systems (Masubuchi (2006, 2007)).

In this paper, we consider synthesis method of gain-scheduled controllers based on the criterion of Masubuchi (2005) by representing LPV system in the descriptor form. Restrictions of descriptor realizations forced in Masubuchi et al. (2004) are removed with the new criterion. We show parameter-dependent linear matrix equations (LMEs) and LMIs for synthesis via change-of-variables. This basic result is further investigated in applying it to LPV systems that are originally in state-space realization, resulting a synthesis procedure that derives a state-space gain-scheduled controller directly from a solution to the LME-LMIs for the descriptor LPV system. This does not involve perturbing any matrix-valued functions of the scheduling parameter in Masubuchi et al. (2003). We also show LMIs for specifications on exponential decay of initial-state responses of the gain-scheduled control system and mention multi-objective control synthesis. A numerical example is provided to illustrate the synthesis procedure proposed in the paper.

2. GAIN-SCHEDULING SYNTHESIS FOR GENERAL DESCRIPTOR LPV SYSTEMS

2.1 LPV systems in the descriptor form

Let us consider the following representation of LPV systems:

$$\begin{cases} E\dot{x} &= \mathcal{A}x + \mathcal{B}_1 w + \mathcal{B}_2 u, \\ z &= \mathcal{C}_1 x + \mathcal{D}_{11} x + \mathcal{D}_{12} u, \\ y &= \mathcal{C}_2 x + \mathcal{D}_{21} w, \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the descriptor variable, $w \in \mathbf{R}^{m_1}$ is the external input, $u \in \mathbf{R}^{m_2}$ is the control input, $z \in \mathbf{R}^{p_1}$ is the controlled output and $y \in \mathbf{R}^{p_2}$ is the measured output. The coefficient matrix E of $\dot{x}$ is assumed to be a constant

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$n \times n$-matrix, which is possibly and typically singular, and let $r = \text{rank}E$. Denote by $\theta(t)$ the scheduling parameter which is differentiable and satisfies $\theta(t) \in \Theta$ and $\dot{\theta}(t) \in \Omega$ for some given subsets $\Theta, \Omega \subset \mathbf{R}^N$. Let $\boldsymbol{\Theta}$ be the set of such functions θ: $t \in \mathbf{R} \to \Theta$. The coefficient matrices other than E are functions of θ.

We define a version of internal stability for linear time-invariant descriptor systems as well as underlying definitions for it. Then the definition of an exponential stability of LPV descriptor systems follows.

Definition 1. Let E, $A \in \mathbf{R}^{n\times n}$. The pencil $sE - A$ is *regular* if $\det(sE-A)$ is not identically zero. Suppose that $sE - A$ is regular. The *exponential modes* of $sE - A$ are the finite eigenvalues of $sE - A$, namely, $s \in \mathbf{C}$ such that $\det(sE - A) = 0$. Let a vector v_1 satisfy $Ev_1 = 0$. Then the infinite eigenvalues associated with the generalized eigenvectors v_k satisfying $Ev_k = Av_{k-1}$, $k = 2, 3, 4, \ldots$ are *impulsive modes* of $sE-A$. The pencil $sE-A$ is *impulse-free* if it is regular and has no impulsive modes. The pencil $sE-A$ is said to be *admissible* if it is regular, impulse-free and has no unstable exponential modes.

Definition 2. Consider the system (1) with $w = 0$, $u = 0$. The system (1) is said to be *exponentially stable* if the pencil $sE - \mathcal{A}$ is admissible for all $\theta \in \Theta$ and there exist positive constants M and α for which $\|x(t)\| \le Me^{-\alpha t}\|x(0)\|$ holds for any $\theta(\cdot) \in \boldsymbol{\Theta}$, $x(0)$ and $t \ge 0$.

2.2 Formulation of the synthesis problem for general descriptor LPV systems

Let us describe the control system. Consider a controller in the descriptor from:

$$\begin{cases} E_c\dot{x}_c &= \mathcal{A}_c x_c + \mathcal{B}_c y, \\ \quad u &= \mathcal{C}_c x_c + \mathcal{D}_c y, \end{cases} \tag{2}$$

where $x_c \in \mathbf{R}^{n_c}$, $E_c \in \mathbf{R}^{n_c\times n_c}$ and $\text{rank}E_c = r_c$. The closed-loop system is represented as

$$\begin{cases} E_{cl}\dot{x}_{cl} &= \mathcal{A}_{cl}x_{cl} + \mathcal{B}_{cl}w, \\ \quad z &= \mathcal{C}_{cl}x_{cl} + \mathcal{D}_{cl}w, \end{cases} \tag{3}$$

where $x_{cl}^\mathsf{T} = \begin{bmatrix} x^\mathsf{T} & x_c^\mathsf{T} \end{bmatrix} \in \mathbf{R}^{n_{cl}}$, $n_{cl} = n + n_c$. For a given plant (1), our goal in this section is to find a controller (2) for which the closed-loop system (3) is exponentially stable and satisfies the following dissipativity or IQC specification:

$$\int_0^\infty \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^\mathsf{T} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^\mathsf{T} & \Pi_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} \mathrm{d}t < 0 \tag{4}$$

for any nonzero $w \in \mathcal{L}_2[0,\infty)$ if $x(0) = 0$. We assume $\Pi_{11} \ge 0$ and let $\Gamma_{11} \in \mathbf{R}^{p_1\times q}$ be a matrix such that $\Gamma_{11}\Gamma_{11}^\mathsf{T} = \Pi_{11}$. We define $\Pi_{1*} = [\,\Pi_{12}\ \Gamma_{11}\,]$, $\Psi = [\,I_{m_1}\ 0_{m_1\times q}\,]$, $\Pi_{2a} = \text{diag}\{\Pi_{22}, -I_q\}$ and use the notation:

$$\begin{bmatrix} \mathcal{A} & \widetilde{\mathcal{B}}_1 & \mathcal{B}_2 \\ \widetilde{\mathcal{C}}_1 & \widetilde{\mathcal{D}}_{11} & \widetilde{\mathcal{D}}_{12} \\ \mathcal{C}_2 & \widetilde{\mathcal{D}}_{21} & 0 \end{bmatrix} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & \Pi_{1*}^\mathsf{T} & 0 \\ 0 & 0 & I_{p_2} \end{bmatrix} \times \begin{bmatrix} \mathcal{A} & \mathcal{B}_1 & \mathcal{B}_2 \\ \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{C}_2 & \mathcal{D}_{21} & 0 \end{bmatrix} \begin{bmatrix} I_n & 0 & 0 \\ 0 & \Psi & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix},$$

$$\begin{bmatrix} \widetilde{\mathcal{A}}_{cl} & \widetilde{\mathcal{B}}_{cl} \\ \widetilde{\mathcal{C}}_{cl} & \widetilde{\mathcal{D}}_{cl} \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{cl} & \mathcal{B}_{cl}\Psi \\ \Pi_{1*}^\mathsf{T}\mathcal{C}_{cl} & \Pi_{1*}^\mathsf{T}\mathcal{D}_{cl}\Psi \end{bmatrix}.$$

The specifications of the dissipativity and the exponential stability are fulfilled whenever the following matrix equations and inequalities hold for some continuously differentiable matrix-valued functions $\mathcal{Y}_{cl}$: $\Theta \to \mathbf{R}^{n_{cl}\times n_{cl}}$ and $\mathcal{Z}_{cl}$: $\Theta \to \mathbf{R}^{(m_1+q)\times n_{cl}}$ for which

$$E_{cl}\mathcal{Y}_{cl}^\mathsf{T} = \mathcal{Y}_{cl}E_{cl}^\mathsf{T} \ge 0, \quad E_{cl}\mathcal{Z}_{cl}^\mathsf{T} = 0, \tag{5}$$

$$\mathbf{He}\left(\begin{bmatrix} \mathcal{A}_{cl} & \widetilde{\mathcal{B}}_{cl} \\ \widetilde{\mathcal{C}}_{cl} & \widetilde{\mathcal{D}}_{cl} \end{bmatrix} \begin{bmatrix} \mathcal{Y}_{cl}^\mathsf{T} & \mathcal{Z}_{cl}^\mathsf{T} \\ 0 & I_{m_1+q} \end{bmatrix} \right) + \text{diag}\{-E_{cl}\dot{\mathcal{Y}}_{cl}^\mathsf{T}, \Pi_{2a}\} \ll 0, \tag{6}$$

where $\mathbf{He}A = A + A^\mathsf{T}$ and "$\ll 0$" means that there exists a positive scaler c such that the left hand side is less than $-cI$ for all $\theta(\cdot) \in \boldsymbol{\Theta}$. Note that $\mathcal{Y}_{cl}$ is nonsingular if it satisfies (6). Moreover, being time-invariant, the system (3) is admissible and satisfies the dissipativity constraint (4) if and only if (5)-(6) holds for constant matrices (Masubuchi (2005)). We also note that this LMI condition guarantees the exponential stability of the closed-loop system.

2.3 LME-LMI condition for synthesis

Based on the above KYP-type inequalities generalized to descriptor systems, we present a parameter-dependent LME-LMI condition by which we can obtain a gain-scheduled controller in the descriptor form (2).

Theorem 1. The conditions (5) and (6) hold for some $\mathcal{Y}_{cl}$ and $\mathcal{Z}_{cl}$ if and only if there exist matrix-valued functions $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$, $\mathcal{W}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, $\mathcal{J}$, where $\mathcal{X}$ and $\mathcal{Y}$ are continuously differentiable, that satisfy

$$\begin{bmatrix} E & 0 \\ 0 & E^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & I \\ I & \mathcal{X} \end{bmatrix} = \begin{bmatrix} \mathcal{Y} & I \\ I & \mathcal{X}^\mathsf{T} \end{bmatrix} \begin{bmatrix} E^\mathsf{T} & 0 \\ 0 & E \end{bmatrix} \ge 0, \tag{7}$$

$$E^\mathsf{T}\mathcal{W} = 0, \quad E\mathcal{Z}^\mathsf{T} = 0, \tag{8}$$

$$\begin{aligned} &\mathbf{He}\left[\begin{array}{c|c} \begin{bmatrix} \mathcal{A} & \widetilde{\mathcal{B}}_1 \\ \widetilde{\mathcal{C}}_1 & \widetilde{\mathcal{D}}_{11} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & \mathcal{Z}^\mathsf{T} \\ 0 & I \end{bmatrix} + \begin{bmatrix} \mathcal{B}_2 \\ \widetilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{F}^\mathsf{T} & \begin{bmatrix} \mathcal{A} & \widetilde{\mathcal{B}}_1 \\ \widetilde{\mathcal{C}}_1 & \widetilde{\mathcal{D}}_{11} \end{bmatrix} + \begin{bmatrix} \mathcal{B}_2 \\ \widetilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{J}^\mathsf{T} \begin{bmatrix} \mathcal{C}_2 & \widetilde{\mathcal{D}}_{21} \end{bmatrix} \\ \hline \mathcal{H}^\mathsf{T} & \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{A} & \widetilde{\mathcal{B}}_1 \\ \widetilde{\mathcal{C}}_1 & \widetilde{\mathcal{D}}_{11} \end{bmatrix} + \mathcal{G}^\mathsf{T} \begin{bmatrix} \mathcal{C}_2 & \widetilde{\mathcal{D}}_{21} \end{bmatrix} \end{array} \right] \\ &+ \left[\begin{array}{cc|cc} -\dot{\mathcal{Y}}E^\mathsf{T} & 0 & 0 & 0 \\ 0 & \Pi_{2a} & 0 & \Pi_{2a} \\ \hline 0 & 0 & \dot{\mathcal{X}}^\mathsf{T}E & 0 \\ 0 & \Pi_{2a} & 0 & \Pi_{2a} \end{array} \right] \ll 0 \end{aligned} \tag{9}$$

