Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Fault Detection of Distributed Networked Control Systems with Access Constraints

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Abstract: This paper considers the fault detection problem of distributed networked control systems (DNCS) with limited data transmission rate. In order to deal with the limited bandwidth of the network, two steps are taken. The first one is the periodic communication sequence which is introduced to the two-level DNCS to ensure that only some specified subsystems rather than all of them are connected to the central fault diagnosis unit at a certain time; the second one is that the signals which are transmitted to the central unit from each subsystem are not the inputs and outputs but the residuals which are smaller. Because periodic communication sequence's introduction changes the observability of the system, a theorem is provided to discuss the observability of DNCS as well as to give a new system model under observable condition. On the basis of residuals transmitted from subsystems to the central unit, some steps are taken to get the inputs and outputs which are necessary for fault detection. Then, according to the new system model and obtained inputs and outputs, the central fault diagnosis unit based on the periodic system theory is designed under this communication pattern. Finally, a numerical example is provided to illustrate the effectiveness of the proposed method.

1. INTRODUCTION

It is increasingly popular to close control loops over a control network to form a networked control system (NCS). As an integration of sensors, controllers, actuators and field bus, NCS delivers enormous distinct advantages such as modular and flexible system design (e.g., distributed processing and interoperability), simple and fast implementation (e.g., reduced system wiring and powerful configuration tools), ease of system diagnosis and maintenance, and increased system agility (Walsh et al., 2001; Tipsuwan et al., 2003). Nowadays, NCS can be found in many fields including manufacturing automation factories, electric factories, robots, advanced aircraft and electrified transportation(Xia et al.,2005; Zhang and D.Hristu,2006).

To authors' knowledge, many existing results on fault detection of NCS were shown in (Fang et al., 2006); however, the researches on fault diagnosis of DNCS where the multilevel communication architectures are able to meet new requirements such as mobility, modularity, control and diagnosis decentralization and/or distribution, autonomy, redundancy, quick and easy maintenance, are very limited (Ding and Zhang, 2007;Ding and Zhang, 2006; Zhang and Ding ,2006; Sauter et al., 2006;Zong et al). The system structure of two-level DNCS, the information exchange between the central unit and subsystems, and the observer-based fault diagnosis are considered in detail by (Ding and Zhang, 2006). A method based on system structural analysis is then proposed to provide fault detectability and fault isolability conditions, and an algorithm which allows distributing the FDI task on local autonomous nodes is provided in (Sauter et al., 2006).

The overall behaviour of the DNCS is influenced by the limited bandwidth of the network, which is defined as the maximal amount of meaningful data that can be transmitted per unit time (Lian and Moyne et al., 2001; Ji et al., 2005). If the network is overloaded, the transmission delays and the packet loss rate begin to increase significantly, as there is more traffic on the network than what can be transmitted (Zhang and Ding, 2006). To reduce the network traffic in each subsystem, the periodic communication sequence is applied to realize the switch between different sensors and actuators in (Ding and Zhang, 2007; Ding and Zhang, 2006). With the distributed networked control system becoming more and more larger and lots of subsystems being connected to the central fault diagnosis unit, it is necessary to reduce the network traffic between the central fault diagnosis unit and subsystems, therefore the packet-based transmission mechanism is introduced to the DNCS in (Zong et al).In this paper, the periodic communication sequence which makes sure that at a certain time only some specified subsystems have access to the network is used to reduce the network traffic between the subsystems and the central fault diagnosis unit instead of in the subsystem like that in (Ding and Zhang, 2007; Ding and Zhang, 2006), and the method used in (Ding and Zhang, 2007; Ding and Zhang, 2006) is hard to be applied in this situation. It is known that the introduction of periodic communication sequence changes the observability of the DNCS.Therefore, before designing the observer-based fault detection method in the central unit, the observability of the system is analyzed firstly and the modified system model is obtained under the observable condition. Finally, in term of the modified system model, the model-based fault detection method based on the periodic system theory is presented.

The paper is organized in five sections as follows. The two-level DNCS with periodic communication sequence is introduced and the observability of the system is discussed in Section 2. Section 3 refers to the observer-based fault diagnosis for the proposed system. Section 4 describes a case study to validate the fault diagnosis method. Finally, some concluding remarks end the paper.

2. DNCS WITH PERIODIC COMMUNICATION SEQUENCES

2.1 Two-level DNCS

As shown in Fig.1, the two-level DNCS is composed of the central unit and banks of subsystem which include sensors, controllers, actuators, and they interact with each other through the Ethernet.

Assume that each subsystem is described by the following dynamic model:

$$\begin{aligned} \dot{x}_i(t) &= A_{ii}x_i(t) + \sum_{j=1, j\neq i}^{n} A_{ij}x_j(t) + B_i u_i(t) + E_1^i d(t) + F_1^i f(t) \\ y_i(t) &= C_i x_i(t) + D_i u_i(t) + E_2^i d(t) + F_2^i f(t) \quad i = 1,2,...,n \end{aligned} \tag{1}$$

where the superscript i denotes the matrices or vectors related to the ith subsystem, $x^i \in R^n$, $u^i \in R^n$, $y^i \in R^n$, denotes the state vector, the actuator input vector and the measurable output vector of the ith subsystem, $d \in R^n$, $f \in R^n$ denotes the unknown disturbance vector and the fault vector. $A_{ii}, A_{ij}, B_i, C_i, D_i, E_1^i, E_2^i, F_1^i, F_2^i$, $i, j = 1,...,n$ are known matrices of appropriate dimensions. n denotes the total number of the subsystems in the DNCS

