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Constraint Control of Recycle Systems with Input Multiplicities

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Abstract: Recycle systems exhibit steady state input multiplicities due to interactions of the units, even though individual units are quite simple. When handling constraints, if prospective constraint variables show such nonlinearity, control problems may arise because the steady state gain changes its sign. Using the reactor/separator system with two material recycles as a process example, a robust constraint handling controller is designed by confining the input into the large gain directions. Such directions are obtained as the Pareto optimal front of the multi-objective optimization problem which minimizes energy consumption in each unit. Performance of the designed controller is demonstrated through simulations.

Keywords: Process control; nonlinear systems; instability; optimization; predictive control

1. INTRODUCTION

Recycle systems are very common in chemical plants, where integration of units is favored for economic and environmental reasons. It has been known that control system design for processes with recycle presents some specific difficulties, because all the units linked by recycle streams must be accounted for simultaneously [Papadourakis et al. (1987)].

In industrial practices, well developed linear control techniques such as PID control and linear model predictive control have been widely used. These linear control techniques suffice for chemical processes which are operated in a relatively small operating region where the assumptions of linearity hold. Linear model predictive control is most often designed for maximum profitability, in which case, control system design only around some fixed process constraints is required. For recycle systems, although some specific difficulties arise, linear control techniques can achieve satisfactory performance as long as the operating range is limited and the plant wide perspective is retained in the control system design [Luyben (1994); Wu and Yu (1996); Larsson et al. (2003)].

Meanwhile, with growing concern for the environment and ever increasing global competition, chemical processes are required to be more efficient, and forced to operate in regions where the assumptions of linearity tend to break down. One of the challenges in process control under these circumstances is that control systems have to be robust and flexible enough to cope with nonlinearities; extra design consideration will be needed if a linear control technique is to be utilized.

Among the nonlinearities exhibited by chemical processes, steady state input multiplicity is known to pose one of the most difficult control problems [Koppel (1982)], because there are more than one set of inputs associated with a given set of outputs, which implies that steady state gain (in multivariable systems, determinant of the steady state gain matrix) changes its sign and a linear controller with integral action becomes unstable where the signs of the controller gain and the process gain are different [Morari (1983)]. As process examples which exhibit input multiplicities, chemical reactors with specific reaction kinetics are relatively well known [Dash and Koppel (1989); Sistu and Bequette (1995); Chen et al. (1995)], while Dash and Koppel (1989) pointed out that the use of a recycle in a simple process flowsheet can also cause this nonlinearity even when the individual process units are quite simple.

In this paper, it is shown that a recycle system, which is represented by the reactor/separator system with two material recycles [Tyreus and Luyben (1993)], indeed exhibits steady state input multiplicities when the system is to be operated over a wider range, and their implication for constraint handling control is elaborated. Nonlinear control techniques may be applied to this kind of processes [Sistu and Bequette (1995); Chen et al. (1995); Seki and Morari (1997)], but Skogestad (2000) showed that self-optimizing control, which is basically linear, is quite efficient in handling processes with input multiplicity, so that a practical solution on the basis of self-optimizing control is introduced and performances of the designed controller are demonstrated through simulations.

2. REACTOR/SEPARATOR SYSTEM WITH MATERIAL RECYCLE

2.1 Description of the Process with Self-optimizing Control

Fig.1 shows the reactor/separator system which consists of a liquid phase CSTR(continuous stirred tank reactor) and two distillation columns interconnected with two material recycles [Tyreus and Luyben (1993)]. The fresh feeds containing pure A and B are fed to the reactor where the irreversible reaction A+B→C takes place. The reactor effluent is sent to the distillation column train where the unconverted reactants A, B and the product C are separated; the unconverted reactants are recycled back to the CSTR, while the product C is withdrawn from the system. The specific design data and basic control system design used for this study can be found in Seki and Naka (2007).

A nonlinear process model in the form of an ordinary differential equation:

$$\dot{x} = f(x, u) \tag{1}$$

$$y = h(x, u) \tag{2}$$

has been developed and used for control system design, where x is the state variable, u is the input, and y is the measurement.

