Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A QFT/EEAS Design of Multivariable Robust Adaptive Controllers

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Abstract: This paper presents a robust adaptive control design methodology for multi-input multi-output (MIMO) plants based on Quantitative Feedback Theory (QFT) and Externally Excited Adaptive System (EEAS), both of which are the novel ideas of Horowitz. Self Oscillating Adaptive Systems (SOAS) are proposed to mainly overcome the problem of large gain variations, which is important in certain applications. To further improve the SOAS design, the idea of EEAS was developed. Finally, combined QFT and EEAS proposed a robust adaptive controller for SISO uncertain plants. However, due to the complex design nature of the proposed combined methodology and the difficulty of an optimal design, this line of Horowitz's research was not followed further. In this paper, to overcome the above mentioned problems the design procedure is reformulated as a set of cost functions and constraints. Genetic Algorithms are then used to solve the optimal design. Also, QFT/EEAS design is extended to multivariable uncertain plants. Sufficient conditions are derived to assure the achievement of given off-diagonal performance. Then, the given main channel performance could be achieved by using SISO QFT/EEAS method. Simulation studies indicate the effective performance of the proposed QFT/EEAS MIMO design methodology. It is shown that the proposed approach can handle large plant parameter uncertainties with lower loop bandwidths.

1. INTRODUCTION

Quantitative Feedback Theory (QFT) is a powerful tool in the design of robust control systems for uncertain plants (Horowitz, 1991).

QFT is initially devised to design robust controllers for highly uncertain, linear time-invariant (LTI), single-input single-output (SISO) systems, through a two–degrees-of-freedom design structure. The extension of SISO QFT design methodology to multivariable systems provides a number of competing techniques, which can be categorized into two classes, being the non-sequential design methodologies (Horowitz, 1979) and sequential design methodologies (Horowitz, 1982; Horowitz and Yaniv, 1986).

In solving an $n \times n$ multivariable design problem, the synthesis problem is converted into n equivalent single-loop MISO problems, where parameter uncertainties, external disturbances and performance tolerances are derived from the original problem, and the coupling effects between MISO subsystems are treated as disturbance inputs. These coupling effects need to be rejected in the QFT design of each subsystem, which can be achieved by the SISO QFT design techniques (Houpis *et al.*, 2006; Yaniv, 1999).

The disturbance-rejection requirements often dominate the tracking-performance requirements, in the design of each subsystem (Wu *et al.*, 2004). On the other hand, in the case of plants with large parameter uncertainties, QFT design technique can lead to controllers with too large bandwidths. These can result in high control gains that may cause actuator saturation, reduce the control-loop performance, and lead to over design.

SOAS is an adaptive scheme which was introduced at Honeywell in the context of adaptive flight control systems (Astrom and Wittenmark, 1995). The adaptive feature of the SOAS is centered on the high gain property introduced by relay feedback. The system represents a type of adaptive control in which there are intentional perturbations, which excite the system all the time. An adaptive QFT approach is proposed in (Horowitz *et al.*, 1974b) using Self Oscillating Adaptive Systems (SOAS) for SISO plants. The resulted methodology is insensitive to large gain variations. This however causes limit cycles in the closed loop plant, which is not desirable in many practical applications.

In an alternative approach, Horowitz proposed Externally Excited Adaptive Systems (EEAS) to replace SOAS (Horowitz *et al.*, 1974a). He showed that it is more flexible than SOAS in satisfying the quasilinearity constraints. This circumvents the need for the oscillation condition and improves the quality of closed loop performance.

He also extended these dithered adaptive systems with additional adaptive loops thereby reducing the oscillation frequency ω_o (Horowitz and Shapiro, 1979) and loop bandwidth requirements (Horowitz *et al.*, 1991).