The overall dynamics of the physical system can be described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E_1 d(t) + F_1 f(t) \\ y(t) &= Cx(t) + Du(t) + E_2 d(t) + F_2 f(t) \end{aligned} \tag{2}$$

where

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad A = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_n \end{bmatrix} \quad C = \begin{bmatrix} C_1 & & 0 \\ & \ddots & \\ 0 & & C_n \end{bmatrix} \quad D = \begin{bmatrix} D_1 & & 0 \\ & \ddots & \\ 0 & & D_n \end{bmatrix}$$

$$E_1 = \begin{bmatrix} E_1^1 \\ \vdots \\ E_1^n \end{bmatrix} \quad F_1 = \begin{bmatrix} F_1^1 \\ \vdots \\ F_1^n \end{bmatrix} \quad E_2 = \begin{bmatrix} E_2^1 \\ \vdots \\ E_2^n \end{bmatrix} \quad F_2 = \begin{bmatrix} F_2^1 \\ \vdots \\ F_2^n \end{bmatrix}$$

If the delay and packet loss are negligible, then by a discretization with sampling period h, the dynamic of system is :

$$\begin{aligned} x(k+1) &= \underline{A}x(k) + \underline{B}u(k) + \underline{E}_d d(k) + \underline{E}_f f(k) \\ y(k) &= \underline{C}x(k) + \underline{D}u(k) + \underline{E}_d d(k) + \underline{E}_f f(k) \end{aligned} \tag{3}$$

where $\underline{A} = e^{Ah}$, $\underline{B} = \int_0^h e^{At} B dt$, $\underline{E}_d = \int_0^h e^{At} E_1 dt$

$\underline{E}_f = \int_0^h e^{At} F_1 dt$, $\underline{C} = C$ $\underline{D} = D$, $\underline{E}_d = E_2$, $\underline{E}_f = F_2$

2.2 DNCS with Periodic Communication Sequence

In order to avoid problems caused by limited bandwidth, the ideal way is to improve the capacity of the network. Alternatively, reducing the flow rate would also reduce the uncertainty caused by the delays and the packet loss. To this aim, two steps are taken. The first one is the periodic communication sequence which is introduced to the two-level DNCS to ensure that at a certain time only part of the subsystems rather than all of them are connected to the central fault diagnosis unit, just as shown in Fig.1; the second one is that the signals which are transmitted to the central unit from each subsystem are not the inputs and outputs but the residuals which are smaller.

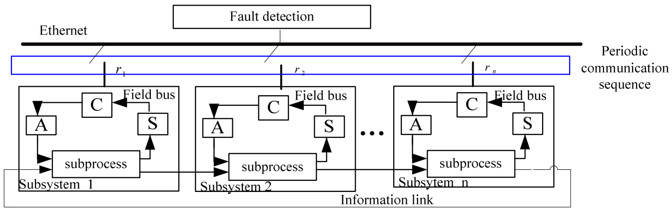

Fig.1 DNCS with periodic communication sequence

We assume that all the subsystem have the same sampling period h, and the communication sequence is $\beta(k) = \{\beta_1(k), \beta_2(k) \cdots \beta_p(k)\}$, where the integer p denotes the number of steps in a communication period $T^s = ph$.

$\beta_i(k) = [\beta_{i,1}(k), \beta_{i,2}(k) \cdots \beta_{i,n}(k)]^T$ $i = 1,2 \cdots p$ where $\beta_{i,j}(k) \in \{0,1\}$ $j = 1,2 \cdots n$ denotes whether the subsystem j is connected to the central unit at step i. If $\beta_{i,j}(k) = 1$, it represents that the subsystem j transmits some signals to the central unit and vice versa. $\beta_{i,j}(k)$ must satisfy the condition: $N_i \le n$, $N_i = \sum_{j=1}^{n} \beta_{i,j}(k)$, $i = 1,2 \cdots p$. When $N_i = n$, the DNCS is the one without any periodic communication sequence. The case $N_i = 1$ is discussed in this paper.

Let $a = \{\beta_{1,q}, \beta_{2,w} \cdots \beta_{p,v}\}$ $q, w \cdots v \le n$ denote all the steps that the subsystems are access to the central unit at a communication period, and a is obtained by selecting all the $\beta_{i,j}(k) = 1, i \le p \text{ and } j \le n$ in sequence from the periodic communication sequence $\beta(k)$.

Example 1: Assume that the communication sequence is $\beta(k) = \{ [0\ 1\ 0]^T, [1\ 0\ 0]^T, [0\ 0\ 1]^T \}$, therefore $p = 3$, $n = 3$ $N_1 = N_2 = N_3 = 1$, $a = \{\beta_{1,2}, \beta_{2,1}, \beta_{3,3}\}$

Let $Q_i(k) = diag(\beta_i(k))$ denotes the "matrix form" of $\beta_i(k)$ (Zhang and Dimitrios, 2006), so we thus have:

$$\begin{aligned} y_i^c &= Q_i(k) y_i^s \text{, where } y_i^s = [y_{i,1}^s, y_{i,2}^s \cdots y_{i,n}^s]^T \\ u_i^c &= Q_i(k) u_i^s \text{, where } u_i^s = [u_{i,1}^s, u_{i,2}^s \cdots u_{i,n}^s]^T \end{aligned} \tag{4}$$

$y_{i,j}^s, j = 1,2 \cdots n$ denotes the output of the ith subsystem at step i, y_i^c denotes the outputs obtained by the central unit.