One of the fresh feeds, F_B, is chosen as an independent variable to directly specify the production rate, while the product compositions $x_{A,2}$ and $x_{B,2}$ are specified at some fixed values and controlled using the composition measurements. The regulatory loops which control inventories and compositions are configured using multi-loop PI controllers with the pairings shown in Fig.1.

On top of the regulatory control system, self-optimizing control [Skogestad (2000)] has been realized by manipulating the remaining degree-of-freedom: the fresh feed F_A and the two recycle streams B_1, D_2 appropriately. For that purpose, the following steady state optimization problem in terms of energy consumption for a fixed throughput is solved:

$$\min_{x,u} J(x, u) = V_1 + V_2 \tag{3}$$

subject to

$$f(x, u) = 0, \tag{4}$$

$$y_R = y_R^{set}, \tag{5}$$

where y_R is the controlled variable in the regulatory control system and y_R^{set} is its setpoint. After finding the optimal operation u^* for various feed rates, correlation between the operational variables F_A, B_1, D_2 and the manipulated variables of the regulatory controllers are evaluated to obtain

$$\begin{aligned} \tilde{F}_A &= a_1 L_1 + b_1 \\ \tilde{B}_1 &= a_2 V_1 + b_2 \\ \tilde{D}_2 &= a_3 V_2 + b_3, \end{aligned} \tag{6}$$

where the coefficients $a_i, b_i, \quad (i = 1, 2, 3)$ are determined through the linear regression of these variables, and $\tilde{(\cdot)}$ denotes the variable is that of the self-optimizing controller.

2.2 Nonlinearity of Recycle Processes - Steady State Input Multiplicities

Recycle processes tend to exhibit nonlinearities, namely steady state input multiplicities, due to interactions of the units, even though nonlinearity of each isolated unit is not very strong. In the process example, increase of

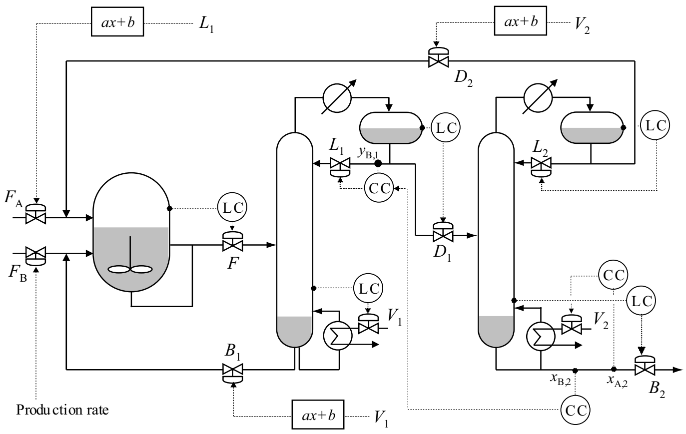

Fig. 1. Reactor/separator system with two material recycles.

the recycle flows may lead to increased consumption of the species in the reactor and increased load on the separators simultaneously. These competing effects result in complicated behavior such as input multiplicity.

Here, a new manipulated variable vector $\delta = (\delta_{F_A}\ \delta_{B_1}\ \delta_{D_2})^T$ is defined as deviation from the self-optimizing control:

$$u_{op} = \tilde{u}_{op} + \delta, \tag{7}$$

where $u_{op} = (F_A\ B_1\ D_2)^T, \tilde{u}_{op} = (\tilde{F}_A\ \tilde{B}_1\ \tilde{D}_2)^T$.