for all $\theta \in \boldsymbol{\Theta}$. If LMEs and LMIs in (7)-(9) hold, there exist also nonsingular $\mathcal{X}$ and $\mathcal{Y}$ that satisfy (7)-(9). The following coefficients give a gain-scheduled controller in (2) that satisfies (5) and (6) and hence the exponential stability and the dissipativity of the closed-loop system.

$$E_c = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{D}_c = \mathcal{J}, \tag{10}$$

$$\mathcal{C}_c = \left(\mathcal{F}^\mathsf{T} - \mathcal{J}^\mathsf{T} \begin{bmatrix} \mathcal{C}_2 & \widetilde{\mathcal{D}}_{21} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & \mathcal{Z}^\mathsf{T} \\ 0 & I \end{bmatrix} \right) \begin{bmatrix} \mathcal{S}^\mathsf{T} & \mathcal{Z}_c^\mathsf{T} \\ 0 & I \end{bmatrix}^{-1}, \tag{11}$$

$$\mathcal{B}_c = \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ -\mathcal{W}_c^\mathsf{T} & I \end{bmatrix}^{-1} \left(\begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{B}_2 \\ \widetilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{J}^\mathsf{T} - \mathcal{G}^\mathsf{T} \right), \tag{12}$$

$$\begin{aligned} \mathcal{A}_c = \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ -\mathcal{W}_c^\mathsf{T} & I \end{bmatrix}^{-1} & \left\{ \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{A} & \widetilde{\mathcal{B}}_1 \\ \widetilde{\mathcal{C}}_1 & \widetilde{\mathcal{D}}_{11} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & \mathcal{Z}^\mathsf{T} \\ 0 & I \end{bmatrix} \right. \\ & + \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{B}_2 \\ \widetilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{F}^\mathsf{T} - \begin{bmatrix} \mathcal{X}^\mathsf{T} \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{X}^{-\mathsf{T}}) E^\mathsf{T} & 0 \\ 0 & 0 \end{bmatrix} \\ & - \mathcal{H}^\mathsf{T} - \left(\begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{B}_2 \\ \hat{\mathcal{D}}_{12} \end{bmatrix} \mathcal{J}^\mathsf{T} - \mathcal{G}^\mathsf{T} \right) \\ & \left. \times \begin{bmatrix} \mathcal{C}_2 & \widetilde{\mathcal{D}}_{21} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & \mathcal{Z}^\mathsf{T} \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} \mathcal{S}^\mathsf{T} & \mathcal{Z}_c^\mathsf{T} \\ 0 & I \end{bmatrix}^{-1}, \end{aligned} \tag{13}$$

where

$$\begin{cases} \mathcal{S} = \mathcal{Y} - \mathcal{X}^{-\mathsf{T}}, \quad \mathcal{W}_c = -\mathcal{S}^{-\mathsf{T}}(\mathcal{Y}^\mathsf{T}\mathcal{W} + \mathcal{Z}^\mathsf{T}), \\ \mathcal{Z}_c^\mathsf{T} = \mathcal{X}^{-1}\mathcal{W} + \mathcal{Z}^\mathsf{T}. \end{cases} \tag{14}$$

Theorem 1 provides a controller of the descriptor form with $n_c = n + m_1 + q$ and $r_c = r$. Through perturbation of $\mathcal{A}_c$ if necessary, we can obtain also a state-space controller with a state variable in $\mathbf{R}^r$. More of this issue is considered in the next section. Theorem 1 generalizes the result of Masubuchi (2006) to LPV synthesis. Unlike in this previous paper, the obtained LMEs and LMIs have the symmetric and separated structure with respect to state-feedback and observer counterparts of dynamic output-feedback synthesis. This will lead to a clear discussion in the next section where we consider application of the LME-LMI condition (7)-(9) to synthesis of gain-scheduled control systems that has a state-space LPV representation.

2.4 Synthesis for specified exponential decay

In this subsection we consider specifications on the decay of initial-state responses of the descriptor variable. Suppose $w = 0$ in the closed-loop system (3).

Definition 3. The descriptor system, assumed to be exponentially stable, has exponential decay of interval $[\alpha, \beta]$, where $0 < \alpha < \beta$, if there exist positive constants c_α and c_β such that

$$c_\beta e^{-\beta t}\|x_{cl}(0)\| \le \|x_{cl}(t)\| \le c_\alpha e^{-\alpha t}\|x_{cl}(0)\| \tag{15}$$

holds for all $\theta(\cdot) \in \mathbf{\Theta}$, $x(0)$ and $t \ge 0$.

