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Decentralized attitude alignment control of spacecraft within a formation without angular velocity measurements ⋆

A. Abdessameud [*] **A. Tayebi** [**]

[*] *Department of Electrical and Computer Engineering, University of Western Ontario, London, Ontario, Canada (e-mail: aabdessa@uwo.ca).*
[**] *Department of Electrical Engineering, Lakehead University, Thunder Bay, Ontario, Canada, (e-mail: atayebi@lakeheadu.ca).*

Abstract: In this paper, we consider the coordinated attitude control problem without velocity measurements. Based on the recently introduced unit quaternion output feedback for the attitude tracking of a rigid body, we present a class of decentralized coordinated control laws to solve the alignment problem for a group of spacecraft within a formation without velocity measurements. The approach consists of introducing an auxiliary system for each spacecraft and for each pair of spacecraft with a communication link. The vector parts of the unit quaternion, representing the discrepancies between the output of the auxiliary systems and the attitude tracking error as well as the relative attitude errors between spacecraft, are used in the control law instead of the angular velocity and the relative angular velocity vectors. The spacecraft attitudes are guaranteed to converge to a desired attitude (possibly time-varying), while keeping the flight formation during the transient. Simulation results of a scenario of four spacecraft are provided to show the effectiveness of the proposed control scheme.

1. INTRODUCTION

The coordination and control of formation of multiple vehicles has received significant attention in recent years and various strategies have been proposed. These approaches can be categorized according to their control architecture as Multiple-Input Multiple-output, leader-follower, virtual structure and behavioral architectures. The latter providing the most useful tool in the coordinated attitude control problem, Scharf et al. (2004), Ren and Beard (2004), Vandyke et al. (2006).

One advantage of the behavioral approach is that explicit formation feedback is included through the communication between neighbors in the formation. Another important feature of the behavioral approach is that it is a decentralized implementation that can achieve more flexibility, reliability and robustness than centralized approaches. This can be seen from the works of Balch and Arkins (1998), Lawton et al. (2000a), Lawton et al. (2000b), Lawton and Beard (2002), Ren and Beard (2004) and Ren (2007). In Lawton et al. (2000a), the authors present the so-called *coupled dynamics controller* to solve the coordinated attitude control problem, where behavior-based formation control strategies needed to maintain relative attitude during elementary formation maneuvers are derived. The coupled dynamics controller consists of an attitude alignment part and a formation keeping part, and uses a bidirectional ring communication topology. This work has been extended in Lawton et al. (2000b) and Lawton and Beard (2002).

More recently, In Ren (2007), the author extends the decentralized virtual structure proposed by Ren and Beard (2004) and the work of Lawton and Beard (2002) to a more general formation scenario, where a distributed attitude alignment is proposed for a team of deep space formation flying spacecraft through local information exchange, and three scenarios were considered. In Vandyke et al. (2006), globally significant kinematic error variables are defined and used in the development of a class of decentralized attitude alignment laws that guarantee global asymptotic stability of the attitude of spacecraft within a formation.

In the above mentioned approaches, in order to ensure the asymptotic stability of the formation, it is required that each spacecraft knows its own angular velocity and the angular velocity of its neighbors. In Lawton and Beard (2002), this requirement is removed by the introduction of a passivity based stabilizing control law which is an extension of the passivity based velocity-free attitude regulation scheme proposed in Lizarralde et al. (1996). Several global velocity-free attitude regulation schemes are available in the literature. The extension of those schemes to the tracking problem without velocity measurement is not an obvious task and, to the best of our knowledge, the existing results are only local or semi-global (see, for instance, Caccavale et al. (1999), Costic et al. (2000)). The approach presented in Tayebi (2007) provides a new quaternion based solution to the attitude tracking problem without velocity measurements and guarantees almost global asymptotic stability.

Based on the work of Tayebi (2007), the main contribution of this paper is to modify the work of Vandyke et al. (2006)

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and Ren (2007), and design a velocity-free decentralized coordinated attitude control scheme such that a group of spacecraft converge to their desired time varying attitude while maintaining the same relative attitude during formation maneuvers with a non zero desired angular velocity. We introduce an auxiliary system for each spacecraft and for each pair of spacecraft with a communication link. The vector parts of the unit quaternion describing the discrepancy between the output of these auxiliary systems and the attitude tracking error as well as the relative attitude errors between spacecraft, are used in the control law to generate the necessary damping that would have been generated by the angular velocities and the relative angular velocities of the spacecraft. Almost global asymptotic stability results are obtained.