For illustrative purposes, the input δ is confined to the 2-dimensional space spanned by the two input singular vectors v_1 and v_2 of the steady state gain matrix G between δ and $(V_1\ V_2)^T$ around the nominal operating point:

$$\begin{aligned} G &= U \Sigma V^T, \\ U &= (u_1\ u_2), \qquad u_i \in \Re^2 \\ V &= (v_1\ v_2\ v_3), \qquad v_i \in \Re^3 \end{aligned} \tag{8}$$

and the steady state responses of V_1 and V_2 are shown as contour plots in Fig. 2 for various values of the input vector δ:

$$\delta = \theta_1 v_1 + \theta_2 v_2, \qquad \theta_1, \theta_2 \in \Re.$$

In fact, the output singular vectors are $u_1 = (1/\sqrt{2}\ -1/\sqrt{2})^T$, $u_2 = (1/\sqrt{2}\ 1/\sqrt{2})^T$, and the second largest singular value is $\sigma_2 = 0$, because at the nominal operating condition where $\theta_1 = \theta_2 = 0$, $V_1 + V_2$ is minimized. There is a strong directionality in the steady state responses, which leads to existence of input multiplicities; in the direction of u_2, which reduces the energy consumptions of the two columns simultaneously, the gain is small and easily changes its sign.

It should be noted that the manipulated variable choice (7) is not the origin of the input multiplicity; the intrinsic nature of the process that the minimum for $V_1 + V_2$ is unconstrained is the root cause. Even if F_A, B_1 and D_2 are directly chosen as the manipulated variables, the input multiplicity still persists in the input/output relation between these manipulated variables and V_1, V_2. Even a nonlinear control scheme which tries to regulate $V_1 + V_2$ at its unconstrained minimum using $V_1 + V_2$ directly as a controlled variable would encounter a difficult problem, whereas the self-optimizing control that is constructed by

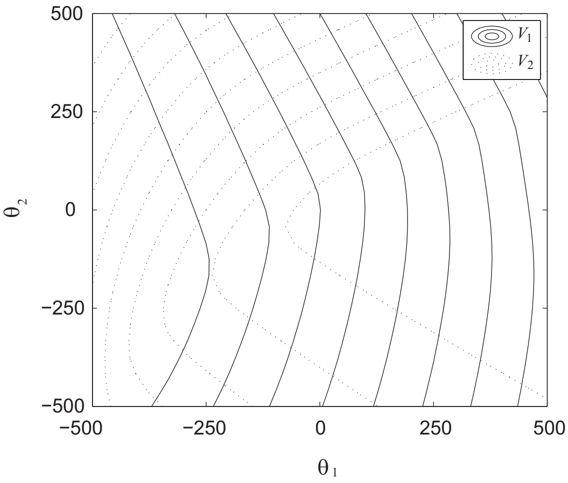

Fig. 2. Contour plots for the steady state responses for the energy consumptions of the two columns. Solid: V_1, dotted: V_2

the simple ratio scheme (6) is readily able to keep the process around the unconstrained optimum.

Now, consider a control scenario in which mode of operations may be switched between feed maximization and energy minimization, according to economical and other situations. Energy minimization can be easily achieved by the self-optimizing control, while in the case of feed maximization, the vapor boilups V_1, V_2 and the reflux flows L_1, L_2 would be the prospective constraint variables which limit the production rate. For feed maximization, regulation of these variables around their constraints are required and the input multiplicities may cause a severe control problem, since the sign of the determinant of the steady state gain matrix changes in the operating region.

3. CONTROL SYSTEM DESIGN

3.1 Restriction on Input Directions

In the context of the linear control theory, the problem of input multiplicities may be related to ill-conditioned multivariable systems. With the well-known distillation column composition control problem, simultaneous regulation of the top and bottom compositions poses a severe robustness problem [Skogestad et al. (1988)]. In such a case, directions of the manipulated variables may be restricted in the larger gain direction.

Following the same line, we may give up manipulating the operational variable vector in the direction which simultaneously reduces the energy consumption of the two distillation columns.

