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Robust Controller Synthesis for the Attenuation of Non-stationary Sinusoidal Disturbances with Uncertain Frequencies ⋆

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Abstract: Attenuation of sinusoidal disturbances with uncertain and arbitrarily time-varying frequencies is considered. The disturbances are modeled as the outputs of an autonomous exogenous system, whose system matrix depends on some uncertain parameters and is yet skew-symmetric for all admissible parameter values. A procedure is then developed for the synthesis of a linear time-invariant controller that guarantees a desired level of attenuation at steady-state as well as sufficiently fast transient response in the face of all admissible parameter variations. The procedure is based on solving a convex optimization problem in which the variables are subject to a set of linear matrix inequality as well as equality constraints. The order of the controller is equal to the order of the plant plus the order of the exogenous system.

Keywords: Robust control, regulation, linear parameter-varying systems, robust linear matrix inequalities, convex optimization.

1. INTRODUCTION

Rejection of sinusoidal or periodic disturbances is a common problem in various engineering systems ranging from disk drives, Sacks et al. [1996], to CD players, Lee [1998], Dettori [2001], helicopters, Arcara et al. [2000] and steel casting, Manayathara et al. [1996]. With the disturbances generated by a known autonomous and unstable exogenous system from unknown initial conditions, it is well-established in the theory of asymptotic regulation (see Saberi et al. [2000], Byrnes et al. [1997]) when and how such a problem can be solved in a stationary setting. In this case, the solution essentially amounts to replicating in the feedback loop the dynamics of the exogenous system as required by the Internal Model Principle of Francis et al. [1974]. The classical asymptotic regulation theory, however, does not offer an immediate solution when the disturbances have a non-stationary and uncertain nature, i.e. when their frequencies or periods can change in time. Hence, part of the recent interest concerning sinusoidal/periodic disturbance rejection is on reducing the sensitivity of the design against changes in the period/frequency. This can be realized either by robust controller synthesis techniques, Lee and Chung [1998], Tsao et al. [2000], Li and Tsao [2001], Steinbuch [2002], Osburn and Franchek [2004], Kim and Tsao [2004], Steinbuch et al. [2007], Dietz et al. [2007], or by adaptive methods, Bodson and Douglas [1997], Bodson [2001], Guo and Bodson [2005], Serrani et al. [2001] as specialized to sinusoidal disturbance rejection. When the frequency is measurable or estimable online, as is the case -for instance- in systems with rotational machinery, linear parameter-varying (LPV) controller synthesis techniques can also be applied for robust and adaptive non-stationary sinusoidal disturbance attenuation, Dettori [2001], Du et al. [2003], Hüttner et al. [2005], Kulkarni et al. [2005], Gruenbacher et al. [2007]. Within the LPV control framework, it even becomes possible to systematically and simultaneously handle other performance objectives, as has been illustrated by Köroğlu and Scherer [2007].

In this paper, we are concerned with robust attenuation of sinusoidal disturbances of uncertain and time-varying frequencies by a linear time-invariant (LTI) controller. We formulate in Section 2 the sinusoidal disturbance attenuation problem more in the spirit of the classical regulation theory with the help of a neutrally-stable exo-system. The exo-system depends on uncertain and possibly time-varying parameters, which correspond to the variations in the frequencies. With inspirations from Hu et al. [2005], we adopt a generalized notion of asymptotic regulation and impose requirements on the steady-state disturbance attenuation level as well as the transient response. We first describe in Section 3 a characterization of the controllers that solve the nominal version of the problem as derived in Köroğlu and Scherer [2008]. The controllers are identified by a generic structure, which in fact originates from the solution of the exact asymptotic regulation problem (see Scherer et al. [1997], Stoorvogel et al. [2000]). We adopt the same structure when studying the robust version of the problem in Section 4, where a convex solution is provided for the problem considered in this paper. An illustrative example is also provided Section 5, after which some concluding remarks are drawn.

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2. PROBLEM FORMULATION

This paper is concerned with the attenuation of multi-sinusoidal disturbances with uncertain and possibly time-varying frequencies. These disturbances are viewed as the outputs of an autonomous system of the form

$$\dot{v} = A_{\rm e}(\delta)v;\ A_{\rm e}(\delta) = -A_{\rm e}(\delta)^T \in \mathbb{R}^{l\times l}, \qquad (1)$$

where $\delta = [\,\delta_1 \,\cdots\, \delta_r\,]^T$ represents a vector of uncertain and possibly time-varying parameters. As a simple yet sufficiently representative example, let us consider

$$A_{\rm e} = \begin{bmatrix} 0 & -\varpi(t) \\ \varpi(t) & 0 \end{bmatrix}, \varpi(t) = (1+\delta(t))\omega_0, \qquad (2)$$

where $\omega_0 \geq 0$ corresponds to a nominal frequency. With

$$\phi(t) = \int_0^t \varpi(\tau)d\tau = \omega_0 t + \omega_0 \int_0^t \delta(\tau)d\tau, \qquad (3)$$

it is straightforward to verify for this example that

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} \cos(\phi(t)) & -\sin(\phi(t)) \\ \sin(\phi(t)) & \cos(\phi(t)) \end{bmatrix} \begin{bmatrix} v_1(0) \\ v_2(0) \end{bmatrix}, \qquad (4)$$

which reveals the motivations behind viewing the systems described by (1) as the generators of non-stationary sinusoidal disturbances. Systems that generate multi-sinusoidal disturbances can be obtained -for instance- by using block-diagonal system matrices with sub-blocks of the form given in (2). The uncertain parameters in our setting basically reflect the deviations of the frequencies from their nominal values and are assumed to vary in time in a compact region $\mathcal{R} \subset \mathbb{R}^\eta$. The admissible parameter trajectories are hence identified as $\mathcal{T}_\mathcal{R} \triangleq \{\delta(\cdot) : [0,\infty) \to \mathcal{R}\}$. Note that, irrespective of the parameter trajectory, the state of the system in (1) evolves with a constant norm, i.e. $\|v(t)\|^2 \triangleq v(t)^T v(t) = \|v(0)\|, \forall t \geq 0$ (which can easily be established as $d\|v(t)\|^2/dt = v(t)^T \mathfrak{He}(A_{\rm e}(\delta(t)))v(t) = 0$, where $\mathfrak{He}\, A_{\rm e} \triangleq A_{\rm e} + A_{\rm e}^T$). This is a property which we particularly rely on in the problem formulation.

