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Indirect fuzzy adaptive control for active shimmy damping

Gaétan Pouly * Thai-Hoang Huynh *
Jean-Philippe Lauffenburger * Michel Basset *

* *ENSISA Lumière/MIPS Laboratory*
12, rue des frères Lumière
68093 MULHOUSE Cedex, France

Abstract: In the context of aircraft, shimmy is an oscillatory phenomenon of the landing gear mainly due the tire dynamics and the landing gear structural dynamics. This phenomenon, which can result in severe structural damages of the landing gear, is here actively damped by an indirect fuzzy adaptive controller. The difficulties to model the ground/wheel interface require the use of an adaptive controller that can modify its behavior in accordance with the plant dynamics. Thus, the proposed controller uses a fuzzy system to estimate the plant dynamics, and then implements this estimate to generate the control law. Based on Lyapunov's theory, it is shown that the proposed adaptive control solution guarantees that the tracking errors will asymptotically converge to zero even if approximation errors appear in the estimation. Simulation results show that the proposed control law creates a realistic control input which properly damps the oscillations. This work is supported by the European DRESS project (Distributed and Redundant Electromechanical nose gear Steering System).

1. INTRODUCTION

Shimmy, the lateral vibration of towed wheel, is a well-known example of self-excited nonlinear oscillation. This phenomenon is influenced by many parameters, from the landing gear structure to the tire properties. It induces oscillations responsible for severe damages of the landing gear. The intimate relation between tire mechanics and shimmy problems is well known, but it is troublesome due to the difficulties of modeling the tire. However, in the available literature, an important number of models have been developed to describe shimmy phenomena. These are based on different ways of modeling the elasticity of tires Somieski [1997], Stépán [1991].
Because of the risk of landing gear damages, different solutions have been proposed to damp the unstable shimmy oscillations. Classical solutions suggest to improve the stability by modifying the nose landing gear (NLG) Besselink [2000] (adding masses to change the gravity center, increasing the damping constant with a shimmy damper, ...). One of the main drawbacks of such passive damping solutions is that the damping characteristics may vary under changing load conditions or ground/wheel interfaces.

Recently, active damping solutions have been investigated for general oscillated systems and different possibilities have been studied. First, simple controllers such as PD controllers are developed to damp oscillation of linear second order type systems Houlston et al. [2006]. Afterward, modern control theories such as optimal control, adaptive control, robust control and fuzzy controller or neural networks have been used to design damping controller for more complex oscillatory systems . Concerning active shimmy damping of the NLG, only a few papers are available in the literature Goodwine and Stépán [2000].

This work presents a controller using the feedback linearization method. However, the applicability of feedback linearization is limited due to the requirement of a detailed knowledge of the system in order to synthesize a precise nonlinear controller. To cope with the drawbacks of feedback control, an adaptive nonlinear control is proposed. These types of algorithms have had an important interest over the last years and recently, the theory of fuzzy logic has been incorporated in the conventional adaptive control solution. Two approaches could be distinguished in the design of a fuzzy adaptive controller: direct or indirect solutions. For the direct solution, the fuzzy system is adjusted directly to ensure the control objectives, while the indirect adaptive approach uses fuzzy systems to estimate the plant dynamics and then calculate the control law Spooner and Passino [1996].

In this paper, an indirect fuzzy adaptive controller is described to perform an active damping of the NLG shimmy phenomenon. First, the difficulties to model the tire and its nonlinearities induced at the ground/wheel interface explain the use of a fuzzy system to estimate the plant dynamics. Secondly, the particularities of the model, with a constant and independent state control gain, enable to use the indirect adaptive solution without apparition of singularities.

This paper is organized as follows: the nonlinear NLG model is described in Section 2. Section 3 presents the fuzzy system and the adaptive control algorithm with its respective stability analysis. Simulation results with the proposed control solution are shown in Section 4. Finally, some concluding remarks are given in Section 5.