The first idea is to confine the movement of δ into the direction of the input singular vector v_1 corresponding to the largest singular value, which has been obtained through the singular value decomposition of the gain matrix G shown in (8). Describing the operational vector δ as

$$\delta = \theta v_1, \qquad \theta \in \Re, \tag{9}$$

steady state responses of the prospective constraint variables are shown in Fig. 3 for various operating conditions

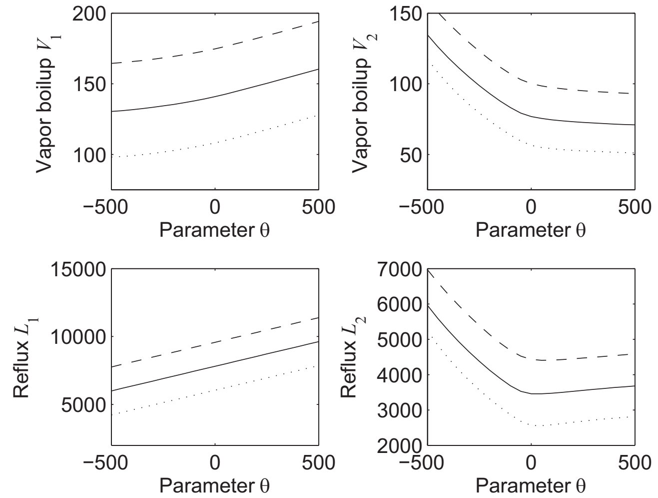

Fig. 3. Steady state relation between θ and prospective constraint variables. Input direction (9) obtained from the steady state gain analysis. Dotted: $F_B = 0.8\bar{F}_B$, solid: $F_B = \bar{F}_B$, dashed:$F_B = 1.2\bar{F}_B$.

(in the figure, $\bar{F}_B$ is the nominal feed rate). Although input multiplicities are not found in the responses of V_1, V_2 and L_1, the sign of the gain changes with L_2. The sign change in one of the elements of the steady state gain matrix does not immediately imply instability of closed loops in multivariable control system, but a stability problem may be expected if the constraint for L_2 becomes active.

In order to find a better input direction which covers a wider operating region, the following multi-objective optimization problem is considered:

$$\min_{x,u} J_{mobj}(x,u) = (V_1 \ V_2)^T,$$

$$\text{subject to } (4), (5),$$

where energy consumption of each distillation column is selected as the performance index; the Pareto optimal front will be used as the input direction in which the manipulated variable moves are restricted.

Figure 4 shows the Pareto optimal front obtained for various values of the fresh feed rate. Operations on the Pareto optimal front yield, for example, minimization of V_1 for a fixed V_2, which is quite reasonable in terms of constraint control. Moreover, reflux flows and vapor boilups of distillation columns are physically correlated, so that minimization of V_1 most often leads to reduction in L_1, which may be also convenient in constraint handling.

Fortunately, it has been found that the Pareto optimal front in terms of the operational variable vector δ can be approximated by a nonlinear 1-dimensional vector space. To make controller implementation (slightly) easier, the nonlinear vector is linearized around the nominal operating condition and the operational vector is described as

$$\delta = \theta v_p, \qquad |v_p| = 1, \quad v_p \in \Re^3, \tag{10}$$

where v_p is the linearized vector for the Pareto optimal front. Note that the operation for the minimum energy consumption lies on the Pareto optimal front with $\theta = 0$.

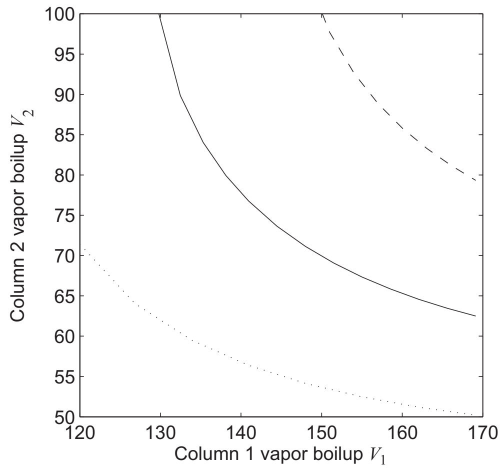

Fig. 4. Pareto optimal front for various feed rates. Dotted: $F_B = 0.9\bar{F}_B$, solid: $F_B = \bar{F}_B$, dashed:$F_B = 1.1\bar{F}_B$.