From formula (4), we can find that the periodic communication sequence affects the observability of the system, so before designing the observer-based fault detection method, we must guarantee the observability of DNCS with periodic communication sequence.

2.3 Observability of DNCS with Periodic Communication Sequence

The following theorem is used to discuss the observability of DNCS with periodic communication sequence.

Theorem 1 For the system with periodic communication sequence $\beta(k)=\left\{\beta_1(k),\beta_2(k)\cdots\beta_p(k)\right\}$ and $a=\left\{\beta_{1,q},\beta_{2,w}\cdots\beta_{p,v}\right\}$, We can call it z -step observable(Zhang and Dimitrios, 2006), if the following conditions are met :(1) $z=mp,\ m\in\mathbb{N}$

(2) $rank\left[(C')^T \ \ (C'A')^T \ \cdots (C'(A')^{n-1})^T\right]^T = n$

where

$$C'=[C(q,:)\ \ C(w,:)*A\ \cdots C(v,:)*A^{p-1}$$
$$C(q,:)*A^p\ \ C(w,:)*A*A^p\ \cdots(v,:)*A^{p-1}*A^p$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$C(q,:)*(A^p)^{m\text{-}1}\ \ C(w,:)*A*(A^p)^{m\text{-}1}\ \cdots(v,:)*A^{p-1}*(A^p)^{m\text{-}1}\ ]^T$$

$A'=(A^p)^m$ $\quad C(1,:)$ represents the first row of C.

Proof: The periodic communication sequence's introduction transforms the DNCS into periodic time-varying system; so many existing methods are failed to be used in DNCS with periodic communication sequence to validate its observability. Therefore, the lifting technique is used to obtain the time-invariant system model from working at the basic period h to the observable period $T^o = zh$.

①For DNCS without any periodic communication sequence, the DNCS model is changed from working at the basic period h to the communication period $T^s = ph$.

From formula (3), we can get the following result:

$$x(kT^s+T^s)=x(kT^s+ph)=\underline{A}^p x(kT^s)+\sum_{i=0}^{p-1}\underline{A}^i\underline{B}u(kT^s+(p-i-1)h)$$
$$+\sum_{i=0}^{p-1}\underline{A}^i\underline{E}_d d(kT^s+(p-i-1)h)+\sum_{i=0}^{p-1}\underline{A}^i\underline{E}_f f(kT^s+(p-i-1)h)$$

and

$$y(kT^s+(p-1)h)=\underline{C}\ \underline{A}^{p-1}x(kT^s)+\sum_{i=0}^{p-2}\underline{C}\underline{A}^i\underline{B}u(kT^s+(p-2-i)h)$$
$$+\sum_{i=0}^{p-2}\underline{C}\underline{A}^i\underline{E}_d d(kT^s+(p-2-i)h)+\sum_{i=0}^{p-2}\underline{C}\underline{A}^i\underline{E}_f f(kT^s+(p-2-i)h)$$
$$+\underline{D}u(kT^s+(p-1)h)+\underline{F}_f f(kT^s+(p-1)h)$$

by the same reason, we can obtain that :

$$y(kT^s+wh)=\underline{C}\ \underline{A}^w x(kT^s)+\sum_{i=0}^{w-1}\underline{C}\underline{A}^i\underline{B}u(kT^s+(w-1-i)h)$$
$$+\sum_{i=0}^{w-1}\underline{C}\underline{A}^i\underline{E}_d d(kT^s+(w-1-i)h)+\sum_{i=0}^{w-1}\underline{C}\underline{A}^i\underline{E}_f f(kT^s+(w-1-i)h)$$
$$+\underline{D}u(kT^s+wh)+\underline{F}_f f(kT^s+wh)$$
$$0\le\omega<p$$

Finally, DNCS model is obtained at the communication period $T^s = ph$:

$$\begin{aligned}x^c(k+1)&=A^c x^c(k)+B^c u^c(k)+E_d^c d^c(k)+E_f^c f^c(k)\\ y^c(k)&=C^c x^c(k)+D^c u^c(k)+F_d^c d^c(k)+F_f^c f^c(k)\end{aligned}\qquad(5)$$

where

$$u^c=\left[u_1^T(kT^s)\cdots u_n^T(kT^s)\vdots\ \cdots\ \vdots u_1^T(kT^s+(p-1)h)\cdots u_n^T(kT^s+(p-1)h)\right]^T$$
$$y^c=\left[y_1^T(kT^s)\cdots y_n^T(kT^s)\vdots\ \cdots\ \vdots y_1^T(kT^s+(p-1)h)\cdots y_n^T(kT^s+(p-1)h)\right]^T$$

$$D^c=\begin{bmatrix}\underline{D} & 0 & \cdots & 0 & 0\\ \underline{C}\underline{B} & \underline{D} & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \underline{C}\underline{A}^{p-3}\underline{B} & \underline{C}\underline{A}^{p-4}\underline{B} & \cdots & \underline{D} & 0\\ \underline{C}\underline{A}^{p-2}\underline{B} & \underline{C}\underline{A}^{p-3}\underline{B} & \cdots & \underline{C}\underline{B} & \underline{D}\end{bmatrix}\quad A^c=\underline{A}^p$$