The disturbance attenuation problem is formulated for a plant with dynamics

$$G: \begin{bmatrix} \dot{x} \\ e \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_{\rm r}(\delta) & B \\ \hline C_{\rm r} & D_{\rm r}(\delta) & D_{\rm rc} \\ C & D_{\rm cr} & 0 \end{array}\right] \begin{bmatrix} x \\ v \\ u \end{bmatrix}, \qquad (5)$$

where $x(t) \in \mathbb{R}^k$ denotes the state vector, while $u(t) \in \mathbb{R}^n$ is the vector of control inputs that are to be used to regulate the outputs $e(t) \in \mathbb{R}^r$ based on the measurements $y(t) \in \mathbb{R}^m$. We assume that:

A.1 $\underbrace{\begin{bmatrix} B_{\rm r}(\delta) \\ A_{\rm e}(\delta) \end{bmatrix}}_{\tilde{B}_{\rm r}(\delta)} = \underbrace{\begin{bmatrix} B_{\rm r}^0 \\ A_{\rm e}^0 \end{bmatrix}}_{\tilde{B}_{\rm r}^0} + \underbrace{\begin{bmatrix} B_{\rm r}^1(\delta) \\ A_{\rm e}^1(\delta) \end{bmatrix}}_{\tilde{B}_{\rm r}^1(\delta)}$ and $D_{\rm r}(\delta) = D_{\rm r}^0 + D_{\rm r}^1(\delta)$, with $\tilde{B}_{\rm r}^1(\delta)$ and $D_{\rm r}^1(\delta)$ depending continuously on δ and satisfying $\tilde{B}_{\rm r}^1(0) = 0$, $D_{\rm r}^1(0) = 0$;

A.2 $(A\,|B)$ is stabilizable ($\exists F : A+BF$ is Hurwitz);

A.3 $\left(\dfrac{\tilde{A}}{\tilde{C}}\right) = \left(\begin{array}{cc} A & B_{\rm r}^0 \\ 0 & A_{\rm e}^0 \\ \hline C & D_{\rm cr} \end{array}\right)$ is detectable ($\exists \tilde{L} : \tilde{A}+\tilde{L}\tilde{C}$ is Hurwitz).

With an LTI controller of the form

$$K: \begin{bmatrix} \dot{\xi} \\ u \end{bmatrix} = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \begin{bmatrix} \xi \\ y \end{bmatrix}, \qquad (6)$$

the closed-loop dynamics are described by

$$\begin{aligned} \dot{\chi} &= \underbrace{\begin{bmatrix} A+BD_K C & BC_K \\ B_K C & A_K \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} x \\ \xi \end{bmatrix}}_{\chi} + \underbrace{\begin{bmatrix} B_{\rm r}(\delta)+BD_K D_{\rm cr} \\ B_K D_{\rm cr} \end{bmatrix}}_{\mathcal{B}_{\rm r}(\delta)} v, \\ e &= \underbrace{[C_{\rm r}+D_{\rm rc}D_K C \;\; D_{\rm rc}C_K]}_{\mathcal{C}_{\rm r}}\chi + \underbrace{[D_{\rm r}(\delta)+D_{\rm rc}D_K D_{\rm cr}]}_{\mathcal{D}_{\rm r}(\delta)} v. \end{aligned} \qquad (7)$$

We consider designing a fixed controller K such that:

C.1. (**Internal Stability**) The feedback system formed by G and K is asymptotically stable (i.e. $\mathcal{A}$ is Hurwitz);

C.2. (**Robust Generalized Asymptotic Regulation to a Level $\kappa > 0$ with a Decay Rate $\rho > 0$**) There exists a $\varphi \in \mathbb{R}_+$ such that $\|e(t)\|^2 < \varphi\|\hat{\chi}(0)\|^2 e^{-2\rho t} + \kappa^2\|v(0)\|^2, \forall t\geq 0, \forall \delta(\cdot)\in\mathcal{T}_\mathcal{R}$, where $\hat{\chi}^T \triangleq [\,\chi^T\ v^T\,]$.

Remark 1. A larger class of disturbances can be considered with exo-systems for which there exists a positive-definite matrix P such that $A_{\rm e}^T(\delta)P + PA_{\rm e}(\delta) = 0, \forall\delta \in \mathcal{R}$. Such cases can easily be subsumed to our framework through the state transformation $\nu = P^{1/2}v$, since $P^{1/2}A_{\rm e}(\delta)P^{-1/2}$ is then skew-symmetric.

3. NOMINAL GENERALIZED ASYMPTOTIC REGULATION

In this section, we describe the solution to the nominal version of the problem as derived by Köroğlu and Scherer [2008]. The key step that leads to the solution of this problem is the introduction of a state transformation of the form $\varkappa = \chi + \Psi v$, based on a matrix variable Ψ. In this fashion, we obtain an alternative description of the closed-loop dynamics as

$$\begin{aligned} \dot{\varkappa} &= \mathcal{A}\varkappa + \underbrace{(\mathcal{B}_{\rm r}(\delta) + \Psi A_{\rm e}(\delta) - \mathcal{A}\Psi)}_{\mathcal{B}_{\rm a}(\Psi,\delta)} v, \\ e &= \mathcal{C}_{\rm r}\varkappa + \underbrace{(\mathcal{D}_{\rm r}(\delta) - \mathcal{C}_{\rm r}\Psi)}_{\mathcal{D}_{\rm a}(\Psi,\delta)} v. \end{aligned} \qquad (8)$$