Because of the design complexity, this line of Horowitz's research has not been deeply pursued. Finally, Horowitz proposed combined QFT/EEAS controller design for SISO uncertain plants (Horowitz *et al*., 1974a). To further develop the Horowitz's ideas in the combined design, and to facilitate this complex design procedure, in (Khaki-Sedigh *et al*., 2005) relevant cost functions and constraints are introduced, and the problem is formulated as a nonlinear optimization problem. This problem is then solved using stochastic optimization techniques that result in an optimal design (Khaki-Sedigh *et al*., 2005). The extension of the combined QFT/EEAS design technique for uncertain MIMO plants, is proposed in (Namaki-Shoushtari *et al*., 2005), where the MIMO control problem is simplified into equivalent MISO design problems and the controller is diagonal. The systems on the diagonal of the closed loop system have the input tracking and disturbance rejection requirements. While the systems corresponding to the off-diagonal parts, can only fulfill disturbance rejection requirements. Then, the MISO problems are solved based on SISO QFT/EEAS design method.

This paper extends the robust adaptive design of controller using the QFT/EEAS approach to multi-input multi-output (MIMO) plants in a more precise framework. Sufficient conditions are first derived to attain the given off-diagonal performance. Then, the given performance on the main channels is satisfied taking into account these additional constraints. In order to further improve the controller performance, the design steps involved are stated as design objectives and the problem is formulated as a constrained nonlinear optimization problem. Finally, simulation studies on a design example from Horowitz (Horowitz, 1979) which is used in (Khaki-Sedigh and Lucas, 2000) are employed to show the effectiveness of the proposed method. A comparison with direct MIMO QFT clearly indicates the advantages gained by the proposed design.

2. DESIGN PRELIMINARIES

QFT is a well established methodology for the design of robust control systems (Horowitz, 1991). The main steps involved in the QFT design can be summarized as: template generation, bound computation, loop shaping and pre-filter design. These steps are formulated in an optimal framework and solved using random optimization techniques in (Khaki-Sedigh and Lucas, 2000).

Also, EEAS is an adaptive control methodology as shown in Fig. 1 for SISO systems. $P(s) = k\, P_h(s)$ represents the uncertain plant with varying parameters, where k is the high-frequency gain of the plant, that is $\lim_{s\to\infty} P_h(s) = s^{-e_p}$ and e_p is the excess of poles over zeros of the plant. In this structure $F(s)$, $G_a(s)$, $G_b(s)$, $G_c(s)$ are linear compensators, whose values are to be chosen.

EEAS is closely related to the SOAS which is obtained by injecting a high frequency sinusoid to measure the gain of the process and to set the controller gain. This gives the designer more freedom than SOAS, because the frequency of the excitation can be chosen more easily (Astrom and Wittenmark, 1995).

The main features of this strategy are: (Horowitz *et al*., 1974a)

1- It consists of a two-degrees-of freedom feedback system with linear elements, except for one nonlinear element N, whose characteristic is taken as static, odd, with hard saturation when its input is sufficiently large.

2- If a fast and large (relative to control signals frequencies and amplitudes) periodic signal is applied which saturates N twice per period, then the system response to the control signals is essentially linear.

3- It exhibits the valuable property of zero sensitivity to changes in the plant high-frequency gain factor k, ($P(s) = k\, P_h(s)$). This suggests that EEAS is much superior to the linear feedback system.

EEAS is one of the very few adaptive control schemes for which a quantitative feedback theory exists. QFT enables a direct design to satisfy numerical specifications in the face of given ranges of plant parameter values (Horowitz *et al*., 1974a). This is due to the decoupling of the system response to the dither signal from its response to the slower and smaller control signals. The system is basically linear time invariant (LTI) for the latter signals. Also, the adaptive action of the nonlinearity is decoupled from the benefits of the LTI component of the feedback loop.

