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New Approaches to H_∞ Controller Designs for Discrete T-S Fuzzy System *

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Abstract: This paper proposed new approaches to stabilization analysis and H_∞ performance for a class of discrete Takagi-Sugeno (T-S) fuzzy model. The main results given here concern their H_∞ controllers design using PDC-like control laws and nonquadratic Lyapunov functions. New relaxed conditions and linear matrix inequality-based design methods are proposed that allow outperforming previous found in the literature. Finally, an example is given to demonstrate the efficiency of the proposed approaches.

1. INTRODUCTION

Over the past decade, there have been significant research efforts devoted to stability analysis and systematic design of fuzzy control law of Takagi-Sugeno(T-S) fuzzy model(Tanaka & Wang, 2001; Zhou et al, 2005; Kim & Lee,2000 and reference therein). Some relaxed stability and stabilization conditions for T-S fuzzy models were presented to reduce the conservatism of basic conditions(Johansson, Rantzer, & Arzen, 1999). More recently, a number of works on stability analysis and control synthesis of fuzzy models based on nonquadratic Lyapunov function has appeared(Choi & Park, 2003; Guerra et al, 2002; Guerra, & Vermeiren, 2004). It was shown that, with the use of nonquadratic Lyapunov functions, less conservatism control results can be obtained than with the use of a single Lyapunov quadratic function(Guerra, & Vermeiren, 2004; Zhou, Feng, Lam & Xu, 2005). However, in(Guerra, & Vermeiren, 2004), a common additional matrix variable is introduced to obtain a PDC control law, which may be a new source of conservatism. The other work(Zhou, Feng, Lam & Xu, 2005) developed a H_∞ controller based on nonquadratic Lyapunov function with no relaxtion procedure considered.

This paper focuses on the stabilization and H_∞ performance of discrete T-S fuzzy models based on a relaxed approach towards the use of nonquadratic Lyapunov functions and PDC-like control law. A new fuzzy controller design method is proposed. It is shown that the solution of the controller design problem can be obtained by solving a set of linear matrix inequalities(LMIs), which can be implemented by using MATLAB software. The novel features of our result are that the Lyapunov function as well as control law are different from the one used in(Zhou, Lam & Zheng, 2007), which gives less conservative results.

The organization of this paper is as follows. The problem formulation and preliminary results are given in section 2. In section 3, The nonquadratic approaches based on two new Lyapunov functions and corresponding PDC-like control laws are proposed. Sufficient conditions for stabilization and H_∞ performance are presented, which will then be employed in section 4 to develop two H_∞ controllers. a numerical example is given to illustrate the effectiveness of the approaches in section 5. Finally, the remark concluding is made in section 6.

2. PROBLEM FORMULATION AND PRELIMINARIES

A affine nonlinear system can be represented as a T-S fuzzy model(Takagi & Sugeno, 1985), which is composed of s plant rules that can be represented as
Plant rule i: IF $\xi_1(k)$ is W_{i1} and $\cdots$ and $\xi_r(k)$ is W_{ir} , THEN

$$\begin{aligned} x_{k+1} &= A_i x_k + B_{1i}\omega_k + B_{2i}u_k \\ z_k &= C_i x_k + D_{1i}\omega_k + D_{2i}u_k, \quad i \in S = \{1, 2, \ldots, s\} \end{aligned}$$

where W_{ij} is a fuzzy set, $\xi_k=[\xi_1(k), \ldots, \xi_r(k)]^T$ is the premise variable vector, $x_k \in \mathcal{R}^n$ is the state, $\omega_k \in \mathcal{R}^p$ is the disturbance input and $\omega_k \in l_2[0,\infty)$, $u_k \in \mathcal{R}^m$ is the control input, $z_k \in \mathcal{R}^q$ is the regulated output, and A_i, B_{1i}, B_{2i}, C_i, D_{1i} and D_{2i} are known matrices with appropriate dimensions.

It is assumed in this paper that the premise variables ξ_k does not explicitly depend on the input variable u_k and the disturbance ω_k. Given a pair of (x_k, u_k), the final outputs of the fuzzy system are inferred as follows

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$$\Sigma: x_{k+1} = \sum_{i=1}^{s} h_i(\xi_k)(A_i x_k + B_{1i}\omega_k + B_{2i}u_k) \quad (1)$$

$$z_k = \sum_{i=1}^{s} h_i(\xi_k)(C_i x_k + D_{1i}\omega_k + D_{2i}u_k) \quad (2)$$

with $\varpi_i(\xi_k) = \prod_{j=1}^{r} W_{ij}(\xi_j(k))$; $h_i(\xi_k) = \frac{\varpi_i(\xi_k)}{\sum_{j=1}^{s}\varpi_j(\xi_k)}$ $W_{ij}(\xi_j(k))$ is the grade of membership of $\xi_j(k)$ in W_{ij}. It is easy to see that $\varpi_i(\xi_k) \geq 0$,$(i \in S)$,$\sum_{j=1}^{s}\varpi_j(\xi_k) > 0$,$\forall k$. Therefore,

$$h_i(\xi_k) \geq 0, \ (i \in S) \qquad \sum_{j=1}^{s} h_j(\xi_k) = 1 \quad (3)$$

for all k. In what follows, we will drop the argument of $h_i(\xi_k)$ for simplicity.

