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A general adaptive robust nonlinear motion controller combined with disturbance observer

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Abstract: A general adaptive robust nonlinear motion controller combined with disturbance observer (DOB) for positioning control of a nonlinear single-input-single-output (SISO) mechanical system is proposed. Theoretical performance such as transient performance, ultimate tracking error bound and mean square tracking error bound are analyzed rigorously, and simulation results are provided to support the theoretical results.

Keywords: Disturbance observer; Input-to-state stability; Robust control; Adaptive control.

1. INTRODUCTION

In this report, we propose a general adaptive robust nonlinear motion controller combined with DOB. With the help of nonlinear damping terms, the input-to-state stability (ISS) property [3] of the overall nonlinear control system is ensured. Basically, the boundedness of the internal signals are ensured by the robustifying nonlinear damping terms. The DOB is employed to compensate the lumped uncertainties without the necessity of parameterization. The adaptive laws are employed to furthermore account for fast-changing uncertainties which the DOB cannot handle sufficiently.

Our major contribution is to incorporate the DOB technique and the adaptive technique which have been considered as two contrastively different approaches in the literature, into one controller under the framework of ISS property. Moreover, transient performance, ultimate tracking error bound and mean square tracking error bound are analyzed rigorously, with transparent physical meaning. Finally, simulation results are provided to support the theoretical results.

2. STATEMENT OF THE PROBLEM

Consider the following SISO nonlinear mechanical system.

$$\dot{x}_1 = x_2 \tag{1a}$$

$$\dot{x}_2 = F(\boldsymbol{x}) + d(\boldsymbol{x},t) + G(\boldsymbol{x})u \tag{1b}$$

where, $\boldsymbol{x} = [x_1, x_2]^T$, x_1 and x_2 are the position and velocity respectively, u is the control input; $G(\boldsymbol{x})$ and $F(\boldsymbol{x})$ are the modelable nonlinear functions; and $d(\boldsymbol{x},t)$ is the completely unknown term composed of unmodelled nonlinearities and disturbances.

Denoting the nominal nonlinearities based on prior knowledge as $F_0(\boldsymbol{x})$ and $G_0(\boldsymbol{x})$, we can model $F(\boldsymbol{x})$ and $G(\boldsymbol{x})$ as

$$F(\boldsymbol{x}) = F_0(\boldsymbol{x}) + \Delta_F(\boldsymbol{x}),\ G(\boldsymbol{x}) = G_0(\boldsymbol{x}) + \Delta_G(\boldsymbol{x}) \tag{2}$$

where the modelling errors $\Delta_F(\boldsymbol{x})$ and $\Delta_G(\boldsymbol{x})$ can be approximated by the linear-in-the-parameters networks. Then we have

$$\begin{aligned} \widehat{F}(\boldsymbol{x},\boldsymbol{w}_F) &= F_0(\boldsymbol{x}) + \widehat{\Delta}_F(\boldsymbol{x},\boldsymbol{w}_F) \\ \widehat{G}(\boldsymbol{x},\boldsymbol{w}_G) &= G_0(\boldsymbol{x}) + \widehat{\Delta}_G(\boldsymbol{x},\boldsymbol{w}_G) \end{aligned} \tag{3}$$

where

$$\widehat{\Delta}_F(\boldsymbol{x},\boldsymbol{w}_F) = \boldsymbol{\phi}_F^T(\boldsymbol{x})\boldsymbol{w}_F,\ \ \widehat{\Delta}_G(\boldsymbol{x},\boldsymbol{w}_G) = \boldsymbol{\phi}_G^T(\boldsymbol{x})\boldsymbol{w}_G \tag{4}$$

The regressor vectors $\boldsymbol{\phi}_F(\boldsymbol{x}), \boldsymbol{\phi}_G(\boldsymbol{x})$ and parameter vectors $\boldsymbol{w}_F, \boldsymbol{w}_G$ are defined as

$$\begin{aligned} \boldsymbol{\phi}_F(\boldsymbol{x}) &= [\phi_{F1}(\boldsymbol{x}), \cdots, \phi_{FN_F}(\boldsymbol{x})]^T \\ \boldsymbol{\phi}_G(\boldsymbol{x}) &= [\phi_{G1}(\boldsymbol{x}), \cdots, \phi_{GN_G}(\boldsymbol{x})]^T \\ \boldsymbol{w}_F &= [w_{F1}, \cdots, w_{FN_F}]^T \\ \boldsymbol{w}_G &= [w_{G1}, \cdots, w_{GN_G}]^T \end{aligned} \tag{5}$$

(2)∼(5) lead to the following relations.

$$\begin{aligned} F(\boldsymbol{x}) &= \widehat{F}(\boldsymbol{x},\widehat{\boldsymbol{w}_{F}}_t) + \left(\Delta_F(\boldsymbol{x}) - \widehat{\Delta}_F(\boldsymbol{x},\widehat{\boldsymbol{w}_{F}}_t)\right) \\ &= \widehat{F}(\boldsymbol{x},\widehat{\boldsymbol{w}_{F}}_t) + \eta_F(\boldsymbol{x},\widehat{\boldsymbol{w}_{F}}_t) \\ &= \widehat{F}(\boldsymbol{x},\widehat{\boldsymbol{w}_{F}}_t) + \eta_F(\boldsymbol{x},\boldsymbol{w}_F^*) - \boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{F}}_t \end{aligned} \tag{6a}$$

$$\begin{aligned} G(\boldsymbol{x}) &= \widehat{G}(\boldsymbol{x},\widehat{\boldsymbol{w}_{G}}_t) + \left(\Delta_G(\boldsymbol{x}) - \widehat{\Delta}_G(\boldsymbol{x},\widehat{\boldsymbol{w}_{G}}_t)\right) \\ &= \widehat{G}(\boldsymbol{x},\widehat{\boldsymbol{w}_{G}}_t) + \eta_G(\boldsymbol{x},\widehat{\boldsymbol{w}_{G}}_t) \\ &= \widehat{G}(\boldsymbol{x},\widehat{\boldsymbol{w}_{G}}_t) + \eta_G(\boldsymbol{x},\boldsymbol{w}_G^*) - \boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{G}}_t \end{aligned} \tag{6b}$$

