

# COLLUSION HELPS ABATE ENVIRONMENTAL EXTERNALITIES: A DYNAMIC APPROACH

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**Abstract:** We investigate the bearings product market collusion on the abatement of polluting emissions in a Cournot oligopoly where production entails a negative environmental externality. We model the problem as a differential game and investigate the feedback solution of two alternative settings: a fully noncooperative oligopoly and a cartel maximising the discounted profits of all firms in the industry. Our analysis proves that the output reduction entailed by collusive behaviour may have a beneficial effect on steady state welfare, as a result of the balance between a higher market price and a lower amount of polluting emissions. This result opens a new perspective on the debate about the management of environmental externalities, which so far has mainly focussed on the design of Pigouvian taxation. *Copyright* © 2008 IFAC

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## 1. INTRODUCTION

The control of polluting emissions damaging the environment is a hot issue and is receiving an increasing amount of attention in the current literature in the field of environmental economics. Most of the existing contributions investigate the design of optimal Pigouvian taxation aimed at inducing firms to reduce damaging emissions, both in monopoly and oligopoly settings.<sup>1</sup> Accordingly, the established approach to this problem consists in taking the social optimum, where a benevolent planner chooses a production plan for the firms in the industry so as to maximise social welfare, as a benchmark against which the performance of the profit-seeking firms has to be assessed. This produces corrective policy measures which, ideally, should take the form of tax schemes able to reproduce the same social welfare level associated with the first best.

Another stream of literature analyses the feasibility of tradeable pollution permits, which, however, may lead to the monopolization of the industry.<sup>2</sup> As we shall see in the remainder of the paper, in industries affected by negative externalities monopolization is not as bad as it is usually thought to be.

To the best of our knowledge, the potential benefits of appropriate competition policy measures, unrelated to

taxation or pollution permits, has been disregarded so far. Clearly, competition policy are not conceived as part of the toolkit of environmental policies (and, at present, competition policy has indeed no role therein), since environmental policy does have its own set of instruments to abate polluting emissions. However, our aim is precisely to outline, in an admittedly unconventional and even provocative way, the beneficial effects that collusion or monopolization (and, as an ancillary case, horizontal merger waves) may exert in a market where the production of the final output has the undesirable property of generating negative environmental externalities. We use an established differential game approach to this issue<sup>3</sup> to prove that, under specific conditions involving the relative size of intertemporal parameters, a cartel maximising industry profits is socially preferable to a non-cooperative oligopoly following feedback Nash rules. The basic intuition behind this result is the trade-off between the welfare damage associated to any price increase (such as the one usually brought about by collusion) and the desirable reduction in the emission as well as the stock of pollution implied by any output contraction. Such a trade-off is of course absent in industries where no external effects take place: here the consequences of collusion or cartelization are surely negative and the antitrust stance against collusive behaviour is fully justified. However, current antitrust laws around the globe never take explicitly into account

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<sup>1</sup> See Bergstrom *et al.* (1987), Karp and Livernois (1992, 1994) and Benckroun and Long (1998, 2002), *inter alia*.

<sup>2</sup> To this regard, see Newbery (1990) and von der Fehr (1993), *inter alia*.

<sup>3</sup> For a technical introduction to and a thorough overview of differential games applied to environmental economics, see Dockner *et al.* (2000, ch. 12).

the aforementioned trade-off and the related (potentially beneficial) consequences of cartels on the quality of the environment.

The basic model is in section 2. The non-cooperative equilibrium and the cartel equilibrium are outlined in section 3. Section 4 contains the welfare analysis and the related implications for competition policy. Concluding remarks are in section 5.

## 2. THE SETUP

The basic structure of the model is borrowed from Benckekroun and Long (1998, 2002). Consider an oligopoly market over an infinite (continuous) time horizon,  $t \in [0, \infty)$ . Firms supply a homogeneous good, whose market demand function is

$$p(t) = a - Q(t) \quad (1)$$

at any time  $t \in [0, \infty)$ , with  $a > 0$  being a positive constant parameter measuring the reservation price and  $Q(t) = \sum_{i=1}^N q_i(t)$  being the sum of all firms' output levels. Production takes place at constant returns to scale (CRS), with a marginal cost  $c \in (0, a)$  constant and common to all firms, so that firm  $i$ 's instantaneous cost function is  $C_i(t) = cq_i(t)$ . Every firm has to pay a fixed fee  $F > 0$  at  $t = 0$ . The production of the final output goes along with a negative environmental externality taking the form of a flow of polluting emissions  $E(t) = Q(t)$ . The stock of pollution,  $S(t)$ , evolves over time according to the following dynamics:

$$\dot{S}(t) = Q(t) - \delta S(t) \quad (2)$$

where  $\delta > 0$  is the decay rate of the stock. The instantaneous external effect  $\Theta(t)$  generated by pollution is made of two components generated by the flow  $Q(t)$  and the stock  $S(t)$ , respectively:

$$\Theta(t) = \varepsilon Q(t) + \gamma \frac{S^2(t)}{2}; \quad \varepsilon, \gamma > 0. \quad (3)$$

Consumer surplus  $CS(t)$  is measured by the area below the demand function and above market price  $p(t)$ , minus the externality  $\Theta(t)$ :

$$CS(t) = \frac{Q^2(t)}{2} - \varepsilon Q(t) - \gamma \frac{S^2(t)}{2}. \quad (4)$$

It is worth noting that a contraction of output has ambiguous consequences over consumer surplus, due to the presence of a negative externality proportional to the output: on the one hand, shrinking output goes along with increasing market price, which is harmful; on the other hand, it entails reducing the environmental externality, which is desirable. The balance between these components will play a key role in the remainder of the analysis.

Social welfare, defined as the sum of industry profits and consumer surplus, writes as follows:

$$SW(t) = \sum_{i=1}^N \pi_i(t) + \frac{Q^2(t)}{2} - \varepsilon Q(t) - \gamma \frac{S^2(t)}{2} \quad (5)$$

where  $\pi_i(t) = [p(t) - c]q_i(t)$  is firm  $i$ 's instantaneous profit function.

In the remainder of the paper, we investigate two cases: (i) the non-cooperative game where firms compete à la Cournot-Nash to maximise individual profits; (ii) the cartel case where firms explicitly cooperate to maximise joint profits. In case (i), firm  $i$  chooses  $q_i(t)$  to maximise the discounted individual profit flow:

$$J_i(t) = \int_0^\infty \pi_i(t) e^{-\rho t} dt \quad (6)$$

s.t. the state dynamics (2) and the initial condition  $S(0) = S_0$ . Parameter  $\rho > 0$  represents the constant discount factor common to all firms in the industry. In case (ii), the industry chooses its aggregate production so as to maximise the discounted flow of joint profits:

$$J_C(t) = \int_0^\infty \Pi(t) e^{-\rho t} dt, \quad (7)$$

where

$$\Pi(t) = \sum_{i=1}^N \pi_i(t) \quad (8)$$

under the same constraints. Subscript  $C$  stands for *cartel*.

## 3. EQUILIBRIUM ANALYSIS

Here we characterise the feedback equilibria of the two models, starting with the non-cooperative game. The Bellman equation of firm  $i$  is:<sup>4</sup>

$$\rho V_i(S) = \max_{q_i} \left\{ \pi_i + \frac{\partial V_i(S)}{\partial S} [Q - \delta S] \right\} \quad (9)$$

where, in view of the linear-quadratic structure of the problem, we may assume  $V_i(S) = g + hS + kS^2$ ,  $g, h, k$  being the unknown coefficients. The first order condition (FOC) is:

$$a - c + h - 2q_i - \sum_{j \neq i} q_j + kS = 0. \quad (10)$$

In view of the *ex ante* symmetry across firms, we impose  $q_j = q_i = q$  and solve (10) to obtain the optimal individual output  $q^* = (a - c + h + kS) / (N + 1)$ , to be plugged into (9), simplifying the latter as follows:

$$\Phi(k) S^2 + \Psi(h, k) S + \Omega(g, h) = 0, \quad (11)$$

where  $\Phi(k)$ ,  $\Psi(h, k)$  and  $\Omega(g, h)$  must be simultaneously equal to zero w.r.t.  $g, h$  and  $k$ . This yields:

$$h = -\frac{(N^2 + 1)(a - c)k}{2N^2k - (\delta + \rho)(N + 1)^2} \quad (12)$$

and<sup>5</sup>

$$k = \frac{(N + 1)^2(2\delta + \rho)}{2N^2}. \quad (13)$$

Then, imposing  $\dot{S} = 0$ , we find the steady state level of the stock of pollution:

$$S^* = \frac{(a - c)[2\delta + \rho(N^2 + 1)]}{\delta(N + 1)[2\delta + \rho(N + 1)]}. \quad (14)$$

The resulting steady state levels of equilibrium individual output and profits are:

$$q^* = \frac{(a - c)[2\delta + \rho(N^2 + 1)]}{N(N + 1)[2\delta + \rho(N + 1)]} \quad (15)$$

<sup>4</sup> In the remainder of the paper we will omit the time argument for the sake of brevity.