$$B^c=[\underline{A}^{p-1}\underline{B}\ \ \underline{A}^{p-2}\underline{B}\cdots\underline{B}\ ]\ \ C^c=[\underline{C}\ \ \underline{C}\underline{A}\ \ \underline{C}\underline{A}^2\ \ \cdots\underline{C}\underline{A}^{p-1}]^T$$

$$F_d^c=\begin{bmatrix}\underline{F}_d & 0 & \cdots & 0 & 0\\ \underline{C}\underline{E}_d & \underline{F}_d & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \underline{C}\underline{A}^{p-1}\underline{E}_d & \underline{C}\underline{A}^{p-2}\underline{E}_d & \cdots & \underline{F}_d & 0\\ \underline{C}\underline{A}^{p-1}\underline{E}_d & \underline{C}\underline{A}^{p-2}\underline{E}_d & \cdots & \underline{C}\underline{E}_d & \underline{F}_d\end{bmatrix}$$

$$E_d^c=[\underline{A}^{p-1}\underline{E}_d\ \ \underline{A}^{p-2}\underline{E}_d\ \cdots\underline{E}_d]\ \ E_f^c=[\underline{A}^{p-1}\underline{E}_f\ \ \underline{A}^{p-2}\underline{E}_f\ \cdots\underline{E}_f]$$

$$F_f^c=\begin{bmatrix}\underline{F}_f & 0 & \cdots & 0 & 0\\ \underline{C}\underline{E}_f & \underline{F}_f & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \underline{C}\underline{A}^{p-2}\underline{E}_f & \underline{C}\underline{A}^{p-2}\underline{E}_f & \cdots & \underline{F}_f & 0\\ \underline{C}\underline{A}^{p-1}\underline{E}_f & \underline{C}\underline{A}^{p-2}\underline{E}_f & \cdots & \underline{C}\underline{E}_f & \underline{F}_f\end{bmatrix}$$

where A^T denotes the transpose of A.

②For DNCS without any periodic communication sequence, the DNCS model is changed from working at the communication period $T^s = ph$ to the observable period $T^o = zh$.

The DNCS model at the communication period $T^o = zh$ is obtained by taking the same steps as that in ①.

$$\begin{aligned}\underline{x}^c(k+1)&=\underline{A}^c\underline{x}^c(k)+\underline{B}^c\underline{u}^c(k)+\underline{E}_d^c\underline{d}^c(k)+\underline{E}_f^c\underline{f}^c(k)\\ \underline{y}^c(k)&=\underline{C}^c\underline{x}^c(k)+\underline{D}^c\underline{u}^c(k)+\underline{F}_d^c\underline{d}^c(k)+\underline{F}_f^c\underline{f}^c(k)\end{aligned}\qquad(6)$$

where

$$\underline{u}^c=\left[(u^c(kT'))^T\ \ (u^c(kT'+ph))^T\cdots\ (u^c(kT'+(m-1)ph))^T\right]^T$$
$$\underline{y}^c=\left[(y^c(kT'))^T\ \ (y^c(kT'+ph))^T\cdots\ (y^c(kT'+(m-1)ph))^T\right]^T$$
$$\underline{A}^c=(A^c)^m\ \ \underline{B}^c=[(A^c)^{m-1}B^c\ \ \ (A^c)^{m-2}B^c\cdots B^c]$$
$$\underline{C}^c=[C^c\ \ C^cA^c\ \ C^c(A^c)^2\ \ \cdots C^c(A^c)^{m-1}]^T$$

$$\underline{D}^c=\begin{bmatrix}D^c & 0 & \cdots & 0 & 0\\ C^c\underline{B} & D^c & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ C^c(A^c)^{m-2}B^c & C^c(A^c)^{m-3}B^c & \cdots & D^c & 0\\ C^c(A^c)^{m-1}B^c & C^c(A^c)^{m-2}B^c & \cdots & C^cB^c & D^c\end{bmatrix}$$

$$\underline{F}_d^c=\begin{bmatrix}F_d^c & 0 & \cdots & 0 & 0\\ C^c\underline{E}_d & F_d^c & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ C^c(A^c)^{m-2}E_d^c & C^c(A^c)^{m-3}E_d^c & \cdots & F_d^c & 0\\ C^c(A^c)^{m-1}E_d^c & C^c(A^c)^{m-2}E_d^c & \cdots & C^cE_d^c & F_d^c\end{bmatrix}$$

$$\underline{F}_f^c=\begin{bmatrix}F_f^c & 0 & \cdots & 0 & 0\\ C^c\underline{E}_f & F_f^c & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ C^c(A^c)^{m-2}E_f^c & C^c(A^c)^{m-3}E_f^c & \cdots & F_f^c & 0\\ C^c(A^c)^{m-1}E_f^c & C^c(A^c)^{m-2}E_f^c & \cdots & C^cE_f^c & F_f^c\end{bmatrix}$$

$\underline{E}_d^c = [(A^c)^{m-1} E_d^c \;\; (A^c)^{m-2} E_d^c \cdots E_d^c] \;\; \underline{E}_f^c = [(A^c)^{m-1} E_f^c \;\; (A^c)^{m-2} E_f^c \cdots E_f^c]$

③The effect of the periodic communication sequence on the system model is considered to get the modified system model. In term of the set $a = \{\beta_{1,q}, \beta_{2,w} \cdots \beta_{p,v}\}$, modified system model is obtained by cancelling all the parameters related to the ones that are not in the $a = \{\beta_{1,q}, \beta_{2,w} \cdots \beta_{p,v}\}$.