where

$$\begin{aligned}\eta_F(\boldsymbol{x},\boldsymbol{w}_F) &= \Delta_F(\boldsymbol{x}) - \widehat{\Delta}_F(\boldsymbol{x},\boldsymbol{w}_F)\\ \eta_G(\boldsymbol{x},\boldsymbol{w}_G) &= \Delta_G(\boldsymbol{x}) - \widehat{\Delta}_G(\boldsymbol{x},\boldsymbol{w}_G)\end{aligned} \tag{7}$$

$$\begin{aligned}\boldsymbol{w}_F^* &= \arg\min_{\boldsymbol{w}_F}\left\{\sup_{\boldsymbol{x}\in\Omega_X}|\eta_F(\boldsymbol{x},\boldsymbol{w}_F)|\right\}\\ \boldsymbol{w}_G^* &= \arg\min_{\boldsymbol{w}_G}\left\{\sup_{\boldsymbol{x}\in\Omega_X}|\eta_G(\boldsymbol{x},\boldsymbol{w}_G)|\right\}\end{aligned} \tag{8}$$

$$\widetilde{\boldsymbol{w}_{Ft}} = \widehat{\boldsymbol{w}_{Ft}} - \boldsymbol{w}_F^*, \quad \widetilde{\boldsymbol{w}_{Gt}} = \widehat{\boldsymbol{w}_{Gt}} - \boldsymbol{w}_G^* \tag{9}$$

We impose the following standing assumptions for $\boldsymbol{x}$ on the desired domain of operation Ω_X.

Assumption 1. The lower and upper bounds of the parameter vectors are known *a priori*:

$$\underline{\boldsymbol{w}}_F \le \boldsymbol{w}_F^* \le \overline{\boldsymbol{w}}_F, \quad \underline{\boldsymbol{w}}_G \le \boldsymbol{w}_G^* \le \overline{\boldsymbol{w}}_G \tag{10}$$

Assumption 2. $G(\boldsymbol{x}) > 0$ and the parameter bounds satisfy

$$\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right) = G_0(\boldsymbol{x}) + \widehat{\Delta}_G\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right) > 0 \tag{11}$$

where $\underline{\boldsymbol{w}}_G \le \widehat{\boldsymbol{w}_{Gt}} \le \overline{\boldsymbol{w}}_G$.

Assumption 3. The adaptively updated $\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)$ satisfies

$$\frac{\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)}{G(\boldsymbol{x})} \le {}^{\exists}M_{\widehat{G}} < \infty, \quad \frac{\left|\widehat{\eta}_G\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)\right|}{G(\boldsymbol{x})} \le {}^{\exists}M_{\widetilde{G}} < \infty \tag{12}$$

Assumption 4. There exist known continuous bounding functions $\overline{F}(\boldsymbol{x}) > 0$ and $\overline{d}(\boldsymbol{x},t) > 0$ such that

$$\begin{aligned}&\frac{\left|\widehat{\eta}_F\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right)\right|}{\overline{F}(\boldsymbol{x})} \le {}^{\exists}M_{\widetilde{F}} < \infty, \quad \frac{|d(\boldsymbol{x},t)|}{\overline{d}(\boldsymbol{x},t)} \le {}^{\exists}M_d < \infty\\ &\frac{\left|\widehat{F}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right)\right|}{\overline{F}(\boldsymbol{x})} \le {}^{\exists}M_{\widehat{F}} < \infty\end{aligned} \tag{13}$$

Assumption 5. The reference trajectory $y_r(t)$ is appropriately chosen as a sufficiently smooth function such that $\dot{y}_r$ and $\ddot{y}_r$ are known and

$$\mathcal{D}_{y_r} = \left\{y_r,\dot{y}_r,\ddot{y}_r \middle| [y_r,\dot{y}_r]^T \in \Omega_Y \subset \Omega_X, |\ddot{y}_r| \le {}^{\exists}\overline{\ddot{y}}_r < \infty\right\} \tag{14}$$

3. CONTROLLER DESIGN

The proposed controller is designed in a backstepping procedure, composed of two steps.

Step 1: Define the position error and velocity error signals respectively as

$$z_1 = x_1 - y_r, \quad z_2 = x_2 - \alpha_1 \tag{15}$$

where α_1 is the virtual input to stabilize z_1.

Then from (1a) we have subsystem $\mathcal{S}1$:

$$\mathcal{S}1: \quad \dot{z}_1 = \alpha_1 + z_2 - \dot{y}_r \tag{16}$$

The virtual input α_1 is designed based on the common PI control technique:

$$\alpha_1 = -c_{1p}z_1 - c_{1i}\int_0^t z_1 dt + \dot{y}_r \tag{17}$$

where $c_{1p}, c_{1i} > 0$.

The next step is to stabilize the velocity error z_2.

Step 2: From (1b), (6) and (15) we have subsystem $\mathcal{S}2$:

$$\begin{aligned}\mathcal{S}2: \quad \dot{z}_2 = \ &\widehat{F}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right) - \dot{\alpha}_1 + \widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)u\\ &+ d(\boldsymbol{x},t) + \eta_F\left(\boldsymbol{x},\boldsymbol{w}_F^*\right) - \boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}}\\ &+ \eta_G\left(\boldsymbol{x},\boldsymbol{w}_G^*\right)u - \boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Gt}}u\end{aligned} \tag{18}$$

Denoting the uncertain terms as the lumped disturbance w, we have

$$w = \dot{z}_2 - \left(\widehat{F}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right) + \widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)u - \dot{\alpha}_1\right) \tag{19a}$$

$$= d(\boldsymbol{x},t) + \eta_F\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right) + \eta_G\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)u \tag{19b}$$

$$\begin{aligned}= \ &d(\boldsymbol{x},t) + \eta_F\left(\boldsymbol{x},\boldsymbol{w}_F{}^*\right) - \boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}}\\ &+ \eta_G\left(\boldsymbol{x},\boldsymbol{w}_G{}^*\right)u - \boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Gt}}u\end{aligned} \tag{19c}$$

