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An Adaptive Statistical Approach To Flutter Detection

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Abstract: Flutter is a critical instability phenomenon for aircrafts. In previous investigations, the authors have proposed several online statistical subspace-based algorithms for flutter monitoring. Each algorithm monitors some stability criterion (damping, flutter margin...) w.r.t. a fixed reference flight point using the online CUSUM test. The drawback of this technique is that the flutter detection corresponds to a light trend of the criterion toward instability and thus the estimated flutter airspeed is conservative. In this paper, a new moving reference version is proposed which intends to give a better estimation of the flutter airspeed. Application on simulation data shows the relevance of the new algorithm. *Copyright © 2008 IFAC.*

Keywords: flutter monitoring, subspace-based algorithms, damping coefficient, CUSUM tests, flight dynamics.

1. INTRODUCTION

The validation of new aircraft prototypes requires flight flutter testing to design flutter free boundaries of the flight envelope. This process remains very expensive in time and money in spite of the research progress (Wright [1992]) and the flutter is still reported on many recent accidents as the crash cause (Gero [1999]). Indeed, the flutter is a complex aeroelastic instability phenomenon that results from unfavorable coupling between aerodynamical and structural forces. First investigations were conducted by Lanchester [1916], Von Baumhauer and Koning [1923] to understand the flutter onset mechanism and important practical solutions were provided by Keldysh [1938]. The main analysis tool is to monitor the aeroelastic modes for changing aerodynamic conditions (airspeed, altitude...) under in-flight perturbations (noise, turbulence...) and artificial excitations (control surface pulses, thrusters...), see Kehoe [1995] for details. Modal identification from flight data has been widely used to estimate stability criteria (damping coefficient, flutter margin...) and track modal changes toward instability. The approach consists in solving either in time (Basseville et al. [2001]) or frequency (Guillaume et al. [2003]) domain the modal space equations for increasing system orders and then selecting the physical modes by means of stabilization diagrams. The high computational cost of running identification at each flight point limits the online application of this identification approach.

The use of a detection algorithm to monitor the trend of a stability criterion rather than estimating its value appears to be a promising solution for online flutter monitoring. In Mevel et al. [2005], a fast statistical detection algorithm builds on a residual associated with subspace-based identification and the online CUSUM test. This algorithm can detect the flutter when the damping coefficient decreases below some critical value. In this approach, the subspace-based residual is firstly calibrated on data at a certain flight point and then the CUSUM test runs sample-wise for new data. The test checks the orthogonality of the newly collected covariance data Hankel matrix to a left kernel computed at the reference structure. The approach was extended to monitor other stability criteria: flutter margin in Zouari et al. [2006], pairs of time-varying frequencies in Basseville et al. [2006], and modeshapes correlation in Zouari et al. [2007].

Besides time reduction, this detection approach is more robust to the bias/variance tradeoff than identification. Moreover, experiments showed good performances in detecting the deviation of the system w.r.t. the reference toward instability. However, the reference corresponds in practice to normal flight conditions far enough from flutter where significant amount of sensor data can be recorded to perform modal identification. It results that the estimation of the detection of flutter is conservative, namely alarms are raised too early w.r.t. the true flutter airspeed. Typically, 15% safety margin is considered in airworthiness regulations between flight envelope and flutter. Thus conservative flutter estimation restricts the flight domain expansion of the aircraft. A solution for reducing conservatism in flutter prediction is proposed in Lind and Brenner [1998, 2000] in the framework of robust stability analysis with a combined model and data-based approach called flutterometer.

This paper introduces a new version of the detection algorithm above. This algorithm considers a moving window update of the reference left kernel during online test. Updating the reference can be an interesting solution for less conservative flutter detection because it adjusts the test to neglect small trends to instability and react to important criterion changes near the critical zone.

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The paper is organized as follows. Section 2 details the subspace-based residual approach with moving reference. The online flutter monitoring algorithm using the CUSUM test is presented in section 3. In section 4, the aircraft wing model is described and results on simulation are reported. Finally, concluding remarks are drawn in section 5.

