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Multiobjective optimization for automatic tuning of robust Model Based Predictive Controllers

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Abstract: In this paper a general procedure for tuning multivariable model predictive controllers (MPC) with constraints is presented. It has been applied to tune the control system of an activated sludge process control in a wastewater treatment plant. Control system parameters are obtained by solving a mixed sensitivity optimization problem, defined in terms of the H_∞ norms of different weighted closed loop transfer functions matrices of the system, and a set of constraints, some of them expressed using the l_1 norm. The use of multiple linearized models for the control allows for the specification of many robust performance criteria. The mathematical optimization for tuning all controller parameters is tackled in two iterative steps due to the existence of integer and real numbers. First, integer parameters are obtained using a special type of random search, and secondly a sequential programming method is used to tune the real parameters.

1. Introduction

Model Based Predictive Control (MPC or MBPC) has become the leading form of advanced multivariable control in the process industries. The popularity of MPC is due to the successful results, the natural way of incorporating constraints, and its simplicity for operators.

MPC controllers have been tuned traditionally through a number of different parameters including prediction horizon, number of computer input moves, input and output weights in the objective function, and, in some cases, artificially imposed input/output constraints. The tuning task can be particularly difficult if the system is multivariable, since the whole set represents a formidable array of possible tuning combinations and also because many of these parameters have overlapping effects on the closed-loop performance and robustness. In these cases the advantages of using automatic MPC tuning methods is clear.

In the literature, many works deal with the automatic tuning of MPC, but most of them without considering horizons in a systematic way or leaving apart some particular aspects of the problem ([2],[1],[6]). In [4] we have already proposed a methodology for the on-line automatic tuning of the whole set of parameters of linear Model Based Predictive Control Systems, and it was carried out by minimizing dynamical indexes as performance measures. An important drawback of this work is that within the optimization procedure dynamical simulations have to be carried, making the procedure extremely slow.

Frequency domain methods for tuning linear optimal controllers have been studied since the beginning of 1980's (see Doyle and Francis [3] for a review) and they are a good alternative to speed up MPC automatic tuning procedures. In [12] there is an example of how they can be employed using nominal linearized models.

At the view of previous works, in this paper we propose a new approach for the optimal automatic tuning of MPC that uses multiple models and it is based on the frequency domain robust control and optimization theory [9]. The most relevant aspects of the proposal are:

- It is based on the resolution of a mixed sensitivity optimization problem defined in terms of the H_∞ norms of different weighted closed loop transfer functions of the system and a set of controllability and operation constraints, expressed by means of the l_1 norm. Multiple models have been considered to give robustness to the obtained MPC.
- The optimal tuning searches for linear MPC parameters by solving a MINLP/DAE optimization problem.
- The use of the proposed automatic tuning approach within an Integrated Design framework is straightforward.

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- The approach has been validated on a simulated example based on a real wastewater treatment plant. Real scenarios have been considered in the simulated model by means of real data records of the main disturbances to make a more realistic analysis of the results.

The paper is organized as follows. First, the method for automatic tuning of the MPC is posed and explained in detail. Second, the activated sludge process model, selected for validation, is described. Third, the control problem and the application of the tuning method are stated. Then, some results are presented, to end up with some conclusions.

2. MPC formulation

The MPC considered is based on a linear state space model of the plant and calculates manipulated variables by solving the following on-line constrained optimization problem subject to constraints on inputs, predicted outputs and changes in manipulated variables.

$$\min_{\Delta u} V(k) = \sum_{i=Hw}^{Hp} W_y \cdot (\hat{y}(k+i \mid k) - r(k+i \mid k))^2 + \\ + \sum_{i=0}^{Hc-1} W_u \cdot (\Delta \hat{u}(k+i \mid k))^2 \tag{1}$$

where k denotes the current sampling point, $\hat{y}(k+i|k)$ is the predicted output vector at time $k+i$, depending of measurements up to time k, $r(k+i|k)$ is the reference trajectory, $\Delta\hat{u}$ are the changes in the manipulated variables, H_p is the upper prediction horizon, H_w is the lower prediction horizon, H_c is the control horizon, W_u is a vector representing the weights of the change of manipulated variables and W_y is a vector representing the weights of the errors of set-points tracking.