If the closed-loop is guaranteed to be stable and if Ψ is chosen to render $\mathcal{B}_{\rm a}(\Psi,0) = 0$, the state evolution of (8) becomes autonomous as well as stable, and the asymptotic influence of v on the output to be regulated is then determined only by the feed-through term $\mathcal{D}_{\rm a}(\Psi,0)$. With a partition of the form $\Psi^T = [\,\Pi^T\ \Phi^T\,]$ compatible with $\mathcal{A}$, we can easily obtain

$$\mathcal{B}_{\rm a}(\Psi,\delta) = \begin{bmatrix} B_{\rm r}(\delta) + \Pi A_{\rm e}(\delta) - A\Pi - B\Gamma \\ \Phi A_{\rm e}(\delta) - A_K\Phi - B_K(C\Pi - D_{\rm cr}) \end{bmatrix}, \qquad (9)$$

$$\mathcal{D}_{\rm a}(\Psi,\delta) = D_{\rm r}(\delta) - C_{\rm r}\Pi - D_{\rm rc}\Gamma, \qquad (10)$$

after introducing Γ as

$$\Gamma \triangleq C_K\Phi + D_K(C\Pi - D_{\rm cr}). \qquad (11)$$

Further investigations based on these formulas lead us to the following result from Köroğlu and Scherer [2008]:

Theorem 2. Consider the generalized asymptotic regulation problem within the setting described in Section 2 for $\kappa = \kappa_0$ and assume that $\mathcal{R} = \{0\}$. There exists a controller that solves this problem for a sufficiently small ρ, if and only if there exist $\Pi \in \mathbb{R}^{k\times l}$ and $\Gamma \in \mathbb{R}^{n\times l}$ that satisfy

$$B_{\rm r}^0 + \Pi A_{\rm e}^0 - A\Pi - B\Gamma = 0, \qquad (12)$$

$$\begin{bmatrix} \kappa_0 I & \left(D_{\rm r}^0 - C_{\rm r}\Pi - D_{\rm rc}\Gamma\right)^T \\ D_{\rm r}^0 - C_{\rm r}\Pi - D_{\rm rc}\Gamma & \kappa_0 I \end{bmatrix} \succ 0. \qquad (13)$$

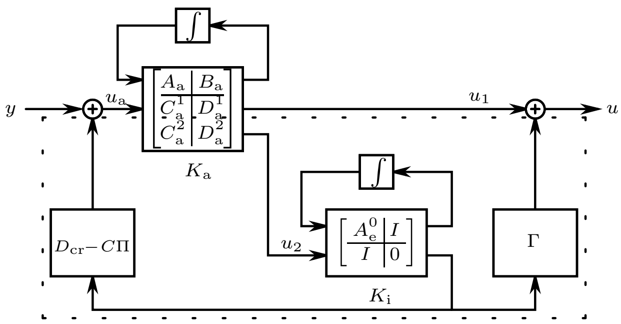

Fig. 1. A structured controller for asymptotic regulation.

Any controller that solves this problem admits a realization of the form

$$K:\begin{bmatrix}\dot{\xi}\\ u\end{bmatrix}=\left[\begin{array}{cc|c} A_{\rm e}^0+D_{\rm a}^2(D_{\rm cr}-C\Pi) & C_{\rm a}^2 & D_{\rm a}^2\\ B_{\rm a}(D_{\rm cr}-C\Pi) & A_{\rm a} & B_{\rm a}\\ \hline \Gamma+D_{\rm a}^1(D_{\rm cr}-C\Pi) & C_{\rm a}^1 & D_{\rm a}^1\end{array}\right]\begin{bmatrix}\xi\\ y\end{bmatrix},\quad (14)$$

and can be implemented as in Figure 1, where $K_{\rm i}$ is a controller that replicates the dynamics of the nominal exosystem and $K_{\rm a}$ is an accompanying controller with which the feedback-loop formed by G and K is stabilized.

4. ROBUST GENERALIZED ASYMPTOTIC REGULATION

In this section, we derive a solution to the robust generalized asymptotic regulation problem as formulated in Section 2. Our approach is based on adopting the structure of the controller that emerges naturally from the nominal version of the problem and deriving a convex optimization problem to determine suitable values for the involved variables (i.e. $\Pi,\Gamma,A_{\rm a},B_{\rm a},C_{\rm a},D_{\rm a}$). The optimization problem is derived from the parameter-dependent version of the matrix inequality condition adapted from Hu et al. [2005]:

Lemma 3. There exists a solution to the generalized asymptotic regulation problem as formulated in Section 2, if there exist $\mathcal{X}=\mathcal{X}^T$, $R=R^T$ and Q for which

$$\hat{\mathcal{L}}_{\rm s}(\delta)=\mathfrak{He}\underbrace{\begin{bmatrix}\mathcal{X} & Q\\ Q^T & R\end{bmatrix}}_{\hat{\mathcal{X}}}\underbrace{\begin{bmatrix}\mathcal{A}+\rho I & \mathcal{B}_{\rm r}(\delta)\\ 0 & A_{\rm e}(\delta)+\rho I\end{bmatrix}}_{\hat{\mathcal{A}}(\delta)}\preccurlyeq 0,\forall\delta\in\mathcal{R},(15)$$

$$\hat{\mathcal{L}}_{\rm r}(\delta)=\begin{bmatrix}\mathcal{X} & Q & \mathcal{C}_{\rm r}^T\\ Q^T & \kappa I+R & \mathcal{D}_{\rm r}(\delta)^T\\ \mathcal{C}_{\rm r} & \mathcal{D}_{\rm r}(\delta) & \kappa I\end{bmatrix}\succ 0,\forall\delta\in\mathcal{R}.(16)$$

The generalized asymptotic regulation condition is then satisfied with any φ for which $\hat{\mathcal{X}}\preccurlyeq\kappa^{-1}\varphi I$.