2. SUBSPACE-BASED RESIDUAL FOR MODAL MONITORING WITH MOVING REFERENCE

Given sensor data Y of an aircraft, the vibration-based structural monitoring problem is studied as monitoring the eigenstructure of the state transition matrix F of a linear dynamic system (Ewins [2000]):

$$\begin{cases} X_{k+1} = F\ X_k + V_{k+1} \\ Y_k = H\ X_k \end{cases} \tag{1}$$

namely the roots (λ, ϕ_λ) of :

$$\det(F - \lambda I) = 0, \quad (F - \lambda I)\ \phi_\lambda = 0 \tag{2}$$

The frequency f and damping coefficient d are recovered from eigenvalue λ through:

$$f = \alpha/2\pi\tau, \quad d = |\beta|/\sqrt{\alpha^2+\beta^2} \\ \text{where} \quad \alpha = |\arctan \Im(\lambda)/\Re(\lambda)|\ , \ \beta = \ln|\lambda| \tag{3}$$

and τ is the sampling frequency. Let the $(\lambda, \varphi_\lambda)$'s be stacked into: $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\Phi \end{pmatrix}$, where Λ is the vector whose elements are the eigenvalues λ, Φ is the matrix whose columns are the mode-shapes $\varphi_\lambda \triangleq H\phi_\lambda$'s, and vec is the column stacking operator. It is assumed that a reference parameter θ_0 is available, identified on data recorded on a reference flight point, using output-only covariance-driven subspace-based identification algorithm. It consists in computing the SVD of the empirical Hankel matrix $\widehat{\mathcal{H}}^0_{p+1,q}$ ($p+1$ block rows and q block columns) filled with covariances. Given a new data sample, the detection problem is to decide whether it is still well described by θ_0 or not.

Based on the subspace interpretation of the SVD, the modal signature θ_0 can be characterized by:

$$S^T(\theta_0)\ \widehat{\mathcal{H}}^0_{p+1,q} = 0 \tag{4}$$

The left kernel $S(\theta_0)$ is an orthonormal matrix S such that:

$$S^T \mathcal{O}_{p+1}(\theta_0) = 0 \tag{5}$$

where $\mathcal{O}_{p+1}(\theta_0)$ is the observability matrix computed in modal basis from θ_0:

$$\mathcal{O}_{p+1}(\theta_0) = \begin{pmatrix} \Phi_0 \\ \Phi_0\Delta_0 \\ \vdots \\ \Phi_0\Delta_0^p \end{pmatrix} \tag{6}$$

with $\Delta_0 = diag(\Lambda_0)$. Matrix S depends implicitly on θ_0 and is not unique, but can be treated as a function $S(\theta_0)$.

In order to decide whether θ is still well described by θ_0 or not, the residual corresponding to (4) is defined in Basseville et al. [2000] as :

$$\zeta_n(\theta_0) \triangleq \sqrt{n}\ \text{vec}\left(S(\theta_0)^T \widehat{\mathcal{H}}^n_{p+1,q}\right) \tag{7}$$

where $\widehat{\mathcal{H}}^n_{p+1,q}$ filled with covariances of new data from the (possibly unstable) system that writes:

$$\widehat{\mathcal{H}}^n_{p+1,q} \simeq 1/n \sum_{i=q}^{n-p} \mathcal{Y}^+_{i,p+1} \mathcal{Y}^{-T}_{i,q} \tag{8}$$

where: $\mathcal{Y}^+_{k,p+1} \triangleq \begin{pmatrix} Y_k \\ \vdots \\ Y_{k+p} \end{pmatrix}, \ \mathcal{Y}^-_{k,q} \triangleq \begin{pmatrix} Y_k \\ \vdots \\ Y_{k-q+1} \end{pmatrix}$.

Testing if $\theta = \theta_0$ holds true or not – or equivalently deciding that residual ζ_n is *significantly* different from zero – can be achieved with a statistical local approach by assuming close hypotheses:

$$\mathbf{H}_0 :\ \theta = \theta_0 \quad \text{and} \quad \mathbf{H}_1 :\ \theta = \theta_0 + \Upsilon/\sqrt{n} \tag{9}$$

where vector Υ is unknown, but fixed.