The MPC prediction model used in this paper is a linear discrete state space model of the plant obtained by linearizing the model equations [8].

When the MPC controller is linear and unconstrained, it can be represented by the block diagram of figure 1, i.e.

$$u = K_1(r - y) + K_3 d \tag{2}$$

where K_i are the transfer functions between the control signal and the different inputs (r,y,d), and they depend on the control system tuning parameters (W_u, H_p, H_c). Consequently, the closed loop response can be obtained from

$$y = \frac{GK_1}{1+GK_1} r + \frac{1}{1+GK_1} \tilde{d} \tag{3}$$

where $\tilde{d} = (GK_3 + G_d)\, d$ (4)

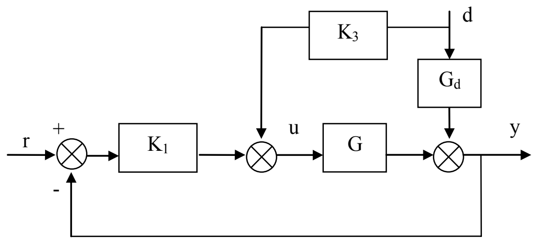

Fig. 1: Equivalent closed loop system

In order to define the automatic tuning problem, we define the sensitivity function S' between the load disturbances (d) and the outputs (y) and M' the Control Sensitivity transfer function defined between the load disturbances (d) and the control signals (u) when the reference is zero. Their calculation is straightforward applying block algebra to diagram of figure 1:

$$S'(s) = \frac{y(s)}{d(s)} = \frac{K_3 G + G_d}{1+GK_1} \quad M'(s) = \frac{u(s)}{d(s)} = \frac{K_3 - K_1 G_d}{1+GK_1} \tag{5}$$

3. Automatic tuning of MPC

3.1. Mixed sensitivity optimization problem

The problem of finding an optimal MPC is stated as a mixed sensitivity optimization problem that takes into account both disturbance rejection and control effort objectives in the same tuning function. The problem definition is then

$$\min_{K_1,K_3} \|N\|_\infty = \max_w |N(jw)| \quad \text{where} \quad N = \begin{pmatrix} W_p S' \\ W_{esf} s \cdot M' \end{pmatrix} \tag{6}$$

subject to the set of constraints explained below. K_1 and K_3 are the MPC control compensators (see block diagram of Figure 1) which depend on the tuning parameter vector defined by $c = (H_p, H_c, W_u)$. W_p and W_{esf} are suitable weights for optimization. Note that control efforts rather than magnitudes of control are included in the objective function by considering the derivative of the transfer function M'.

3.2. Performance constraints

In order to ensure that disturbances are properly rejected we impose

$$\|W_p \cdot S'\|_\infty < 1 \tag{7}$$

W_p is selected for the specification of load disturbances rejection, what means that its inverse must be smaller in magnitude than the inverse of disturbances spectrums.

3.3. Limits on control and output variables

The maximum value of the control (u_{max}) and the output variable (y_{max}) for the worst case of disturbances can be constrained to be less than certain limits by means of its l_1 norm and the following conditions:

$$\|M'\|_1 < u_{\max} \quad \|S'\|_1 < y_{\max} \tag{8}$$

3.4. Multiobjective optimization approach

The optimization problems for optimal automatic tuning can be defined as multiobjective optimization problems by defining the following objectives:

$$f_1 = \|N\|_\infty \,;\; f_2 = \|M'\|_1 \,;\; f_3 = \|S'\|_1 \tag{9}$$

with the respective goals f_1^*, f_2^*, f_3^*. In order to keep satisfying constraints (8) when the solutions do not get the objectives exactly, goals are chosen in the following way:

$$f_2^* < u_{\max} \,;\; f_3^* < y_{\max} \tag{10}$$

3.5. Multiple models for robustness

The statement of the problem presented can be modified to include not only the nominal model but also linearized models around a set of working points. For instance, to obtain robust performance in the face of non linearities, the constraint (7) can be rewritten to in the following way:

$$\|W_p \cdot S_i'\|_\infty < 1 \qquad i = 1,\ldots,\mathrm{N} \tag{11}$$

where S_i' are the sensitivity functions obtained with those linearized models, being N the number of multiple models considered.

3.6. Algorithm description and implementation

The main problem when solving this optimization problem is that involves real and integer variables (control and prediction horizons). In this work we propose a two iterative steps algorithm that combines a random search based on the Solis method [11] for tuning the horizons, and the classical goal attainment multiobjective method for tuning weights W_u., implemented in MATLAB function *fgoalattain*. Similar two steps approaches are presented in [5].

The random search basically generates new horizons by adding and subtracting random integers to the current point, and selects that with the lower cost. The gaussians variance to obtain those new points decreases with algorithm iterations. The multiobjective algorithm for the real part is stated as a sequential quadratic programming problem that minimizes parameter γ, that represents the deviation of objectives (f_i) from goals (f_i^*)

$$\min_{x,\gamma\in\mathbb{R}} \gamma \quad \text{s.t.} \;\; f_i(c) - w_i\gamma \le f_i^* \tag{12}$$

where w_i are the weigths for every objective. In this work the values of these weights are such that the importance of all objectives is the same.

The controller implementation is based on the MPC Toolbox of MATLAB and some modifications of Maciejowski [8].

4. Activated sludge process and Model Based Predictive Controller

4.1. Plant description

The plant layout is represented in Figure 2, consisting of one aerobic tank and one secondary settler [13]. The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor to maintain there an adequate concentration of microorganisms.

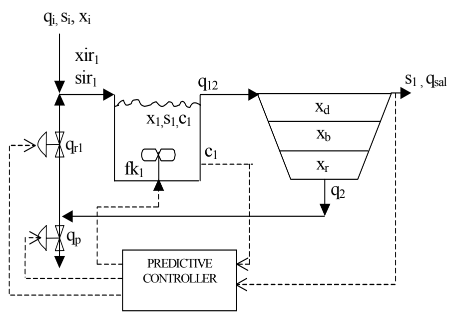

Fig. 2: Plant and controller layout

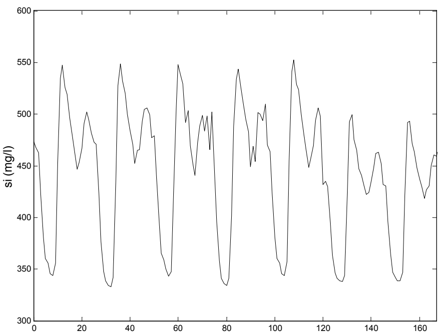

Fig. 3: Substrate disturbances at the influent (s_i)

The whole set of variables is presented also in Figure 2. Generically, "x" is used for the biomass concentrations (mg/l), "s" for the organic substrate concentrations (mg/l), "c" for the oxygen concentrations (mg/l) and "q" for flow rates (m^3/h). The complete set of differential equations and model parameters are given in [10].

4.2. Control problem

The control of this process aims to keep the substrate at the output (s_1) below a legal value despite the large variations of the flow rate and the substrate concentration in the incoming water (q_i and s_i), which are the input disturbances and one of the main problems when trying to control the plant properly. Another control objective is to keep dissolved oxygen concentration (c_1) around 2 mg/l, concentration that is necessary for the proper working of activated sludge process.

The set of disturbances used in dynamic simulations (Figure 3) has been determined by COST 624 program and its benchmark.