2. SPACECRAFT DYNAMICS

The individual spacecraft in the formation are modeled as rigid bodies. The equations of motion of the j^{th} spacecraft are

$$I_{f_j}\dot{\omega}_j = \tau_j - \omega_j \times I_{f_j}\omega_j, \tag{1}$$

$$\dot{\bar{q}}_j = \frac{1}{2}\bar{q}_j \odot \bar{\omega}_j \tag{2}$$

where $\bar{\omega}_j = \left(\omega_j^T\ ,\ 0\right)^T$, and ω_j denotes the angular velocity of the j^{th} spacecraft expressed in the body-fixed frame $\mathcal{F}_j$. $I_{f_j} \in \mathbb{R}^{3\times 3}$ is a symmetric positive definite constant inertia matrix of the j^{th} spacecraft with respect to $\mathcal{F}_j$. The vector τ_j is the external torque applied to the j^{th} spacecraft expressed in $\mathcal{F}_j$. The unit quaternion $\bar{q}_j = \left(q_j^T, q_{j,4}\right)^T$, composed of a vector component $q_j \in \mathbb{R}^3$ and a scalar component $q_{j,4} \in \mathbb{R}$, represents the orientation of the j^{th} spacecraft frame, $\mathcal{F}_j$, with respect to the inertial frame, $\mathcal{F}_i$, which are subject to the constraint

$$q_j^T q_j + q_{j,4}^2 = 1 \tag{3}$$

The rotation matrix that brings $\mathcal{F}_i$ onto $\mathcal{F}_j$, denoted by $R(\bar{q}_j) \in \mathbb{R}^{3\times 3}$, is defined as follows

$$R(\bar{q}_j) = (q_{j,4}^2 - q_j^T q_j)I_3 + 2q_j q_j^T - 2q_{j,4}q_j\times \tag{4}$$

where $\times$ is the vector cross product and I_3 is the 3×3 identity matrix. The quaternion multiplication between two unit quaternion, $\bar{q}_j$ and $\bar{q}_k$, is defined by the following non-commutative operation

$$\bar{q}_j \odot \bar{q}_k = \left(q_{j,4}q_k + q_{k,4}q_j + q_j \times q_k\ ,\ q_{j,4}q_{k,4} - q_j^T q_k\right) \tag{5}$$

The inverse or conjugate of a unit quaternion is defined by, $\bar{q}_j^{\,-1} = (-q_j^T, q_{j,4})^T$, with the quaternion identity given by $(0,0,0,1)^T$, Shuster (1993).

Our objective is to design a control scheme for each spacecraft such that all the spacecraft converge to the desired attitude, $\bar{q}^d(t) = (q^d(t), q_4^d(t))$, with a desired angular velocity $\omega^d(t)$, while maintaining the same relative attitude during formation maneuvers. We assume that $\omega^d(t)$ as well as its first and second time-derivatives are bounded.

We first define the attitude tracking error for spacecraft j as follows:

$$\bar{\delta q}_j = (\bar{q}^d)^{-1} \odot\ \bar{q}_j \tag{6}$$

which is governed by the dynamics

$$\dot{\bar{\delta q}}_j = \begin{pmatrix} \dot{\delta q}_j \\ \dot{\delta q}_{j,4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \delta q_{j,4}\ I_3 + \delta q_j \times \\ -\ \delta q_j^T \end{pmatrix} \delta\omega_j, \tag{7}$$

$$\delta\omega_j = \omega_j - R(\bar{\delta q}_j)\ \omega^d \tag{8}$$

The attitude error between the j^{th} and the k^{th} spacecraft, namely $\bar{q}_{jk}$, is defined as:

$$\bar{q}_{jk} = (\bar{q}_k)^{-1} \odot \bar{q}_j, \tag{9}$$

$$\dot{\bar{q}}_{jk} = \begin{pmatrix} \dot{q}_{jk} \\ \dot{q}_{jk,4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} q_{jk,4}\ I_3 + q_{jk}\times \\ -\ q_{jk}^T \end{pmatrix} \omega_{jk} \tag{10}$$

with $\omega_{jk} = \omega_j - R(\bar{q}_{jk})\omega_k$. The vector ω_{jk} is the relative angular velocity of $\mathcal{F}_j$ with respect to $\mathcal{F}_k$ expressed in $\mathcal{F}_j$, $\bar{q}_{jk}$ represents the rotation from $\mathcal{F}_k$ to $\mathcal{F}_j$ and $R(\bar{q}_{jk})$ is the rotation matrix related to $\bar{q}_{jk}$.