Consider the second inequality in (15) that specifies the upperbound to the norm of responses. The following inequality with (5) is sufficient for this condition to hold:

$$\begin{aligned} \mathbf{He} & \left(\begin{bmatrix} \mathcal{A}_{cl} + \alpha E_{cl} & \widetilde{\mathcal{B}}_{cl} \\ \widetilde{\mathcal{C}}_{cl} & \widetilde{\mathcal{D}}_{cl} \end{bmatrix} \begin{bmatrix} \mathcal{Y}_{cl}^\mathsf{T} & \mathcal{Z}_{cl}^\mathsf{T} \\ 0 & I \end{bmatrix} \right) \\ & - \mathrm{diag}\{\dot{\mathcal{Y}}_{cl} E_{cl}^\mathsf{T}, \varepsilon I\} \ll 0, \end{aligned} \tag{16}$$

where ε is a scalar. Based on this inequality we obtain a formulation of an LME-LMI condition for synthesis in the same manner as (7)-(9):

Corollary 2. The condition (16) with (5) holds for some $\mathcal{Y}_{cl}$, $\mathcal{Z}_{cl}$ and ε if and only if there exists matrix-valued functions $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$, $\mathcal{W}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, $\mathcal{J}$, where $\mathcal{X}$ and $\mathcal{Y}$ are continuously differentiable, and ε that satisfy (7), (8) and

$$\mathcal{M}_\alpha + \left[\begin{array}{cc|cc} -\dot{\mathcal{Y}} E^\mathsf{T} & 0 & 0 & 0 \\ 0 & -\varepsilon I & 0 & -\varepsilon I \\ \hline 0 & 0 & \dot{\mathcal{X}}^\mathsf{T} E & 0 \\ 0 & -\varepsilon I & 0 & -\varepsilon I \end{array} \right] \ll 0 \tag{17}$$

for all $\theta \in \mathbf{\Theta}$, where

$$\mathcal{M}_\alpha := \mathbf{He} \left[\begin{array}{c|c} \begin{bmatrix} \mathcal{A}_\alpha & \tilde{\mathcal{B}}_1 \\ \tilde{\mathcal{C}}_1 & \tilde{\mathcal{D}}_{11} \end{bmatrix} \begin{bmatrix} \mathcal{Y}^\mathsf{T} & \mathcal{Z}^\mathsf{T} \\ 0 & I \end{bmatrix} + \begin{bmatrix} \mathcal{B}_2 \\ \tilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{F}^\mathsf{T} & \begin{bmatrix} \mathcal{A}_\alpha & \tilde{\mathcal{B}}_1 \\ \tilde{\mathcal{C}}_1 & \tilde{\mathcal{D}}_{11} \end{bmatrix} \\ \hline \mathcal{H}^\mathsf{T} + \begin{bmatrix} \alpha E & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \mathcal{X}^\mathsf{T} & 0 \\ \mathcal{W}^\mathsf{T} & I \end{bmatrix} \begin{bmatrix} \mathcal{A}_\alpha & \tilde{\mathcal{B}}_1 \\ \tilde{\mathcal{C}}_1 & \tilde{\mathcal{D}}_{11} \end{bmatrix} + \mathcal{G}_1^\mathsf{T} \begin{bmatrix} \mathcal{C}_2 & \tilde{\mathcal{D}}_{21} \end{bmatrix} \end{array} \right] \tag{18}$$

and $\mathcal{A}_\alpha = \mathcal{A} + \alpha E$. If this LME-LMI condition is true, a gain-scheduled controller (2) given by (10)-(13) and (14) satisfies the exponential decay condition $\|x_{cl}(t)\| \le c_\alpha e^{-\alpha t}\|x_{cl}(0)\|$ of the closed-loop system.

For the other inequality in (15), the following LMI:

$$\mathcal{M}_\beta + \left[\begin{array}{cc|cc} -\dot{\mathcal{Y}} E^\mathsf{T} & 0 & 0 & 0 \\ 0 & \delta I & 0 & \delta I \\ \hline 0 & 0 & \dot{\mathcal{X}}^\mathsf{T} E & 0 \\ 0 & \delta I & 0 & \delta I \end{array} \right] \gg 0 \tag{19}$$

with (7), (8) are sufficient, where δ is a scaler. A merit of (17) and (19) is that they are applicable to multi-objective synthesis with the dissipativity and exponential decay specifications. In spite of possible conservatism in taking common variables in LMIs for more than one specifications, combining those LMIs provides a simple computational method to derive a gain-scheduled controller satisfying multiple specifications. We show in Subsection 3.2 that several components of the variables in these LME-LMI conditions can be non-common among multiple specifications.

3. APPLICATION OF THEOREM 1 TO SYNTHESIS FOR STATE-SPACE LPV SYSTEMS WITH RATIONAL COEFFICIENTS

3.1 Formulation

So far, we have presented a synthesis method of gain-scheduled controllers for plants in the general descriptor form (1). The solution in the previous subsection basically solves the synthesis problem. However, it requires possible perturbation of several matrices, namely $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{A}_c$, which are functions of the scheduling parameter θ. Though this can be executed somehow, we have to check the control system with the derived controller through perturbation still satisfies the specifications adequately. This involves additional numerical problems to examine functions of θ.

Below we show a method that avoids this inconvenient situation, assuming that descriptor LPV plant is equivalent to a state-space LPV system. This is typical if the descriptor LPV plant is derived from a state-space one by augmenting the state variable to a descriptor variable in order to simplify the coefficient matrices. By the following procedures to formulate a descriptor LPV plant and to construct a state-space gain-scheduled controller directly from a solution to (7)-(9), this LME-LMI condition with affine coefficients is more useful for synthesis of state-space LPV systems with rational coefficients.