The objective of this paper is to design state feedback controllers such that the following specifications are met for controlled discrete T-S fuzzy system Σ.

($\Xi 1$): The closed-loop system is asymptotically stable for any fuzzy basis functions $\{h_i\}_{i=1}^{s}$ satisfying (3) with disturbance-free, i.e.$\omega_k \equiv 0$.
($\Xi 2$): The L_2-gain between the exogenous input ω_k and the regulated output z_k of the closed-loop system is less than γ, that is, for any nonzero $\omega_k \in l_2[0,\infty)$ and zero initial condition $x_0 = 0$, $\|z_k\|_2 < \gamma\|\omega_k\|_2$

In the sequel, we will refer system satisfying ($\Xi 1$) and ($\Xi 2$) to as stable with H_∞ norm bound γ. To develop the required results, the following lemmas are needed.

Lemma 1. (Zhou, Lam & Zheng, 2007): If $P_j > 0$, then

$$A_i^T P_j A_l + A_l^T P_j A_i \leq A_i^T P_j A_i + A_l^T P_j A_l$$

Lemma 2. (De Oliveira et al., 1999): With matrices of appropriated dimensions, if there is $P > 0$, such that

$$\begin{bmatrix} -\Gamma & * \\ \Psi\Phi & -\Psi - \Psi^T + P \end{bmatrix} < 0$$

then $$\Phi^T P \Phi - \Gamma < 0$$

3. H_∞ PERFORMANCE ANALYSIS

In this section, with two approaches, we will give some new relaxed conditions of stability with H_∞ norm bound γ for the system Σ.

3.1 Approach 1

For this approach, we choose the following PDC-like control law

$$u_k = -(\sum_{i=1}^{s} h_i F_i)(\sum_{i=1}^{s} h_i P_i)^{-1} x_k \quad (4)$$

where $P_i > 0$, F_i $(i \in S)$ are matrices with appropriate dimensions. Thus, the closed-loop system of Σ is given by

$$\Sigma_1^c: x_{k+1} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (G_{ij}^{\xi} x_k + B_{1i}\omega_k) \quad (5)$$

$$z_k = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (M_{ij}^{\xi} x_k + D_{1i}\omega_k) \quad (6)$$

with $G_{ij}^{\xi} = A_i - B_{2i}F_j(\sum_{i=1}^{s} h_i P_i)^{-1}$; $M_{ij}^{\xi} = C_i - D_{2i}F_j$ $(\sum_{i=1}^{s} h_i P_i)^{-1}$

Theorem 1. The closed-loop system Σ_1^c is stable with H_∞ norm bound γ, if there exist matrices $\{P_i > 0\}_{i\in S}$, $\{F_i\}_{i\in S}$ and $\{Q_{ij} : Q_{ij}^T = Q_{ij}\}_{i,j\in S}$ such that

$$\begin{bmatrix} (\bar{G}_{ij}^{\xi})^T & (\bar{M}_{ij}^{\xi})^T \\ B_{1i}^T & D_{1i}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{G}_{ij}^{\xi} & B_{1i} \\ \bar{M}_{ij}^{\xi} & D_{1i} \end{bmatrix} + Q_{ij}$$

$$- \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1s} \\ Q_{21} & Q_{22} & \cdots & Q_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{s1} & Q_{s2} & \cdots & Q_{ss} \end{bmatrix} > 0 \quad (8)$$

here $\bar{G}_{ij}^{\xi} = A_i(\sum_{i=1}^{s} h_i P_i) - B_{2i}F_j$; $\bar{M}_{ij}^{\xi} = C_i(\sum_{i=1}^{s} h_i P_i)$ $-D_{2i}F_j$

Proof. The considered Lyapunov function is

$$V_k = x_k^T(\sum_{i=1}^{s} h_i P_i)^{-1} x_k \quad (9)$$

when $\omega_k = 0$, system (5) becomes

$$x_{k+1} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j G_{ij}^{\xi} x_k \quad (10)$$