Therefore, the lumped disturbance w can be obtained as (19a). However, since calculation of $\dot{z}_2$ by direct differentiation is usually contaminated with high frequency noise, we have to pass (19a) through a low-pass filter $Q(s)$ to obtain the estimate of w as

$$\widehat{w} = Q(s)w \tag{20}$$

$$Q(s) = \frac{1}{1+\lambda s}, \quad \overline{Q}(s) = 1 - Q(s) = \frac{\lambda s}{1+\lambda s} \tag{21}$$

This is the so called DOB in the literature [1, 2, 4]. The benefit of compensating the control input by $\widehat{w}$ is obvious. Replacing u in (18) by $u = (v - \widehat{F}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right) + \dot{\alpha}_1 - \widehat{w})/\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)$ and assuming $\widehat{w} \approx w$, we have

$$\dot{z}_2 \simeq v \tag{22}$$

where v is a nominal linear input. A simple controller can therefore be designed. The simplest design is, for example, to let $v = -c_2 z_2$.

However, actually we can only expect $\widehat{w} \approx w$ at low-frequencies. If the disturbance and model mismatch are fast-changing, the estimation error $w - \widehat{w}$ cannot be neglected and even can destroy the stability of the closed-loop in the case of large model mismatch [4].

To stabilize the subsystem $\mathcal{S}2$, we design the following controller.

$$\begin{aligned}&u = u_l + u_w + u_r + u_e\\ &u_l = \frac{\alpha_{20}}{\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)}, \quad u_w = \frac{-\widehat{w}}{\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)}\\ &u_r = \frac{-\sum_{i=1}^{5} u_{di}z_2}{\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)}, \quad u_e = \frac{-(e_G + e_F)}{\widehat{G}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Gt}}\right)}\end{aligned} \tag{23}$$

where

$$\begin{aligned}\alpha_{20} &= -c_2z_2 + \dot{\alpha}_1 - \widehat{F}\left(\boldsymbol{x},\widehat{\boldsymbol{w}_{Ft}}\right)\\ u_{d1} &= \kappa_{21}\overline{F}(\boldsymbol{x})\\ u_{d2} &= \kappa_{22}\alpha_{2d}\\ u_{d3} &= \kappa_{23}\overline{d}(\boldsymbol{x},t)\\ u_{d4} &= \kappa_{24}|\widehat{w}|\\ u_{d5} &= \kappa_{25}|e_G + e_F|\\ \alpha_{2d} &= |-c_2z_2 + \dot{\alpha}_1| + \overline{F}(\boldsymbol{x})\end{aligned} \tag{24}$$

and $c_2, \kappa_{21}, \kappa_{22}, \kappa_{23}, \kappa_{24}, \kappa_{25} > 0$.

u_l is a feedback controller with model compensation. u_w is a compensating term by the DOB's output. $u_{d1}z_2$, $u_{d2}z_2$ and $u_{d3}z_2$ are nonlinear damping terms to counteract

$\Delta_F(\boldsymbol{x}) - \widehat{\Delta}_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}})$, $\Delta_G(\boldsymbol{x}) - \widehat{\Delta}_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})$ and $d(\boldsymbol{x}, t)$ respectively. $u_{di}(i = 1, \cdots, 3)$ are designed as time-varying control gains so that they grow at least as the same order as the corresponding uncertain terms to be counteracted.

$u_{d4}z_2$ is a nonlinear damping term to ensure boundedness of z_2 when $\widehat{w}$ is used. As will be seen in (25) and (26), u_e is introduced to compensate the terms e_G and e_F defined in (26). Notice that e_G and e_F stem from the fact that adaptive laws are not applicable directly to the terms $\overline{Q}(s)\big[\boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Gt}}\big]$ and $\overline{Q}(s)\big[\boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}}\big]$, but are applicable to the terms $\big\{\overline{Q}(s)\boldsymbol{\phi}_G^T(\boldsymbol{x})\big\}\widetilde{\boldsymbol{w}_{Gt}}$ and $\big\{\overline{Q}(s)\boldsymbol{\phi}_F^T(\boldsymbol{x})\big\}\widetilde{\boldsymbol{w}_{Ft}}$. Finally, $u_{d5}z_2$ is a nonlinear damping term to ensure boundedness of z_2 when u_e is used. The roles of the nonlinear damping terms will be shown later through stability analysis. See (35) and (36).

Applying u to $\mathcal{S}2$, we have

$$\begin{aligned}
\dot{z}_2 &= -c_2z_2 + \widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})\,u_r + \eta_F(\boldsymbol{x}, \boldsymbol{w}_F^*) - \boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}} \\
&\quad + d(\boldsymbol{x}, t) + \eta_G(\boldsymbol{x}, \boldsymbol{w}_G^*)u - \boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Gt}}u \\
&\quad - Q(s)\Big(\eta_F(\boldsymbol{x}, \boldsymbol{w}_F^*) - \boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}} + d(\boldsymbol{x}, t) \\
&\quad + \eta_G(\boldsymbol{x}, \boldsymbol{w}_G^*)u - \boldsymbol{\phi}_G^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Gt}}u\Big) - (e_G + e_F) \\
&= -c_2z_2 + \widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})\,u_r + \overline{Q}(s)\eta_F(\boldsymbol{x}, \boldsymbol{w}_F^*) \\
&\quad + \overline{Q}(s)d(\boldsymbol{x}, t) + \overline{Q}(s)\big[\eta_G(\boldsymbol{x}, \boldsymbol{w}_G^*)u\big] \\
&\quad - \big[\overline{Q}(s)\boldsymbol{\phi}_F^T(\boldsymbol{x})\big]\widetilde{\boldsymbol{w}_{Ft}} - \big\{\overline{Q}(s)\big[\boldsymbol{\phi}_G^T(\boldsymbol{x})u\big]\big\}\widetilde{\boldsymbol{w}_{Gt}}
\end{aligned} \tag{25}$$

$$\begin{aligned}
e_F &= \big\{\overline{Q}(s)\boldsymbol{\phi}_F^T(\boldsymbol{x})\big\}\widetilde{\boldsymbol{w}_{Ft}} - \overline{Q}(s)\big[\boldsymbol{\phi}_F^T(\boldsymbol{x})\widetilde{\boldsymbol{w}_{Ft}}\big] \\
e_G &= \big\{\overline{Q}(s)\big[\boldsymbol{\phi}_G^T(\boldsymbol{x})u\big]\big\}\widetilde{\boldsymbol{w}_{Gt}} - \overline{Q}(s)\big[\boldsymbol{\phi}_G^T(\boldsymbol{x})u\widetilde{\boldsymbol{w}_{Gt}}\big]
\end{aligned} \tag{26}$$