Proof. With $\mathcal{V}(\hat{\chi})\triangleq\hat{\chi}^T\hat{\mathcal{X}}\hat{\chi}$, we observe from (15) that

$$d\left(e^{2\rho t}\mathcal{V}(\hat{\chi}(t))\right)/dt=\hat{\chi}(t)^T\hat{\mathcal{L}}_{\rm s}(\delta(t))\hat{\chi}(t)\le 0,$$

and hence $\mathcal{V}(\hat{\chi}(t))\le\mathcal{V}(\hat{\chi}(0))e^{-2\rho t}\le\kappa^{-1}\varphi\|\hat{\chi}(0)\|^2e^{-2\rho t}$, $\forall t\ge 0,\forall\delta(\cdot)\in\mathcal{T}_{\mathcal{R}}$, along the trajectories of the closed-loop system. Recalling the positive definiteness of $\mathcal{X}$ (i.e $\exists\epsilon>0:\epsilon I\preccurlyeq\mathcal{X}$), we infer for $v(0)=0$ that $\epsilon\|x(t)\|^2\le\chi(t)^T\mathcal{X}\chi(t)\le\chi(0)^T\mathcal{X}\chi(0)e^{-2\rho t}\le\kappa^{-1}\varphi\|\chi(0)\|^2e^{-2\rho t}$, which establishes the internal stability. For arbitrary $v(0)$, we have from (16) with $\psi(t)^T=\begin{bmatrix}\hat{\chi}(t)^T & -\kappa^{-1}e(t)^T\end{bmatrix}$ that

$$\psi(t)^T\hat{\mathcal{L}}_{\rm r}(\delta(t))\psi(t)=\mathcal{V}(\hat{\chi}(t))+\kappa\|v(t)\|^2-\kappa^{-1}\|e(t)\|^2>0,$$

which guarantees generalized asymptotic regulation. □

By suitable congruence transformations, we can equivalently express the conditions of Lemma 3 as the existence of an invertible matrix $\mathcal{Y}$, an arbitrary matrix $\Psi=\mathcal{X}^{-1}Q$ and symmetric matrices $\mathcal{X}$, $P=R-\Psi^T\mathcal{X}\Psi$, for which

$$\mathfrak{He}\begin{bmatrix}\mathcal{Y}^T\mathcal{X}(\mathcal{A}+\rho I)\mathcal{Y} & \mathcal{Y}^T\mathcal{X}\mathcal{B}_{\rm a}(\Psi,\delta)\\ 0 & P\left(A_{\rm e}(\delta)+\rho I\right)\end{bmatrix}\preccurlyeq 0,\forall\delta\in\mathcal{R},\quad(17)$$

$$\begin{bmatrix}\mathcal{Y}^T\mathcal{X}\mathcal{Y} & 0 & \mathcal{Y}^T\mathcal{C}_{\rm r}^T\\ 0 & \kappa I+P & \mathcal{D}_{\rm a}(\Psi,\delta)^T\\ \mathcal{C}_{\rm r}\mathcal{Y} & \mathcal{D}_{\rm a}(\Psi,\delta) & \kappa I\end{bmatrix}\succ 0,\forall\delta\in\mathcal{R}.\quad(18)$$

With the candidate controllers parameterized as in (14), the challenge thus becomes finding suitable $\mathcal{Y}$ and Ψ with which conditions (17) and (18) are rendered tractable.

Since other options typically hinder tractability of (17) due to the term $\mathcal{Y}^T\mathcal{X}\mathcal{B}_{\rm a}(\Psi,\delta)$, the choice of Ψ would naturally be of the form $\Psi=\begin{bmatrix}\Pi^T & \Phi^T\end{bmatrix}^T$ with $\Phi=I_{(k+l)\times k}=I_{k\times(k+l)}^T\triangleq\begin{bmatrix}I_k & 0_{k\times l}\end{bmatrix}$, as inherited from the solution of the nominal problem. Recall that, when considering nominal generalized regulation, the matrix variables Π and Γ are required to satisfy (12) as well as (13). Although this need not be the case for the robust version of the problem, it turns out that respecting (12) opens the path towards a nice solution, whereas satisfaction of (13) is not required unless there are concerns about the behavior of the system when $\delta(t)=0,\forall t\ge 0$. For the convenience of our presentation, we try to explicate this by introducing an extended plant whose dynamics are described by

$$\begin{aligned}\dot{\hat{x}}&=\underbrace{\begin{bmatrix}A & B\Gamma\\ 0 & A_{\rm e}^0\end{bmatrix}}_{\hat{A}(\Gamma)}\underbrace{\begin{bmatrix}x\\ \xi_{\rm i}\end{bmatrix}}_{\hat{x}}+\underbrace{\begin{bmatrix}B_{\rm r}(\delta)\\ 0\end{bmatrix}}_{\hat{B}_{\rm r}(\delta)}v+\underbrace{\begin{bmatrix}B & 0\\ 0 & I\end{bmatrix}}_{\hat{B}}\underbrace{\begin{bmatrix}u_1\\ u_2\end{bmatrix}}_{\tilde{u}},\\ e&=\underbrace{\begin{bmatrix}C_{\rm r} & D_{\rm rc}\Gamma\end{bmatrix}}_{\hat{C}_{\rm r}(\Gamma)}\hat{x}+D_{\rm r}(\delta)v+\underbrace{\begin{bmatrix}D_{\rm rc} & 0\end{bmatrix}}_{\tilde{D}_{\rm rc}}\tilde{u},\\ u_{\rm a}&=\underbrace{\begin{bmatrix}C & D_{\rm cr}-C\Pi\end{bmatrix}}_{\hat{C}(\Pi)}\hat{x}+D_{\rm cr}v.\end{aligned}\quad(19)$$