This work is supported by the European DRESS project (Distributed and Redundant Electromechanical nose gear Steering System). The goal of this project is to research,

develop and validate a distributed and redundant electrical steering system technology for an aircraft nose landing gear, that will provide improved competitiveness and improved aircraft safety. One of the objective of the DRESS project, in direct link with this paper, is to analyse the impact of the new electrical steering system on the shimmy phenomenon and to study the different method of shimmy damping.

2. DYNAMICS OF THE SHIMMY PHENOMENON

In this paper, the simplified NLG model presented in Somieski [1997] is further developed for active damping design by integrating an actuator. The considered system consists of the mechanical dynamics of an actuator, the torsional dynamics of the NLG, and the forces and moments describing the tire's elasticity. The diagram of this model is illustrated in figure 1:

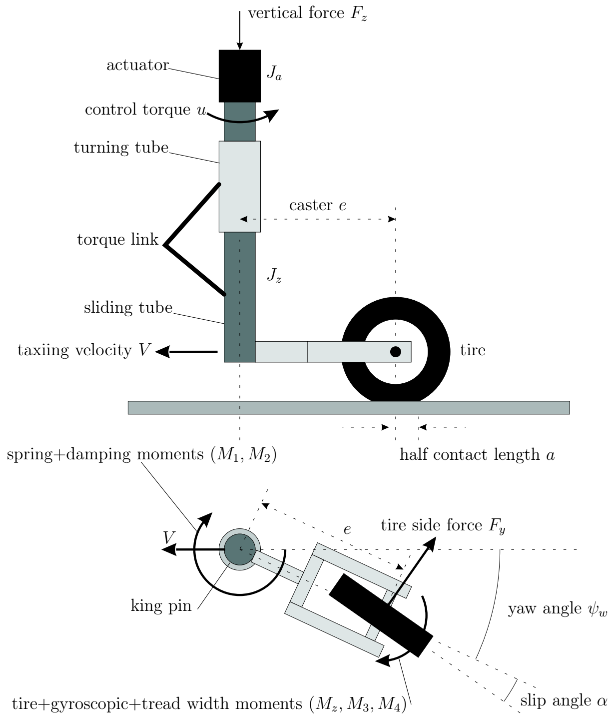

Fig. 1. Nose Landing Gear model as presented by Somieski [1997]

2.1 Nonlinear mathematical model

The input of the model is the control torque u that must be provided by an actuator, and the output of the model is the angle ψ_w of the wheel about its vertical rotating axis. Let us suppose that the link between the actuator and the turning tube is rigid, this means that the angle of the actuator output ψ_a is equal to the angle of the turning tube. Applying Newton's second law to the rotating movements of the actuator and the NLG leads to the following equations:

$$\begin{cases} J_a\ddot{\psi}_a = u - B_a\dot{\psi}_a - M_1 - M_2 \\ J_z\ddot{\psi}_w = M_1 + M_2 + M_3 + M_4 \end{cases} \tag{1}$$

where $M_1 = k_s(\psi_a - \psi_w)$ is the torsional moment provided by the torque link, $M_2 = k_d(\dot{\psi}_a - \dot{\psi}_w)$ is the damping moment from viscous friction in the bearings of the oil-pneumatic shock absorber, M_3 is the tire moment caused by the lateral tire deformations due to side slip and M_4 is the tire damping moment related to the yaw rate, J_a is the inertia of the actuator, J_z is the inertia of the NLG and B_a is the viscous friction constant of the actuator. The following equations summarize the nonlinear characteristics of the tire, which are discussed in detail in Somieski [1997]:

$$M_3 = M_z - eF_y \tag{2}$$

$$F_y = \begin{cases} c_{F\alpha}\alpha F_z & for\,|\alpha| \le \delta \\ c_{F\alpha}\delta F_z sign(\alpha) & for\,|\alpha| > \delta \end{cases} \tag{3}$$

$$M_z = \begin{cases} c_{M\alpha}F_z\dfrac{\alpha_g}{180}\sin\left(\dfrac{180}{\alpha_g}\alpha\right) & for\,|\alpha| \le \alpha_g \\ 0 & for\,|\alpha| > \alpha_g \end{cases} \tag{4}$$

$$M_4 = \frac{\kappa}{v}\dot{\psi} \tag{5}$$

$$\dot{y}_l + \frac{v}{\sigma}y_l = v\psi_w + (e-a)\dot{\psi}_w \tag{6}$$

$$\alpha \approx \arctan\alpha = \frac{y_l}{\sigma} \tag{7}$$

where M_z is the self aligning torque, F_y is the side force, F_z is the vertical force, v is the aircraft ground speed, y_l is the lateral displacement of the wheel and α is the slip angle of the wheel, e is the caster length, a is half of the contact length and c_{f_α}, c_{M_α}, κ, δ, α_g, σ are constants defined in Somieski [1997].