The modified system model is:

$$\begin{aligned} x_m^c(k+1) &= A_m^c x_m^c(k) + B_m^c u_m^c(k) + E_{d,m}^c d_m^c(k) + E_{f,m}^c f_m^c(k) \\ y_m^c(k) &= C_m^c x_m^c(k) + D_m^c u_m^c(k) + F_{d,m}^c d_m^c(k) + F_{f,m}^c f_m^c(k) \end{aligned} \tag{7}$$

where $A_m^c = \underline{A}^c$

$u_m^c = \left[u_q^T(kT^o) \cdots u_v^T(kT^o + (p-1)h) \vdots \cdots \vdots u_q^T(kT^o + (m-1)ph) \cdots u_v^T(kT^o + (mp-1)h) \right]^T$

$y_m^c = \left[y_q^T(kT^o) \cdots y_v^T(kT^o + (p-1)h) \vdots \cdots \vdots y_q^T(kT^o + (m-1)ph) \cdots y_v^T(kT^o + (mp-1)h) \right]^T$

$$\begin{aligned} B_m^c = [&\underline{B}^c(:,q) \;\; \underline{B}^c(:,n+w) \cdots \underline{B}^c(:,(p-1)n+v) \\ &\underline{B}^c(:,np+q) \;\; \underline{B}^c(:,np+n+w) \cdots \underline{B}^c(:,np+(p-1)n+v) \\ &\vdots \qquad \vdots \qquad \vdots \\ &\underline{B}^c(:,(m-1)np+q) \;\; \underline{B}^c(:,(m-1)np+n+w) \cdots \\ &\underline{B}^c(:,(m-1)np+(p-1)n+v)] \end{aligned}$$

$$\begin{aligned} C_m^c = [&\underline{C}^c(q,:) \;\; \underline{C}^c(n+w,:) \cdots \underline{C}^c((p-1)n+v,:) \\ &\underline{C}^c(np+q,:) \;\; \underline{C}^c(np+n+w,:) \cdots \underline{C}^c(np+(p-1)n+v,:) \\ &\vdots \qquad \vdots \qquad \vdots \\ &\underline{C}^c((m-1)np+q,:) \;\; \underline{C}^c((m-1)np+n+w,:) \cdots \\ &\underline{C}^c((m-1)np+(p-1)n+v,:)]^T \end{aligned}$$

$$\begin{aligned} D_m' = [&\underline{D}^c(:,q) \;\; \underline{D}^c(:,n+w) \cdots \underline{D}^c(:,(p-1)n+v) \\ &\underline{D}^c(:,np+q) \;\; \underline{D}^c(:,np+n+w) \cdots \underline{D}^c(:,np+(p-1)n+v) \\ &\vdots \qquad \vdots \qquad \vdots \\ &\underline{D}^c(:,(m-1)np+q) \;\; \underline{D}^c(:,(m-1)np+n+w) \cdots \\ &\underline{D}^c(:,(m-1)np+(p-1)n+v)] \end{aligned}$$

$$\begin{aligned} D_m^c = [&D_m'(q,:) \;\; D_m'(n+w,:) \cdots D_m'((p-1)n+v,:) \\ &D_m'(np+q,:) \;\; D_m'(np+n+w,:) \cdots D_m'(np+(p-1)n+v,:) \\ &\vdots \qquad \vdots \qquad \vdots \\ &D_m'((m-1)np+q,:) \;\; D_m'((m-1)np+n+w,:) \cdots \\ &D_m'((m-1)np+(p-1)n+v,:)]^T \end{aligned}$$

Other parameters in system model are changed according to the steps above. For example, $E_{d,m}^c$ and $E_{f,m}^c$ is obtained by taking the same steps of B_m^c.

④ A new system model is provided which is equivalent to formula (7) but sometimes it is obtained easily. On the base of the steps in step③, a simple system model is introduced.

$$\begin{aligned} x_s^c(k+1) &= A_s^c x_s^c(k) + B_s^c u_s^c(k) + E_{d,s}^c d_s^c(k) + E_{f,s}^c f_s^c(k) \\ y_s^c(k) &= C_s^c x_s^c(k) + D_s^c u_s^c(k) + F_{d,s}^c d_s^c(k) + F_{f,s}^c f_s^c(k) \end{aligned} \tag{8}$$

where $A_s^c = (A^p)^m$

$$\begin{aligned} B_{s,i}^c = [&(A^p)^{i-1} * A^{p-1} * B(:,v) \cdots (A^p)^{i-1} * A * B(:,w) \;\; (A^p)^{i-1} * B(:,q) \\ &(A^p)^{i-2} * A^{p-1} * B(:,v) \cdots (A^p)^{i-2} * A * B(:,w) \;\; (A^p)^{i-2} * B(:,q) \\ &\vdots \qquad \vdots \qquad \vdots \\ &A^{p-1} * B(:,v) \cdots A * B(:,w) \;\; B(:,q)] \end{aligned}$$

$$\begin{aligned} C_{s,i}^c = [&C(q,:) \;\; C(w,:) * A \cdots C(v,:) * A^{p-1} \\ &C(q,:) * A^p \;\; C(w,:) * A * A^p \cdots (v,:) * A^{p-1} * A^p \\ &\vdots \qquad \vdots \qquad \vdots \\ &C(q,:) * (A^p)^{i-1} \;\; C(w,:) * A * (A^p)^{i-1} \cdots (v,:) * A^{p-1} * (A^p)^{i-1} \;]^T \end{aligned}$$