We can observe with the help of Figure 1 that our problem can be reformulated as the design of $K_{\rm a}$ for this extended plant. Assuming (12) is satisfied, the state-transformation

$$\hat{x}=\tilde{T}(\tilde{x}-\tilde{E}^Tv);\ \tilde{T}=\begin{bmatrix}\tilde{\Pi}\\ \tilde{E}\end{bmatrix}\triangleq\begin{bmatrix}I & \Pi\\ 0 & I\end{bmatrix},\quad(20)$$

leads us to an alternative realization of the form

$$\begin{aligned}\dot{\tilde{x}}&=\underbrace{\begin{bmatrix}A & B_{\rm r}^0\\ 0 & A_{\rm e}^0\end{bmatrix}}_{\tilde{A}}\tilde{x}+\underbrace{\begin{bmatrix}B_{\rm r}^1(\delta)\\ A_{\rm e}^1(\delta)\end{bmatrix}}_{\tilde{B}_{\rm r}^1(\delta)}v+\underbrace{\begin{bmatrix}B & -\Pi\\ 0 & I\end{bmatrix}}_{\tilde{B}(\Pi)}\tilde{u},\\ e&=\underbrace{\begin{bmatrix}C_{\rm r} & C_{\rm r}\Pi+D_{\rm rc}\Gamma\end{bmatrix}}_{\tilde{C}_{\rm r}(\Pi,\Gamma)}\tilde{x}+\underbrace{(D_{\rm r}(\delta)-C_{\rm r}\Pi-D_{\rm rc}\Gamma)}_{\Lambda(\Pi,\Gamma,\delta)}v+\tilde{D}_{\rm rc}\tilde{u},\\ u_{\rm a}&=\underbrace{\begin{bmatrix}C & D_{\rm cr}\end{bmatrix}}_{\tilde{C}}\tilde{x}.\end{aligned}\quad(21)$$

We note in this alternative representation that the measured output is not influenced by the infinite-energy disturbance, which means that the controller will not feed back any disturbance input. In the nominal generalized regulation problem, the state is not influenced either and hence a solution can easily found by constructing a (-for instance- observer-based) controller $K_{\rm a}$ that stabilizes the extended plant. The robust version of the problem longs for a more delicate design procedure for K_{a}, by which e is to be desensitized to the variations in δ as well as to the resulting disturbances v.

The way to a tractable design procedure is paved by a convenient choice of $\mathcal{Y}$ followed by a transformation of the accompanying controller parameters. This transformation is obtained by modifying the one proposed before by Scherer et al. [1997] for integrating the exact regulation objective into LMI-based controller synthesis. The modification is done in a way to reduce computational complexity and allow for direct construction of reduced order K_{a}'s (of order k rather than $k+l$). With the Lyapunov matrix $\mathcal{X}$ and its inverse partitioned compatibly with the extended plant and the accompanying controller as

$$\mathcal{X} = \begin{bmatrix} \hat{X} & \hat{U} \\ \hat{U}^T & H \end{bmatrix} = \begin{bmatrix} \tilde{T}^{-T}\tilde{X}\tilde{T}^{-1} & \tilde{T}^{-T}\tilde{U} \\ \tilde{U}^T\tilde{T}^{-1} & S^{-1}+\tilde{U}^T\tilde{X}^{-1}\tilde{U} \end{bmatrix}, \tag{22}$$

$$\mathcal{X}^{-1} = \begin{bmatrix} \hat{Y} & \hat{V} \\ \hat{V}^T & S \end{bmatrix} = \begin{bmatrix} \tilde{T}\tilde{Y}\tilde{T}^T & \tilde{T}\tilde{V} \\ \tilde{V}^T\tilde{T}^T & H^{-1}+\tilde{V}^T\tilde{Y}^{-1}\tilde{V} \end{bmatrix}, \tag{23}$$

the variable transformation that we propose relies on choosing

$$\mathcal{Y} = \left[\begin{array}{c|c} \hat{Y}I_{(k+l)\times k} & \tilde{T} \\ \hline \hat{V}^T I_{(k+l)\times k} & 0 \end{array}\right] = \left[\begin{array}{c|cc} Y & I & \Pi \\ Z & 0 & I \\ \hline V^T & 0 & 0 \end{array}\right]. \tag{24}$$

We can obtain by exploiting $\mathcal{X}\mathcal{X}^{-1} = I$ that

$$\mathcal{Y}^T\mathcal{X} = \begin{bmatrix} I_{k\times(k+l)} & 0 \\ \tilde{X}\tilde{T}^{-1} & \tilde{U} \end{bmatrix}, \; \mathcal{Y}^T\mathcal{X}\mathcal{Y} = \begin{bmatrix} Y & \tilde{\Pi} \\ \tilde{\Pi}^T & \tilde{X} \end{bmatrix}. \tag{25}$$

Further manipulations that also recall $\Psi = \begin{bmatrix} \Pi^T & I & 0 \end{bmatrix}^T$ with (12) lead to

$$\left[\begin{array}{c|c} \mathcal{Y}^T\mathcal{X}\mathcal{A}\mathcal{Y} & \mathcal{Y}^T\mathcal{X}\mathcal{B}_{\mathrm{a}}(\Psi,\delta) \\ \hline \mathcal{C}_{\mathrm{r}}\mathcal{Y} & \mathcal{D}_{\mathrm{a}}(\Psi,\delta) \end{array}\right] = \left[\begin{array}{cc|c} AY+B\Gamma Z & \tilde{\Pi}\tilde{A} & \tilde{\Pi}\tilde{B}_{\mathrm{r}}^1(\delta) \\ \tilde{X}\tilde{A}\tilde{Y}\tilde{\Pi}^T & \tilde{X}\tilde{A} & \tilde{X}\tilde{B}_{\mathrm{r}}^1(\delta) \\ \hline C_{\mathrm{r}}Y+D_{\mathrm{rc}}\Gamma Z & \tilde{C}_{\mathrm{r}}(\Pi,\Gamma) & \Lambda(\Pi,\Gamma,\delta) \end{array}\right]$$
$$+ \left[\begin{array}{cc} 0 & \tilde{\Pi}\tilde{B}(\Pi) \\ \tilde{U} & \tilde{X}\tilde{B}(\Pi) \\ \hline 0 & \tilde{D}_{\mathrm{rc}} \end{array}\right] \begin{bmatrix} A_{\mathrm{a}} & B_{\mathrm{a}} \\ C_{\mathrm{a}} & D_{\mathrm{a}} \end{bmatrix} \left[\begin{array}{cc|c} \tilde{V}^T\tilde{\Pi}^T & 0 & 0 \\ \tilde{C}\tilde{Y}\tilde{\Pi}^T & \tilde{C} & 0 \end{array}\right]. \tag{26}$$