To let the adaptive parameters stay in the prescribed range we adopt the following adaptive laws with projection.

$$\dot{\widehat{w_{Fnt}}} = \begin{cases} 0 & \text{for } \widehat{w_{Fnt}} = \underline{w_F}_n,\ \big\{\overline{Q}(s)\phi_{Fn}(\boldsymbol{x})\big\}z_2 < 0 \\ 0 & \text{for } \widehat{w_{Fnt}} = \overline{w_F}_n,\ \big\{\overline{Q}(s)\phi_{Fn}(\boldsymbol{x})\big\}z_2 > 0 \\ \gamma_F\ \big\{\overline{Q}(s)\phi_{Fn}(\boldsymbol{x})\big\}z_2 & \text{otherwise} \end{cases} \tag{27}$$

where $n = 1, \cdots, N_F$, $\gamma_F \geq 0$.

$$\dot{\widehat{w_{Gnt}}} = \begin{cases} 0 & \text{for } \widehat{w_{Gnt}} = \underline{w_G}_n,\ \big\{\overline{Q}(s)\big[\phi_{Gn}(\boldsymbol{x})u\big]\big\}z_2 < 0 \\ 0 & \text{for } \widehat{w_{Gnt}} = \overline{w_G}_n,\ \big\{\overline{Q}(s)\big[\phi_{Gn}(\boldsymbol{x})u\big]\big\}z_2 > 0 \\ \gamma_G\ \big\{\overline{Q}(s)\big[\phi_{Gn}(\boldsymbol{x})u\big]\big\}z_2 & \text{otherwise} \end{cases} \tag{28}$$

where $n = 1, \cdots, N_G$, $\gamma_G \geq 0$.

It can be verified that the adaptive laws satisfy

$$\begin{aligned}
&\underline{\boldsymbol{w}}_F \leq \widehat{\boldsymbol{w}_{Ft}} \leq \overline{\boldsymbol{w}}_F, \quad \underline{\boldsymbol{w}}_G \leq \widehat{\boldsymbol{w}_{Gt}} \leq \overline{\boldsymbol{w}}_G \\
&\widetilde{\boldsymbol{w}_{Ft}}^T\dot{\widetilde{\boldsymbol{w}_{Ft}}} - \gamma_F\big\{\overline{Q}(s)\boldsymbol{\phi}_F^T(\boldsymbol{x})\big\}\widetilde{\boldsymbol{w}_{Ft}}z_2 \leq 0 \\
&\widetilde{\boldsymbol{w}_{Gt}}^T\dot{\widetilde{\boldsymbol{w}_{Gt}}} - \gamma_G\big\{\overline{Q}(s)\big[\boldsymbol{\phi}_G(\boldsymbol{x})u\big]\big\}\widetilde{\boldsymbol{w}_{Gt}}z_2 \leq 0
\end{aligned} \tag{29}$$

Remark 1. Inspection of (27) and (28) reveals interesting physical features of the adaptive laws. Since the complementary filter $\overline{Q}(s)$ is high-pass, in the case of slowly-changing signals, the amplitudes of $\big\{\overline{Q}(s)\phi_{Fn}(\boldsymbol{x})\big\}_{n=1}^{n_F}$ and $\big\{\overline{Q}(s)\big[\phi_{Gn}(\boldsymbol{x})u\big]\big\}_{n=1}^{n_G}$ are trivial so that the adaptive laws become to be "lazy". Contrastively, the DOB in (20) is more efficient for low-frequency uncertainties.

4. STABILITY ANALYSIS

4.1 ISS property analysis of each subsystem

Applying α_1 to the subsystem $\mathcal{S}1$, we have

$$\dot{z}_1 = z_2 - c_{1p}z_1 - c_{1i}\int_0^t z_1 dt \tag{30}$$

Denote the Laplace operator as s. Then the subsystem $\mathcal{S}1$ can be expressed as

$$z_1 = \frac{sz_2}{s^2 + c_{1p}s + c_{1i}} \tag{31}$$

This can be rewritten into the state-space form:

$$\dot{\boldsymbol{z}}_{1a} = \boldsymbol{A}\ \boldsymbol{z}_{1a} + \boldsymbol{B}\ z_2 \tag{32}$$

where $\boldsymbol{z}_{1a} = [\int_0^t z_1 dt, z_1]^T$,

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -c_{1i} & -c_{1p} \end{bmatrix}, \quad \boldsymbol{B} = [0\ 1]^T \tag{33}$$

The ISS property of the subsystem $\mathcal{S}1$ is described in the following lemma.