Monitoring the sensitivity of the residual to modal changes relative to uncertainties and noise requires the definition of the mean deviation (Jacobian) and covariance matrices:

$$\mathcal{J}_n(\theta_0, \theta) \triangleq -1/\sqrt{n}\ \ \partial/\partial\theta\ \mathbf{E}_\theta\ \zeta_n(\theta)\ |_{\theta=\theta_0} \tag{10}$$

$$\Sigma_n(\theta_0, \theta) \triangleq \mathbf{E}_\theta(\zeta_n\ \zeta_n^T) \tag{11}$$

where $\mathbf{E}_\theta$ is the expectation when the actual system parameter is θ. The Jacobian and covariance matrices should be updated during flutter detection because flight data have time-varying statistics (Brenner [2003]).

Provided that $\Sigma_n(\theta_0, \theta)$ is positive definite, and for all Υ, the residual ζ_n in (7) is asymptotically Gaussian distributed when assuming θ as in (9):

$$\Sigma_n^{-1/2}(\theta_0, \theta)\,(\zeta_n(\theta_0) - \mathcal{J}_n(\theta_0, \theta)\Upsilon) \xrightarrow[n\to\infty]{} \mathcal{N}(0, I) \tag{12}$$

Thus a deviation $\Upsilon \neq 0$ in the system parameter θ is reflected into a change in the mean value of the residual ζ_n.

It may be numerically preferable to have identity covariances for all values θ, and thus handle the *scale-normalized* residual :

$$\overline{\zeta}_n(\theta_0) \triangleq \mathcal{J}_n(\theta_0, \theta)^T\ \Sigma_n^{-1}(\theta_0, \theta)\ \zeta_n(\theta_0) \tag{13}$$

From (12), $\overline{\zeta}_n(\theta_0)$ is asymptotically Gaussian:

$$\left(\overline{\zeta}_n(\theta_0) - \overline{\Sigma}_n(\theta_0, \theta)^{1/2}\Upsilon\right) \xrightarrow[n\to\infty]{} \mathcal{N}(0, I) \tag{14}$$

where $\overline{\Sigma}_n(\theta_0, \theta) \triangleq \mathcal{J}_n(\theta_0, \theta)^T \Sigma_n^{-1}(\theta_0, \theta) \mathcal{J}_n(\theta_0, \theta)$.

For an online detection algorithm, a data-driven computation for the residual is preferable. This results from (8). Assuming $n > p+q$, $\overline{\zeta}_n(\theta_0)$ writes as the *cumulative sum*:

$$\overline{\zeta}_n(\theta_0) \simeq \sum_{k=q}^{n-p} Z_k(\theta_0)/\sqrt{n} \quad \text{where} \tag{15}$$

$$Z_k(\theta_0) \triangleq \mathcal{J}_k(\theta_0, \theta)^T \Sigma_k^{-1}(\theta_0, \theta) \text{vec}(S(\theta_0)^T \mathcal{Y}^+_{k,p+1} \mathcal{Y}^{-T}_{k,q}) \tag{16}$$

From (14) and (15), $\sum_{k=q}^{n-p} Z_k(\theta_0)/\sqrt{n}$ is asymptotically Gaussian distributed, with *zero mean* under $\mathbf{H}_0$ and $\overline{\Sigma}_n(\theta_0, \theta)^{1/2}\Upsilon$ under $\mathbf{H}_1$. For n large enough and $k = 1, ..., n$, the sample-based residual $Z_k(\theta_0)$ is also asymptotically Gaussian distributed with *zero mean* under the reference, and the $Z_k(\theta_0)$'s are *independent* (Benveniste et al. [1990]). Furthermore, a change in θ is reflected into a change in the mean vector ν of $Z_k(\theta_0)$.