It is important to note that there are two nonlinearities in the model related to the elasticity of the tires. These nonlinearities may cause limit cycles in the system. Hence, the nose landing gear is rather difficult to control.

2.2 State space representation

The state space representation of the nose landing gear model is needed to design the adaptive damping controller. By choosing the state variables $x_1 = \psi_w$, $x_2 = \dot{\psi}_w$, $x_3 = y_l$, $x_4 = \psi_a$, $x_5 = \dot{\psi}_a$ and considering the control torque u, the nonlinear dynamics presented above can be expressed as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{k_s(x_4 - x_1)}{J_z} + \dfrac{k_d(x_5 - x_2)}{J_z} + f_1(x_3) + f_2(x_2) \\ \dot{x}_3 = vx_1 + (e-a)x_2 - \dfrac{v}{\sigma}x_3 \\ \dot{x}_4 = x_5 \\ \dot{x}_5 = -\dfrac{B_a x_5}{J_a} - \dfrac{k_s(x_4 - x_1)}{J_a} - \dfrac{k_d(x_5 - x_2)}{J_a} + \dfrac{1}{J_a}u \end{cases} \tag{8}$$

where:

$$f_1(x_3) = \frac{M_3(\alpha)}{J_z} = \frac{M_3(y_l/\sigma)}{J_z} \tag{9}$$

$$f_2(x_2) = \frac{M_4(\dot{\psi}_w/v)}{J_z} \tag{10}$$

The output of the system is $y = \psi_w = x_1$. Then, the third derivative of the output is:

$$\begin{aligned} y^{(3)} = &\frac{k_s(\dot{x}_4 - \dot{x}_1)}{J_z} + \frac{k_d(\dot{x}_5 - \dot{x}_2)}{J_z} + \cdots \\ &\cdots + \dot{f}_1(x_3)\dot{x}_3 + \dot{f}_2(x_2)\dot{x}_2 \end{aligned} \tag{11}$$

Substituting the derivatives of the state variables (8) into (11), it is obvious that the input u appears on the right hand side of the result. This means that the system has the relative degree of 3, and can be described by the following equation:

$$y^{(3)} = a(x) + b(x)u \tag{12}$$

where $x = [x_1, x_2, \ldots, x_5]^T$ is the system's state vector, $a(x)$ and $b(x)$ are nonlinear functions. The explicit descriptions of these two nonlinear functions can be obtained after some mathematical manipulations. However, even if the exact expressions are calculated, they might not accurately describe the dynamics of the system when it is operating, because of time-varying parameters such as vertical force or tire characteristics. For this reason, $a(x)$ is considered as an unknown function.

3. INDIRECT FUZZY ADAPTIVE CONTROL

3.1 Problem formulation

Let us consider the class of SISO nonlinear systems described by the following state equations:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{13}$$

where $x = [x_1, x_2, ..., x_n]^T \in \Re^n$, $u \in \Re$ and $y \in \Re$ are respectively the system states, input and output; $f(x) \in \Re^n$, $g(x) \in \Re^n$ and $h(x) \in \Re$ are smooth functions describing the dynamic of the system. If the system has the relative degree of r ($r \leq n$), then its output can be expressed Sastry and Bodson [1989]:

$$y^{(r)} = a(x) + b(x)u \tag{14}$$

where $a(x) = L_f^r h(x)$ and $b(x) = L_g L_f^{r-1} h(x) \neq 0$ such that $L_f h(x)$ is the Lie derivative of the function $h(x)$ with respect to $f(x)$. Moreover, the particularity of the Somieski's model is seen while developing equation (11). It is possible to formulate the control gain B by a constant value:

$$B = \frac{k_d}{J_a J_z} \tag{15}$$

The control gain is thus independent of the state variables. Considering this constant, equation (14) is simplified and becomes:

$$y^{(r)} = a(x) + Bu \tag{16}$$

The aim is then to design a feedback control law to drive the system output y tracking a given reference output y_m Spooner and Passino [1996], considering the following assumptions: all the states of the system are measurable and available for feedback, the reference output $y_m(t)$ and its derivatives up to the r$^{\text{th}}$ order are measurable and bounded. In this study, $a(x)$ is an unknown nonlinear function which must be estimated to calculate the feedback linearization control law. In this situation, fuzzy system enables to find an estimation of this unknown function.

3.2 Fuzzy system

In this paper, a MISO (multi input, single output) fuzzy logic system mapping from an input vector $x = [x_1, x_2, ..., x_n]^T \in \Re^n$ to an output $\hat{a}(x) \in \Re$ is considered. Let F_i^{ki}, $k_i = 1, ..., p_i$, be the fuzzy sets defined on the i$^{\text{th}}$ input. Using the standard fuzzy systems Passino and Yurkovich [1998], the fuzzy logic system is characterized by a set of $P = \prod_{i=1}^{n} p_i$ fuzzy rules $R^1, ..., R^P$ such that:

$$R^k: \quad \text{If}(x_1\, is\, F_1^{k_1} \text{ and ... and } x_n\, is\, F_n^{k_n}) \text{ then } c_k \tag{17}$$

where c_k is the crisp output for the k$^{\text{th}}$ rule.
Using product norm to implement the *and* operation, and weighted average method for defuzzification, the output of the fuzzy system can be expressed as:

$$\hat{a}(x) = \frac{\sum_{i=1}^{P} c_i \mu_i(x)}{\sum_{i=1}^{P} \mu_i(x)} \tag{18}$$

where $\mu_i = \prod_{k=1}^{n} \mu_{F_i^{ki}}(x_k)$, $k_i \in 1, 2, ..., p_i$. Moreover, $\mu_{F_i^{ki}}(x_k)$ is the membership function of the fuzzy set $F_i^{k_i}$ and is a Gaussian function.
Moreover, (18) can be expressed by:

$$\hat{a}(x) = c^T \varsigma \tag{19}$$

where $c^T := [c_1 \ldots c_P]$ and $\varsigma^T := [\mu_1 \ldots \mu_P] / \left[\sum_{i=1}^{P} \mu_i\right]$.
The unknow function $a(x)$ can be formulated by:

$$a(x) = c^{*T} \varsigma + d(x) \tag{20}$$

where $d(x)$ is the optimal approximation error of $a(x)$ by the fuzzy system and c^* is the best value of the parameter c:

$$c^* := \arg\min_c \left[\sup_x \left| c^T \varsigma - a(x) \right| \right] \tag{21}$$

We can thus prove that the fuzzy system (17) can approximate a smooth nonlinear function with arbitrary small error if the number of fuzzy rules is large enough Kosko [1994].

The parameter error vector $\phi(t)$ represents the difference between the current estimated parameter and the best value of this parameter and is defined by:

$$\phi(t) = c(t) - c^*. \tag{22}$$

3.3 Fuzzy adaptive control

The adaptive solution consider in this paper is represented in figure 2. This algorithm uses the following adaptive control law:

$$u = u_{ce} + u_{si} \tag{23}$$

where the "certainty equivalence" control term u_{ce} is used to estimate the feedback linearization control term. The "sliding mode" control term u_{si} enables to overcome the modeling error due to the limited number of fuzzy rules.