Lemma 1. If the virtual input α_1 is applied to the subsystem $\mathcal{S}1$, and if z_2 is made uniformly bounded at the next step, then the subsystem $\mathcal{S}1$ is ISS, i.e., for $^\exists\lambda_0, ^\exists\rho_0 > 0$,

$$|\boldsymbol{z}_{1a}(t)| \leq \lambda_0 e^{-\rho_0 t}|\boldsymbol{z}_{1a}(0)| + \frac{\lambda_0}{\rho_0}\left[\sup_{0\leq\tau\leq t}|z_2(\tau)|\right]$$

To establish the ISS property of the subsystem $\mathcal{S}2$, we first rewrite (25):

$$\begin{aligned}
\dot{z}_2 &= -c_2z_2 + \widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})(u_r + u_w + u_e) + d(\boldsymbol{x}, t) \\
&\quad + \eta_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}}) + \eta_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})(u_l + u_w + u_r + u_e) \\
&= -c_2z_2 + G(\boldsymbol{x})\,u_r + \eta_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}}) + d(\boldsymbol{x}, t) \\
&\quad + \eta_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})\,u_l - \frac{G(\boldsymbol{x})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}(\widehat{w} + e_G + e_F) \\
&= -c_2z_2 + \widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})\,u_r + w - (\widehat{w} + e_G + e_F)
\end{aligned} \tag{34}$$

Then we have

$$\begin{aligned}
\frac{d}{dt}\left(\frac{z_2^2}{2}\right) &= -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + D_2\right]z_2^2 + d_2z_2 \\
&\leq -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + D_2\right]|z_2|\,[|z_2| - \mu_2]
\end{aligned} \tag{35}$$

where

$$\mu_2(t) = \frac{|d_2|}{\dfrac{c_2}{2} + D_2} \tag{36}$$

$$\begin{aligned}
d_2 &= \frac{\eta_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}\alpha_{20} + \eta_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}}) \\
&\quad + d(\boldsymbol{x}, t) - \frac{G(\boldsymbol{x})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}(\widehat{w} + e_F + e_G) \\
D_2 &= \frac{G(\boldsymbol{x})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}\Big(\kappa_{21}\overline{F}(\boldsymbol{x}) + \kappa_{22}\alpha_{2d} \\
&\quad + \kappa_{23}\overline{d}(\boldsymbol{x}, t) + \kappa_{24}|\widehat{w}| + \kappa_{25}|e_F + e_G|\Big)
\end{aligned} \tag{37}$$

According to Assumptions 1∼5, it is obvious that each term in the numerator of μ_2 is counteracted by a nonlinear damping term in the denominator which grows at least as the same order as the corresponding term in the numerator, so that μ_2 is uniformly bounded. Furthermore, from (35) we have

$$|z_2| \geq \mu_2(t) \Rightarrow \frac{d}{dt}\left(z_2^2\right) \leq -c_2z_2^2 \tag{38}$$

and hence

$$|z_2(t)| \le |z_2(0)|e^{-c_2 t/2} + \sup_{0\le\tau\le t} \mu_2(\tau) \tag{39}$$

Therefore the uniform boundedness of z_2 can be ensured by the nonlinear damping terms. Thus, Lemma 1 holds, which implies $|\boldsymbol{z}_{1a}|$ is bounded. Since the reference trajectory y_r, $\dot{y}_r$ and $\ddot{y}_r$ are uniformly bounded (Assumption 5), we can, therefore, conclude that all the internal signals of the two subsystems are uniformly bounded.

In the above analysis, the main attention is to show the boundedness of the internal signals of the closed-loop. No analysis yet has been done for the attenuation effects of $w - \widehat{w}$. Without such an analysis, we cannot clearly see how the DOB's output $\widehat{w}$ can bring an improvement. We now attempt to make such an effort.

Then, keeping that all the internal signals are bounded in mind, we rewrite (35) by using (34):

$$\begin{aligned}\frac{d}{dt}\left(\frac{z_2^2}{2}\right) &= -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + D_{2w}\right] z_2^2 + wz_2 \\ &\quad -(\widehat{w} + e_G + e_F)z_2 \\ &\le -\frac{c_2}{2}z_2^2 - \left[\frac{c_2}{2} + D_{2w}\right] |z_2| \left[|z_2| - \mu_{2w}\right]\end{aligned} \tag{40}$$

where

$$\begin{aligned}\mu_{2w}(t) &= \frac{|w-\widehat{w}| + |e_G + e_F|}{\frac{c_2}{2} + D_{2w}} = \frac{|\overline{Q}(s)w| + |e_G + e_F|}{\frac{c_2}{2} + D_{2w}} \\ D_{2w} &= \kappa_{21}\overline{F}(\boldsymbol{x}) + \kappa_{22}\alpha_{2d} + \kappa_{23}\overline{d}(\boldsymbol{x},t) \\ &\quad + \kappa_{24}|\widehat{w}| + \kappa_{25}|e_F + e_G|\end{aligned} \tag{41}$$

Notice that we can write $w - \widehat{w}$ as

$$\begin{aligned}w - \widehat{w} &= \overline{Q}(s)\Big(d(\boldsymbol{x},t) + \eta_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}}) + \eta_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})\, u\Big) \\ &= \frac{\eta_G(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}(-D_{2w}z_2 + \alpha_{20} - e_F - e_G) \\ &\quad + \eta_F(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Ft}}) + d(\boldsymbol{x},t) - \frac{G(\boldsymbol{x})}{\widehat{G}(\boldsymbol{x}, \widehat{\boldsymbol{w}_{Gt}})}\widehat{w}\end{aligned} \tag{42}$$

It should be commented here that μ_{2w} has very transparent physical meaning. At low-frequencies, we can expect $w - \widehat{w} \approx 0$. And any nonzero $w - \widehat{w}$ at high-frequencies is counteracted by $c_2/2 + D_{2w}$ so that z_2 is quite robust against $w - \widehat{w}$.

Remark 2. As mentioned previously, the terms e_G and e_F defined in (26) are inevitablly due to the time-varying effects of the adaptively updated parameters. In the case of $\gamma_F = \gamma_G = 0$, i.e., the adaptive laws are switched off, we have $e_G = e_F = 0$.

Remark 3. In the case of $\gamma_F = \gamma_G = 0$, the controller is reduced to a fixed robust controller with DOB [4], and thus the boundedness of μ_2 and μ_{2w} still holds under Assumptions 1∼5.