By some algebraic manipulations, the increment of V_k along the solution of (10) is given by

$$\begin{aligned} \Delta V_k|_{(10)} &= \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \\ & x_k^T\{(G_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}(G_{pq}^{\xi}) - (\sum_{i=1}^{s} h_i P_i)^{-1}\}x_k \\ &= x_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \frac{1}{2} \\ & \{(G_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}(G_{pq}^{\xi}) - 2(\sum_{i=1}^{s} h_i P_i)^{-1} \\ & +(G_{pq}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}(G_{ij}^{\xi})\}x_k \end{aligned}$$

where $h_i^+ = h_i(\xi_{k+1})$. By lemma 1, one has

$$\begin{aligned} \Delta V_k|_{(10)} &\leq x_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \frac{1}{2} \\ & \{(G_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}(G_{ij}^{\xi}) - 2(\sum_{i=1}^{s} h_i P_i)^{-1} \end{aligned}$$

$$+(G_{pq}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}(G_{pq}^{\xi})\}x_k$$
$$= x_k^T \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (\sum_{i=1}^{s} h_i P_i)^{-T}\{(\bar{G}_{ij}^{\xi})^T$$
$$(\sum_{i=1}^{s} h_i^+ P_i)^{-1}\bar{G}_{ij}^{\xi} - P_i)\}(\sum_{i=1}^{s} h_i P_i)^{-1} x_k \quad (11)$$

It follows from (7) that

$$\begin{bmatrix} (\bar{G}_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}\bar{G}_{ij}^{\xi} - P_i + (\bar{M}_{ij}^{\xi})^T\bar{M}_{ij}^{\xi} & (\bar{G}_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}B_{1i} + (\bar{M}_{ij}^{\xi})^T D_{1i} \\ B_{1i}^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}\bar{G}_{ij}^{\xi} + D_{1i}^T\bar{M}_{ij}^{\xi} & B_{1i}^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}B_{1i} + D_{1i}^T D_{1i} - \gamma^2 I \end{bmatrix} + Q_{ij} < 0 \quad (12)$$

Now partition matrices for all $i, j \in S$ conformably with the first matrix in (12)

$$Q_{ij} = \begin{bmatrix} Q_{ij}^{(11)} & Q_{ij}^{(12)} \\ Q_{ij}^{(21)} & Q_{ij}^{(22)} \end{bmatrix} \quad (13)$$

Inequalities (12) and (13) imply that

$$(\bar{G}_{ij}^{\xi})^T(\sum_{i=1}^{s} h_i^+ P_i)^{-1}\bar{G}_{ij}^{\xi} - P_i + Q_{ij}^{(11)} < 0 \quad (14)$$

It follows from (11) and (14) that

$$\Delta V_k|_{(10)} \le -x_k^T \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j (\sum_{i=1}^{s} h_i P_i)^{-T} Q_{ij}^{(11)} (\sum_{i=1}^{s} h_i P_i)^{-1} x_k$$
$$= -x_k^T \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j \bar{Q}_{ij}^{(11)} x_k$$
$$= -\begin{bmatrix} h_1 x_k \\ h_2 x_k \\ \vdots \\ h_s x_k \end{bmatrix}^T \begin{bmatrix} \bar{Q}_{11}^{(11)} & \bar{Q}_{12}^{(11)} & \cdots & \bar{Q}_{1s}^{(11)} \\ \bar{Q}_{21}^{(11)} & \bar{Q}_{22}^{(11)} & \cdots & \bar{Q}_{2s}^{(11)} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Q}_{1s}^{(11)} & \bar{Q}_{2s}^{(11)} & \cdots & \bar{Q}_{ss}^{(11)} \end{bmatrix} \begin{bmatrix} h_1 x_k \\ h_2 x_k \\ \vdots \\ h_s x_k \end{bmatrix} \quad (15)$$

here $\bar{Q}_{ij}^{(11)} = (\sum_{i=1}^{s} h_i P_i)^{-T} Q_{ij}^{(11)} (\sum_{i=1}^{s} h_i P_i)^{-1}$.