Let us consider the following LPV system:

$$\left\{\begin{aligned} \dot{x}_s &= \mathcal{A}_s x_s + \mathcal{B}_{1s} w + \mathcal{B}_{2s} u, \\ z &= \mathcal{C}_{1s} x_s + \mathcal{D}_{11s} w + \mathcal{D}_{12s} u, \\ y &= \mathcal{C}_{2s} x_s + \mathcal{D}_{21s} w, \end{aligned}\right. \tag{20}$$

where $x_s \in \mathbf{R}^r$ is the state variable and the coefficient matrices such as $\mathcal{A}_s$ are assumed to be rational functions of the scheduling parameter θ. One can represent this system by an equivalent descriptor LPV equation (1), where the both representations are related to each other as follows. Without loss of generality, let

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{21}$$

in (1) and partition the other coefficient matrices accordingly as

$$\left[\begin{array}{c|cc} \mathcal{A} & \mathcal{B}_1 & \mathcal{B}_2 \\ \hline \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{C}_2 & \mathcal{D}_{21} & 0 \end{array}\right] = \left[\begin{array}{cc|cc} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{21} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{12} & \mathcal{B}_{22} \\ \hline \mathcal{C}_{11} & \mathcal{C}_{12} & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{C}_{21} & \mathcal{C}_{22} & \mathcal{D}_{21} & 0 \end{array}\right], \tag{22}$$

where $\mathcal{A}_{11} \in \mathbf{R}^{r\times r}$. Then, from the rational coefficient matrices in the state-space LPV system (20), we *always* derive an equivalent descriptor LPV system (1) so that the coefficient matrices satisfy

$$\begin{aligned} \left[\begin{array}{c|cc} \mathcal{A}_s & \mathcal{B}_{1s} & \mathcal{B}_{2s} \\ \hline \mathcal{C}_{1s} & \mathcal{D}_{11s} & \mathcal{D}_{12s} \\ \mathcal{C}_{2s} & \mathcal{D}_{21s} & 0 \end{array}\right] &= \left[\begin{array}{c|cc} \mathcal{A}_{11} & \mathcal{B}_{11} & \mathcal{B}_{21} \\ \hline \mathcal{C}_{11} & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{C}_{21} & \mathcal{D}_{21} & 0 \end{array}\right] \\ &\quad - \left[\begin{array}{c} \mathcal{A}_{12} \\ \hline \mathcal{C}_{12} \\ \mathcal{C}_{22} \end{array}\right] \mathcal{A}_{22}^{-1} \left[\, \mathcal{A}_{21} \,|\, \mathcal{B}_{12} \;\; \mathcal{B}_{22} \,\right], \end{aligned} \tag{23}$$

where we emphasize that (23) involves a constraint: $\mathcal{C}_{22}\mathcal{A}_{22}^{-1}\mathcal{B}_{22} = 0$. We also prepare notation of these matrices including parameters from Π_{ij} that represents the dissipativity specification:

$$\begin{aligned} \begin{bmatrix} \mathcal{A}_s & \widetilde{\mathcal{B}}_{1s} & \mathcal{B}_{2s} \\ \widetilde{\mathcal{C}}_{1s} & \widetilde{\mathcal{D}}_{11s} & \widetilde{\mathcal{D}}_{12s} \\ \mathcal{C}_{2s} & \widetilde{\mathcal{D}}_{21s} & 0 \end{bmatrix} &= \begin{bmatrix} I_n & 0 & 0 \\ 0 & \Pi_{1*}^{\mathsf{T}} & 0 \\ 0 & 0 & I_{p_2} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \mathcal{A}_s & \mathcal{B}_{1s} & \mathcal{B}_{2s} \\ \mathcal{C}_{1s} & \mathcal{D}_{11s} & \mathcal{D}_{12s} \\ \mathcal{C}_{2s} & \mathcal{D}_{21s} & 0 \end{bmatrix} \begin{bmatrix} I_n & 0 & 0 \\ 0 & \Psi & 0 \\ 0 & 0 & I_{m_2} \end{bmatrix}, \\ \left[\begin{array}{cc|c|c} \mathcal{A}_{11} & \mathcal{A}_{12} & \widetilde{\mathcal{B}}_{11} & \mathcal{B}_{21} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \widetilde{\mathcal{B}}_{12} & \widetilde{\mathcal{B}}_{22} \\ \hline \widetilde{\mathcal{C}}_{11} & \widetilde{\mathcal{C}}_{12} & \widetilde{\mathcal{D}}_{11} & \widetilde{\mathcal{D}}_{12} \\ \hline \mathcal{C}_{21} & \mathcal{C}_{22} & \widetilde{\mathcal{D}}_{21} & 0 \end{array}\right] &= \left[\begin{array}{c|c|c} I_n & 0 & 0 \\ \hline 0 & \Pi_{1*}^{\mathsf{T}} & 0 \\ \hline 0 & 0 & I_{p_2} \end{array}\right] \\ &\quad \times \left[\begin{array}{cc|c|c} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{21} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{12} & \mathcal{B}_{22} \\ \hline \mathcal{C}_{11} & \mathcal{C}_{12} & \mathcal{D}_{11} & \mathcal{D}_{12} \\ \hline \mathcal{C}_{21} & \mathcal{C}_{22} & \mathcal{D}_{21} & 0 \end{array}\right] \left[\begin{array}{c|c|c} I_n & 0 & 0 \\ \hline 0 & \Psi & 0 \\ \hline 0 & 0 & I_{m_2} \end{array}\right]. \end{aligned}$$

3.2 Construction of a state-space controller when the original plant has state-space representation

Our strategy is to obtain a state-space gain-scheduled controller realization, without any perturabtion of matrices, from a solution of the LME-LMI condition in Theorem 1 in terms of the descriptor system (1). Looking into the LME-LMI condition in Theorem 1 in view of coefficient matrices of (20), we derive the following:

Proposition 1. Assume that the coefficients of the descriptor plant (1) satisfy (21)-(23). Let

$$\mathcal{X} = \begin{bmatrix} \mathcal{X}_s & 0 \\ \mathcal{X}_{21} & \mathcal{X}_{22} \end{bmatrix}, \quad \mathcal{Y} = \begin{bmatrix} \mathcal{Y}_s & \mathcal{Y}_{12} \\ 0 & \mathcal{Y}_{22} \end{bmatrix}, \tag{24}$$

where $\mathcal{X}_s$, $\mathcal{Y}_s$ are symmetric. Then (7), (8) and (9) hold iff

$$\begin{bmatrix} \mathcal{Y}_s & I \\ I & \mathcal{X}_s \end{bmatrix} \gg 0 \tag{25}$$

and (8), (9) are true. From a solution to the latter, define

$$\begin{aligned} \left[\begin{array}{c|cc} \mathcal{J}_s & \mathcal{G}_{1s} & \mathcal{G}_{2s} \\ \hline \mathcal{F}_{1s} & \mathcal{H}_{11s} & \mathcal{H}_{22s} \\ \mathcal{F}_{2s} & \mathcal{H}_{21s} & \mathcal{H}_{22s} \end{array}\right] &:= \left[\begin{array}{c|ccc} I & 0 & 0 & 0 \\ \hline 0 & I & -\mathcal{A}_{12}\mathcal{A}_{22}^{-1} & 0 \\ 0 & 0 & -\widetilde{\mathcal{C}}_{12}\mathcal{A}_{22}^{-1} & I \end{array}\right] \\ &\quad \times \left[\begin{array}{c|c} \mathcal{J} & \mathcal{G} \\ \hline \mathcal{F} & \mathcal{H} \end{array}\right] \left[\begin{array}{c|cc} I & 0 & 0 \\ \hline 0 & I & 0 \\ 0 & -\mathcal{A}_{22}^{-1}\mathcal{A}_{21} & -\mathcal{A}_{22}^{-1}\mathcal{B}_{12} \\ 0 & 0 & I \end{array}\right]. \end{aligned} \tag{26}$$

Then with the following controller the closed-loop system is exponentially stable and satisfies the dissipativity:

$$\begin{aligned} \mathcal{A}_c = \Bigg\{ &\begin{bmatrix} \mathcal{A}_s & \widetilde{\mathcal{B}}_{1s} \\ \widetilde{\mathcal{C}}_{1s} & \widetilde{\mathcal{D}}_{11s} \end{bmatrix} \begin{bmatrix} \mathcal{Y}_s & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \mathcal{B}_{2s} \\ \widetilde{\mathcal{D}}_{12s} \end{bmatrix} \left[\, \mathcal{F}_{1s}^{\mathsf{T}} \;\; \mathcal{F}_{2s}^{\mathsf{T}} \,\right] \\ &- \begin{bmatrix} \mathcal{X}_s^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \mathcal{H}_{11s}^{\mathsf{T}} & \mathcal{H}_{21s}^{\mathsf{T}} \\ \mathcal{H}_{12s}^{\mathsf{T}} & \mathcal{H}_{22s}^{\mathsf{T}} \end{bmatrix} - \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{X}_s^{-1}) & 0 \\ 0 & 0 \end{bmatrix} \\ &- \left(\begin{bmatrix} \mathcal{B}_2 \\ \widetilde{\mathcal{D}}_{12} \end{bmatrix} \mathcal{J}^{\mathsf{T}} - \begin{bmatrix} \mathcal{X}_s^{-1}\mathcal{G}_{1s}^{\mathsf{T}} \\ \mathcal{G}_{2s}^{\mathsf{T}} \end{bmatrix} \right) \\ &\times \left[\, \mathcal{C}_{2s} \;\; \widetilde{\mathcal{D}}_{21s} \,\right] \begin{bmatrix} \mathcal{Y}_s & 0 \\ 0 & I \end{bmatrix} \Bigg\} \begin{bmatrix} \mathcal{S}_s^{-1} & 0 \\ 0 & I \end{bmatrix}, \end{aligned} \tag{27}$$

$$\mathcal{B}_c = \begin{bmatrix} \mathcal{B}_{2s} \\ \widetilde{\mathcal{D}}_{12s} \end{bmatrix} \mathcal{J}^{\mathsf{T}} - \begin{bmatrix} \mathcal{X}_s^{-1}\mathcal{G}_{1s}^{\mathsf{T}} \\ \mathcal{G}_{2s}^{\mathsf{T}} \end{bmatrix}, \tag{28}$$

$$\mathcal{C}_c = \left(\left[\, \mathcal{F}_{1s}^{\mathsf{T}} \;\; \mathcal{F}_{2s}^{\mathsf{T}} \,\right] - \mathcal{J}^{\mathsf{T}} \left[\, \mathcal{C}_2\mathcal{Y}_s \;\; \widetilde{\mathcal{D}}_{21} \,\right] \right) \begin{bmatrix} \mathcal{S}_s^{-1} & 0 \\ 0 & I \end{bmatrix}, \tag{29}$$

$$\mathcal{D}_c = \mathcal{J}^{\mathsf{T}}, \quad E_c = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{S}_s = \mathcal{Y}_s - \mathcal{X}_s^{-1}, \tag{30}$$

where (25) implies nonsingularity of $\mathcal{X}_s$, $\mathcal{Y}_s$ and $\mathcal{S}_s$. Moreover, this descriptor controller has no impulsive modes and hence equivalent to a state-space LPV system.