Remark 4. At the present stage, we are mainly concentrated on the ensurance of the boundedness of μ_2 and μ_{2w}. If we do not adopt the compensation term u_e in (23), $e_G + e_F$ does not appear in the numerators of μ_2 and μ_{2w}, and thus the boundedness of μ_2 and μ_{2w} is still ensured. Empirically, the amplitudes of e_G and e_F given in (26) are trivial in most cases since $\widehat{\boldsymbol{w}}_F$ and $\widehat{\boldsymbol{w}}_G$ do not change so fast compared to the high-passed signals, and thus the control performance do not degenerate significasntly when u_e in (23) is removed. However, for the convenience to show how the adaptive laws bring improved ultimate bound and mean square bound of z_2 theoretically, it is recommended to employ u_e so that the time-varying effects of the adaptively updated parameters are fully compensated. See (25).

Remark 5. When the DOB is not used, i.e., $Q(s) = 0$, we have $\widehat{w} = 0, e_F = 0, e_G = 0$, and thus u_{d4}, u_{d5}, u_e in (23) can be removed. In this case, the boundedness of μ_2 and μ_{2w} is still ensured, owing to the nonlinear damping terms.

Finally, we have

$$|z_2| \ge \mu_{2w}(t) \Rightarrow \frac{d}{dt}\left(z_2^2\right) \le -c_2 z_2^2 \tag{43}$$

and hence the result of Lemma 2.

Lemma 2. Let Assumptions 1∼5 hold. If the control input u is applied to the subsystem $\mathcal{S}2$, then the subsystem $\mathcal{S}2$ is ISS and the error signal $z_2(t)$ is uniformly bounded as

$$|z_2(t)| \le |z_2(0)|e^{-c_2 t/2} + \sup_{0\le\tau\le t} \mu_{2w}(\tau)$$

Notice that lemmas 1 and 2 imply that the internal signals of the two subsystems are bounded, i.e., the boundedness are ensured by the nonlinear damping terms no matter if the adaptive laws are activated.

4.2 Error bounds achieved by DOB and adaptive laws

We first consider the subsystem $\mathcal{S}1$ in (32) and (33). Since there exists a positive definite symmetric matrix $\boldsymbol{P}$ satisfying $\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q}$ for any positive definite symmetric matrix $\boldsymbol{Q}$, we have

$$\begin{aligned}\frac{d}{dt}\left(\frac{\boldsymbol{z}_{1a}^T\boldsymbol{P}\boldsymbol{z}_{1a}}{2}\right) &= -\frac{1}{2}\boldsymbol{z}_{1a}^T\boldsymbol{Q}\boldsymbol{z}_{1a} + \boldsymbol{z}_{1a}^T\boldsymbol{P}\boldsymbol{B}z_2 \\ &\le -\frac{\lambda_{Qmin}}{2}|\boldsymbol{z}_{1a}|^2 + |\boldsymbol{z}_{1a}||\boldsymbol{P}\boldsymbol{B}||z_2| \\ &\le -\frac{\lambda_{Qmin}}{4}|\boldsymbol{z}_{1a}|^2 + \frac{1}{\lambda_{Qmin}}|\boldsymbol{P}\boldsymbol{B}|^2|z_2|^2\end{aligned} \tag{44}$$

where λ_{Qmin} is the minimal eigenvalue of $\boldsymbol{Q}$. Then we have

Lemma 3. If α_1 is applied to the subsystem $\mathcal{S}1$, and if z_2 is made uniformly ultimately bounded with ultimate bound $\overline{z}_2^u$ at the next step, the error signal $\boldsymbol{z}_{1a}(t)$ is uniformly ultimately bounded such that

$$|\boldsymbol{z}_{1a}(t)| \le C_1\overline{z}_2^u, \quad ^\exists C_1 > 0, \quad \text{as } t \ge {}^\exists T_1 > 0$$

and the mean square of $\boldsymbol{z}_{1a}(t)$ satisfies

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T |\boldsymbol{z}_{1a}|^2 dt \le \frac{4|\boldsymbol{P}\boldsymbol{B}|^2}{\lambda_{Qmin}^2}\lim_{T\to\infty}\frac{1}{T}\int_0^T |z_2|^2 dt$$

Furthermore, to analyze how the adaptive laws help to improve $|z_2|$, we impose one more assumption:

Assumption 6. The networks are sufficiently complex such that the approximation errors are sufficiently small on the desired domain of operation Ω_X, i.e., there exist $\boldsymbol{w}_F^*$ and $\boldsymbol{w}_G^*$ satisfying

$$\sup_{\boldsymbol{x}\in\Omega_X} |\eta_F(\boldsymbol{x}, \boldsymbol{w}_F^*)| \le {}^\exists\varepsilon_F, \quad \sup_{\boldsymbol{x}\in\Omega_X} |\eta_G(\boldsymbol{x}, \boldsymbol{w}_G^*)| \le {}^\exists\varepsilon_G \tag{45}$$

Then we define the following Lyapunov function for $\gamma_F, \gamma_G > 0$.

$$V_2 = \frac{z_2^2}{2} + \frac{\widetilde{\boldsymbol{w}_{Ft}}^T \widetilde{\boldsymbol{w}_{Ft}}}{2\gamma_F} + \frac{\widetilde{\boldsymbol{w}_{Gt}}^T \widetilde{\boldsymbol{w}_{Gt}}}{2\gamma_G} \tag{46}$$