$B_s^c = B_{s,m}^c \quad C_s^c = C_{s,m}^c$

$$D_s^c = \begin{bmatrix} D^s & 0 & \cdots & 0 & 0 \\ C_{s,1}^c B_{s,1}^c & D^s & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{s,1}^c A^{m-3} B_{s,1}^c & C_{s,1}^c A^{m-4} B_{s,1}^c & \cdots & D^s & 0 \\ C_{s,1}^c A^{m-2} B_{s,1}^c & C_{s,1}^c A^{m-3} B_{s,1}^c & \cdots & C_{s,1}^c B_{s,1}^c & D^s \end{bmatrix}$$

⑤ It is obvious that the formula (7) or (8) represents the time-invariant system, so the regular observability criterion for discrete system can be applied to the system denoted by the model above (Wilson, 1996). It is well known that the system above is observable if the following condition is met:

$$rank\left[(C_s^c)^{\mathrm{T}} \;\; (C_s^c A_s^c)^{\mathrm{T}} \cdots (C_s^c (A_s^c)^{n-1})^{\mathrm{T}} \right]^T = n \tag{9}$$

3. FAULT DIGANOSIS

3.1 Inputs and Outputs

It is known that inputs and output are necessary for fault diagnosis, but in our system in order to keep little traffic on Ethernet, we just transmit the residual of each subsystem to the central unit, so we should know the inputs and outputs firstly to design the fault diagnosis method.

Before getting the inputs and outputs, we should introduce new parameters $a_j, j = 1,2 \cdots n$ which are obtained by collecting elements in $a = \{\beta_{1,q}, \beta_{2,w} \cdots \beta_{p,v}\}$, and these elements represent that the same subsystem is access to the central unit. For example, $a = \{\beta_{1,1}, \beta_{2,2} \;\beta_{3,1} \;\beta_{4,1} \;\beta_{5,2} \;\beta_{6,1} \;\beta_{7,1} \;\beta_{8,2} \;\beta_{9,1}\}$, we can get $a_1 = \{\beta_{1,1}, \beta_{3,1}, \beta_{4,1}, \beta_{6,1}, \beta_{7,1}, \beta_{9,1}\}$,and $a_2 = \{\beta_{2,2}, \beta_{5,2}, \beta_{8,2}\}$.

For convenience, we take $a_\delta = \{\beta_{g,\delta} \cdots \beta_{f,\delta}, \beta_{\lambda,\delta} \cdots \beta_{\tau,\delta}\}$ for example, and we assume that the observer is predefined for the subsystem δ as follows;

$$\begin{aligned} \hat{x}_\delta(k+1) &= A_{\delta,\delta}\hat{x}(k) + B_\delta u_\delta(k) + L_\delta(y_\delta(k) - \hat{y}_\delta(k)) \\ \hat{y}_\delta(k) &= C_\delta \hat{x}(k) + D_\delta u_\delta(k) \end{aligned} \tag{10}$$

where $\hat{x}_\delta(k+1)$ is short for $\hat{x}_\delta((k+1)h)$

Formula (11) is used to get $\hat{x}_\delta(k+\lambda)$ with the premise that all the parameters related to $\beta_{f,\delta}$ are known such as $\hat{x}(k+f)$, $u(k+f)$, $r(k+f)$ and so on.

$$\begin{aligned} \hat{x}_\delta(k+\lambda) = (A_{\delta,\delta})^{\lambda-f}\hat{x}(k+f) + \sum_{j=0}^{\lambda-f-1}(A_{\delta,\delta})^j B_\delta u_\delta(k+\lambda-j-1) \\ + \sum_{j=0}^{\lambda-f-1}(A_{\delta,\delta})^j L_\delta r_\delta(k+\lambda-j-1) \end{aligned} \tag{11}$$

When a new period of periodic communication sequence starts, the following formula is used to get $\hat{x}_\delta(k+ip+g)$ related to $\beta_{g,\delta}$ with the premise that all the parameters related to $\beta_{\tau,\delta}$ are known at the last period.

$$\hat{x}_\delta(k+ip+g) = (A_{\delta,\delta})^{p-\tau+g}\hat{x}(k+(i-1)p+\tau) + \sum_{j=0}^{p-\tau+g-1}(A_{\delta,\delta})^j B_\delta u_\delta(k+ip+g-j-1) + \sum_{j=0}^{p-\tau+g-1}(A_{\delta,\delta})^j L_\delta r_\delta(k+ip+g-j-1) \quad (12)$$

When the estimation of state like $\hat{x}_\delta(k+\lambda)$ is known it is easy to get $\hat{y}_\delta(k+\lambda)$ and $u_\delta(k+\lambda)$

$$y_\delta(k+\lambda) = C_\delta\hat{x}(k+\lambda) + D_\delta u_\delta(k+\lambda) + r(k+\lambda) \quad (13)$$

$$u_\delta(k+\lambda) = K_\delta\hat{x}(k+\lambda)$$

where K_δ is predefined.