2.1 Quasilinear Representation

The quasilinear properties of EEAS permit the extension of the quantitative linear feedback theory to this system. Let the input to nonlinear element N (Fig. 1) be $x = A\sin(\omega_o t + \theta) + x_r + x_d + x_\eta := x_o + x_f$ with independent terms. (x_o is the oscillation component due to the excitation signal v. The forced component x_f consists of x_r, x_d due to inputs r, d and x_η due to sensor noise η(t)). Under certain conditions the output of nonlinear element N is closely approximated by: (Horowitz *et al*., 1974a)

$$\begin{aligned} &y = N_o A\sin(\omega_o t + \theta) + N_f\, x_f \overset{\Delta}{=} y_o + y_f \\ &N_f = 0.5\, N_o \quad , \quad N_o = M / A \quad , \quad M, \text{ a constant.} \end{aligned} \tag{1}$$

These conditions are: (Horowitz *et al*., 1974a)

$$\begin{aligned} &\max_{d,r,P,t} \left| x_f(t) \right| \le A_1 / \alpha \\ &\max\ (\omega_{dx}, \omega_{rx}) \le \beta\omega_o \end{aligned} \tag{2}$$

Where, ω_{ix} is the bandwidth of x_f component due to input i (i: d, r), $A_1 = A_{\min} = \min_{P,t} A$ and α, β are constants depending on the acceptable accuracy; $\alpha = \beta^{-1} = 3$ is suggested in (Horowitz *et al*., 1974a). For example, if N is an ideal relay with output $\pm M_0$ then $M = 4M_0/\pi$ and the error in Equation (1) is 5-10 percent.
If the quasilinear conditions are satisfied, then

$$\frac{C_r}{R} \overset{\Delta}{=} T_r = F\frac{L_f}{1+L_f} \quad , \quad L_f = G_a G_b G_c N_f P = 0.5 L_o \tag{3}$$

Where, L_o and L_f are the transmission loop gains for x_o and x_f (the forward path from x_o and x_f to output $c(t)$), respectively. In (Horowitz *et al.*, 1974a) it is shown that, T_r is insensitive to plant gain variations.

To employ an EEAS design, the following constraints and cost functions are introduced:

1- To quench any self-induced limit cycle, if N is an ideal relay, $Arg\, L_o(j\omega_\pi) = -180^\circ$, then no limit cycle will exist at ω_π if: (Horowitz *et al.*, 1974a)

$$\left| L_o(j\omega_\pi) \right| = \frac{M}{A} \left| G_a G_b G_c k P_h (j\omega_\pi) \right| < \rho, \text{ (= 1.17 for ideal relay)}$$

$$1.17 - \left| L_o(j\omega_\pi) \right| > 0 \tag{4}$$

2- To limit $\left| C(j\omega_o) \right|$ to an acceptable value m; with $(k_{\max} := k_2)$

$$\left| C(j\omega_o) \right|_{\max} = \max_k \frac{A_o \left| G_c k P_h (j\omega_o) \right|}{\left| 1 + L_o(j\omega_o) \right|} = \frac{A_o k_2 \left| G_c P_h (j\omega_o) \right|}{\left| 1 + L_o(j\omega_o) \right|} \le m,$$

$$m - \left| C(j\omega_o) \right|_{\max} \ge 0. \tag{5}$$

3- To satisfy the quasilinear conditions; (Horowitz *et al.*, 1974a)

$$\left| L_o(j\omega_o) \right| - \frac{2\alpha k_2}{m k_1} \left| (b + j\omega_o) Z_e(j\omega_o) \right| \ge 0 \tag{6}$$

Where, Z_e is assumed to be the extreme plant output, Let sub-e denote the corresponding extreme signals, so $X_e = Z_e / G_c G_b k_1 P_h N_f = qb/(s+b)$ is chosen to be the model of extreme value of $x_f(t)$ (defined as $\max_{r,d,P,t} \left| x_f(t) \right|$) in the ω domain, such that $qb \le A_1/\alpha$ [from Equation (2)], this choice is justified in (Horowitz *et al.*, 1974a), and the plant high-frequency gain k belongs to $[k_1, k_2]$. For further details refer to (Horowitz *et al.*, 1974a).

An optimal quantitative synthesis procedure is addressed for the EEAS (Khaki-Sedigh *et al.*, 2005) for SISO systems, permitting systematic optimal design to achieve given performance tolerances over specified plant uncertainty.