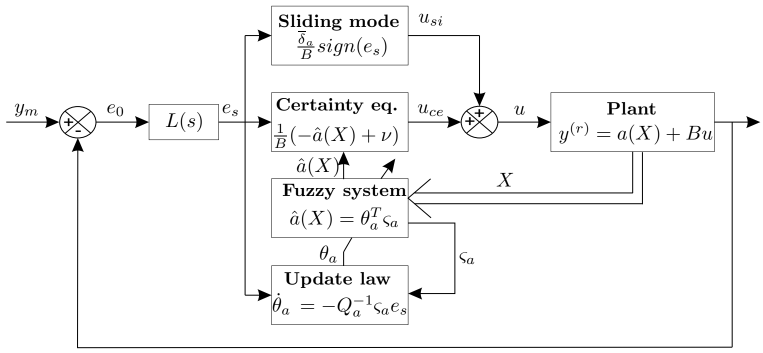

Fig. 2. Indirect Fuzzy Adaptive control Scheme

Certainty equivalence control term The "certainty equivalence" control term is defined by:

$$u_{ce} = \frac{1}{B}\left(-\hat{a}(x) + \upsilon\right) \tag{24}$$

where $\upsilon(t) := y_m^{(r)} + \eta e_s + \bar{e}_s$ and $\bar{e}_s := \dot{e}_s - e_o^{(r)}$. The tracking error is defined as $e_s := k^T e$ with $e := [e_0\ \dot{e}_0\ \ldots\ e_0^{(r-1)}]$, $k := [k_0\ \ldots\ k_{r-2}\ 1]$ and $e_0 = y_m - y$. The elements of k are chosen such that $L(s) := s^{r-1} + k_{r-2}s^{r-2} + \ldots + k_1 s + k_0$ is Hurwitz.

Sliding mode control term The "sliding mode" control term used in this control law is:

$$u_{si} = \frac{D}{B} sign(e_s) \tag{25}$$

It has been chosen to enable the proof of the stability. The constant $D \in \Re$ is defined such that:

$$|d(x)| \leq D \tag{26}$$

It represents a known bound of the error estimation due to the fuzzy system. This term is used in this control law to counteract the modeling error between the real nonlinear function $a(x)$ and its estimate $\hat{a}(x)$.

Adaptation Algorithm The estimation of the function $\hat{a}(x)$ needs to be updated to follow the right function. Then, the following fuzzy system update law is chosen:

$$\dot{c}(t) = -Q^{-1}\varsigma e_s \tag{27}$$

with Q a square positive semidefinite matrix.

3.4 Lyapunov stability

Aeronautical constraints are very strict and it is fundamental to be assured that the control solution is stable. The properties of the indirect fuzzy adaptive controller are presented in the following theorem.

Theorem 1. Stability and tracking error results: Considering the system defined in (16) and assuming the following assumption:

A1 : the error estimation due to the fuzzy system is bounded (Equation (26)).

It can be concluded that:

C1 : the plant output and its derivatives up to (r-1) order are bounded.
C2 : the control signal is bounded.
C3 : the output error e_0 will converge to zero.

Proof.

The *r-derivative* of the output error can be written as:

$$\begin{aligned} e_0^{(r)} &= y_m^{(r)} - y^{(r)} \\ &= y_m^{(r)} - a(x) - B(u_{ce} + u_{si}) \\ &= -a(x) + \hat{a}(x) - \eta e_s - \bar{e}_s - Bu_{si} \end{aligned} \tag{28}$$

The tracking error equation becomes:

$$\begin{aligned} \dot{e}_s + \eta e_s &= -Bu_{si} - a(x) + \hat{a}(x) \\ &= -Bu_{si} + (\phi^T\varsigma - d(x)) \end{aligned} \tag{29}$$

Considering the Lyapunov function candidate:

$$V = \frac{1}{2}e_s^2 + \frac{1}{2}\phi^T Q \phi \tag{30}$$

where $Q \in \Re^{d\times d}$ (d=dimϕ) is a positive definite matrix. Differentiating $V(t)$ with respect to time leads to:

$$\dot{V} = -\eta e_s^2 - Bu_{si}e_s + (\hat{a}(x) - a(x))e_s + \phi^T Q \dot{\phi} \tag{31}$$

Considering equations (22) and (27), the derivative of the parameter error vector becomes:

$$\dot{\phi} = \dot{c} \tag{32}$$

Consequently $\dot{V}$ becomes:

$$\dot{V} = -\eta e_s^2 - Bu_{si}e_s + (\hat{a}(x) - a(x))e_s - \phi^T\varsigma e_s \tag{33}$$