The crucial observation at this point is that the matrices in (17) and (18) can be rendered affine in a set of free variables, simply by absorbing the bilinear terms $\tilde{X}\tilde{A}\tilde{Y}\tilde{\Pi}^T$, $B\Gamma Z$ and $D_{\mathrm{rc}}\Gamma Z$ into the transformed controller parameters as

$$\begin{bmatrix} \tilde{J} & \tilde{M} \\ N & D \end{bmatrix} = \begin{bmatrix} \tilde{U} & \tilde{X}\tilde{B}(\Pi) \\ 0 & I_{n\times(n+l)} \end{bmatrix} \begin{bmatrix} A_{\mathrm{a}} & B_{\mathrm{a}} \\ \tilde{C}_{\mathrm{a}} & \tilde{D}_{\mathrm{a}} \end{bmatrix} \begin{bmatrix} \tilde{V}^T\tilde{\Pi}^T & 0 \\ \tilde{C}\tilde{Y}\tilde{\Pi}^T & I \end{bmatrix} + \begin{bmatrix} \tilde{X}\tilde{A}\tilde{Y}\tilde{\Pi}^T & 0 \\ \Gamma Z & 0 \end{bmatrix}. \tag{27}$$

This leads us to the following solution for the robust disturbance attenuation problem considered in this paper:

Theorem 4. There exists an LTI controller that solves the robust generalized asymptotic regulation problem formulated in Section 2, if there exist $Y = Y^T \in \mathbb{R}^{k\times k}$, $\tilde{X} = \tilde{X}^T \in \mathbb{R}^{(k+l)\times(k+l)}$, $P = P^T \in \mathbb{R}^{l\times l}$, $\Pi \in \mathbb{R}^{k\times l}$, $\Gamma \in \mathbb{R}^{n\times l}$, $\tilde{J} \in \mathbb{R}^{(k+l)\times k}$, $\tilde{M} \in \mathbb{R}^{(k+l)\times m}$, $N \in \mathbb{R}^{n\times k}$ and $D \in \mathbb{R}^{n\times m}$ such that

$$\tilde{\Pi}\tilde{B}_{\mathrm{r}}^0 - A\Pi - B\Gamma = 0, \tag{28}$$

and for all $\delta \in \mathcal{R}$, we have

$$\mathfrak{He}\begin{bmatrix} (A+\rho I)Y+BN & \tilde{\Pi}\tilde{A}+\rho\tilde{\Pi}+BD\tilde{C} & \tilde{\Pi}\tilde{B}_{\mathrm{r}}^1(\delta) \\ \tilde{J}+\rho\tilde{\Pi}^T & \tilde{X}(\tilde{A}+\rho I)+\tilde{M}\tilde{C} & \tilde{X}\tilde{B}_{\mathrm{r}}^1(\delta) \\ 0 & 0 & P(A_{\mathrm{e}}(\delta)+\rho I) \end{bmatrix} \preccurlyeq 0, \tag{29}$$

$$\mathfrak{He}\begin{bmatrix} 0.5Y & 0.5\tilde{\Pi} & 0 & 0 \\ 0.5\tilde{\Pi}^T & 0.5\tilde{X} & 0 & 0 \\ 0 & 0 & 0.5(\kappa I+P) & 0 \\ C_{\mathrm{r}}Y+D_{\mathrm{rc}}N & \tilde{C}_{\mathrm{r}}(\Pi,\Gamma)+D_{\mathrm{rc}}D\tilde{C} & \Lambda(\Pi,\Gamma,\delta) & 0.5\kappa I \end{bmatrix} \succ 0, \tag{30}$$

where $\tilde{\Pi} \triangleq [\, I \; \Pi \,]$, $\tilde{C}_{\mathrm{r}}(\Pi,\Gamma) \triangleq [\, C_{\mathrm{r}} \; C_{\mathrm{r}}\Pi + D_{\mathrm{rc}}\Gamma \,]$ and $\Lambda(\Pi,\Gamma,\delta) \triangleq D_{\mathrm{r}}(\delta) - C_{\mathrm{r}}\Pi - D_{\mathrm{rc}}\Gamma$. With $I_{k\times(k+l)} = I_{(k+l)\times k}^T \triangleq [\, I_k \; 0_{k\times l} \,]$, $\tilde{E} \triangleq [\, 0_{l\times k} \; I_l \,]$ and

$$W = Y - \tilde{\Pi}\tilde{X}^{-1}\tilde{\Pi}^T, \tag{31}$$

$$\tilde{Y} = \tilde{X}^{-1} + I_{(k+l)\times k} W I_{k\times(k+l)}, \tag{32}$$

$$Z = \tilde{E}\tilde{X}^{-1}\tilde{\Pi}^T, \tag{33}$$

a controller that solves the problem can then be constructed as

$$\left[\begin{array}{c|c} A_{\mathrm{a}} & B_{\mathrm{a}} \\ \hline C_{\mathrm{a}}^1 & D_{\mathrm{a}}^1 \\ C_{\mathrm{a}}^2 & D_{\mathrm{a}}^2 \end{array}\right] = \left[\begin{array}{cc} \tilde{\Pi} & -B \\ \hline 0 & I \\ \tilde{E} & 0 \end{array}\right] \begin{bmatrix} \tilde{X}^{-1}\tilde{J} - \tilde{A}\tilde{Y}\tilde{\Pi}^T & \tilde{X}^{-1}\tilde{M} \\ N - \Gamma Z & D \end{bmatrix} \left[\begin{array}{c|c} -W^{-1} & 0 \\ \tilde{C}\tilde{Y}\tilde{\Pi}^T W^{-1} & I \end{array}\right]. \tag{34}$$

Proof. In order to finalize the proof, we need to construct a positive-definite $\mathcal{X}$ and an invertible $\mathcal{Y}$, using the matrix variables that satisfy (28)-(30). For this, it suffices to find $\tilde{Y}, \tilde{U}, \tilde{V}$ and S with which $\tilde{X}\tilde{Y} + \tilde{U}\tilde{V} = I$, $\tilde{X}\tilde{V} + \tilde{U}S = 0$ and $\mathcal{Y}$ is invertible. With W and $\tilde{Y}$ being as introduced in (31) and (32), these conditions are satisfied by $\tilde{U} = \tilde{X}I_{(k+l)\times l}U$, $\tilde{V} = -I_{(k+l)\times l}WU^{-T}$ and $S = U^{-1}WU^{-T}$, where U is an arbitrary matrix except for being invertible. Note that in this case $\mathcal{Y}^{-1}$ can be obtained explicitly as

$$\mathcal{Y}^{-1} = \begin{bmatrix} 0 & -W^{-1}U \\ I & \tilde{Y}\tilde{\Pi}^T W^{-1}U \end{bmatrix}.$$