3. COMBINED MIMO QFT/EEAS DESIGN

3.1 Problem formulation

With no loss of generality and for simplicity, the design process is developed for 2 × 2 MIMO plants. The procedure can be extended to general MIMO case. Consider the feedback structure shown in Fig. 2. The transfer function matrix $P(s) = [p_{ij}(s)]$ represents the LTI uncertain 2 × 2 plant to be controlled. The $G_i = diag[g_{i1}(s), g_{i2}(s)],\ i: a, b, c$ and $F(s) = diag\ [f_{11}(s), f_{22}(s)]$, which are assumed diagonal, represent the feedback compensators and the prefilter matrices, respectively. Also, the nonlinear elements N_1 and N_2 are assumed to be ideal relays with outputs $\pm M_{01}$ and $\pm M_{02}$. Moreover, v_1 and v_2 are the excitation signals.

Let $T_{C/R}(s)$ be the input-output relation from input $R(s)$ to output $C(s)$, which is clearly derived as

$$T_{C/R}(s) = [I + P(s)G(s)]^{-1}\ P(s)G(s)F(s) \tag{7}$$

Where, $G = G_c G_b N_f G_a$ ($N_f = diag(n_{f1}, n_{f2})$, are describing functions of the relays).

Due to uncertainty, $P \in \{\mathcal{P}\}$ is a set of possible plants and it is assumed here that the plant set is finite or can be adequately approximated by a finite set so that numerical algorithms can be developed. The combined QFT/EEAS control design task is to find $G_i(s)$ and $F(s)$ with proper rational and stable elements and the (output of) ideal relay N_1 and N_2, in order to satisfy a client's performance specifications $\forall P \in \{\mathcal{P}\}$. For example, tracking specifications may require that $\forall P \in \{\mathcal{P}\}$,

$$Tl_{ij}(\omega) \le \left| T_{C/R}(j\omega) \right|_{ij} \le Tu_{ij}(\omega) \quad i, j = 1, 2 \tag{8}$$

Where, $Tu_{ij}(\omega)$ and $Tl_{ij}(\omega)$ are the upper and lower specifications. For simplicity, this paper will concentrate on tracking performance in Equation (9), but there may be other specifications on sensitivity, sensor noise to input sensitivity, as well as engineering considerations such as those in direct MIMO-QFT. At high frequencies the benefits of feedback are negligible. High frequency specifications will result in large bandwidth with very little closed-loop performance improvement. It is thus recommended that the specifications to be enforced to the lowest possible frequency ω_h (the Horowitz frequency) (Yaniv, 1999). In addition, an implicit design objective is the minimization of the loop bandwidths when sensor noise attenuation is concerned.

3.2. Development of the Design Process

In Fig.2, if the quasilinear conditions are satisfied, the output is closely approximated by:

$$\begin{aligned} C &= [I + P G_c\, G_b\, N_f\, G_a]^{-1} P G_c\, G_b\, N_f\, G_a\, F\ R \\ &\quad + [I + P G_c\, G_b\, N_o\, G_a]^{-1} P G_c\ \underline{V} \end{aligned} \tag{9}$$

Where, $R = [r_1\ r_2]^T$ and $\underline{V} = [v_1\ v_2]^T$ are command input and excitation signals, respectively. N_f and N_o, are diagonal, and consist of the describing functions of nonlinear elements with respect to R and $\underline{V}$. When P is nonsingular, Equation (10) can be rewritten as:

$$\begin{aligned} C &= [P^{-1} + G_c\, G_b\, N_f\, G_a]^{-1}\ G_c\, G_b\, N_f\, G_a\, F\ R \\ &\quad + [P^{-1} + G_c\, G_b\, N_o\, G_a]^{-1}\ G_c\ \underline{V} \end{aligned} \tag{10}$$