The general structure of a multivariable controller applied to the activated sludge process can be seen in figure 4. Three manipulated variables are considered: recycling flow (qr_1), purge flow (q_p) and aeration factor (fk_1); and three outputs: substrate (s_1), biomass (x_1) and dissolved oxygen (c_1) in the reactor. Here the biomass is only a constrained variable for a good performance of the process and it is not controlled. In this work we will focus on substrate control, although the methodology proposed is general and could be also extended to oxygen control.

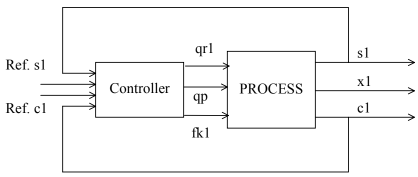

Fig. 4: General controller structure

5. Tuning Results

The controller considered is a linear MPC with constraints applied to the nonlinear plant model, with sample period of T=0.5 hours, suitable for representing the process dynamics. Disturbances s_i and q_i are assumed to be measured and scaled to make methodology improvement clearer. Biomass concentration x_1 is only a constrained variable. The selected plant is fixed with dimensions V_1=7668 m^3 (reactor volume) and A=2970.88 m^2 (settler area) and a steady state point defined by s_1=58.445 and qr_1=220.

5.1. H_∞ mixed sensitivity problem with multiple models

In this point some results are shown when several linear models are considered in the MPC tuning procedure in order to give some robustness to operating variations. Weights W_p and W_{esf} are kept constant.

The first case of study of MPC tuning considering multiple models consists of including performance constraints for two new models obtained changing the nominal operation point 20 mg/l around s_1=58.445 mg/l. The optimal MPC obtained produces better disturbance rejection than the controller obtained considering only one model for those plants working even on the edge of the region (s_1+20, s_1-20 mg/l). Simulations and numerical results are shown in figure 5 and Table I, for the case of the working point s_1-20 (which is the worst case)

TABLE I
RESULTS FOR MULTIPLE MODELS CHANGING S_1

	Single model	Multiple models
Operating point	**s_1-20 mg/l**	**s_1-20 mg/l**
W_u	0.008	0.003
H_p	7	10
H_c	2	5
Max(qr1_linear)	3033.6	3683
Max(s1_linear)	48.5	44.21
$\|N\|_\infty$	2.48	3.02
$\|M'\|_1$	3284	5258
$\|W_p S'\|_\infty$	1.84	0.9899
Computational time (min)	2.21	2.89
Operating point	**s_1-10 mg/l**	**s_1-10 mg/l**
Max(qr_1)	1454.1	1511.3
Max(s_1)	54.95	53.21

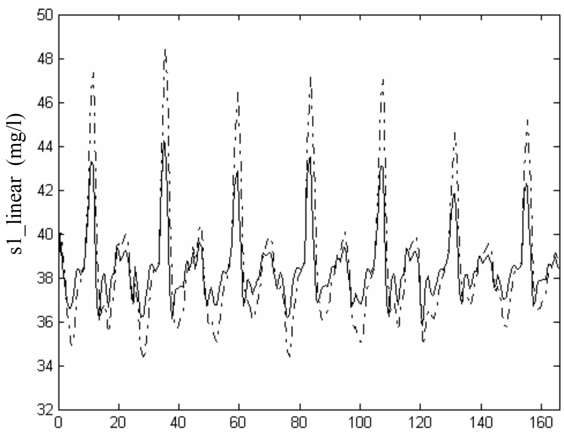

Fig. 5: Substrate (s1_linear) for plant and MPC tuned with multiple models (solid line) or single model (dashed dotted line) for the linearized system.