The following equations relating the relative states of the j^{th} and k^{th} spacecraft are derived easily

$$\bar{q}_{jk} = (\bar{\delta q}_k)^{-1} \odot \bar{\delta q}_j \tag{11}$$

$$\omega_{jk} = \delta\omega_j - R(\bar{q}_{jk})\delta\omega_k \tag{12}$$

$$R(\bar{q}_{kj})^T = R(\bar{q}_{jk}) \tag{13}$$

$$q_{kj} = -q_{jk} = -R(\bar{q}_{kj})\ q_{jk} \tag{14}$$

With the above definitions, our objective will be to design a control scheme such that the vectors δq_j, q_{jk}, $\delta\omega_j$ and ω_{jk} tend to zero asymptotically as time tends to infinity. Actually, in Ren (2007) and Vandyke et al. (2006) this problem was successfully solved in the full information case, when both the attitude and angular velocity of each spacecraft are available for feedback. Our interest is to provide a solution to this problem when the angular velocity is not measurable/available for feedback.

3. AUXILIARY SYSTEMS

In this section, we extend the work of Tayebi (2007) to formation control without velocity measurements. Consider the auxiliary system for each individual spacecraft defined as

$$\dot{\bar{p}}_j = \frac{1}{2}\bar{p}_j \odot \bar{\beta}_j \tag{15}$$

with $\bar{\beta}_j = (\beta_j^T\ ,\ 0)^T$ and $\beta_j \in \mathbb{R}^3$ to be defined later. The mismatch between the auxiliary system output and the attitude tracking error for the j^{th} spacecraft is defined by the unit quaternion

$$\bar{\delta p}_j = (\bar{p}_j)^{-1} \odot \bar{\delta q}_j, \tag{16}$$

$$\dot{\bar{\delta p}}_j = \begin{pmatrix} \dot{\delta p}_j \\ \dot{\delta p}_{j,4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \delta p_{j,4}\ I_3 + \delta p_j \times \\ -\ \delta p_j^T \end{pmatrix} \Omega_j \tag{17}$$

with

$$\Omega_j = \delta\omega_j - R(\bar{\delta p}_j)\beta_j \tag{18}$$

where $R(\bar{\delta p}_j)$ is the rotation matrix related to $\bar{\delta p}_j$.

We also define the following auxiliary system between spacecraft j and k

$$\dot{\bar{p}}_{jk} = \frac{1}{2}\bar{p}_{jk} \odot \bar{\beta}_{jk} \tag{19}$$

with $\bar{\beta}_{jk} = (\beta_{jk}^T\ ,\ 0)^T$ and $\beta_{jk} \in \mathbb{R}^3$ to be defined later. We define the unit quaternion describing the discrepancy between the auxiliary system output and the relative attitude error between the j^{th} and k^{th} spacecraft as

$$\bar{\delta p}_{jk} = (\bar{p}_{jk})^{-1} \odot \bar{q}_{jk}, \tag{20}$$

$$\dot{\bar{\delta p}}_{jk} = \begin{pmatrix} \dot{\delta p}_{jk} \\ \dot{\delta p}_{jk,4} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \delta p_{jk,4}\ I_3 + \delta p_{jk}\times \\ -\ \delta p_{jk}^T \end{pmatrix} \Omega_{jk} \tag{21}$$

with

$$\Omega_{jk} = \omega_{jk} - R(\bar{\delta p}_{jk})\beta_{jk} \tag{22}$$

Our main contribution is to use a combination of the vector parts of (20), for $j, k = 1, ...n$, in the coordinated control law instead of the actual relative angular velocities between spacecraft, leading to almost global asymptotic stability result.