By (8) and (13), the following inequality holds

$$\begin{bmatrix} Q_{11}^{(11)} & Q_{12}^{(11)} & \cdots & Q_{1s}^{(11)} \\ Q_{21}^{(11)} & Q_{22}^{(11)} & \cdots & Q_{2s}^{(11)} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1s}^{(11)} & Q_{2s}^{(11)} & \cdots & Q_{ss}^{(11)} \end{bmatrix} > 0 \quad (16)$$

Pre-multiplying $diag\{(\sum_{i=1}^{s} h_i P_i)^{-T}, (\sum_{i=1}^{s} h_i P_i)^{-T}, \cdots, (\sum_{i=1}^{s} h_i P_i)^{-T}\}$ and post-multiplying $diag\{(\sum_{i=1}^{s} h_i P_i)^{-1}, (\sum_{i=1}^{s} h_i P_i)^{-1}, \cdots, (\sum_{i=1}^{s} h_i P_i)^{-1}\}$ to (16) gives

$$\begin{bmatrix} \bar{Q}_{11}^{(11)} & \bar{Q}_{12}^{(11)} & \cdots & \bar{Q}_{1s}^{(11)} \\ \bar{Q}_{21}^{(11)} & \bar{Q}_{22}^{(11)} & \cdots & \bar{Q}_{2s}^{(11)} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Q}_{1s}^{(11)} & \bar{Q}_{2s}^{(11)} & \cdots & \bar{Q}_{ss}^{(11)} \end{bmatrix} > 0 \quad (17)$$

(15) combines together with (17) implies that system(10) is stable. Next, our objective is to derive the H_∞ norm bound, i.e.$\|z_k\|_2 < \gamma\|\omega_k\|_2$. Let

$$J_N = \sum_{k=0}^{N-1}(z_k^T z_k - \gamma^2 \omega_k^T \omega_k)$$

For any nonzero $\omega_k \in l_2[0, \infty)$ and zero initial condition $x_0 = 0$, one has

$$J_N = \sum_{k=0}^{N-1}(z_k^T z_k - \gamma^2 \omega_k^T \omega_k)$$
$$= \sum_{k=0}^{N-1}(z_k^T z_k - \gamma^2 \omega_k^T \omega_k + \Delta V_k|_{(5)}) - V_N$$

where $\Delta V_k|_{(5)}$ defines the increment of V_k along system (5). It is noted that

$$z_k^T z_k - \gamma^2 \omega_k^T \omega_k$$
$$= \varphi_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \left\{ \begin{bmatrix} (M_{ij}^{\xi})^T \\ D_{1i}^T \end{bmatrix} \begin{bmatrix} M_{pq}^{\xi} & D_{1p} \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \varphi_k$$

$$\Delta V_k|_{(5)} = \varphi_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \left\{ \begin{bmatrix} (G_{ij}^{\xi})^T \\ B_{1i}^T \end{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} \begin{bmatrix} G_{pq}^{\xi} & B_{1p} \end{bmatrix} - \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & 0 \end{bmatrix} \right\} \varphi_k$$

where $\varphi_k = [x_k \ \ \omega_k]^T$, then one has

$$J_N \le \sum_{k=0}^{N-1} \varphi_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \frac{1}{2}$$
$$\left\{ \begin{bmatrix} (G_{ij}^{\xi})^T & (M_{ij}^{\xi})^T \\ B_{1i}^T & D_{1i}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} G_{pq}^{\xi} & B_{1p} \\ M_{pq}^{\xi} & D_{1p} \end{bmatrix} \right.$$
$$+ \begin{bmatrix} (G_{pq}^{\xi})^T & (M_{pq}^{\xi})^T \\ B_{1p}^T & D_{1p}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} G_{ij}^{\xi} & B_{1i} \\ M_{ij}^{\xi} & D_{1i} \end{bmatrix}$$
$$\left. -2 \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \varphi_k$$

By lemma 1, the following inequality holds

$$J_N \le \sum_{k=0}^{N-1} \varphi_k^T \sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{p=1}^{s}\sum_{q=1}^{s} h_i h_j h_p h_q \frac{1}{2}$$

$$\left\{ \begin{bmatrix} (G_{ij}^{\xi})^T & (M_{ij}^{\xi})^T \\ B_{1i}^T & D_{1i}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} G_{ij}^{\xi} & B_{1i} \\ M_{ij}^{\xi} & D_{1i} \end{bmatrix} \right.$$

$$+ \begin{bmatrix} (G_{pq}^{\xi})^T & (M_{pq}^{\xi})^T \\ B_{1p}^T & D_{1p}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} G_{pq}^{\xi} & B_{1p} \\ M_{pq}^{\xi} & D_{1p} \end{bmatrix}$$

$$\left. -2 \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \varphi_k$$

$$= \sum_{k=0}^{N-1} \varphi_k^T \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-T} & 0 \\ 0 & I \end{bmatrix}$$

$$\left\{ \begin{bmatrix} (\bar{G}_{ij}^{\xi})^T & (\bar{M}_{ij}^{\xi})^T \\ B_{1i}^T & D_{1i}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{G}_{ij}^{\xi} & B_{1i} \\ \bar{M}_{ij}^{\xi} & D_{1i} \end{bmatrix} \right.$$

$$\left. - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right\} \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \varphi_k \tag{18}$$