3.2 Observer-based Fault Diagnosis

Based on formula (11), (12) and (13), the inputs and outputs of the subsystems obtained above in the central FDI unit are employed to design the observer:

$$\begin{aligned}\hat{x}(k+1) &= A_s^c\hat{x}(k) + B_s^c\underline{u}(k) + L(y_s^c(k) - \underline{\hat{y}}(k))\\ \underline{\hat{y}}(k) &= C_s^c\hat{x}(k) + D_s^c\underline{u}(k)\\ r(k) &= W(y_s^c(k) - \underline{\hat{y}}(k))\end{aligned} \quad (14)$$

where r is the residual which is zero when there is no fault and non-zero when fault occurs. L is the observer gain matrix. W is the weighting matrix.

Based on the (8) and (14), the formulation is obtained:

$$\begin{aligned}r(z) &= W(G_{r,d}d(z) + G_{r,f}f(z))\\ G_{r,d} &= F_{d,s}^c + C_s^c(zI - A_s^c + LC_s^c)^{-1}(E_{d,s}^c - LF_{d,s}^c)\\ G_{r,f} &= F_{f,s}^c + C_s^c(zI - A_s^c + LC_s^c)^{-1}(E_{f,s}^c - LF_{f,s}^c)\end{aligned} \quad (15)$$

In order to get a compromise between the sensitivity and the robustness, the fault diagnosis problem is transferred to the optimization problem:

$$\min_{L,W} J = \min_{L,W}\frac{\|WG_{r,d}\|_\infty}{\|WG_{r,f}\|_\infty} \quad (16)$$

To get the optimal solution of (16), the following lemma is employed

Lemma: Given the LTI system (8), assume that $(A_s^c, E_{d,s}^c, C_s^c, F_{d,s}^c)$ is detectable and has no transmission zeros on the unit circle, no unreachable modes on the unit circle, and no unobservable modes at the origin. Then $W = W_0$ $L = -L_0^T$ solve the optimization problem (16), where W_0 is the left inverse of a full-column rank matrix H satisfying $HH^T = C_s^c X(C_s^c)^T + F_{d,s}^c(F_{d,s}^c)^T$, and (X, L_0) is the stabilizing solution to the DTARS（Discrete-Time Algebraic Riccati System）

$$\begin{bmatrix} A_s^cX(A_s^c)^T - X + E_{d,s}^c(E_{d,s}^c)^T & A_s^cX(C_s^c)^T + E_{d,s}^c(F_{d,s}^c)^T \\ C_s^cX(A_s^c)^T + F_{d,s}^c(E_{d,s}^c)^T & C_s^cX(C_s^c)^T + F_{d,s}^c(F_{d,s}^c)^T\end{bmatrix}\begin{bmatrix} I \\ L_0\end{bmatrix} = 0 \quad (17)$$

The proof is given in (Zhang et al., 2002), but it does not mention how to solve the DTARS.

The follow formulation is obtained based on (17)

$$\begin{aligned}A_s^cX(A_s^c)^T - X + E_{d,s}^c(E_{d,s}^c)^T + (A_s^cXC_s^c + E_{d,s}^c(F_{d,s}^c)^T)L_0 &= 0\\ C_s^cX(A_s^c)^T + (F_{d,s}^c)^T(E_{d,s}^c)^T + (C_s^cX(C_s^c)^T + F_{d,s}^c(F_{d,s}^c)^T)L_0 &= 0\end{aligned} \quad (18)$$

Make some changes to (18)

$$A_s^cX(A_s^c)^T - X - (A_s^cX(C_s^c)^T + E_{d,s}^c(F_{d,s}^c)^T)(C_s^cX(C_s^c)^T + F_{d,s}^c(F_{d,s}^c)^T)^{-1}(C_s^cX(A_s^c)^T + F_{d,s}^c(E_{d,s}^c)^T) + E_{d,s}^c(E_{d,s}^c)^T = 0 \quad (19)$$

The optimal solution X of (19) is obtained by the stand function DARE in MATLAB, and

$$L_0 = -(C_s^cX(C_s^c)^T + F_{d,s}^c(F_{d,s}^c)^T)^{-1}(C_s^cX(A_s^c)^T + F_{d,s}^c(E_{d,s}^c)^T)$$

4. CASE STUDY

Consider the following two-level DNCS consisting of two interconnected subsystems, the simple form of the case in (Patton, 2007).

$$\begin{aligned}\dot{x}(k+1) &= Ax(k) + Bu(k) + Ed(k) + Ff(k)\\ y(k) &= Cx(k) + Du(k)\end{aligned}$$

where $x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23}\end{bmatrix}^T$ $u = \begin{bmatrix} u_1 & u_2\end{bmatrix}^T$

$$A = \left[\begin{array}{ccc|ccc} -0.5 & 0 & 0 & 0 & 0 & 0\\ 0 & -0.1 & 0 & -0.1 & 0 & 0\\ 0 & 0 & -0.02 & 0 & 0 & 0.1\\ \hline 0.1 & 0 & 0 & -0.2 & 0 & 0\\ 0 & -0.1 & 0 & 0 & -0.1 & 0\\ 0.1 & 0 & -0.1 & 0 & 0 & -0.02\end{array}\right]$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1\end{bmatrix}^T \quad E = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1\end{bmatrix}^T \quad D = \begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}$$

$$C = \begin{bmatrix} 0.1 & 0.1 & 0.2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.2 & 0.1 & 0.1\end{bmatrix} \quad F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}^T$$

Assume that the basic sampling time is $h = 0.1$, and the periodic communication sequence is $\beta(k) = \left\{[0\ \ 1]^T, [1\ \ 0]^T, [1\ \ 0]^T, [0\ \ 1]^T\right\}$, so we get that $p = 4$, $n = 2$, $a = \{\beta_{1,2}, \beta_{2,1}, \beta_{3,1}, \beta_{4,2}\}$, $a_1 = \{\beta_{2,1}, \beta_{3,1}\}$ $a_2 = \{\beta_{1,2}, \beta_{4,2}\}$ and the system is 4-step observable.