4. ATTITUDE ALIGNMENT CONTROL

Based on the coupled dynamics control strategy presented in Lawton et al. (2000a), the proposed decentralized coordinated attitude control law consists of two terms, in order to achieve two different objectives/behaviors, and is given by

$$\tau_j = \tau_j^1 + \tau_j^2 \tag{23}$$

where the first term aims to track a desired attitude and angular velocity, in order to achieve the goal-seeking behavior, while the second is used to achieve the formation-keeping behavior by ensuring the spacecraft alignment in the formation and maintaining the same angular rate between spacecraft.

As the first control action for the j^{th} spacecraft, τ_j^1, we consider the velocity-free attitude tracking control law, developed for a single spacecraft in Tayebi (2007), given by

$$\begin{aligned}\tau_j^1 = I_{f_j} R(\bar{\delta q_j})\dot{\omega}^d + R(\bar{\delta q_j})\omega^d \times I_{f_j} R(\bar{\delta q_j})\omega^d \\ -\ \alpha_{1j}\delta q_j - \alpha_{2j}\delta p_j\end{aligned} \tag{24}$$

where α_{1j} and α_{2j} are the attitude tracking control gains satisfying

$$\alpha_{1j} > 0, \qquad \alpha_{2j} > 0 \tag{25}$$

The control action for the formation-keeping behavior for the j^{th} spacecraft, τ_j^2, is defined as

$$\tau_j^2 = -\sum_{k=1}^{n} k_{jk}^p\ q_{jk} - \sum_{k=1}^{n} k_{jk}^d\ (\delta p_{jk} - R(\bar{q}_{jk})\delta p_{kj}) \tag{26}$$

where n is the number of spacecraft in the formation, and k_{jk}^p, k_{jk}^d are the formation-keeping behavior gains such that $k_{jj}^p = k_{jj}^d = 0$ and

$$k_{jk}^p = k_{kj}^p > 0, \qquad k_{jk}^d = k_{kj}^d > 0 \tag{27}$$

if spacecraft j and k communicate with one another, otherwise they are equal to zero, for $j, k = 1, ...n$.

Note that this control action is a modification of the one used in Vandyke et al. (2006) and Ren (2007) in the full information case, where we consider the vector part of (20) to generate the necessary damping that would have been generated when using the relative angular velocity between the j^{th} and the k^{th} spacecraft.

Remark 1. Note that the choice of the gains k_{jk}^p and k_{jk}^d determines the coordination architecture considered. The magnitude of a nonzero k_{jk}^p and/or k_{jk}^d determines the strength of the connection between spacecraft. Therefore, various coordination architectures can be used by different choices of these gains, Vandyke et al. (2006). It is also important to note that it is not interesting to take them all zero, since each spacecraft will be controlled individually.

To this point, we can state the following theorem.

Theorem 2. Consider the formation given in (1)-(2) under the control law (23) with (24) and (26), with restrictions (25) and (27), and let the inputs of the auxiliary systems (15) and (19) be respectively

$$\beta_j = \Gamma_j \delta p_j \quad , \quad \beta_{jk} = \Gamma_{jk}\delta p_{jk} \tag{28}$$

with $\Gamma_j = \Gamma_j^T > 0$ and $\Gamma_{jk} = \Gamma_{jk}^T > 0$. If the control gains satisfy

$$\alpha_{1j} > 2\sum_{k=1}^{n} k_{jk}^p \tag{29}$$

for $j = 1, ...n$, then all the signals are globally bounded and $q_j(t) \to q_k(t) \to q^d(t)$ and $\omega_j(t) \to \omega_k(t) \to \omega^d(t)$ asymptotically.

Proof. The dynamics of the j^{th} spacecraft angular velocity tracking error is given by

$$\begin{aligned} I_{f_j}\dot{\delta\omega}_j = \ & \tau_j - \left(\delta\omega_j + R(\bar{\delta q}_j)\omega^d\right) \times I_{f_j}\left(\delta\omega_j + R(\bar{\delta q}_j)\omega^d\right) \\ & + I_{f_j}\left(\delta\omega_j \times R(\bar{\delta q}_j)\omega^d - R(\bar{\delta q}_j)\dot{\omega}^d\right) \end{aligned} \tag{30}$$

after few algebraic manipulations, and using the cross product properties, one can show that

$$\begin{aligned}\delta\omega_j^T I_{f_j}\dot{\delta\omega}_j = \delta\omega_j^T(\tau_j - I_{f_j}R(\bar{\delta q_j})\dot{\omega}^d \\ -R(\bar{\delta q_j})\omega^d \times I_{f_j}R(\bar{\delta q_j})\omega^d)\end{aligned} \tag{31}$$