Using the notation, $P^{-1} = [1/q_{ij}]$, matrix P^{-1} is partitioned to the form (Houpis *et al*., 2006)

$$P^{-1}=[1/q_{ij}]=\Lambda+B \quad (11)$$

Where, Λ is the diagonal part and B is the balance of P^{-1}, thus $\lambda_{ii}=1/q_{ii}$, $b_{ii}=0$, and $b_{ij}=1/q_{ij}$ for $i\neq j$. Then the following identity can be derived:

$$\begin{aligned}[P^{-1}+G]^{-1} &= [\Lambda+B+G]^{-1}=[I+\Lambda^{-1}G+\Lambda^{-1}B]^{-1}\,\Lambda^{-1}\\ &=[I+[I+\Lambda^{-1}G]^{-1}\,\Lambda^{-1}B]^{-1}\,[I+\Lambda^{-1}G]^{-1}\,\Lambda^{-1}\end{aligned} \quad (12)$$

Finally, using the above simplifications yields output C in Fig.2 as: (From Equation (10))

$$\begin{aligned}\begin{pmatrix}c_1\\ c_2\end{pmatrix}&=\begin{pmatrix}1 & \dfrac{q_{11}/q_{12}}{1+g_{c1}\,g_{b1}\,n_{f1}\,g_{a1}\,q_{11}}\\ \dfrac{q_{22}/q_{21}}{1+g_{c2}\,g_{b2}\,n_{f2}\,g_{a2}\,q_{22}} & 1\end{pmatrix}^{-1}\\ &\begin{pmatrix}\dfrac{g_{c1}\,g_{b1}\,n_{f1}\,g_{a1}\,q_{11}}{1+g_{c1}\,g_{b1}\,n_{f1}\,g_{a1}\,q_{11}}\,f_{11} & 0\\ 0 & \dfrac{g_{c2}\,g_{b2}\,n_{f2}\,g_{a2}\,q_{22}}{1+g_{c2}\,g_{b2}\,n_{f2}\,g_{a2}\,q_{22}}\,f_{22}\end{pmatrix}\begin{pmatrix}r_1\\ r_2\end{pmatrix}\\ &+\begin{pmatrix}1 & \dfrac{q_{11}/q_{12}}{1+g_{c1}\,g_{b1}\,n_{o1}\,g_{a1}\,q_{11}}\\ \dfrac{q_{22}/q_{21}}{1+g_{c2}\,g_{b2}\,n_{o2}\,g_{a2}\,q_{22}} & 1\end{pmatrix}^{-1}\\ &\begin{pmatrix}\dfrac{g_{c1}\,q_{11}}{1+g_{c1}\,g_{b1}\,n_{o1}\,g_{a1}\,q_{11}} & 0\\ 0 & \dfrac{g_{c2}\,q_{22}}{1+g_{c2}\,g_{b2}\,n_{o2}\,g_{a2}\,q_{22}}\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix}\end{aligned} \quad (13)$$

With this manipulation, minimizing the off-diagonal terms in Equation (13) can be used as sufficient conditions to reduce the channel interactions. Hence, to reduce the interactions due to command inputs, it is sufficient that:

$$\frac{q_{ii}/q_{ij}}{1+g_{ci}\,g_{bi}\,n_{fi}\,g_{ai}\,q_{ii}}<<1, \quad (i\neq j), i,j=1,2 \quad \omega\leq\omega_h \quad (14)$$

To reduce the interactions due to excitation signals, it is sufficient that:

$$\frac{q_{ii}/q_{ij}}{1+g_{ci}\,g_{bi}\,n_{oi}\,g_{ai}\,q_{ii}}<<1, \quad (i\neq j), i,j=1,2 \quad \omega=\omega_{oi} \quad (15)$$

Also, the off-diagonal performance specification given in (8) is satisfied if the following constraints are met;

$$\frac{Tl_{ij}}{Tl_{ii}}\leq\frac{q_{ii}/q_{ij}}{1+g_{ci}\,g_{bi}\,n_{fi}\,g_{ai}\,q_{ii}}\leq\frac{Tu_{ij}}{Tu_{ii}}, \quad (i\neq j), i,j=1,2 \quad \omega\leq\omega_h \quad (16)$$

Finally, the given performance bounds on $|T_{C/R}|_{ij}$ is re-allocated on a reasonable basis, such that not only the main channel performance can be achieved by using the single-input single-output (SISO) QFT/EEAS method (Khaki-Sedigh *et al*., 2005), but also the trade-offs of performance bounds between the loops become transparent during the system synthesis.