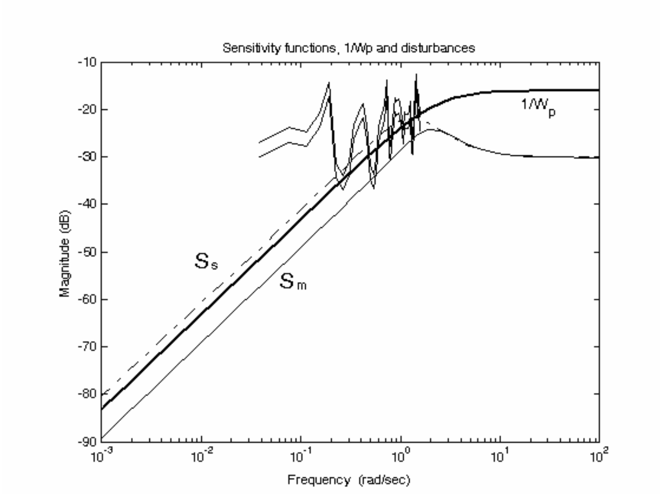

Fig. 6: Sensitivity function for MPC tuned with multiple models (solid line, S_m) and single model (dash dotted line, S_s), together with weight W_p^{-1} and the inverse spectrum of influent disturbances

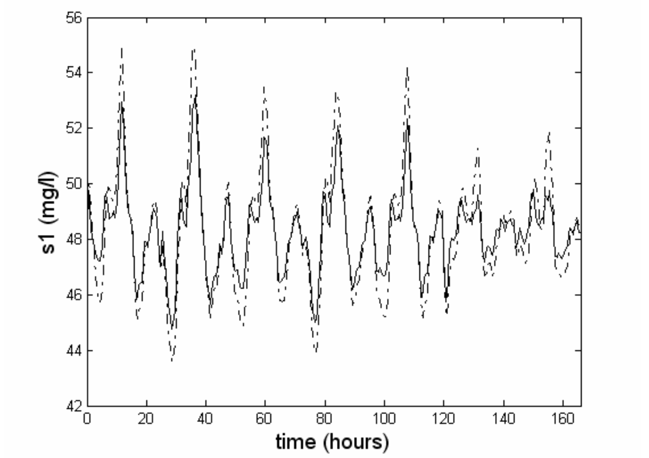

Fig. 7: Substrate responses (s1) for plant and MPC tuned with multiple models (solid line) or single model (dashed dotted line) for the non linear system.

In Fig. 6 a comparison of the sensitivity functions for both closed loop systems is shown, and it is clear that only the MPC tuned with multiple models satisfies the constraint imposed by W_p^{-1}. Finally, responses simulating directly the non linear closed loop systems around the point (s_1-10 mg/l) have been performed (see Fig. 7).

The second case consists of considering multiple models obtained changing the nominal operation point for the plant influent (s_i±120 mg/l, q_i±230 m^3/h). The optimal MPC obtained considering multiple models produces also better disturbance rejection even for worst case plants in the region (s_i+120 mg/l, q_i+230 m^3/h). For this working point numerical results are shown in Table II.

TABLE II
RESULTS FOR MULTIPLE MODELS CHANGING S_I,Q_I

	Single model	Multiple models
Operating point	**s_i+120, q_i+230**	**s_i+120, q_i+230**
W_u	0.008	0.0042
H_p	7	9
H_c	2	3
Max(qr1_linear)	1976.9	2040.8
Max(s1_linear)	66.629	63.279
$\|N\|_\infty$	1.8593	1.9626
$\|M'\|_1$	3293.2	4272
$\|W_pS'\|_\infty$	1.3547	0.999
Operating point	**s_i+60, q_i+115**	**s_i+60, q_i+115**
Max(qr1)	1242.2	1225.3
Max(s1)	64.01	63.02