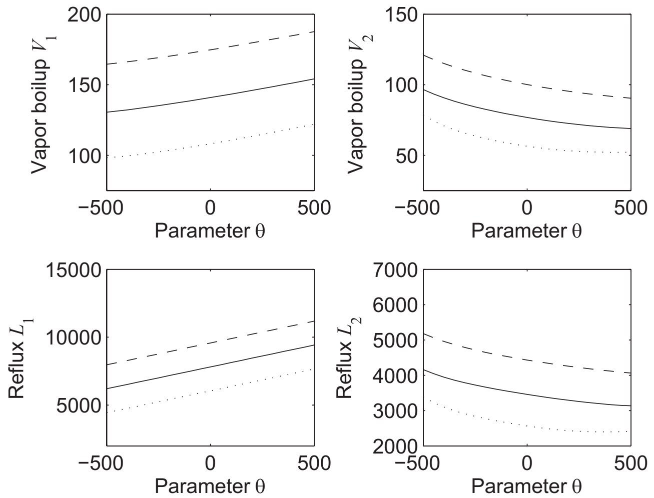

Fig. 5. Steady state relation between θ and prospective constraint variables. Input direction (10) obtained through the linearization of the Pareto optimal front. Dotted: $F_B = 0.8\bar{F}_B$, solid: $F_B = \bar{F}_B$, dashed:$F_B = 1.2\bar{F}_B$.

Figure 5 shows the steady state relation between θ and the prospective constraint variables; steady state input multiplicities are effectively eliminated.

3.2 Constraint Handling Controller

To handle constraints, a linear model predictive control technique is employed. A quadratic problem is introduced to determine the steady state target based on the predicted future response of the process.

Controlled and manipulated variables The manipulated variables are

$$u_{mpc} = (F_B \ \theta)^T$$

and the controlled variables, for which constraints are considered, are

$$y_{mpc} = (V_1 \ L_1 \ V_2 \ L_2)^T.$$

Model A linear approximation of the process around the nominal operating condition is used as the process model:

$$\begin{aligned} \dot{\xi} &= A\xi + Bu_{mpc} \\ \dot{\zeta} &= 0 \\ \hat{y}_{mpc} &= C\xi + \zeta, \end{aligned} \tag{11}$$

where (A, B, C) are the model matrices obtained through step testing or the local linearization of the process model, ξ is the state variable, $\zeta \in \Re^4$ is the output disturbance, and $\hat{y}_{mpc}$ is the measurement estimate.

State estimation and prediction of steady state responses A state estimator is constructed for the linear process model (11) as

$$\dot{\xi} = A\xi + Bu_{mpc} + K_\xi(y_{mpc} - \hat{y}_{mpc}) \tag{12}$$

$$\dot{\zeta} = K_\zeta(y_{mpc} - \hat{y}_{mpc}), \tag{13}$$

where K_ξ and K_ζ are the estimator gain matrices.

Then, the steady state output response, under the assumption that the manipulated variable u_{mpc} and the disturbance variable ζ are held constant at the current values, can be predicted by

$$\hat{y}^\infty_{mpc} = -CA^{-1}Bu_{mpc} + \zeta$$

Target calculation At each sampling time, the following quadratic program is solved to determine the manipulated variable increment Δu_{mpc}:

$$\begin{aligned} \min_{\Delta u_{mpc}} J_{mpc} = (r - u^\infty_{mpc})^T R_1 (r - u^\infty_{mpc}) \\ + \Delta u^T_{mpc} R_2 \Delta u_{mpc} \end{aligned} \tag{14}$$

subject to

$$\begin{aligned} u^\infty_{mpc} &= u_{mpc} + \Delta u_{mpc} \\ y^{LL}_{mpc} &\le \hat{y}^\infty_{mpc} - CA^{-1}B\Delta u_{mpc} \le y^{UL}_{mpc} \end{aligned} \tag{15}$$

where $r = (F^{set}_B \ 0)^T$ is a "setpoint" to the manipulated variable with F^{set}_B being a target value for the fresh feed rate, $R_1, R_2 \in \Re^{2\times 2}$ are positive definite weight matrices, and $y^{LL}_{mpc}, y^{UL}_{mpc}$ are the lower and upper limits of the controlled variables respectively.