As discussed in the introduction, it is more appropriate to handle important modal changes in θ due to flutter instability by updating with new data the left kernel matrix $S(\theta_0)$ that characterizes the fixed reference θ_0. This matrix is empirically computed from a Hankel matrix built with L samples using (4) and the recursive residual is computed after a lag of t samples. For a sample $n > L+t$, the adaptive left kernel matrix denoted by $\widehat{S}_n$ solves:

$$\widehat{S}_n^T \sum_{k=n+q-L-t}^{n-p-t} \mathcal{Y}_{k,p+1}^{+} \mathcal{Y}_{k,q}^{-T} = 0 \tag{17}$$

The Jacobian and covariance are now independant of θ_0 and denoted by $\mathcal{J}_n(\theta)$ and $\Sigma_n(\theta)$. The residual defined in (16) then writes:

$$Z_n(\theta) \triangleq \mathcal{J}_n(\theta)^T \Sigma_n^{-1}(\theta) \text{vec}(\widehat{S}_n^T \mathcal{Y}_{n,p+1}^{+} \mathcal{Y}_{n,q}^{-T}) \tag{18}$$

This adaptive residual compares new sample data to a left kernel matrix built on past data with a constant lag t. Theoretically, considering an adaptive left kernel is incompatible with the asymptotic Gaussian distribution of $\overline{\zeta}_n$ in (14). But this assumption can hold true when assuming that this left kernel sustains small changes along a large data set. By this way, $Z_n(\theta)$ keeps the same statistical properties and the change in its mean vector ν indicates a change in θ w.r.t. its past estimate on the sequence $(Y_{n-t-L}, ..., Y_{n-t})$. Monitoring $Z_n(\theta)$ ensures an *instantaneous* tracking of modal changes adapted to the flutter context.

3. ONLINE FLUTTER MONITORING ALGORITHM

3.1 Damping monitoring

The flutter monitoring problem is addressed as testing for each mode the decrease of the damping coefficient below some critical value. The hypotheses test writes:

$$\widetilde{\mathbf{H}}_0^n : d \geq d_0^n + \varepsilon/2 \text{ and } \widetilde{\mathbf{H}}_1^n : d < d_0^n - \varepsilon/2 \tag{19}$$

where d_0^n is the reference damping value for the sequence $(Y_{n-t-L}, ..., Y_{n-t})$ and ε represents a margin between the reference and the critical value of the damping.

For monitoring the damping coefficient d, the residual sensitivity should be parameterized accordingly. As the same properties of independence and change in the mean of Z_n hold true whatever the Jacobian involved in (16) is, the residual dependant on d writes:

$$Z_n(d) \triangleq \mathcal{J}_n^T(d)\ \Sigma_n^{-1}(\theta) \text{vec}(\widehat{S}_n^T \mathcal{Y}_{n,p+1}^{+} \mathcal{Y}_{n,q}^{-T}) \tag{20}$$

$$\text{where } \mathcal{J}_n(d) \triangleq \mathcal{J}_n(\theta)\ \mathcal{J}_{\theta d} \tag{21}$$

and $\mathcal{J}_{\theta d}$ is the sensitivity of θ w.r.t. d. Details about $\mathcal{J}_{\theta d}$ computation can be found in Basseville et al. [2004].

For each mode, as d is a scalar, the Jacobian $\mathcal{J}_n(d)$ is a vector, and the residual $\overline{\zeta}_n(d)$ is now a scalar number with variance $\overline{\Sigma}_n(d) = \mathcal{J}_n(d)^T \Sigma_n^{-1} \mathcal{J}_n(d) > 0$. From (14), a deviation in d is reflected by a deviation *of the same sign* in the mean of $\overline{\zeta}_n(d)$. $Z_n(d)$ in (20) is also scalar, and a change from $d \geq d_0^n + \varepsilon/2$ to $d < d_0^n - \varepsilon/2$ is reflected by a significant *decrease* in the mean value ν of $Z_n(d)$. Thus the detection of the flutter onset occurs when ν decreases below some critical value. Monitoring ν is achieved with the statistical Cusum test (Basseville and Nikiforov [1993]) to test between the hypotheses: $\widetilde{\mathbf{H}}_0^n : \nu \geq +\ \nu_m/2$ and $\widetilde{\mathbf{H}}_1^n : \nu < -\ \nu_m/2$ where ν_m is the minimum magnitude of change to be detected:

$$R_n(d) \triangleq \sum_{k=L+t}^{n} \overline{\Sigma}_k(d)^{-1/2}\ (Z_k(d) + \nu_m) \tag{22}$$

$$T_n(d) \triangleq \max_{L+t \leq k \leq n} R_k(d)$$

$$g_n(d) \triangleq T_n(d) - R_n(d) \tag{23}$$

The flutter onset decision associated with $\widetilde{\mathbf{H}}_1^n$ is taken when $g_n(d) \geq \varrho$ for some threshold ϱ.

3.2 Estimating the key matrices

The proposed algorithm is implemented to ensure online flutter detection. The computation of the Jacobian $\mathcal{J}_n(d)$ and the covariance inverse Σ_n^{-1} in (20) should then be well optimized.

A consistent estimate of the Jacobian in (10) writes (Basseville et al. [2004]):

$$\widehat{\mathcal{J}}_n(\theta) = (I_s \otimes \widehat{S}_n)(\widehat{\mathcal{H}}_{p+1,q}^n \mathcal{O}_{p+1}^{\dagger T}(\theta) \otimes I_{(p+1)r}) \mathcal{O}'_{p+1}(\theta) \tag{24}$$

where $\mathcal{O}_{p+1}^{\dagger T}(\theta)$ is the pseudo-inverse of $\mathcal{O}_{p+1}(\theta)$,
$s = (p+1)r - 2m$ is the rank of $\widehat{S}_n$ with r the number of sensors and m the number of modes, $\otimes$ is the Kronecker product operator, and

$$\mathcal{O}'_{p+1}(\theta) \triangleq \partial/\partial\theta \text{ vec}(\mathcal{O}_{p+1}(\theta)) \tag{25}$$

$$= \left(\begin{array}{ccc|ccc} \Lambda_1'^{(p)} \otimes \varphi_1 & & 0 & \Lambda_1^{(p)} \otimes I_1 & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \Lambda_m'^{(p)} \otimes \varphi_m & 0 & & \Lambda_m^{(p)} \otimes I_m \end{array} \right)$$

with $\Lambda_i^{(p)T} \triangleq \begin{pmatrix} 1 & \lambda_i & \lambda_i^2 & \ldots & \lambda_i^p \end{pmatrix}$, $\Lambda_i'^{(p)T} \triangleq \begin{pmatrix} 0 & 1 & 2\lambda_i & \ldots & p\lambda_i^{p-1} \end{pmatrix}$ for $1 \leq i \leq m$.

$\mathcal{O}_{p+1}^{\dagger T}(\theta)$, $\mathcal{O}'_{p+1}(\theta)$, and $\mathcal{J}_{\theta d}$ in (24) and (21) are computed once for an initial reference $\theta = \theta_0$. Given that only the sign of the damping deviation is of interest, possible uncertainty due to modal changes can be tolerated.

To estimate the covariance in (11), data are partitioned into blocks with size K. For l data blocks, the covariance matrix writes:

$$\widehat{\Sigma}_l = \frac{1}{l} \sum_{i=1}^{l} (\widehat{\zeta}_i \widehat{\zeta}_i^T - \check{\zeta}_l \check{\zeta}_l^T) \tag{26}$$

where the residual $\widehat{\zeta}_i$ is computed on the block i and $\check{\zeta}_l$ is the residual empirical mean over l blocks. For each new block, this covariance estimate can be updated recursively:

$$\widehat{\Sigma}_l = \frac{l-1}{l} \widehat{\Sigma}_{l-1} + \frac{l-1}{l^2} (\widehat{\zeta}_l - \check{\zeta}_{l-1})(\widehat{\zeta}_l - \check{\zeta}_{l-1})^T \tag{27}$$

where

$$\check{\zeta}_l = \frac{l-1}{l} \check{\zeta}_{l-1} + \frac{\widehat{\zeta}_l}{l} \tag{28}$$