<sup>5</sup> The other solution,  $h = k = 0$ , can be disregarded for obvious reasons.

$$\pi^* = 2(a - c)^2 (\delta + \rho) [2\delta + \rho (N^2 + 1)] / (N + 1)^2 [2\delta + \rho (N + 1)]^2 \quad (16)$$

while social welfare amounts to:

$$SW^* = \{ (a - c) [2\delta + \rho (N^2 + 1)] \times [(a - c) (2\delta^3 (2N + 1) + \delta^2 \rho (1 + N (N + 4)) - \gamma (2\delta + \rho (N^2 + 1))) - 2\delta^2 \varepsilon (N + 1) (2\delta + \rho (N + 1))] \} / [2\delta^2 (N + 1)^2 (2\delta + \rho (N + 1))^2] \quad (17)$$

Having characterised the feedback Cournot-Nash equilibrium, we may address the issue of the joint profit-maximising cartel, whose Bellman equation is:

$$\rho V_C(S) = \max_q \{ \Pi + \frac{\partial V_C(S)}{\partial S} [nq - \delta S] \} \quad (18)$$

where subscript  $C$  stands for *cartel* and  $V_C(S)$  is the value function in the collusive case. In the above expression, the symmetry condition  $q_i = q$  for all  $i$  has been imposed at the outset, so that the FOC is taken w.r.t. the output  $q$  of the generic cartel member:

$$N(a - c - 2Nq + h_C + k_C S) = 0, \quad (19)$$

where  $h_C$  and  $k_C$  are unknown coefficients appearing in the cartel's value function  $V_C(S) = g_C + h_C S + k_C S^2$ . Obviously, given the CRS assumption, the cartel is observationally equivalent to a pure monopoly replacing the population of  $N$  firms. This yields, at the steady state equilibrium:

$$S_C = \frac{a - c}{2\delta}; q_C = \frac{a - c}{2N}; \Pi_C = \frac{(a - c)^2}{4} \quad (20)$$

$$SW_C = (a - c) [\delta^2 (3(a - c) - 4\varepsilon) - \gamma(a - c)] / (8\delta^2) \quad (21)$$

In both cases, it can be shown that the steady state, either  $(S^*, q^*)$  or  $(S_C, q_C)$  is a saddle point. The details of the stability analysis are omitted for brevity.<sup>6</sup>

#### 4. WELFARE ASSESSMENT

The next step consist in a welfare comparison across the two regimes, in order to draw some policy implications. A preliminary observation in this respect is that, while usually we would expect the cartel to operate a contraction of industry output as compared to the non-cooperative oligopoly, (which is the standard reason why a cartel is socially undesirable and therefore prohibited by current antitrust laws around the world), here this does not hold in general, as it can be quickly ascertained on the basis of (15) and (20):

$$q^* - q_C \propto S^* - S_C \propto \rho N - \rho - 2\delta > 0 \quad (22)$$

for all  $N > 1 + 2\delta/\rho$ , with  $1 + 2\delta/\rho \geq 2$  for all  $\delta \geq \rho/2$ . We may therefore claim:

*Lemma 1.* If  $\delta \geq \rho/2$ , the cartel produces less than the non-cooperative oligopoly. Accordingly, under the same

<sup>6</sup> See Bencheckroun and Long (1998, 2002).

condition, the stock of pollution associated with collusion is lower than that associated with non-cooperative behaviour.