Consider the following Lyapunov function candidate

$$V = \sum_{j=1}^{n} V_j \tag{32}$$

where V_j is defined as follows

$$\begin{aligned} V_j =& \frac{1}{2}\delta\omega_j^T I_{f_j}\delta\omega_j + 2\alpha_{1j}(1 - \delta q_{j,4}) + 2\alpha_{2j}(1 - \delta p_{j,4}) \\ &+ \sum_{k=1}^{n}\Big(k_{jk}^p(1 - q_{jk,4}) + 2k_{jk}^d(1 - \delta p_{jk,4})\Big) \end{aligned} \tag{33}$$

The time derivative of V_j evaluated along the closed loop dynamics of the j^{th} spacecraft is

$$\begin{aligned} \dot{V}_j =& \delta\omega_j^T(\tau_j - I_{f_j}R(\bar{\delta q_j})\dot{\omega}^d - R(\bar{\delta q_j})\omega^d \times I_{f_j}R(\bar{\delta q_j})\omega^d) \\ &+ \alpha_{1j}\delta\omega_j^T\delta q_j + \alpha_{2j}\Omega_j^T\delta p_j \\ &+ \frac{1}{2}\sum_{k=1}^{n} k_{jk}^p\omega_{jk}^T q_{jk} + \sum_{k=1}^{n} k_{jk}^d\Omega_{jk}^T\delta p_{jk} \end{aligned} \tag{34}$$

Using equations (18) and (23) with (24) and (26) in (34), we will obtain

$$\begin{aligned} \dot{V}_j =& -\alpha_{2j}\delta p_j^T R(\bar{\delta p}_j)\beta_j + \sum_{k=1}^{n}\Big(\frac{1}{2}k_{jk}^p q_{jk}^T\omega_{jk} + k_{jk}^d\Omega_{jk}^T\delta p_{jk}\Big) \\ &- \delta\omega_j^T\sum_{k=1}^{n}\Big(k_{jk}^p\ q_{jk} + k_{jk}^d\left(\delta p_{jk} - R(\bar{q}_{jk})\delta p_{kj}\right)\Big) \end{aligned} \tag{35}$$

Then, the time derivative of V is

$$\dot{V} = \sum_{j=1}^{n}\dot{V}_j \tag{36}$$

Motivated by Ren (2007), and using the expression of ω_{jk} given in (12), we have;

$$\begin{aligned}\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{p}q_{jk}^{T}\omega_{jk} &= \frac{1}{2}\sum_{j=1}^{n}\delta\omega_{j}^{T}\sum_{k=1}^{n}k_{jk}^{p}q_{jk} \\ &- \frac{1}{2}\sum_{k=1}^{n}\delta\omega_{k}^{T}\sum_{j=1}^{n}k_{jk}^{p}R(\bar{q}_{jk})^{T}q_{jk} \\ &= \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{p}\delta\omega_{j}^{T}q_{jk} \end{aligned} \tag{37}$$

where we have used equations (13), (14) and (27) to obtain the above result. Using the expression of Ω_{jk}, given in (22), and (12), (13) and (27), we can write

$$\begin{aligned}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{d}\Omega_{jk}^{T}\delta p_{jk} &= -\sum_{k=1}^{n}\delta\omega_{k}^{T}\sum_{j=1}^{n}k_{jk}^{d}R(\bar{q}_{jk})^{T}\delta p_{jk} \\ &+ \sum_{j=1}^{n}\delta\omega_{j}^{T}\sum_{k=1}^{n}k_{jk}^{d}\delta p_{jk} - \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{d}\beta_{jk}^{T}R(\bar{\delta p}_{jk})^{T}\delta p_{jk} \\ &= \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{d}\delta\omega_{j}^{T}\left(\delta p_{jk} - R(\bar{q}_{jk})\delta p_{kj}\right) \\ &- \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{d}\beta_{jk}^{T}R(\bar{\delta p}_{jk})^{T}\delta p_{jk}\end{aligned} \tag{38}$$