A realization of K_{a} can then be obtained from the inverse of the transformation in (27), which reads for $U = I$ as given by (34). Different choices for U in fact lead to different realizations of the same controller. □

Remark 5. For a multi-objective version of the problem in which n_{r} outputs e_i are to be regulated down to levels κ_i, Theorem 4 admits an immediate extension in which we have (28) and (29) accompanied by n_{r} constraints of the form (30) expressed in terms of the system matrices and κ_i corresponding to the relevant outputs to be regulated.

Remark 6. We observe from (30) that C_{r} and D_{rc} can be allowed to be parameter-dependent as well. Nevertheless, conditions (29) and (30) read as infinitely many matrix inequalities and hence are not per se tractable. There are a variety of relaxations that can be applied to replace them with finitely many conditions (see Scherer [2006] and the references therein). As far as the sinusoidal disturbance attenuation problem is concerned, the uncertainty can simply be described in the form of affine parameter dependence (i.e. $\tilde{B}_{\mathrm{r}}^1(\delta) = \tilde{B}_{\mathrm{r}}^1(\delta \otimes I)$, $D_{\mathrm{r}}^1(\delta) = D_{\mathrm{r}}^1(\delta \otimes I)$). If, moreover, $\mathcal{R}$ is assumed to be a polytotic region identified by a set of vertices $\{\delta^1, \ldots, \delta^q\}$ (i.e. $\mathcal{R} = \{\sum_{j=1}^q \alpha_j\delta^j : \sum_{j=1}^q \alpha_j = 1, \alpha_j \geq 0\}$), conditions (29) and (30) are satisfied throughout $\mathcal{R}$ if and only if they are satisfied at for each $\delta^j, j = 1, \ldots, q$. In the case of quadratic parameter dependence, one can employ the multi-convexity argument of Gahinet et al. [1996]) to arrive at finitely many LMIs.

Remark 7. Thanks to adopting the controller structure derived from the nominal generalized regulation problem, the controller synthesis procedure described in Theorem 4 offers much flexibility, which can be used to guarantee additional objectives. In particular, one can simply add condition (13) to the set of constraints to guarantee generalized asymptotic regulation of level of κ_0 when $\delta = 0$, of course at the possible cost of increasing the level of robust generalized asymptotic regulation. In fact, one can choose κ as parameter-dependent and guarantee robust generalized asymptotic regulation with a changing level as $\kappa(\delta(t))$. It is intuitive to think of a (graceful) regulation level degradation profile by considering a non-negative and non-decreasing function $\mu(\|\delta\|)$ with $\mu(0) = 0$ (e.g. $\mu(\|\delta\|) = \eta\|\delta\|$ or $\mu(\|\delta\|) = \eta\|\delta\|^2$, where $\eta > 0$) and try to achieve generalized asymptotic regulation down to levels $\kappa(\delta(t)) = \kappa_0 + \mu(\|\delta(t)\|)$.

Remark 8. As is visible from C.2, it is essential to make sure that φ is not too large so that the output does not exhibit undesirable peaking during the transient period. As can be observed from Lemma 3, we can realize this by adding an extra constraint of the form

$$\hat{\mathcal{X}} = [\,I\ \Psi\,]^T \mathcal{X} [\,I\ \Psi\,] + [\,0\ I\,]^T P [\,0\ I\,] \prec \begin{bmatrix} \sigma \tilde{T}^{-T}\tilde{T}^{-1} & 0 \\ 0 & \sigma I \end{bmatrix},$$

and bounding the value of σ from above. With $\mathcal{X}$ constructed as in the proof of Theorem 4 being

$$\mathcal{X} = \left[\tilde{T}^{-1}\ \tilde{I}\right]^T \tilde{X} \left[\tilde{T}^{-1}\ \tilde{I}\right] + [\,0\ I_k\,]^T W^{-1} [\,0\ I_k\,],$$

where $\tilde{I} \triangleq I_{(k+l)\times k}$, this condition can be expressed (after an application of the Schur-complement lemma and a congruence transformation) as

$$\begin{bmatrix} \sigma I - \tilde{X} & \tilde{X} & 0 & 0 \\ \tilde{X} & \sigma I - \tilde{X} - \tilde{E}^T P \tilde{E} & I_{(k+l)\times k} & 0 \\ 0 & I_{k\times(k+l)} & Y & \tilde{\Pi} \\ 0 & 0 & \tilde{\Pi}^T & \tilde{X} \end{bmatrix} \succ 0. \qquad (35)$$

In fact, for a given level of κ that is known to be achievable, one can even minimize σ subject to (28), (29), (30) and (35) to obtain a controller with which $\|e(t)\|^2 \leq \sigma(\|\tilde{T}^{-1}\hat{x}(0)\|^2 + \|\xi_a(0)\|^2 + \|v(0)\|^2)e^{-2\rho t} + \kappa^2\|v(t)\|^2$.