Computing at each block the covariance inverse required in (20) is highly time consuming. The matrix inversion lemma

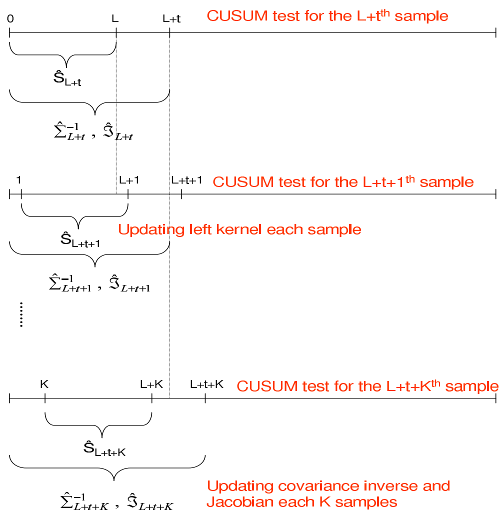

Fig. 1. Scheme of the moving reference algorithm

(Ljung [1999]) is applied to (27) to derive a recursion. For l data blocks, the covariance inverse writes:

$$\widehat{\Sigma}_l^{-1} = \tau_l^{-1}\widehat{\Sigma}_{l-1}^{-1} - \frac{q_l q_l^T}{\gamma_l} \quad (29)$$

where $\tau_l = \frac{l-1}{l}$, $\gamma_l = \frac{l^2}{l-1} + \tau_l^{-1} u_l \widehat{\Sigma}_{l-1}^{-1} u_l^T$, $q_l = \tau_l^{-1}\widehat{\Sigma}_l^{-1} u_l^T$ and $u_l = (\widehat{\zeta}_l - \breve{\zeta}_{l-1})^T$.

For notational convenience, the covariance inverse in (29) is denoted by $\widehat{\Sigma}_n^{-1}$ with $l = \lfloor n/K \rfloor$ blocks ($\lfloor\rfloor$ is the floor operator).

3.3 Summary

The flutter monitoring test is illustrated in figure 1. It can be summarized in the following steps.

Initialization. For an initial airspeed, a modal identification is firstly done to estimate a fixed reference θ_0 and compute the fixed terms of the Jacobian $\widehat{\mathcal{J}}_n(d)$ in (24) and (21). The tuning parameters are chosen: the data sequence length L, lag t, sample block length K, minimum magnitude of change ν_m , and threshold ϱ. Initial estimation of $\widehat{\Sigma}_{L+t}^{-1}$ and $\widehat{\mathcal{J}}_{L+t}(d)$ is performed on the first $L+t$ samples. The left kernel $\widehat{S}_{L+t}$ is estimated on the sequence $(Y_1, ..., Y_L)$ and used with $\widehat{\Sigma}_{L+t}^{-1}$ and $\widehat{\mathcal{J}}_{L+t}(d)$ to compute $R_{L+t}(d)$ in (22).

Recursive loop. To detect a decrease in the damping coefficient, the CUSUM test runs for monitoring $R_n(d)$. For each sample $n \geq L+t$, $\widehat{S}_n$ is estimated on the sequence $(Y_{n-t-L}, ..., Y_{n-t})$ and used with $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n(d)$ to compute $R_n(d)$ and then $g_n(d)$ in (23) until the flutter condition $g_n(d) \geq \varrho$ is satisfied. During this process, $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n(d)$ are updated every K samples.

4. APPLICATION

4.1 Hancock wing model

The aeroelastic model of a rectangular wing introduced in Hancock et al. [1985], is considered for data simulation. The model is a rigid wing with constant chord allowing two degrees-of-freedom in bending and torsion. The equation of motion writes:

$$M\ddot{q} + (D + VB)\dot{q} + (K + V^2C)q = 0 \quad (30)$$

where $q^T = (h \;\; \alpha)$ is the vector of generalized coordinates with h the plunging (vertical displacement) and α the pitching (rotation as a change in the angle of attack), M is the inertial matrix, D the structural damping, K the structural stiffness, B the aerodynamic damping, C the aerodynamic stiffness and V the airspeed. A coupled motion exists between bending and torsion as the center of mass of the wing is located out of its flexural axis. The structural damping D is estimated with the proportional damping assumption (3% damping ratio is fixed for both degrees-of-freedom) and the aerodynamic matrices B and C with the approximation of quasi-steady aerodynamics that neglects their dependency on frequency (Theodorsen [1935]). Eq.(30) can be written in the state space form:

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = A(V) \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad (31)$$

where

$$A(V) = \begin{bmatrix} 0 & I \\ -M^{-1}(K + V^2C) & -M^{-1}(D + VB) \end{bmatrix} \quad (32)$$

With the sampling frequency $\tau = 50$ Hz, the discrete-time eigenvalues (λ_i, $i = 1, 2$) are deduced for each airspeed from the discrete state matrix $F(V) = e^{A(V)/\tau}$ and the modal frequencies f_i and damping coefficients d_i are computed using (3). The evolution of the modal frequencies and damping coefficients with increasing airspeeds is plotted in figure 2. A coupling can be observed between bending and torsion modes when frequencies get closer to each other and the damping coefficients move apart. That illustrates the typical bending-torsion coupling behavior of the flutter. The flutter onset can be estimated when the torsional damping coefficient reaches zero from above at the airspeed $V \simeq 88.5\ m/s$.

4.2 Simulation

Using the control system toolbox of Matlab, time series data are simulated from Hancock model for plunging and pitching. The scenario consists in simulating an aircraft acceleration with transition phase from $V = 20\ m/s$ to $V = 88\ m/s$ (close to flutter). In this speed range, 300 samples (6 seconds) are simulated every $1\ m/s$ step to obtain 2 output time series of length $N = 20700$.

4.3 Results

In order to assess the performances of the approach proposed in this paper, the moving reference approach was applied to simulation data and compared to a previous fixed reference version reported in Mevel et al. [2005] and Basseville et al. [2006]. The parameters of the CUSUM test are $\nu_m = 0.1$ and $\varrho = 100$. For the fixed reference

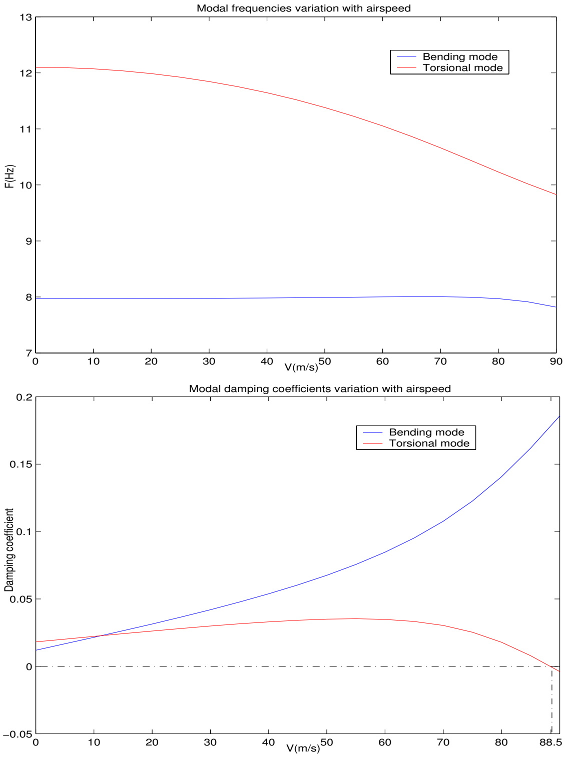

Fig. 2. Modal frequencies (Top) and damping coefficients (Bottom) of the bending (Blue) and torsional (Red) modes as functions of the airspeed.

approach, the left kernel, Jacobian and covariance matrices are computed once for the reference $V = 20\ m/s$ on a large data set and remain constant during the CUSUM test. For the moving reference approach, $L = 2000$ sample sequence is used for the left kernel matrix estimation and the CUSUM test runs after $t = 1000$ samples from this sequence. Starting from an initial estimate on $L + t$ samples, the updating of $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n(d)$ is made after each $K = 50$ samples. The CUSUM test is applied for each mode to detect the decrease of the damping coefficient. The test stops when crossing over the threshold value ϱ. Results for both approaches are displayed in figures 3 and 4.