By using the results of (25) and (29), we have

$$\begin{aligned} \dot{V}_2 &\le -(c_2 + D_{2w})|z_2|^2 + |\overline{Q}(s)w^*||z_2| \\ &= -\frac{(c_2 + D_{2w})}{2}|z_2|^2 - \frac{(c_2 + D_{2w})}{2}|z_2|^2 \\ &\quad + |\overline{Q}(s)w^*||z_2| - \delta_{2m}^2 + \delta_{2m}^2 \\ &\le -\frac{(c_2 + D_{2w})}{2}|z_2|^2 + \delta_{2m}^2 \\ &\le -(c_2 + D_{2w})\left(V_2 - \frac{M_F^2}{2\gamma_F} - \frac{M_G^2}{2\gamma_G}\right) + \delta_{2m}^2 \\ &= -(c_2 + D_{2w})\left(V_2 - \frac{1}{2}\delta_{2u}^2\right) \end{aligned} \tag{47}$$

where

$$\begin{aligned} M_F^2 &= (\overline{\boldsymbol{w}_F} - \underline{\boldsymbol{w}_F})^T(\overline{\boldsymbol{w}_F} - \underline{\boldsymbol{w}_F}) \ge \widetilde{\boldsymbol{w}_{Ft}}^T \widetilde{\boldsymbol{w}_{Ft}} \\ M_G^2 &= (\overline{\boldsymbol{w}_G} - \underline{\boldsymbol{w}_G})^T(\overline{\boldsymbol{w}_G} - \underline{\boldsymbol{w}_G}) \ge \widetilde{\boldsymbol{w}_{Ft}}^T \widetilde{\boldsymbol{w}_{Ft}} \\ \delta_{2m} &= \frac{|\overline{Q}(s)w^*|}{\sqrt{2(c_2 + D_{2w})}} \\ \delta_{2u} &= \sqrt{\frac{M_F^2}{\gamma_F} + \frac{M_G^2}{\gamma_G} + \frac{|\overline{Q}(s)w^*|^2}{(c_2 + D_{2w})^2}} \\ w^* &= d(\boldsymbol{x}, t) + \eta_F(\boldsymbol{x}, \boldsymbol{w}_F^*) + \eta_G(\boldsymbol{x}, \boldsymbol{w}_G^*)\, u \\ &\le d(\boldsymbol{x}, t) + \varepsilon_F + \varepsilon_G u \end{aligned} \tag{48}$$

Then we have the following results.

Lemma 4. Let Assumption 6 and the assumptions and results of lemma 2 hold. If the control input u (23) and the adaptive laws (27) and (28) are applied to the subsystem $\mathcal{S}2$, then the following results hold:

(1) The adaptive parameters satisfy
$$\underline{\boldsymbol{w}_F} \le \widehat{\boldsymbol{w}_{Ft}} \le \overline{\boldsymbol{w}_F}, \quad \underline{\boldsymbol{w}_G} \le \widehat{\boldsymbol{w}_{Gt}} \le \overline{\boldsymbol{w}_G}, \quad \text{for all } t \ge 0$$
(2) The ultimate bound of z_2 is obtained as
$$|z_2| \le \left[\sup_{0 \le \tau \le \infty} |\delta_{2u}(\tau)|\right] \quad \text{as } t \ge {}^{\exists}T_{2u} > 0$$
(3) The mean square error z_2 satisfies
$$\lim_{T \to \infty} \frac{1}{T}\int_0^T |z_2|^2 dt \le \frac{2}{c_2} \lim_{T \to \infty} \frac{1}{T}\int_0^T \delta_{2m}^2 dt$$

Remark 6. Inspection of (47) and (48) reveals clearly the ultimated error bound and mean squares error bound achieved by the DOB and adaptive laws. ε_F and ε_G defined in Assumption 6 imply the best approximation error achieved by the employed networks. If the network complexities are limited, they cannot be made to be zero. Since $d(\boldsymbol{x}, t)$ is an unparameterizable disturbance term, we cannot handle it by parameter adaptation. Therefore, w^* is what we can achieve by the adaptive laws. However, $\overline{Q}(s)w^*$ clearly implies that the low-frequency components of w^* can be removed owing to the DOB.

4.3 Stability of the overall error system

Lemmas 1 and 2 implies that the overall error system is a cascade of two ISS subsystems. Define the error signal vector

$$\boldsymbol{z}(t) = \left[\boldsymbol{z}_{1a}^T(t), z_2(t)\right]^T \tag{49}$$

Then along the same lines of the proof of lemma C.4 in the monograph [3], we can derive the following results.

$$|\boldsymbol{z}(t)| \le \beta_1 e^{-\rho_1 t}|\boldsymbol{z}(0)| + \beta_2 \left[\sup_{0 \le \tau \le t} \mu_{2w}(\tau)\right] \tag{50}$$

where

$$\begin{aligned} \beta_1 &= \sqrt{2}\left(\lambda_0^2 + 3\frac{\lambda_0^2}{\rho_0} + 3\frac{\lambda_0}{\rho_0} + 3\right) \\ \rho_1 &= \min(\rho_0/2, c_2/4) \\ \beta_2 &= \frac{\lambda_0^2}{\rho_0} + \frac{\lambda_0}{\rho_0} + 1 \end{aligned} \tag{51}$$

Furthermore, from Lemmas 3 and 4, we have the mean square bound and ultimate bound of the position tracking error. Additionally, (31) implies that the zero-frequency component of z_1 converges to zero. The results discussed above are summarized as follows.

Theorem 1. The following results hold for the overall error system:

(1) Let the assumptions and results of Lemmas 1 and 2 hold. The overall error system is ISS such that
$$|\boldsymbol{z}(t)| \le \beta_1 e^{-\rho_1 t}|\boldsymbol{z}(0)| + \beta_2 \left[\sup_{0 \le \tau \le t} \mu_{2w}(\tau)\right]$$
(2) Let the assumptions and results of Lemmas 3 and 4 hold. The ultimate bound of the position tracking error can be made sufficiently small such that
$$|z_1(t)| \le \frac{2|\boldsymbol{PB}|}{\lambda_{Qmin}}\left[\sup_{0 \le \tau \le \infty} |\delta_{2u}(\tau)|\right] \quad \text{as } t \ge {}^{\exists}T_u > 0$$
(3) Let the assumptions and results of Lemmas 3 and 4 hold. The mean square bound of the position tracking error can be made sufficiently small such that
$$\lim_{T \to \infty} \frac{1}{T}\int_0^T |z_1|^2 dt \le \frac{8|\boldsymbol{PB}|^2}{c_2 \lambda_{Qmin}^2} \lim_{T \to \infty} \frac{1}{T}\int_0^T \delta_{2m}^2 dt$$
(4) The zero-frequency component of z_1 converges to zero.

5. SIMULATION EXAMPLES

Extensive simulations have been performed for the motion control problem of a linear motor where the friction and periodic ripple disturbances, and unmodelable external distrubance affect the control performance simultaneously.