It follows from (7) and (18) that

$$J_N \leq - \sum_{k=0}^{N-1} \varphi_k^T \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j$$
$$\begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-T} & 0 \\ 0 & I \end{bmatrix} Q_{ij} \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \varphi_k$$
$$= - \sum_{k=0}^{N-1} \begin{bmatrix} h_1 \varphi_k \\ h_2 \varphi_k \\ \vdots \\ h_s \varphi_k \end{bmatrix}^T \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \cdots & \bar{Q}_{1s} \\ \bar{Q}_{21} & \bar{Q}_{22} & \cdots & \bar{Q}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Q}_{1s} & \bar{Q}_{2s} & \cdots & \bar{Q}_{ss} \end{bmatrix} \begin{bmatrix} h_1 \varphi_k \\ h_2 \varphi_k \\ \vdots \\ h_s \varphi_k \end{bmatrix} \tag{19}$$

where

$$\bar{Q}_{ij} = \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-T} & 0 \\ 0 & I \end{bmatrix} Q_{ij} \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-1} & 0 \\ 0 & I \end{bmatrix}$$
$$= \begin{bmatrix} (\sum_{i=1}^{s} h_i P_i)^{-T} Q_{ij}^{(11)} (\sum_{i=1}^{s} h_i P_i)^{-1} & * \\ Q_{ij}^{(21)} \sum_{i=1}^{s} h_i P_i)^{-1} & Q_{ij}^{(22)} \end{bmatrix}$$
$$= \begin{bmatrix} \bar{Q}_{ij}^{(11)} & * \\ \bar{Q}_{ij}^{(21)} & Q_{ij}^{(22)} \end{bmatrix} \quad i, j \in S$$

Pre-multiplying $diag\{(\sum_{i=1}^{s} h_i P_i)^{-T}, I, (\sum_{i=1}^{s} h_i P_i)^{-T}, I, \cdots, (\sum_{i=1}^{s} h_i P_i)^{-T}, I\}$, post-multiplying $diag\{(\sum_{i=1}^{s} h_i P_i)^{-1}, I, (\sum_{i=1}^{s} h_i P_i)^{-1}, I, \cdots, (\sum_{i=1}^{s} h_i P_i)^{-1}, I\}$ to (8) gives

$$\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \cdots & \bar{Q}_{1s} \\ \bar{Q}_{21} & \bar{Q}_{22} & \cdots & \bar{Q}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Q}_{1s} & \bar{Q}_{2s} & \cdots & \bar{Q}_{ss} \end{bmatrix} > 0 \tag{20}$$

(19) combines with (20) implies that for any N, $J_N < 0$, which further gives $\|z_k\|_2 < \gamma \|\omega_k\|_2$, for any nonzero $\omega_k \in l_2[0, \infty)$, $z_k \in l_2[0, \infty)$ and $x_0 = 0$. The proof of theorem 1 is completed.

3.2 Approach 2

Now, reconsider the control law as follows

$$u_k = -(\sum_{i=1}^{s} h_i F_i)(\sum_{i=1}^{s} h_i G_i)^{-1} x_k \tag{21}$$

which leads to the closed-loop system of Σ is

$$\Sigma_2^c : x_{k+1} = \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j (Y_{ij}^{\xi} x_k + B_{1i} \omega_k) \tag{22}$$

$$z_k = \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j (W_{ij}^{\xi} x_k + D_{1i} \omega_k) \tag{23}$$

with $Y_{ij}^{\xi} = A_i - B_{2i} F_j (\sum_{i=1}^{s} h_i G_i)^{-1}; W_{ij}^{\xi} = C_i - D_{2i} F_j (\sum_{i=1}^{s} h_i G_i)^{-1}$.
If the Lyapunov function is chosen as

$$V_k = x_k^T (\sum_{i=1}^{s} h_i G_i)^{-T} (\sum_{i=1}^{s} h_i P_i) (\sum_{i=1}^{s} h_i G_i)^{-1} x_k \tag{24}$$

then, with the idea of proof in theorem 1, the following result can be obtained easily.