Then, from equations (28), (35)-(38) and using the fact that $q^{T}R(\bar{q}) = q^{T}$ for any quaternion $\bar{q}$, the time derivative of V is given by

$$\dot{V} = -\sum_{j=1}^{n}\alpha_{2j}\delta p_{j}^{T}\Gamma_{j}\delta p_{j} - \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^{d}\delta p_{jk}^{T}\Gamma_{jk}\delta p_{jk} \tag{39}$$

which implies that $V(t) \leq V(0)$, and $\bar{\delta q}_{j}$, $\bar{\delta p}_{j}$, $\bar{\delta\omega}_{j}$, $\bar{q}_{jk}$ and $\bar{\delta p}_{jk}$ are globally bounded. In addition, we can verify that $\dot{\delta p}_{j}$ and $\dot{\delta p}_{jk}$ are bounded, and so is $\ddot{V}$. Using Barbalat's lemma we conclude that $\delta p_{j} \to 0$ and $\delta p_{jk} \to 0$, as $t \to \infty$, which implies that $\delta p_{j,4} \to \pm 1$, $\delta p_{jk,4} \to \pm 1$, $\beta_{j} \to 0$, $\beta_{jk} \to 0$, $R(\bar{\delta p}_{j}) \to I_{3}$ and $R(\bar{\delta p}_{jk}) \to I_{3}$.

Now, since $\dot{\omega}^{d}$ is bounded, one can show that $\ddot{\bar{\delta p}}_{j}$ and $\ddot{\bar{\delta p}}_{jk}$ are bounded, and hence $\dot{\bar{\delta p}}_{j} \to 0$ and $\dot{\bar{\delta p}}_{jk} \to 0$, and from equations (17), (18), (21) and (22) we can conclude that $\Omega_{j} \to 0$ and $\Omega_{jk} \to 0$, and consequently, $\delta\omega_{j} \to 0$ and $\omega_{jk} \to 0$. Furthermore, one can easily verify that $\ddot{\delta\omega}_{j}$ is bounded since $\ddot{\omega}^{d}$ is bounded, and so we conclude that $\dot{\delta\omega}_{j} \to 0$.

Using the above results, the closed loop dynamics (30) with (23), (24) and (26) reduces to;

$$\alpha_{1j}\delta q_{j} + \sum_{k=1}^{n}k_{jk}^{p}\; q_{jk} = 0, \quad j = 1, ...n \tag{40}$$

from (5) and (11), we can write

$$\alpha_{1j}\delta q_{j} + \sum_{k=1}^{n}k_{jk}^{p}(\delta q_{k,4}\delta q_{j} - \delta q_{j,4}\delta q_{k} - \delta q_{k}\times\delta q_{j}) = 0 \tag{41}$$

which is equivalent to

$$\begin{aligned}\left(\alpha_{1j} + \sum_{k=1}^{n}k_{jk}^{p}\delta q_{k,4}\right)\delta q_{j} - \delta q_{j,4}\sum_{k=1}^{n}k_{jk}^{p}\delta q_{k} &= \\ -\,\delta q_{j}\times\sum_{k=1}^{n}k_{jk}^{p}\delta q_{k}\end{aligned} \tag{42}$$

Motivated by Lawton and Beard (2002) and Ren (2007), multiply both sides by $\left(\delta q_{j}\times\sum_{k=1}^{n}k_{jk}^{p}\delta q_{k}\right)$, leads to

$$\|\delta q_{j}\times\sum_{k=1}^{n}k_{jk}^{p}\delta q_{k}\|^{2} = 0 \tag{43}$$

from which equation (42) can be rewritten as

$$\left(\alpha_{1j} + \sum_{k=1}^{n}k_{jk}^{p}\delta q_{k,4}\right)\delta q_{j} - \delta q_{j,4}\sum_{k=1}^{n}k_{jk}^{p}\delta q_{k} = 0 \tag{44}$$

for $j = 1, ..., n$.