At each sampling time, the above quadratic program is solved to update the manipulated variable as $u_{mpc} + \Delta u_{mpc}$. Then, the operational variable δ is reconstructed by (10) and the fresh feed increment is implemented on the process.

Expected controller behavior will be as follows:

As long as none of the inequality constraints (15) are violated, the controller tries to realize $r = u^\infty_{mpc}(F_B = F^{set}_B, \theta = 0)$ for $t \to \infty$, thus realizing self-optimizing control; since the control update Δu_{mpc} is penalized by the second term in the objective function (14) at each optimization, the controller asymptotically approaches its target r.

When some of the constraints (15) become active, controller behavior depends on the choice of the weight matrix R_1. If θ is heavily weighted, equivalently deviation

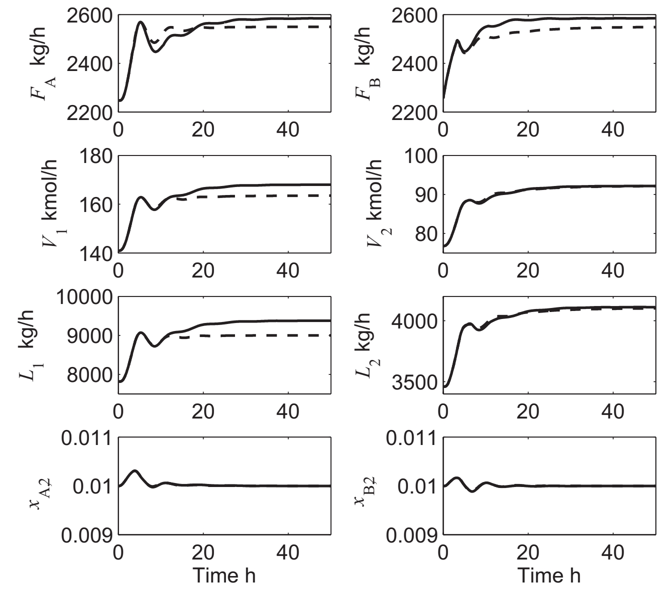

Fig. 6. Simulation results. Solid: feed maximizing controller. Dashed: self-optimizing controller.

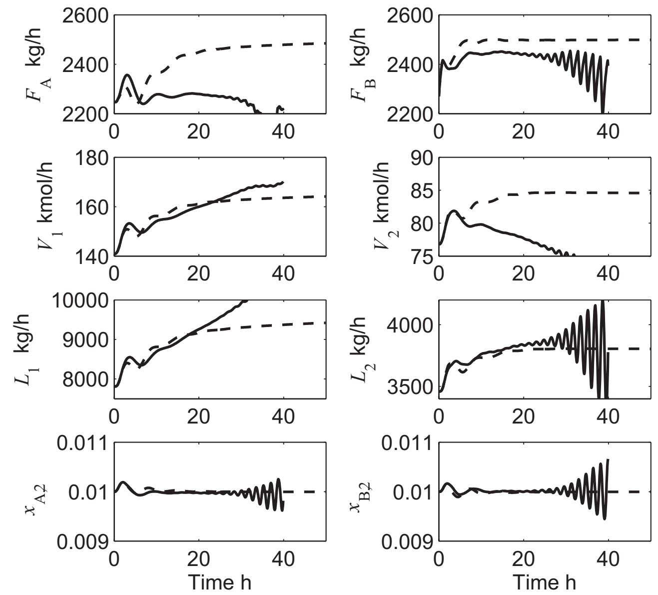

Fig. 7. Simulation results. Solid: feed maximizing controller whose input direction is defined by Eq. (9) (The calculation is terminated around $t = 40$h). Dashed: feed maximizing controller whose input direction is defined by Eq. (10).

from the self-optimizing control is not allowed, the controller stops changing u_{mpc}^{∞} and stays there. If θ is lightly weighted, the controller starts to deviate from the self-optimizing control ($\theta \neq 0$), trying to realize $F_B = F_B^{set}$ while respecting the active constraints, until the next constraint becomes active.