Theorem 2. The closed-loop system Σ_2^c is stable with H_∞ norm bound γ, if there exist matrices $\{P_i > 0\}_{i \in S}$, $\{G_i\}_{i \in S}$, $\{F_i\}_{i \in S}$ and $\{Q_{ij} : Q_{ij}^T = Q_{ij}\}_{i,j \in S}$ such that

$$Q_{ij} + \begin{bmatrix} (\bar{Y}_{ij}^{\xi})^T (\sum_{i=1}^{s} h_i^+ G_i)^{-T} & (\bar{W}_{ij}^{\xi})^T \\ B_{1i}^T (\sum_{i=1}^{s} h_i^+ G_i)^{-T} & D_{1i}^T \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{s} h_i^+ P_i & 0 \\ 0 & I \end{bmatrix}$$
$$\begin{bmatrix} (\sum_{i=1}^{s} h_i^+ G_i)^{-1} \bar{Y}_{ij}^{\xi} & (\sum_{i=1}^{s} h_i^+ G_i)^{-1} B_{1i} \\ \bar{W}_{ij}^{\xi} & D_{1i} \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1s} \\ Q_{21} & Q_{22} & \cdots & Q_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{s1} & Q_{s2} & \cdots & Q_{ss} \end{bmatrix} > 0 \tag{26}$$

where

$$\bar{Y}_{ij}^{\xi} = A_i (\sum_{i=1}^{s} h_i G_i) - B_{2i} F_j; \ \bar{W}_{ij}^{\xi} = C_i (\sum_{i=1}^{s} h_i G_i) - D_{2i} F_j$$

Proof. The procedure is similar to that one in theorem 1 and is thus omitted.

Remark 1. it is necessary to check first the existence of $(\sum_{i=1}^{s} h_i G_i)^{-1}$ with approach 2, which has been done in(Guerra & Vermeiren, 2004).

Remark 2. In Theorem 1 and Theorem 2, we have chosen a nonquadratic Lyapunov function which is the same as that given in (Guerra & Vermeiren, 2004).

4. CONTROLLER DESIGN

Theorem 3. If there exist appropriate dimension matrices $\{P_i > 0\}_{i\in S}$, $\{Q_{ij}^{(11)}\}_{i,j\in S}$, $\{Q_{ij}^{(21)}\}_{i,j\in S}$, $\{Q_{ij}^{(22)}\}_{i,j\in s}$ and $\{F_i\}_{i\in S}$ such that

$$\begin{bmatrix} -P_i + Q_{ij}^{(11)} & * & * & * \\ Q_{ij}^{(21)} & -\gamma^2 I + Q_{ij}^{(22)} & * & * \\ A_i P_l - B_{2i}F_j & B_{1i} & -P_\alpha & * \\ C_i P_l - D_{2i}F_j & D_{1i} & 0 & -I \end{bmatrix} < 0$$

$$i, j, l, \alpha \in S \qquad (27)$$

$$\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1s} \\ Q_{21} & Q_{22} & \cdots & Q_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{s1} & Q_{s2} & \cdots & Q_{ss} \end{bmatrix} > 0 \qquad (28)$$

where

$$Q_{ij} = \begin{bmatrix} Q_{ij}^{(11)} & * \\ Q_{ij}^{(21)} & Q_{ij}^{(22)} \end{bmatrix} \qquad (29)$$

then the closed-loop system Σ_1^c is stable with H_∞ norm bound γ.

Proof. With (27), it follows from Schur complement that

$$\begin{bmatrix} (\bar{G}_{ij}^{\xi})^T & (\bar{M}_{ij}^{\xi})^T \\ B_{1i}^T & D_{1i}^T \end{bmatrix} \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ P_i)^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{G}_{ij}^{\xi} & B_{1i} \\ \bar{M}_{ij}^{\xi} & D_{1i} \end{bmatrix} + Q_{ij}$$

$$- \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \qquad i, j \in S$$

The result then follows from theorem 2.

Theorem 4. If there exist appropriate dimension matrices $\{P_i > 0\}_{i\in S}$, $\{Q_{ij}^{(11)}\}_{i,j\in S}$, $\{Q_{ij}^{(21)}\}_{i,j\in S}$, $\{Q_{ij}^{(22)}\}_{i,j\in S}$, $\{G_i\}_{i\in S}$ and $\{F_i\}_{i\in S}$ such that

$$\begin{bmatrix} -P_i + Q_{ij}^{(11)} & * & * & * \\ Q_{ij}^{(21)} & -\gamma^2 I + Q_{ij}^{(22)} & * & * \\ A_i G_l - B_{2i}F_j & B_{1i} & -G_\alpha - G_\alpha^T + P_\alpha & * \\ C_i G_l - D_{2i}F_j & D_{1i} & 0 & -I \end{bmatrix} < 0$$

$$i, j, l, \alpha \in S \qquad (30)$$

$$\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1s} \\ Q_{21} & Q_{22} & \cdots & Q_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{s1} & Q_{s2} & \cdots & Q_{ss} \end{bmatrix} > 0 \qquad (31)$$

where

$$Q_{ij} = \begin{bmatrix} Q_{ij}^{(11)} & * \\ Q_{ij}^{(21)} & Q_{ij}^{(22)} \end{bmatrix} \qquad (32)$$

then the closed-loop system Σ_2^c is stable with H_∞ norm bound γ.