Following Ren (2007), the above set of equations can be rewritten in matrix form, using the Kronecker product $\otimes$, as

$$(M(t)\otimes I_{3})\,\delta Q = 0 \tag{45}$$

where $\delta Q \in \mathbb{R}^{3n}$ is the column vector composed of δq_{j}, for $j = 1, ..., n$ and $M(t) = [m_{jk}(t)] \in \mathbb{R}^{n\times n}$ is given by

$$m_{jk} = -\delta q_{j,4}k_{jk}^{p},\; m_{jj} = \alpha_{1j} + \sum_{k=1}^{n}k_{jk}^{p}\delta q_{k,4} \tag{46}$$

We can see that the formation has converged only if $\delta Q = 0$. A necessary and sufficient condition for this is that matrix M has full rank. From equations (46), matrix M is strictly diagonally dominant if

$$|m_{jj}| > \sum_{k=1,k\neq j}^{n}|m_{jk}| \tag{47}$$

therefore,

$$|\alpha_{1j} + \sum_{k=1}^{n}k_{jk}^{p}\delta q_{k,4}| > \sum_{k=1,k\neq j}^{n}|\delta q_{j,4}k_{jk}^{p}| \tag{48}$$

which yields

$$|\alpha_{1j} + \sum_{k=1}^{n}k_{jk}^{p}\delta q_{k,4}| > |\delta q_{j,4}|\sum_{k=1,k\neq j}^{n}k_{jk}^{p} \tag{49}$$

taking $\delta q_{j,4} = 1$ and $\delta q_{k,4} = -1$, we have

$$|\alpha_{1j} - \sum_{k=1}^{n}k_{jk}^{p}| > \sum_{k=1,k\neq j}^{n}k_{jk}^{p} \tag{50}$$

Hence, if condition (29) is satisfied, matrix M is strictly diagonally dominant, which implies that the only solution of (45) is $\delta Q = 0$, or $\delta q_{j} = 0$ for $j = 1..., n$.

Finally, we can conclude that $\delta q_{j} \to 0$ and $\delta q_{j,4} \to \pm 1$, or equivalently $q_{j} \to q_{k} \to q^{d}$. Moreover, since $\delta\omega_{j} \to 0$, $\omega_{jk} \to 0$, $R(\bar{\delta q}_{j}) \to I_{3}$ and $R(\bar{q}_{jk}) \to I_{3}$, we conclude that $\omega_{j} \to \omega_{k} \to \omega^{d}(t)$, $\forall j, k = 1, ...n$.

Remark 3. From the above analysis, we can see that condition (29) is restrictive in the sense that priority is given to the goal-seeking behavior over the formation-keeping behavior. In the case where one spacecraft is affected by external disturbances or torque saturation, the above control law does not guarantee perfect alignment during formation maneuvers, since the primary objective for spacecraft is to attain their desired attitude/angular velocity.

Remark 4. We can see from (46) that if we can satisfy that $\delta q_{j,4}$ is positive for all $t \geq 0$, for $j = 1, ...n$, then matrix M will always be strictly diagonally dominant, and the proposed coordinated attitude control scheme will enable the designer to prioritize goal-seeking and formation-keeping behaviors. In fact, in Lawton and Beard (2002), it was shown that this assumption is satisfied for all $t \geq 0$ under some conditions on the initial states.

Remark 5. It is intuitive that increasing the number of connections for each spacecraft will improve the performance of the proposed control scheme. In fact, in Vandyke et al. (2006), a simulation study, in the full information case, showed that improvement in the steady state attitude error can be achieved by increasing the strength of the connections or the number of connections per spacecraft.

5. THE CONSENSUS SEEKING CASE

In this section, we consider the consensus seeking problem without velocity measurement. We consider the case where $\alpha_{1j} = 0$, for $j = 1, ..., n$, in (24), and show that the resulting control strategy allows to achieve a consensus among all spacecraft, *i.e.,* $q_j \to q_k$, $\forall j, k \in \{1, ..., n\}$, and $\omega_j \to \omega^d(t)$, $\forall j \in \{1, ..., n\}$, under some conditions on the communication flow topology. The control gains $k^p_{jk} = k^p_{kj}$ and $k^d_{jk} = k^d_{kj}$ are strictly positive if spacecraft j and k are connected by a communication link, otherwise they are equal to zero.