Equations (20) and (22) enable the following simplification:

$$\dot{V} = -\eta e_s^2 - Bu_{si}e_s + (\phi^T\varsigma - d(x))e_s - \phi^T\varsigma e_s \tag{34}$$

Now the assumptions A1 and the definition of u_{si} (equation 25) allow to write:

$$\begin{aligned} \dot{V} &= -\eta e_s^2 - Dsign(e_s)e_s - d(x)e_s \\ &\leq -\eta e_s^2 - Dsign(e_s)e_s + |d(x)|\,|e_s| \\ &\leq -\eta e_s^2 \end{aligned} \tag{35}$$

This means that $V \in \mathcal{L}_\infty$ and by definition of the Lyapunov function, $e_s \in \mathcal{L}_\infty$ and $\phi \in \mathcal{L}_\infty$.
If $G_i(s)$ is defined by:

$$G_i(s) = \frac{s^i}{L(s)} \tag{36}$$

for $i = 0, \ldots, r-1$, it is simple to show that $G_i(s)$ is stable because $L(s)$ has its $r-1$ roots in the open left half plane. Thus, the error becomes:

$$e_0^{(i)} = G_i(s)e_s \tag{37}$$

with $e_s \in \mathcal{L}_\infty$.
Then, error $e_0^{(i)}$ is bounded for $i = 0, \ldots, r-1$ and $e_0^{(k)} = y_m^{(k)} - y^{(k)}$, so the conclusion is that $y(t), \ldots, y^{(r-1)}(t)$ are bounded.

As proven above, ϕ is bounded so the "certainty equivalence" control term is bounded. Moreover, the "sliding mode" control term is bounded. The conclusion is that u is bounded.

If equation (35) is used:

$$\int_0^\infty \eta e_s^2 dt \le -\int_0^\infty V dt = V(0) - V(\infty) \tag{38}$$

then $e_s \in \mathcal{L}_2$. Moreover previous considerations show that $\hat{a}(x)$, ϕ, $d(x)$ and $\varsigma(x)$ are bounded. From (29) and the fact that e_s and u_{si} are bounded, it is obvious that $\dot{e}_s$ is bounded. Thus, by Barbalat's Lemma, the tracking error e_s will converge to zero and e_0 will converge to zero.

4. ACTIVE SHIMMY DAMPING RESULTS

4.1 Controller design

The active damping controller is applied to the NLG system with the parameters given in Somieski [1997]. The actuator parameters are chosen such that: $J_a = 0.1\,kg.m^2$ and $B_a = 0.1\,Nm/rad/s$.

The active shimmy damping controller is designed on the indirect adaptive fuzzy control algorithm discussed in the previous sections. For the problem of shimmy damping, the aim of the control law is to keep the wheel angle at 0°. Then, the reference output y_m must be set to zero.
The fuzzy system is constructed with 72 rules such that ψ_w and $\dot{\psi}_w$ are defined with three Gaussian membership functions and the other states are defined with two Gaussian membership functions. The membership functions are uniformly distributed on the whole range of each state variable.
The considered NLG model is a 5^{th} order nonlinear system with a relative degree of 3. Then, the tracking error is $e_s(t) = \ddot{e}_0 + k_1\dot{e}_0 + k_0 e_0$, with $k_0 = 0.0001$ and $k_1 = 0.014$. The particularity of this algorithm is the use of a constant value for the control gain. This constant, defined by the model parameters, is equal to $B = \dfrac{k_d}{J_a J_z} = 100$. Moreover, the simulation showing that the maximum value of the estimates $\hat{a}(x)$ is approximately equal to 10,000, it is decided to major the error estimation by 10% of this maximum value, so $D = 1,000$ is obtained.

4.2 Simulation results

To illustrate the performances of the proposed shimmy damping controller, simulations have been made with two different test scenarios.