3.3. Optimal design

Now, the loop-shaping problem is stated as a multi-objective nonlinear constrained optimization problem (Khaki-Sedigh *et al*., 2005; Namaki-Shoshtari *et al*., 2005; Khaki-Sedigh and Lucas, 2000). Relevant cost functions and constraints are introduced, and different important parameters are defined to represent the aims and objectives of an expert.

Since the stability and performance bounds are the constraints which should be satisfied in the design, it is difficult to optimize the QFT/EEAS controller for all the objectives. This also means that it will be non-advantageous if a multi-objective GA is applied, because no compromise may be made to the stability and bound goals. Thus a single composite cost is formed for GA search for the combined QFT/EEAS design, as given by

$$W=\sum_{l=1}^{Nco}\tau_l\;W_l \quad (17)$$

Where, τ_l, l= 1, 2, …, N_{co} (N_{co} is the number of constrains and objectives) are the weighting factors. In general the weights should be reasonably large. In this paper, weighting factors of the constraints are chosen much larger than those of the objectives.

Also, the pre-filter design problem can be transformed into a set of constraints and objectives, and this constrained optimization problem could be solved via Genetic Algorithm in a similar way as in (Khaki-Sedigh *et al*., 2005; Namaki-Shoshtari *et al*., 2005; Khaki-Sedigh and Lucas, 2000).

4. DESIGN EXAMPLE

In this section, a numerical example is used to illustrate the proposed design method. The system to be considered consists of a 2 × 2 MIMO plant with transfer function matrix: ((Horowitz, 1979), (Khaki-Sedigh and Lucas, 2000))

$$P(s)=\begin{bmatrix}\dfrac{\gamma_{11}}{1+s\,\delta_{11}} & \dfrac{\gamma_{12}}{1+s\,\delta_{12}}\\ \dfrac{\gamma_{21}}{1+s\,\delta_{21}} & \dfrac{\gamma_{22}}{1+s\,\delta_{22}}\end{bmatrix}$$

And, a total of nine plant cases are given in Table 1. The first plant is taken as the nominal.

Table 1. Plant Conditions used in example.

NO.	γ_{11}	γ_{22}	γ_{12}	γ_{21}	δ_{11}	δ_{22}	δ_{12}	δ_{21}
1	1	2	0.5	1	1	2	2	3
2	1	2	0.5	1	0.5	1	1	2
3	1	2	0.5	1	0.2	0.4	0.5	1
4	4	5	1	2	1	2	2	3
5	4	5	1	2	0.5	1	1	2
6	4	5	1	2	0.2	0.4	0.5	1
7	10	8	2	4	1	2	2	3
8	10	8	2	4	0.5	1	1	2
9	10	8	2	4	0.2	0.4	0.5	1

Specifications (*for all plants*)

(1)- The tracking specifications, $Tl_{ij} \le \left| T_{C/R}(j\omega) \right|_{ij} \le Tu_{ij}$ ($i, j = 1, 2.$) are basically non-interacting, and are enforced to $\omega_h = 10$ rad/sec.

On-diagonal:

$$Tu_{ii}(\omega) = \left| \frac{25}{s^2 + 6s + 25} \right|_{s=j\omega} \quad and \quad Tl_{ii}(\omega) = \left| \frac{4}{s^2 + 4.4s + 4} \right|_{s=j\omega}$$

Off-diagonal:

$$Tu_{ij}(\omega) = 0.1 \quad and \quad Tl_{ij} = 0$$

(2)- Stability margin: $\left| 1/(1+L_{f_i}) \right| \le 3.5$ dB for all plants.

Where, $L_{f_i} = g_{ci}\, g_{bi}\, n_{f_i}\, g_{ai}\, q_{ii}$, $i = 1, 2$ and i corresponds to the ith loop and this would indicate a gain margin and phase margin of 9.6dB and 39deg respectively (Yaniv, 1999).