Proof. By lemma 2, let

$$\Psi = \begin{bmatrix} \sum_{i=1}^{s} h_i^+ G_i & 0 \\ 0 & I \end{bmatrix}$$

with (30), one has

$$Q_{ij} + \begin{bmatrix} (\bar{Y}_{ij}^{\xi})^T (\sum_{i=1}^{s} h_i^+ G_i)^{-T} & (\bar{W}_{ij}^{\xi})^T \\ B_{1i}^T (\sum_{i=1}^{s} h_i^+ G_i)^{-T} & D_{1i}^T \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{s} h_i^+ P_i & 0 \\ 0 & I \end{bmatrix}$$

$$\times \begin{bmatrix} (\sum_{i=1}^{s} h_i^+ G_i)^{-1} \bar{Y}_{ij}^{\xi} & (\sum_{i=1}^{s} h_i^+ G_i)^{-1} B_{1i} \\ \bar{W}_{ij}^{\xi} & D_{1i} \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0$$

The result then follows from Theorem 2.

Remark 3. The conditions in Theorem 3 can be obtained by letting $G_i = P_i$ in Theorem 4. Thus Theorem 4 has less conservatism than Theorem 3.

5. NUMERICAL EXAMPLE

To illustrate the proposed methods, a design example is worked out. The system under consideration is a nonlinear system, which is similar to that one used in(Guerra, & Vermeiren, 2004).

$$x_{k+1}^{(1)} = x_k^{(1)} - x_k^{(1)} x_k^{(2)} + [5 + x_k^{(1)}]u_k + 0.5\omega_k \quad (33)$$

$$x_{k+1}^{(2)} = -x_k^{(1)} - 0.5x_k^{(2)} + 2x_k^{(1)} u_k - 0.1\omega_k \quad (34)$$

$$z_k = -0.1x_k^{(1)} - 0.5x_k^{(2)} + 0.5u_k + 0.1\omega_k \quad (35)$$

defining $x_k = [x_k^{(1)} \;\; x_k^{(2)}]$. assum that $x_k^{(1)} \in [-\beta, \;\beta]$, the nonlinear system (33) – (35) can be exactly represented by T-S model as

Plant rule 1: IF $x_k^{(1)}$ is about χ_1, THEN

$$x_{k+1} = A_1 x_k + B_{11}\omega_k + B_{21}u_k$$
$$z_k = C_1 x_k + D_{11}\omega_k + D_{21}u_k$$

Plant rule 2: IF $x_k^{(1)}$ is about χ_2, THEN

$$x_{k+1} = A_2 x_k + B_{12}\omega_k + B_{22}u_k$$
$$z_k = C_2 x_k + D_{12}\omega_k + D_{22}u_k$$

with

$$\chi_1 = \frac{x_k^{(1)} + \beta}{2\beta}; \quad \chi_2 = 1 - \chi_1; \quad A_1 = \begin{bmatrix} 1 & -\beta \\ -1 & -0.5 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} 1 & \beta \\ -1 & -0.5 \end{bmatrix}; \quad B_{11} = B_{12} = \begin{bmatrix} -0.03 & 0.01 \\ 0 & 0.01 \end{bmatrix};$$

$$B_{21} = \begin{bmatrix} 5+\beta \\ 2\beta \end{bmatrix}; B_{22} = \begin{bmatrix} 5-\beta \\ -2\beta \end{bmatrix}; C_1 = C_2 = [\,-0.1 \;\; -0.05\,];$$

$$D_{11} = D_{12} = [\,-0.1 \;\; -0.05\,]; D_{21} = D_{22} = 0.5;$$

Let $\beta = 1$ and $\gamma = 0.5$. Using MATLAB LMI Toolbox to solve the LMIs in Theorem 3, a feasible set of solution obtained as follows:

$F_1 = [\,0.6479 \;\; -0.7043\,]$; $\quad F_2 = [\,0.3379 \;\; 0.5238\,]$

$$P_1 = \begin{bmatrix} 4.4852 & -2.9007 \\ -2.9007 & 5.1014 \end{bmatrix}; \quad P_2 = \begin{bmatrix} 4.4683 & -2.9029 \\ -2.9029 & 5.1151 \end{bmatrix}$$

To illustrate the behavior of the control action, Fig.1 depicts the trajectories of the closed-loop system with starting point $x_0 = [-0.9 \;\; 1]^T$. Fig.2 shows that the disturbance ω_k and the output response z_k.