Proof. Obviously (24) and (25) yield (7) and the opposite direction is also true; if (25) is not strictly positive definite one can replace $(\mathcal{Y}_s, \mathcal{X}_s)$ with $(\mathcal{Y}_s + aI, \mathcal{X}_s + aI)$ for small $a > 0$ which does not violate (9). Next, a congruent transformation of (9) shows that

$$\mathbf{He} \left[\begin{array}{c|} \begin{bmatrix} \mathcal{A}_s & \widetilde{\mathcal{B}}_{1s} \\ \widetilde{\mathcal{C}}_{1s} & \widetilde{\mathcal{D}}_{11s} \end{bmatrix} \begin{bmatrix} \mathcal{Y}_s & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \mathcal{B}_{2s} \\ \widetilde{\mathcal{D}}_{12s} \end{bmatrix} \left[\mathcal{F}_{1s}^{\mathsf{T}} \;\; \mathcal{F}_{2s}^{\mathsf{T}} \right] \\ \hline \begin{bmatrix} \mathcal{H}_{11s}^{\mathsf{T}} & \mathcal{H}_{21s}^{\mathsf{T}} \\ \mathcal{H}_{12s}^{\mathsf{T}} & \mathcal{H}_{22s}^{\mathsf{T}} \end{bmatrix} \end{array}\right.$$

$$\left[\begin{array}{c}\begin{bmatrix}\mathcal{A}_s & \widetilde{\mathcal{B}}_{1s}\\ \widetilde{\mathcal{C}}_{1s} & \widetilde{\mathcal{D}}_{11s}\end{bmatrix}+\begin{bmatrix}\mathcal{B}_{2s}\\ \widetilde{\mathcal{D}}_{12s}\end{bmatrix}\mathcal{J}_s^{\mathsf{T}}\begin{bmatrix}\mathcal{C}_{2s} & \widetilde{\mathcal{D}}_{21s}\end{bmatrix}\\ \hline \begin{bmatrix}\mathcal{X}_s & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\mathcal{A}_s & \widetilde{\mathcal{B}}_{1s}\\ \widetilde{\mathcal{C}}_{1s} & \widetilde{\mathcal{D}}_{11s}\end{bmatrix}+\begin{bmatrix}\mathcal{G}_{1s}^{\mathsf{T}}\\ \mathcal{G}_{2s}^{\mathsf{T}}\end{bmatrix}\begin{bmatrix}\mathcal{C}_{2s} & \widetilde{\mathcal{D}}_{21s}\end{bmatrix}\end{array}\right] + \left[\begin{array}{cc|cc}-\dot{\mathcal{Y}}_s & 0 & 0 & 0\\ 0 & \Pi_{2a} & 0 & \Pi_{2a}\\ \hline 0 & 0 & \dot{\mathcal{X}}_s & 0\\ 0 & \Pi_{2a} & 0 & \Pi_{2a}\end{array}\right] \ll 0 \qquad (31)$$

holds for the variables defined in (24) and (26). This is nothing but (9) applied to the original state-space plant (20). Then we see that the descriptor controller (10)-(13) reduces to (27)-(30). Next, let $\mathcal{A}_{c22}$ be the right-lower block of A_c in (27) and $\mathcal{N}$ be the matrix that consists of second and fourth block rows and columns of (31). Then it is easy to see $\mathbf{He}\mathcal{A}_{c22} = [\,I\ -I\,]\mathcal{N}[\,I\ -I\,]^{\mathsf{T}} \ll 0$. Therefore $\mathcal{A}_{c22}$ is nonsingular and thus the controller is impulse-free.

We summarize the synthesis procedure below. This is applicable also to synthesis for exponential decay.

- Let an LPV system be given in the state-space form. Represent it in the descriptor form (1) so that the coefficients satisfy (21)-(23).
- Solve the LMEs and LMIs in (8)-(9)-(25) for $\mathcal{X}$, $\mathcal{Y}$ as in (24) and $\mathcal{Z}$, $\mathcal{W}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, $\mathcal{J}$.
- Set $\mathcal{F}_{is}$, $\mathcal{G}_{js}$, $\mathcal{H}_{ijs}$, $\mathcal{J}_s$ by (26) from the solution to (8)-(9)-(25).
- Set $\mathcal{A}_s$ etc. by (27)-(30) and derive a state-space gain-scheduled controller.

The LME-LMI condition (8)-(9)-(25) includes variables $\mathcal{X}_{21}$, $\mathcal{Y}_{22}$, $\mathcal{Y}_{12}$, $\mathcal{Y}_{22}$ and $\mathcal{Z}$, $\mathcal{W}$ that do not appear in the controller realization (27)-(30). These variables give certain degree of freedom and can reduce conservatism in synthesis. In particular, if one solves the LMIs (9), (17), (19) with (25) for the purpose of multi-objective synthesis to satisfy a dissipativity condition and exponential decay specifications, the variables $\mathcal{X}_{21}$, $\mathcal{Y}_{22}$, $\mathcal{Y}_{12}$, $\mathcal{Y}_{22}$, $\mathcal{Z}$, $\mathcal{W}$ can be non-common between the LMIs (9), (17) and (19).

Employing the descriptor LPV representation for originally state-space LPV systems has a merit to simplify the resulting parameter-dependent LMIs so that the coefficients of the LMIs are only affine functions of the scheduling parameter. The derived LMIs are closely related to those in *dilated LMI* approaches (e.g., Ebihara, & Hagiwara (2002)). Namely, dilating standard Lyapunov and KYP inequalities gives degrees of freedom in the dilated LMI scheme, while applying Lyapunov and KYP-type inequalities to descriptor systems augmented from a state-space system also generates some freedom as shown above. Both of those freedom can be exploited to reduce conservatism in robust synthesis (Kawata et al. (2006)).