Scenario 1: Constant ground speed, pulse disturbance In this simulation, the system is supposed to have a speed of $v = 80\,m/s$. The disturbance is a torque pulse of 1000 Nm during $0.1\,s$ acting directly on the vertical axis at the wheel level. If the damping constant of the NLG is low, $k_d = 10\,Nm/rad/s$, shimmy oscillations occur. Figure 3 shows the shimmy oscillations considering that the turning tube is kept strictly. Figure 4 shows the response of the NLG with the active damping controller in action. It is obvious that no shimmy appears, and the oscillation is damped. However, revealed in figure 4, there is a small bias angle (1.5°) during the time the disturbance is applied. The wheel angle only returns to its original position when the disturbance torque disappears. Notice that the main purpose of the designed controller is not to drive the wheel, but to avoid the shimmy oscillation. The control algorithm just needs the maximum torque of about 600 $N.m$ to effectively prevent the oscillation, while the disturbance magnitude is 1000 $N.m$. In fact, it is possible to choose the design parameters of the adaptive controller so that the wheel angle will return to its nominal position even when the disturbance exists, but in this case, the control torque must be larger than the disturbance.

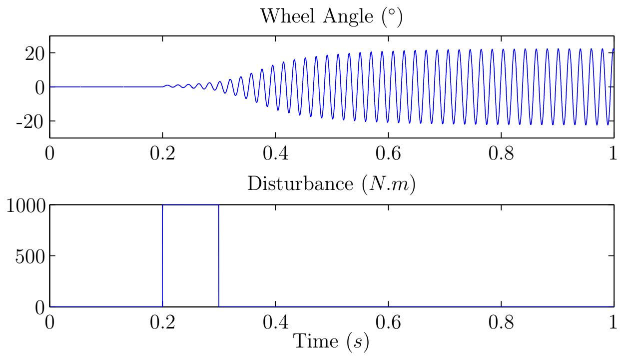

Fig. 3. Shimmy caused by a pulse disturbance

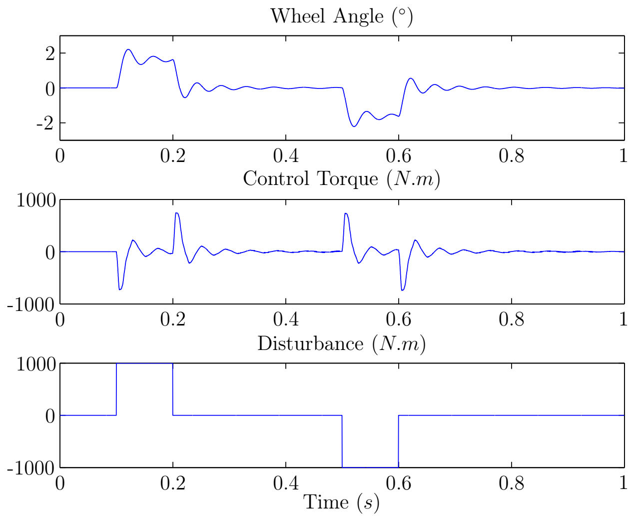

Fig. 4. Active shimmy damping result (scenario 1)

Scenario 2: Constant ground speed, random disturbance The purpose of this test is to investigate the effect of the shimmy active damping controller while the aircraft is running at a maximum speed of $80 m/s$ and considering the influence of the roughness of the runway on the tire. This aspect is modeled by a random disturbance, which is a white noise with zero mean and standard deviation of 100 $N.m$. Without any shimmy damper, this disturbance causes shimmy (Figure 5). With the proposed active damping controller (Figure 6), shimmy does not occur and the variation of the wheel angle is very small (less than $0.2°$). In practice, this small variation cannot cause any damage or malfunction to the NLG.