The process of the nonlinear optimization problem results in the following optimal robust controllers: ($m = 0.1$, b= 1, $\alpha = \beta^{-1} = 3$ are assumed).

$$g_{a1}(s) = \frac{2.39\times10^2}{(s+478.7)}, g_{b1}(s) = \frac{4.8\times10^2\,(s+18.81)}{s\,(s+1214)}, g_{c1}(s) = \frac{0.40}{(s+868.4)}$$

$$g_{a2}(s) = \frac{5.44}{(s+311.4)}, g_{b2}(s) = \frac{1.8\times10^3(s+14.47)}{s\,(s+1733)}, g_{c2}(s) = \frac{0.58}{(s+2207)}$$

Also, the other design parameters are as follows, the relay outputs M_{01}, M_{02}, the amplitudes and the frequencies of the excitation signals v_1, v_2:

$\omega_{o1} = 32.23$ rad/sec , $A_{o1} = 77.06$, $M_{01} = 155.71$, $A_{\min_1} = 5.04\times10^{-4}$

$\omega_{o2} = 30.59$ rad/sec , $A_{o2} = 522.43$, $M_{02} = 413.82$, $A_{\min_2} = 4.86\times10^{-5}$

Because the design specification is 'basically non-interacting', the pre-filter is assumed to be diagonal so as to simplify the design (Boje, 2002). The designed pre-filters are:

$$f_{11}(s) = \frac{3.34\,(s^2 + 58.81\,s + 4930)\,(s^2 + 185.5\,s + 1.28\times10^5)}{(s+376.3)\,(s+20.83)\,(s+1.50)\,(s^2 + 307.9\,s + 1.79\times10^5)},$$

$$f_{22}(s) = \frac{5.94\,(s^2 + 40.66\,s + 7701)\,(s^2 + 385.6\,s + 1.36\times10^5)}{(s+208.1)\,(s+198.2)\,(s+1.30)\,(s^2 + 497.7\,s + 1.16\times10^5)}$$

Fig.3 and Fig.4 show the performance of the closed-loop system in tracking step commands for two plant cases. It is obvious that the combined robust adaptive strategy has reached the desired performance with lower loop bandwidths. The crossover frequency of the first loop gain for all the uncertainty range is $\omega_{c_1} = 21.9$ rad/sec, while in an expert design, given by Horowitz (Horowitz, 1979), the loop crossover frequency varies between 9.96 rad/sec and 205 rad/sec for nine plant cases, also in the optimal design of (Khaki-Sedigh and Lucas, 2000), this belongs to $[6.28\ ,\ 102.7]$ rad/sec . Also, in the second loop the cut off frequency, for all nine plant cases is $\omega_{c_2} = 26.9$ rad/sec, while in Horowitz's design it belongs to [7.31, 54.5] rad/sec, and in the optimized design of (Khaki-Sedigh and Lucas, 2000) it varies between 6.42 rad/sec and 75.8 rad/sec.

6. CONCLUSIONS

QFT and EEAS are two novel ideas proposed by Horowitz for the practical design of control systems. A review of the EEAS development and its use in QFT design proposed by Horowitz is provided. Due to the complex nature of the design, this line of Horowitz's research has not been deeply pursued. Combined QFT/EEAS controller design methodology has been outlined for SISO uncertain plants. To facilitate the Horowitz ideas, the design steps are reformulated in terms of appropriate cost functions and constraints and are solved using Genetic Algorithms. Also, in this paper the methodology is extended to MIMO plants. Proposed design technique can result in controllers with acceptable low loop bandwidth and can be used to reduce sensor noise effects at the plant input. Improvement is significant mainly in those problems with much uncertainty in the high-frequency gain of the plant, since the main characteristic of dithered adaptive systems is that they are insensitive to this uncertainty. Simulation results were used to indicate the practicability and effectiveness of the proposed methodology.