TABLE III
RESULTS FOR MULTIPLE MODELS CHANGING V_1, A

	Single model	Multiple models
Operating point	**V_1-1100 A-1100**	**V_1-1100 A-1100**
W_u	0.008	0.003
H_p	7	7
H_c	2	2
Max(qr1_linear)	2949.6	3331.6
Max(s1_linear)	69.41	63.679
$\|N\|_\infty$	4.09	4.0068
$\|M'\|_1$	6403.9	8448.2
$\|W_pS'\|_\infty$	1.917	0.96753
Operating point	**V_1-550, A-550**	**V_1-550, A-550**
Max(qr_1)	1302.7	1314.3
Max(s_1)	64.866	63.94

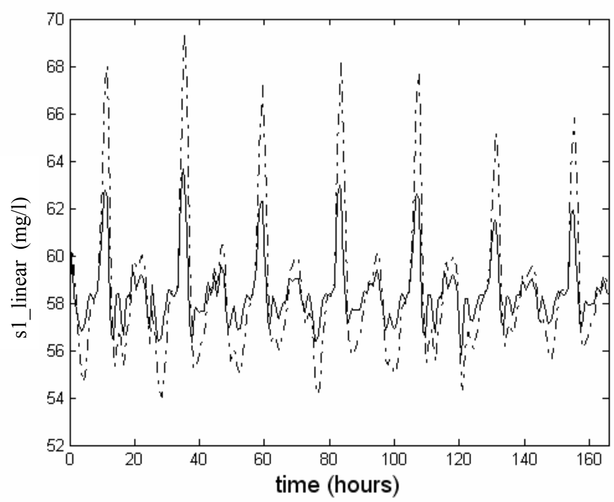

Fig. 8: Substrate responses (s1_linear) for plant and MPC tuned with multiple models (solid line) or single model (dashed dotted line) for the linearized system

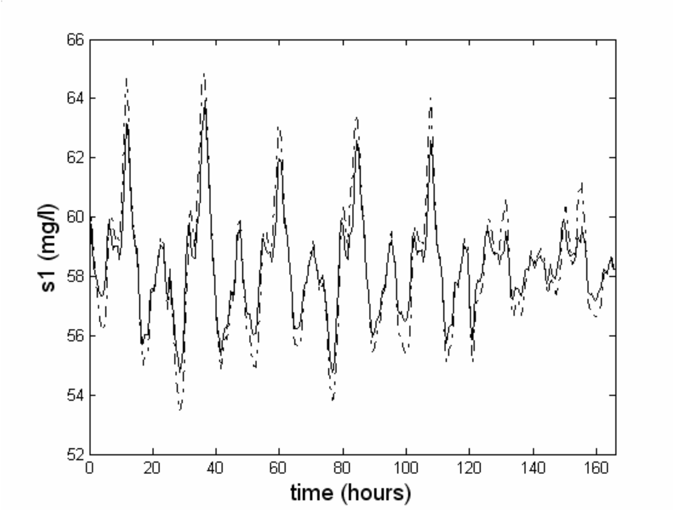

Fig. 9: Substrate responses (s1) for plant and MPC tuned with multiple models (solid line) or single model (dashed dotted line) for the non linear system

In the last case the multiple models have been obtained changing the plant dimensions (V_1±1100 m^3, A±1100 m^2) and the results are similar to the previous cases. This case is particularly interesting because this tuning procedure is intended to be included in an Integrated Design framework in which the plant will change. Numerical results are shown in Table III and substrate evolution for linearized and non linear systems in figures 8 and 9 respectively.

6. Conclusions

In this work a method for tuning robust model predictive controllers has been developed, based on some frequency domain performance indexes. This method has been tested in MPC applied to the activated sludge process, and the closed loop responses for substrate concentration in the reactor show that obtained controllers are properly tuned, taking into account the large magnitude of influent disturbances.

The methodology proposed here is a general one, and any other performance criteria can be considered. The use of multiple linear models also allows for the specification of many robust performance criteria and robust stability conditions.

Finally it is important to show that the developed method is particularly suitable for its inclusion in the resolution of the Integrated Design optimization problem, which determines the optimum controller and the optimum plant at the same time.

Acknowledgments

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