4. A NUMERICAL EXAMPLE

We illustrate the procedure proposed in Subsection 3.2. Consider the following LPV plant in the state-space form:

$$\mathcal{A}_s = \begin{bmatrix} -0.8-2\theta+\theta^2 & 0.5-32\theta \\ \theta & \frac{-1-1.4\theta}{2-1.5\theta} \end{bmatrix},$$

class	(a)	(b)	(c) $N=2$	(d) $N=2$
γ	-	1.2423	1.2291	1.2067

Table 1. Optimal γ v.s. classes of the variables

$$\mathcal{B}_{1s} = \begin{bmatrix} 1+3\theta \\ 0.5 \end{bmatrix}, \quad \mathcal{B}_{2s} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$\mathcal{C}_{1s} = [\,1\ 2\,], \quad \mathcal{C}_{2s} = [\,1\ 1\,],$$
$$\mathcal{D}_{11s} = 0, \quad \mathcal{D}_{12s} = 1, \quad \mathcal{D}_{21s} = 1.$$

Let $\Theta = [0,1]$ and $\Omega = [-\pi,\pi]$. For the state variable $x_s = [\,x_1\ x_2\,]^{\mathsf{T}}$ and the external input w, define $x_3 = \theta x_1 + 3w$, $x_4 = x_2/(2-1.5\theta)$ and set the descriptor variable as $x = [\,x_1\ x_2\ x_3\ x_4\,]^{\mathsf{T}}$. Then we get a descriptor LPV system (1) whose coefficient matrices are affine as follows:

$$E = \mathrm{diag}\{1,1,0,0\},$$
$$\mathcal{A} = \begin{bmatrix} -0.8-2\theta & 0.5-32\theta & \theta & 0 \\ \theta & 0 & 0 & -1-1.4\theta \\ \theta & 0 & -1 & 0 \\ 0 & -1 & 0 & 2-1.5\theta \end{bmatrix},$$
$$\mathcal{B}_1 = [\,1\ 0.5\ 3\ 0\,]^{\mathsf{T}}, \quad \mathcal{B}_2 = [\,1\ 1\ 0\ 0\,]^{\mathsf{T}},$$
$$\mathcal{C}_1 = [\,1\ 2\ 0\ 0\,], \quad \mathcal{C}_2 = [\,1\ 1\ 0\ 0\,],$$
$$\mathcal{D}_{11} = 0, \quad \mathcal{D}_{12} = 1, \quad \mathcal{D}_{21} = 1.$$

Setting $\Pi_{11} = 1$, $\Pi_{22} = -\gamma^2$ and $\Pi_{12} = 0$, we consider L_2-gain from w to z subject to the exponential decay constraint with $[\alpha,\beta] = [0.5, 10]$. We solved the LMEs and LMIs in (7), (8), (9), (17) and (19), with the variables $\mathcal{X}_{21}$, $\mathcal{Y}_{22}$, $\mathcal{Y}_{12}$, $\mathcal{Y}_{22}$, $\mathcal{Z}$, $\mathcal{W}$ being non-common. We sought the unknown variables as (a) constants, (b) affine functions, (c) smoothed spline functions (Masubuchi (1998, 1999)) of θ with N dividing points placed on Θ at equal intervals. Increasing N for spline functions arbitrarily reduces the conservatism in finite-dimensional restriction of the class of the variables. Minimizing γ^2 resulted as shown in Table 1, where the problem was infeasible for (a). The controlled output z and the control input u are shown in Fig. 3 and Fig. 4, respectively, under the disturbance w in Fig. 2 and the scheduling parameter varying as in Fig. 1. The responses decay and converge to zero, where employing spline functions rewards a better responses. Fig. 5 shows the closed-loop poles for 200 samples of frozen θ from Θ. As specified, their real parts belong to $[-10, -0.5]$.

5. CONCLUSION

In this paper, we showed a synthesis method of gain-scheduled controllers by using the new LME-LMI condition for dissipativity of descriptor systems. The proposed method does not restrict descriptor representation and does not require perturbation of parameter-dependent matrices. The synthesis method can include specifications of exponential decay and a numerical example illustrated the proposed procedure applied to a multi-objective design problem.

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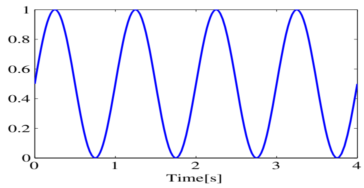

Fig. 1. Scheduling parameter θ

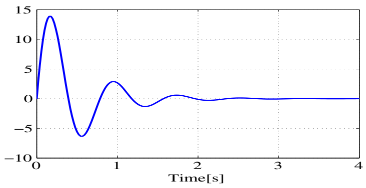

Fig. 2. Disturbance input w

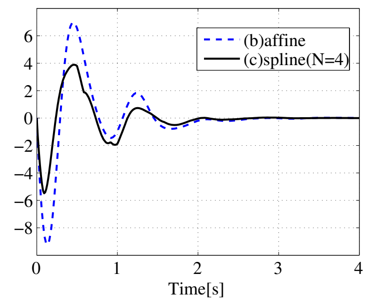

Fig. 3. Controlled output z

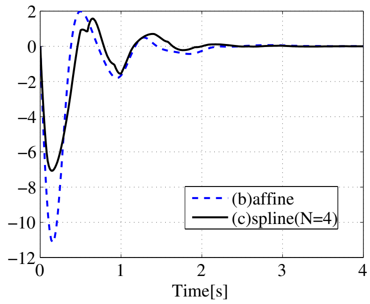

Fig. 4. Control input u

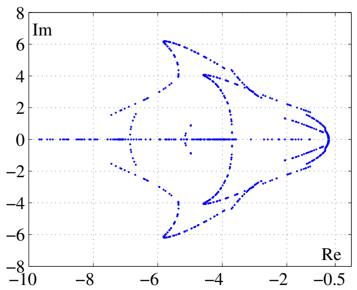

Fig. 5. Closed-loop poles ((c) $N = 4$)

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