For the fixed reference algorithm in figure 3, except some perturbations due to noise, the test for the bending mode (Top) has no reaction while a reaction can be observed for the torsional mode (Bottom) where the flutter alarm is launched. These observations are coherent with the damping coefficient variation in figure 2 where the flutter lies with the torsional mode. It can be also checked that the test reacts approximately at the airspeed range where the damping coefficient becomes below its reference value at $V = 20\ m/s$. That confirms the precision of the subspace-based test in monitoring flutter criteria w.r.t. a reference. However, the test reaction associated with the flutter onset are detected at $V = 65\ m/s$ which can be considered as a conservative estimation comparing to the true flutter airspeed, especially that the torsional damping ratio at this airspeed is about 3%.

For the new algorithm proposed in this paper, it can be firstly observed from figure 4 that the detection results are also coherent with the modal behavior of the system

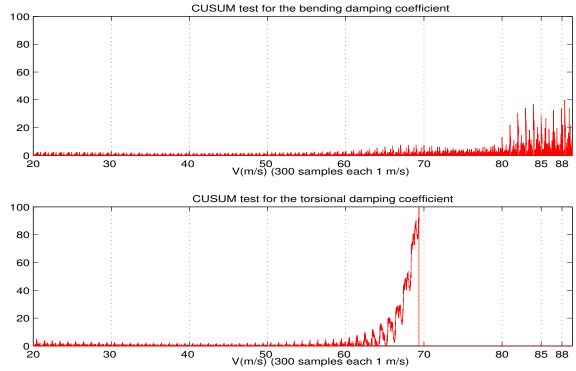

Fig. 3. Previous test with fixed reference at V=20m/s for monitoring the damping coefficient for bending mode (Top) and torsional mode (Bottom).

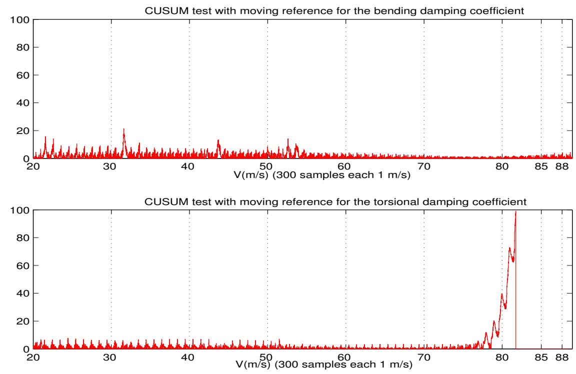

Fig. 4. New test with moving reference for monitoring the damping coefficient for bending mode (Top) and torsional mode (Bottom).

through the test reaction in the torsional mode. The improvement concerns the detection that occurs closer to the flutter zone and starts at $V = 78\ m/s$. The alarm is more realistic than in the previous approach and that gives a larger flight envelope for the aircraft. Even though no abrupt torsional damping drop is observed at the detected airspeed in figure 2, the decreasing rate becomes important and leads to flutter. It can also be noted that the reaction of the test to small damping decrease at the airspeed range preceding the detection remains insignificant (see small reactions in figure 4 at Bottom) because of the reference updating and has no influence on the flutter decision. Obviously, the computational time is higher than in the previous approach but it remains lower than the time series duration (the recording time) and this new algorithm can thus always run online.

5. CONCLUSION

In this paper, a new version of the online subspace-based residual algorithm for flutter monitoring is proposed to update the reference state of the aircraft during the flight. With such an update, the CUSUM test detects the flutter onset only for an important damping coefficient decrease. Results on simulation data show a significant improvement of the flutter detection quality with the new approach. Future investigations will consider more complex aircraft models. In such cases, limitations due to large system orders and sensor number should be overcome, for example with the aid of a recursive left kernel matrix estimation.

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