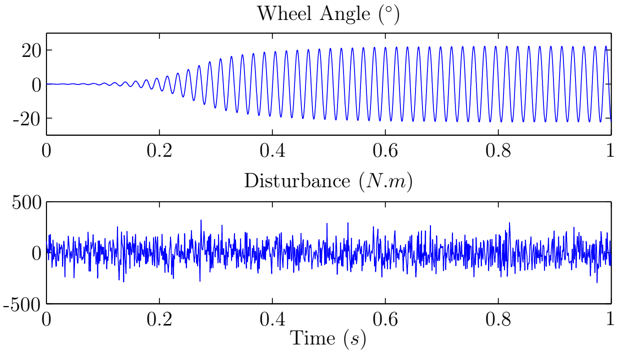

Fig. 5. Shimmy caused by random disturbances

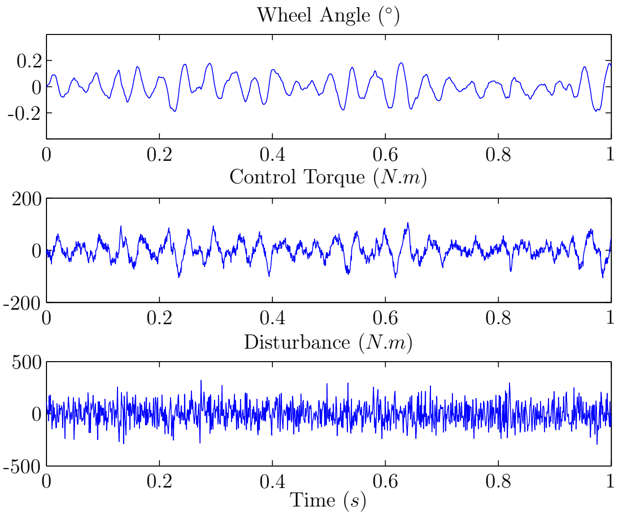

Fig. 6. Active shimmy damping result (scenario 2)

5. CONCLUSION

In this paper, the shimmy, an oscillatory phenomenon which can induce severe structural damages of an aircraft landing gear is introduced. The dynamics of the shimmy phenomenon are modeled according to the mechanical dynamics of an actuator, the torsional dynamics of the nose landing gear, and the forces and moments describing the tire's elasticity. To counteract the oscillatory phenomenon, an active shimmy damping control strategy is developed. This active solution, based on an indirect fuzzy adaptive controller enables to well damp the shimmy. This new solution uses a fuzzy logic system to estimate the unknown and nonlinear dynamics of the system. Then, the control solution consists of an adaptive term and a stabilized term based on the fuzzy estimations. At last, due to the aeronautical constraints, the stability of the algorithm is proved by using the Lyapunov theory.

The performances of the indirect fuzzy algorithm are presented in two particular conditions (pulse disturbance and random disturbance). The results are satisfying taking into consideration the achieved damping results.

However, this active damping has two main drawbacks. First, the tuning of the controller is quite difficult and the initial conditions of the update law have an important influence. Moreover, the control algorithm needs to feedback all the state variables and this requirement might not meet some practical cases. In future works, an algorithm with the feedback of the output of the system must be considered to enable the active shimmy damping.

6. ACKNOWLEDGMENTS

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http://www.dress-project.eu/

REFERENCES

I. J. M. Besselink. *Shimmy of aircraft main landing gears.* PhD thesis, Delft University of Technology, 2000.

B. Goodwine and G. Stépán. Controlling unstable rolling phenomena. *Journal of Vibration and Control*, 6:137–158, 2000.

P.R. Houlston, S.D. Garvey, and A.A. Popov. Modal control of vibration in rotating machines and other generally damped systems. *Journal of Sound and Vibration*, 302:104–116, 2006.

B. Kosko. Fuzzy systems as universal approximators. *IEEE transactions on Computers*, 43:1329–1333, 1994.

K. M. Passino and S. Yurkovich. *Fuzzy Control*. Addison-Wesley, 1998.

Shankar Sastry and Marc Bodson. *Adaptive control Stability, Convergence and Robustness*. 1989.

G. Somieski. Shimmy analysis of a simple aircraft nose landing gear model using different mathematical methods. *Aerospace Science and Technology*, 87:545–555, 1997.

J. T. Spooner and K. M. Passino. Stable adaptive control using fuzzy systems and neural networks. *IEEE Transactions on Fuzzy Systems*, 4:339–359, 1996.

G. Stépán. Chaotic motion of wheel. *Vehicle System Dynamics*, 20:341–351, 1991.