The nonlinear functions in the system model (1) are described as follows.

$$\begin{aligned} &F(\boldsymbol{x}) = F_p(x_1) + F_f(x), \quad G(\boldsymbol{x}) = \frac{1}{M} \\ &d(\boldsymbol{x}, t) = d_f(x_2, \varepsilon) + d_e(t) \end{aligned} \tag{52}$$

$F_p(x_1)$ is the position dependent ripple disturbance modelled by

$$F_p(x_1) = -\frac{2.5\sin(2\pi x_1/P)}{M} - \frac{3.1\sin(4\pi x_1/P + 0.05\pi)}{M} \tag{53}$$

It is assumed that the magnet pitch is known such that $P = 0.03$[m].

$F_f(\boldsymbol{x})$ and $d_f(x_2, \varepsilon)$ are respectively the modelable and unmodelable effects of friction:

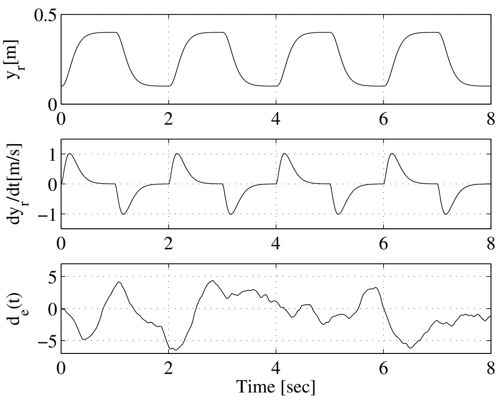

Fig. 1. Reference position y_r and its velocity $\dot{y}_r$, and the unmodelable external disturbance $d_e(t)$.

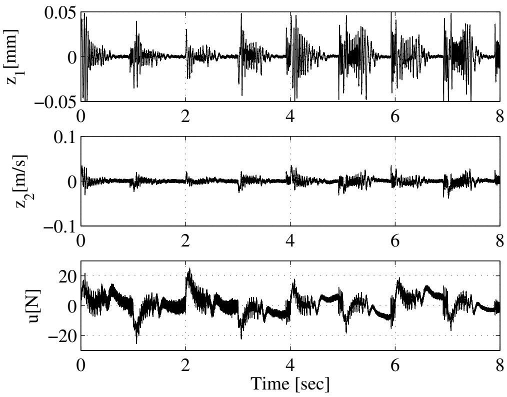

Fig. 2. Results of the adaptive robust nonlinear controller without DOB.

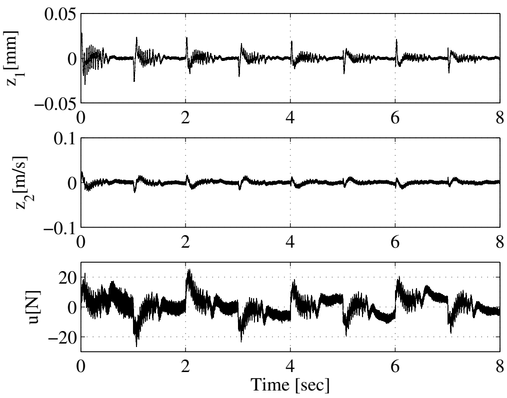

Fig. 3. Results of the adaptive robust nonlinear controller with DOB.

$$F_f(\boldsymbol{x}) = -\frac{\sigma_2 x_2 + \left[F_c + (F_s - F_c)e^{-(x_2/\dot{x}_s)^2}\right]\mathrm{sgn}(x_2)}{M} \tag{54}$$

$$d_f(x_2, \varepsilon) = -\frac{\sigma_0 \varepsilon}{M}\left[1 - \frac{|\sigma_1 x_2|}{F_c + (F_s - F_c)e^{-(x_2/\dot{x}_s)^2}}\right] \tag{55}$$

where $\varepsilon = z - z_s$, and

$$\begin{aligned} \dot{z} &= x_2 - \frac{|x_2|}{h(x_2)}z, \quad z_s = h(x_2)\mathrm{sgn}(x_2) \\ h(x_2) &= \frac{F_c + (F_s - F_c)e^{-(x_2/\dot{x}_s)^2}}{\sigma_0} \end{aligned} \tag{56}$$

It is known that

$$|d_f(x_2, \varepsilon)| \leq \Delta_{d1}|x_2| + \Delta_{d2}, \quad {}^{\exists}\Delta_{d1}, {}^{\exists}\Delta_{d2} > 0 \tag{57}$$

In the above models, the true but unknown values of the physical parameters are given as

$$\begin{aligned} &M = 1[\mathrm{kg}], \quad \sigma_0 = 10^5[\mathrm{N/m}] \\ &\sigma_1 = \sqrt{10^5}[\mathrm{Ns/m}], \quad \sigma_2 = 1[\mathrm{Ns/m}] \\ &F_c = 2[\mathrm{N}], \quad F_s = 4[\mathrm{N}], \quad \dot{x}_s = 0.01[\mathrm{m/s}] \end{aligned} \tag{58}$$

Finally, the unmodelable external disturbance $d_e(t)$ shown in Fig. 1 is generated by passing a stochastic signal through a low-pass filter.

Due to the limit of paper length, the details of the controller design are omitted here. It can be found in Fig. 2 that due to the presence of unmodelable external disturbance $d_e(t)$, the adaptive laws do not bring satisfactory improvement. However, we can find from the results of Fig. 3 that the propoped adaptive robust nonlinear controller inocorporating DOB brings significant improvement to suppress the error amplitudes. The results match the theoretical analysis quite well.

6. CONCLUSIONS

In this paper, a general adaptive robust nonlinear motion controller combined with DOB for positioning control of a nonlinear SISO mechanical system was proposed. Rigorous stability analysis was performed as well. Extensive simulation studies were carried out to support the theoretical analysis given in the paper. Our major academic contribution is to incorporate the DOB technique and adaptive technique which have been considered as two contrastively different approaches in the literature, under the framework of ISS property.

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