The result stated in Lemma 1 is intriguing, as it suggests the possibility that a cartel may indeed outperform a non-cooperative oligopoly on welfare grounds. The reason is intuitive. Any output contraction raises the equilibrium price and brings about an increase in profits as well as a decrease in the part of consumer surplus that has to do strictly with consumption, leaving any external effect aside. The balance of these two effects is always negative as the deadweight loss caused by monopolization or cartelization is not balanced by the increase in industry profits. Here, however, one has to account for the presence of a negative externality appearing in consumer surplus, which introduces a trade-off between the price effect and the external effect: if shrinking the industry output translates into a sufficiently large reduction of the negative externality, then the overall balance may in fact be positive, so as to induce one to reassess market power from a completely unusual angle.

Using (17) and (21), we can verify that  $SW_C - SW^* = 0$  in correspondence of<sup>7</sup>

$$N = 1; N = 1 + \frac{2\delta}{\rho}, \quad (23)$$

with  $SW_C > SW^*$  for all  $N > 1 + 2\delta/\rho$ , if this is an integer, or  $N$  larger than the smallest integer larger than  $1 + 2\delta/\rho$ , otherwise. Accordingly, we have

*Proposition 2.* Collusion (or monopolization) of the industry enhances social welfare as compared to the non-cooperative oligopoly for all  $N > 1 + 2\delta/\rho$ .

A straightforward ancillary result to the above Proposition is the following:

*Corollary 3.* If  $\delta \in [0, \rho/2]$ , then collusion (or monopolization) is welfare-improving as compared to the non-cooperative oligopoly for all  $N \geq 2$ .

A further possibility is to increase welfare through a reduction/increase in the number of firms (and therefore in industry output as well as the associated amount of pollution) via either horizontal mergers or a promotion of the entry process. Using (14) and (17), it can be verified that

$$\frac{\partial S^*}{\partial N} = \frac{\partial SW^*}{\partial N} = 0 \quad (24)$$

in  $N = \pm \sqrt{(2\delta + \rho)/\rho}$ . The negative root,  $N_-$ , can be disregarded for obvious reasons, while the positive one,  $N_+$ , is larger or equal to two for all  $\delta \geq 3\rho/2$ . Moreover,

$$\frac{\partial S^*}{\partial N} < 0 \text{ and } \frac{\partial SW^*}{\partial N} > 0 \quad (25)$$

for all  $N \in [1, \sqrt{(2\delta + \rho)/\rho}]$ , and conversely outside this range. Hence, leaving aside the integer problem, we may claim:

*Proposition 4.* If  $\delta \in [0, 3\rho/2]$ , then any horizontal merger is welfare-improving.

<sup>7</sup> There exist other two roots, that can be disregarded since, for  $\delta, \rho > 0$ , they are either real but lower than one or imaginary.

The proof of this claim is quickly outlined. For all  $\delta \in (0, 3\rho/2]$ , the root  $N_+ \in (1, 2]$ . This implies that, in this range of  $\delta$ ,  $\partial SW^*/\partial N < 0$  for all  $N \geq 2$ . Hence, reducing the number of firms through a horizontal merger (or a wave thereof) surely enhances steady state welfare. Of course, while this monotonicity property holds for sufficiently low values of  $\delta$ , it does not hold beyond such a range, i.e., for all  $\delta \geq 3\rho/2$ . Here, the appropriate competition policy must be designed according to the sign of  $N - N_+$ : if  $N > N_+$ , then mergers are desirable; if instead  $N < N_+$ , then a liberalization aimed at increasing market competition is to be pursued.<sup>8</sup>

## 5. CONCLUSIONS

We have addressed the issue of pollution control in a differential game where firms either compete or collude in output levels, to illustrate the so far neglected perspective whereby collusion (or horizontal mergers) may indeed contribute effectively to reduce environmental externalities. This result stems from a balance between two effects operating in opposite directions, generated by the output contraction carried out by a cartel. One is the well known (and *per se* undesirable) price increase always accompanying collusive behaviour, the other is the decrease in polluting emissions that is specific to industries like the one we have described here, and may turn out to be sufficiently strong to offset the price effect. Taking seriously the policy implications of the foregoing analysis, one should explicitly mention the role of externalities in the current antitrust rules, in order to carry out a careful assessment of the pros and cons of collusive behaviour as well as increasing market concentration through mergers in such industries.

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<sup>8</sup> This can be done by decreasing  $F$ , or eliminating it altogether. A policy of this kind has been recently adopted in Italy in the taxi market. However, the related debate focussed on the price side, without mentioning at all the external effect concerning cars' polluting emissions.

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