5. ILLUSTRATIVE EXAMPLE

In this section, we consider a mass-spring damper system whose dynamics are described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \hline e \\ y \end{bmatrix} = \left[\begin{array}{cccc|cc} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b}{m_1} & \frac{k_1}{m_1} & \frac{b}{m_1} & 0 & \frac{1}{m_1} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{b}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{b}{m_2} & \frac{k_2}{m_2} & -\frac{1}{m_2} \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \hline d \\ u \end{bmatrix},$$

where the disturbance affecting the system is assumed to be of the form $d(t) = \sin(\omega_0(1+\delta(t)))$, $\delta(t) \in [-\beta, \beta]$, as is the first state of (2) for the initial condition $v(0) = [\,0\ -1\,]^T$. For a set of parameters given by $m_1 = 2, m_2 = 0.5, k_1 = 100, k_2 = 150, b = 10, \omega_0 = 4$, we synthesized three different controllers using the procedure of Theorem 4 for three different β values $\beta_1 = 0.18$, $\beta_2 = 0.1$ and $\beta_3 = 0.01$. Thanks to the affine parameter dependence, condition (29) is satisfied for all $\delta \in \mathcal{R}$ if and only if it is satisfied for $\delta = -\beta$ and $\delta = \beta$. We added (35) as well as some additional constraints that bound $\mathcal{X}$ and the norms of the transformed controller parameters from above. By favor of the Yalmip (Löfberg [2004]) interface, we solved the optimization problems in MATLAB® with SeDuMi (Sturm [1999]) and obtained the the minimum κ levels for the three different parameter ranges respectively as $\kappa_1 = 0.4962$, $\kappa_2 = 0.2821$ and $\kappa_3 = 0.0367$. A fourth controller is designed to guarantee generalized asymptotic regulation of level $\kappa = 0.001$ for the zero parameter trajectory $\delta = 0$ as well as best possible level of generalized asymptotic regulation for any parameter variation within the range identified by β_2. This is realized as described in Remark 7 and the minimum achievable κ is obtained as a significantly large value given by 103.5. The transfer functions of the designed controllers are given by

$$K_1 = \frac{28.64(s-2.16\cdot 10^8)(s-45.1)(s+8.1)(s+2.94)(s^2+34.64s+808.7)}{(s+4152)(s+1525)(s+25.49)(s+2.843)(s^2-140.2s+1.173\cdot 10^5)},$$

$$K_2 = \frac{28.59(s-9.36\cdot 10^7)(s-41.56)(s+6.58)(s+4.01)(s^2+32.64s+699.9)}{(s+18.33)(s+3.585)(s^2-228.2s+6.42\cdot 10^4)(s^2+2893s+4.87\cdot 10^6)},$$

$$K_3 = \frac{-16.02(s+9.96\cdot 10^7)(s-35.7)(s^2+9.89s+35.42)(s^2+28.74s+480.5)}{(s^2+13.38s+60.1)(s^2-87.72s+2.04\cdot 10^4)(s^2+934.4s+7.94\cdot 10^6)},$$

$$K_4 = \frac{81.73(s+5.96\cdot 10^6)(s+48.15)(s^2+9.66s+32.42)(s^2+16.11s+592.7)}{(s+2.78\cdot 10^4)(s+39.99)(s^2+15.01s+67.14)(s^2+264.4s+6.045\cdot 10^4)}.$$

For a disturbance input as in Figure 2-a, the outputs obtained with these controllers (starting from zero initial plant and controller states) are presented in Figure 2-b. Note from Figure 2-a that the uncertain parameter first switches between 0.3 and −0.3 with increasing frequency, then remains zero for a while and finally increases from −0.3 up to 0.3 with a constant rate. Although the parameter range is much larger than the ranges considered to design the controllers, the regulation performance does not degrade undesirably even when the parameter is close to its extreme values. In fact the controller K_2, which is

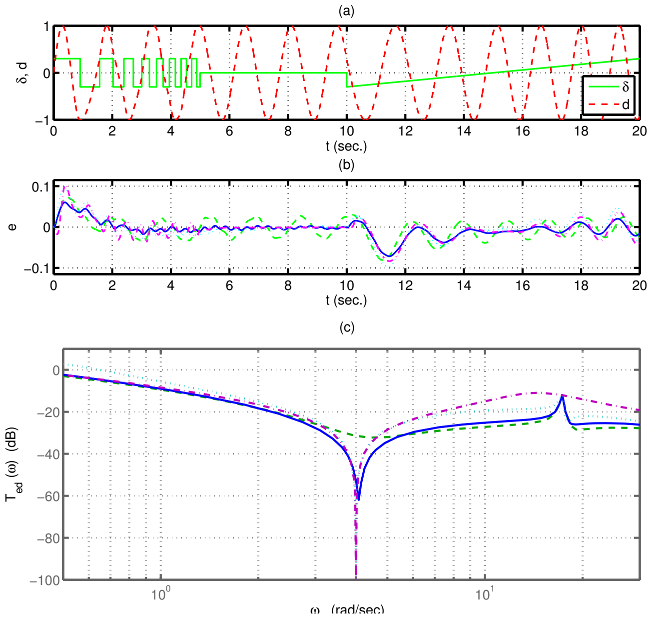

Fig. 2. Example designs and simulations: (a) Parameter trajectory and the disturbance; (b) Outputs, (c) Bode magnitude plots of T_{ed} for various designs (K_1:dashed, K_2:solid, K_3:dotted, K_4:dash-dotted)

designed for the range $\beta_2 = 0.1$ usually outperforms all of the controllers including K_1, which is designed for a larger range. We should, however, note that this observation is restricted to the considered particular parameter trajectory and cannot be expected generically. In fact, even K_4 exhibits quite acceptable performance for this parameter trajectory, although it is designed to guarantee (almost) exact nominal regulation and has poor guarantees against parameter variation. The performance of the controllers for constant parameter trajectories can be analyzed based on the Bode magnitude plots of the transfer function from the disturbance to the error signal, which are displayed in Figure 2-c. All of the four controllers in fact guarantee more than -20dB of attenuation within the frequency range $[2.8, 5.2]$rad/sec. We should however note that no theoretical guarantees can be inferred from the Bode plots when the parameter is varying arbitrarily.

6. CONCLUDING REMARKS

We have developed a novel procedure for the synthesis of an LTI controller that guarantees robust attenuation of non-stationary sinusoidal disturbances. The proposed method provides performance guarantees in contrast to ad-hoc loop-shaping procedures. The key step that leads to a convex solution for our particular problem in fact paves the way to the solutions of a variety of optimal controller synthesis problems under generalized asymptotic regulation constraints. It is open whether and how the performance can be improved by taking into account any available bounds on the rates-of-variation of the parameters.

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