Setting the system sampling time is 0.4s, the modified system model is:

$$\begin{aligned}x_s^c(k+1) &= A_s^cx_s^c(k) + B_s^cu_s^c(k) + E_{d,s}^cd_s^c(k) + E_{f,s}^cf_s^c(k)\\ y_s^c(k) &= C_s^cx_s^c(k) + D_s^cu_s^c(k) + F_{d,s}^cd_s^c(k) + F_{f,s}^cf_s^c(k)\end{aligned}$$

where

$$\begin{aligned}u_s^c &= \left[u_2^T(KT)\ u_1^T(KT+0.1)\ u_1^T(KT+0.2)\ u_2^T(KT+0.3)\right]\\ y_s^c &= \left[y_2^T(KT)\ y_1^T(KT+0.1)\ y_1^T(KT+0.2)\ y_2^T(KT+0.3)\right]\end{aligned}$$

and the observer is obtained according to the section 3:

$$L = \begin{bmatrix} 0 & 0 & 0 & 0\\ -0.0016 & 0.0137 & 0.002 & -0.9804\\ -0.0006 & 0.0072 & -0.0144 & -0.4412\\ 0 & 0 & 0 & 0\\ -0.0023 & 0.0209 & -0.0451 & -2.451\\ 0.0007 & -0.0016 & 0.0294 & 0.2941\end{bmatrix}$$

$$W = \begin{bmatrix} 4.994 & -49.8128 & -0.0294 & 0.9067\\ 0 & 500.6246 & 0.6864 & -6.1261\\ 0 & 0 & 9.8058 & 48.0292\\ 0 & 0 & 0 & 499.9039\end{bmatrix}$$

The simulation is realised in MATLAB/SIMULINK and Truetime, and a fault happens in subsystem 1 at time 15-35s.Because the effect of fault on subsystem 2 is very small, only the subsystem 1 simulation result is shown in Fig.2.From it we can conclude that with the premise that the data transmitted through the network is reduced by half, we still get the acceptable result of fault diagnosis. The simulation validates the effectiveness of the method applied in two-level DNCS.

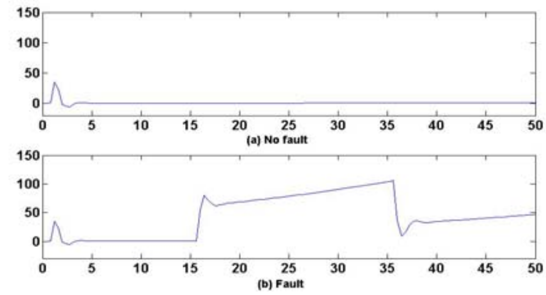

Fig.2 Simulation result

6. CONCLUSIONS

This paper considers the fault detection problem of two-level DNCS. In order to deal with the limited bandwidth of the network and to reduce the network load and thus avoid the uncertainty caused by transmission delays and packet loss, two-level DNCS with periodic communication sequences is presented and central fault diagnosis unit is designed under this communication pattern. The simulation result guarantees that the fault diagnosis result is still acceptable while the total data sent in the network is greatly reduced.

APPENDIX

$$A_s^c = \left[\begin{array}{ccc|ccc} 0.0625 & 0 & 0 & 0 & 0 & 0 \\ -0.0047 & 0.0001 & 0 & 0.0015 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0.0001 \\ \hline 0.0203 & 0 & 0 & 0.0016 & 0 & 0 \\ -0.0009 & 0.0004 & 0 & 0.0011 & 0.0001 & 0 \\ 0 & 0 & -0.0001 & 0 & 0 & 0.0001 \end{array}\right]$$

$$C_s^c = \begin{bmatrix} 0 & 0 & 0 & 0.2 & 0.1 & 0.1 \\ -0.05 & -0.01 & -0.004 & -0.01 & 0 & 0.02 \\ 0.024 & 0.001 & -0.0019 & 0.003 & 0 & -0.0008 \\ 0.0079 & -0.0003 & 0.0001 & -0.002 & -0.0001 & 0.0001 \end{bmatrix}$$

$$B_s^c = \begin{bmatrix} 0 & 0.25 & -0.5 & 0 \\ 0 & 0 & -0.1 & 0 \\ -0.0009 & 0 & 0 & 0 \\ 0 & -0.07 & 0.1 & 0 \\ -0.001 & 0.02 & -0.1 & 1 \\ 0.0006 & 0 & 0 & 1 \end{bmatrix} \quad D_s^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.02 & 0.2 & 1 & 0 \\ 0 & 0.01 & 0 & 1 \end{bmatrix}$$

ACKNOWLEDGEMENTS

This work is partially supported by Chinese Natural Science Foundation 60574055 and Specialized Research Fund for Doctoral Program of Higher Education Grant 20050056037, and Tianjin Natural Science Project Grant 08JCYBJC11900.

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