4. SIMULATION

Simulation calculations are performed using the nonlinear process model (1).

A very crude approximation has been made in designing the controller for the simulation studies: as the linear process model (11), the matrices A and C are chosen as $A = \mathrm{diag}(1/5\ 1/5\ 1/5\ 1/5)$, $C = I_4$, which assume uncoupled 1st order dynamics with the time constant being 5h. The matrix B is then calculated so that the steady state gain matrix $-CA^{-1}B$ coincides with that obtained from the locally linearized model around the nominal operating condition.

4.1 Feed Maximization

Responses to +20% increase in the fresh feed rate F_B are simulated by giving the setpoint as $r = (1.2\bar{F}_B\ 0)^T$. The upper and lower constraints are set as ±20% of the nominal values of the variables.

Comparisons are made for two parameter settings: one with a large weight on θ ($R_1 = \mathrm{diag}(1\ 10^6)$: self-optimizing control), the other with a small weight on θ ($R_1 = \mathrm{diag}(1\ 10^{-6})$: feed maximizing control).

Figure 6 compares the responses of the two controllers. In both cases, as the fresh feed F_B is increased, the upper limit constraint for V_2 becomes active first. The self-optimizing controller, then, stops increasing the feed. On the other hand, the feed maximizing controller keeps increasing the feed rate, while respecting the constraint on V_2 by manipulating δ, until another constraint, in this case the upper limit for L_1, becomes active.

Consequently, the steady state feed rates are $F_B^{\infty} = 2550$ with the self-optimizing controller, and $F_B^{\infty} = 2585$ with the feed maximizing controller.

As shown in this example, modes of operations can be easily switched simply by changing the control parameter, without the need for tedious logic in reconfiguring control loops.

4.2 Instability due to Input Multiplicities

In this case study, the performance of the controller whose input direction is defined by (9) is evaluated. The input weight is chosen as $R_1 = \mathrm{diag}(1\ 10^{-6})$.

Under the same operating conditions as the previous simulation studies, it was found that the controller was capable of achieving stable responses (results not shown), with L_1 and V_2 being the constrained variable. The achieved feed rate was $F_B^{\infty} = 2578$, which is slightly smaller than that of the feed maximizing controller.

However, as implied by Fig. 3, a problem can be expected when the constraint for the reflux L_2 becomes active. To demonstrate such a situation, the upper limit for L_2 is set at a smaller value ($L_2^{UL} = 3800$) for a reachable feed setpoint $F_B^{set} = 2500$, and simulation calculations are performed. Figure 7 shows the result: the closed loop becomes unstable, while the feed maximizing controller can safely handle the same operating condition.

In this case study, since one of the manipulated variables, namely the feed rate F_B, is constrained, the control problem is reduced to a single-input/single-output problem, so that the sign change in the gain $\partial L_2/\partial\theta$ immediately results in instability of the closed loop.

As shown in this simulation, stability of the controller depends on which constraints become active. For chemical processes which exhibit input multiplicities, extra care should be taken in control system design by considering several control scenarios and evaluating stability for each set of constraints.

5. CONCLUSION

It has been shown that the reactor/separator system with two material recycles exhibit steady state input multiplicities which cause control problems, even though nonlinearity of each individual unit may not be very strong.

A practical solution to control system design for the example process has been shown: the operational variable vector is confined into the the Pareto optimal front, which has been obtained by the multi-objective optimization problem posed as minimization of energy consumptions of the comprising units. The resulting controller is linear and easily implementable. The simulation studies have shown that the poorly defined input direction causes stability problem.

Frequent use of recycles in chemical processes suggests that the problem of steady state input multiplicity may not be a rare occurrence. As long as operating ranges are limited, this problem does not occur. However, once processes are required to be operated in a wider range and with a varieties of scenarios for more flexibility, extra design consideration such as shown in this study may become necessary.

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