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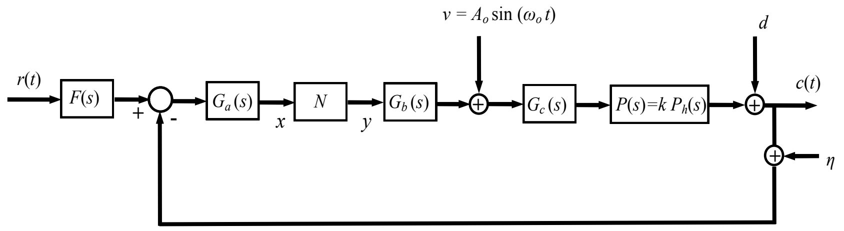

Fig. 1. A canonic EEAS structure $x = A \sin(\omega_o t + \theta) + x_r + x_d + x_\eta$, $y \approx (M/A)\,[A \sin(\omega_o t + \theta) + 0.5\,(x_r + x_d + x_\eta)]$

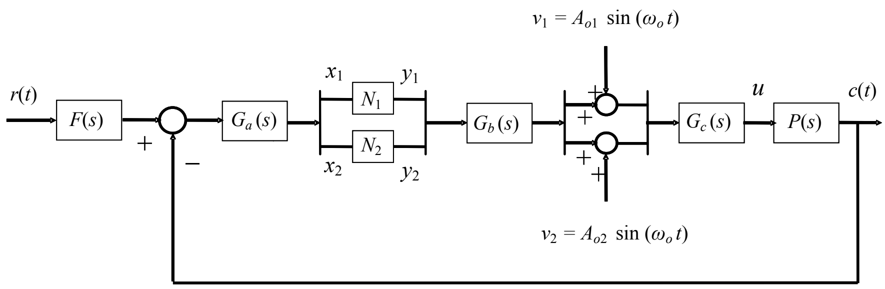

Fig. 2. The combined QFT/EEAS structure for 2 × 2 MIMO plants

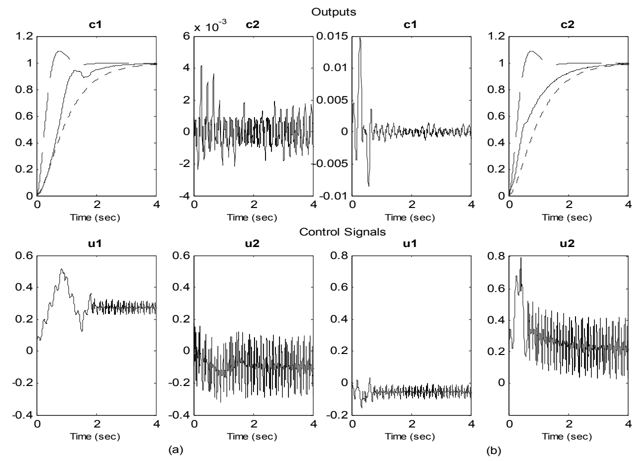

Fig. 3. Time domain simulation for plant parameters as in case 4, (a) the command input is $R = [1 \quad 0]^T$ (b) the command input is $R = [0 \quad 1]^T$.

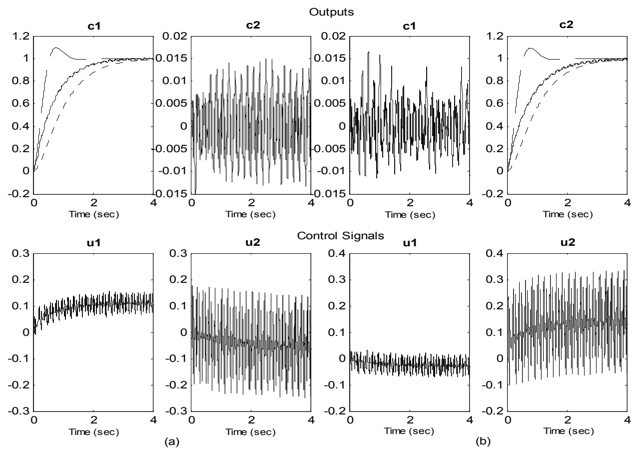

Fig. 4. Time domain simulation for plant parameters as in case 9, (a) the command input is $R = [1 \quad 0]^T$ (b) the command input is $R = [0 \quad 1]^T$