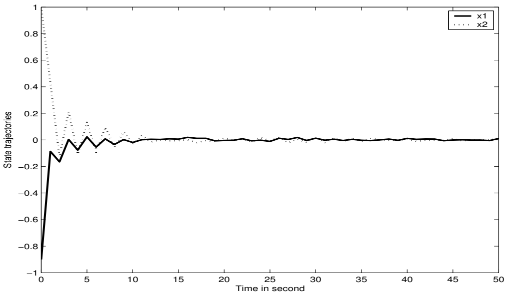

Fig.1 The state trajectories of closed-loop system with control law in Theorem 3

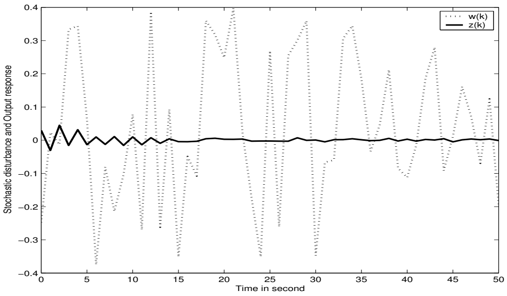

Fig.2 The disturbance ω_k and the output response z_k

The same operation is done to solve the LMIs in Theorem 4, a feasible set of solution is:

$F_1 = [\,0.7044\ -0.8398\,]; F_2 = [\,0.4574\ 0.4432\,]$

$$G_1 = \begin{bmatrix} 5.1554 & -3.1881 \\ -3.1213 & 5.6781 \end{bmatrix}; \ G_2 = \begin{bmatrix} 4.9387 & -3.3358 \\ -3.0070 & 5.8254 \end{bmatrix}$$

Fig.3 depicts the trajectories of the closed-loop system with starting point $x_0 = [-0.9\ \ 1]^T$. Fig.4 show the disturbance ω_k and the output response z_k.

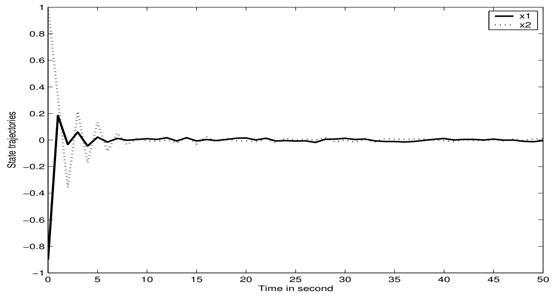

Fig.3 The state trajectories of closed-loop system with control law in Theorem 4

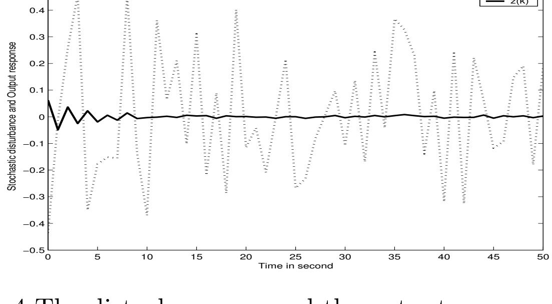

Fig.4 The disturbance ω_k and the output response z_k

Table 1 summarizes the minimum values of the H_∞ norm bound γ_{min} by solving the LMIs in Theorem 3 and Theorem 4 with a given fixed $\beta > 0$. It is noted from Table 1 that for a given fixed $\beta > 0$, the controllers obtained from Theorem 3 and Theorem 4 can guarantee a smaller minimum value of the H_∞ performance level than that one given in (Zhou, Lam & Zheng, 2007). Judging from this example, one can conclude that the results of this paper are of less conservatism than that one in (Zhou, Lam & Zheng, 2007).

Table 1
γ_{min} computed by Th.3 and Th.4 for different β

β	γ_{min}in th.3	γ_{min}in th.4	γ_{min}in(Zhou'07)
0.01	0.0167	0.0167	0.0167
0.10	0.0169	0.0169	0.0169
0.50	0.0184	0.0184	0.0192
1.00	0.054	0.053	0.3322
1.01459	0.065	0.062	99.9499
1.45	infeasible	infeasible	infeasible

6. CONCLUSION

This paper presents stabilization analysis and H_∞ performance for a class of discrete fuzzy systems. By using nonquadratic Lypunov functions associated with PDC-like control law, several relaxed results are obtained based on LMIs. Finally, a numerical example is given to illustrate the main results. The results can also be easily extended to the systems with uncertainties and time-delay.

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