Following the steps of the proof of our theorem, equation (40), in this case, becomes

$$\sum_{k=1}^{n} k^p_{jk}\, q_{jk} = 0, \quad j = 1, ...n \tag{51}$$

For further analysis of (51), it is appropriate to describe the information flow between spacecraft by the weighted undirected graphs $\mathcal{G}_1 = (\mathcal{N}, \mathcal{E}, \mathcal{K}^p)$ and $\mathcal{G}_2 = (\mathcal{N}, \mathcal{E}, \mathcal{K}^d)$, with $\mathcal{N}$ being the set of nodes or vertices, describing the set of spacecraft in the formation, $\mathcal{E}$ the set of unordered pairs of nodes, called edges, describing the set of links between spacecraft, and $\mathcal{K}^{p,d}$ are the set of weights associated to every link of each graph respectively, containing the values $k^{p,d}_{jk}$. Note that $\mathcal{G}_1$ and $\mathcal{G}_2$ differ only on the set of weights and they characterize, respectively, the interaction graphs of q_{jk} and δp_{jk}.

We assign a direction to the graph $\mathcal{G}_1$, by considering one of the nodes to be the positive end of the link, and obtain the directed graph $\tilde{\mathcal{G}}_1 = (\mathcal{N}, \tilde{\mathcal{E}}, \mathcal{K}^p)$, with $\tilde{\mathcal{E}}$ being the set of ordered edges of the graph. The positive end of a link can be chosen arbitrarily. Let $m = |\tilde{\mathcal{E}}|$ be the total number of edges in the graph $\tilde{\mathcal{G}}_1$, which is also equal to the total number of undirected links in $\mathcal{G}_1$. The weighted incidence matrix of $\tilde{\mathcal{G}}_1$ is $D \in \mathbb{R}^{n\times m}$ defined as

$$d_{jl^{(u,v)}} = \begin{cases} +k^p_{uv} & \text{if node } j \text{ is the positive end of link } (u,v) \\ -k^p_{uv} & \text{if node } j \text{ is the negative end of link } (u,v) \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

where $l^{(u,v)} : \tilde{\mathcal{E}} \to \{1, ..m\}$ is a function that associates a number from the set $\{1, ...m\}$ to each link $(u, v) \in \tilde{\mathcal{E}}$.

The rank of D is $n - 1$ if the graph $\tilde{\mathcal{G}}_1$ is connected (*i.e.,* there is a path between any two spacecraft), and it is full column rank if this graph does not contain cycles (*i.e.,* does not contain closed paths). Let Q_u be the column vector stack of all q_{jk}, $\forall (j, k) \in \tilde{\mathcal{E}}$. Using the fact that $q_{jk} = -q_{kj}$, equation (51) is equivalent to

$$(D \otimes I_3)\, Q_u = 0 \tag{53}$$

Under the assumption that the communication graph is connected and contains no cycles, or $|\tilde{\mathcal{E}}| = n - 1$, the only solution of (53) is $Q_u = 0$, that is $q_{jk} = 0, \forall (j, k) \in \tilde{\mathcal{E}}$.

Since the graph is connected, each spacecraft is communicating with at least one other spacecraft, which allows to conclude that $q_{jk} \to 0$, or $R(\bar{q}_{jk}) \to I_3$, for $j, k = 1, ...n$. Finally, since $\delta\omega_j \to 0$ and $\omega_{jk} \to 0$, we conclude that $q_j \to q_k$, and $\omega_j \to \omega_k \to \omega^d(t)$, $\forall j, k \in \{1, ..., n\}$.

6. SIMULATION RESULTS

Using SIMULINK, we consider a scenario where four spacecraft are required to align their attitudes while tracking the desired reference trajectory defined by $\omega^d(t) = 0.1sin(0.1\pi t)(1, 1, 1)^T$ and $\bar{q}^d(0) = (0, 0, 0, 1)^T$. In addition, we consider that each spacecraft communicates only with its two neighbors; that is two connections for each spacecraft. The spacecraft are modeled as rigid bodies whose inertia matrices are taken as $I_{f_j} = diag(20, 20, 30)$. The initial conditions for the four spacecraft are selected to be: $\bar{q}_1(0) = (0, 0, 1, 0)$, $\bar{q}_2(0) = (1, 0, 0, 0)$, $\bar{q}_3(0) = (0, 1, 0, 0)$, $\bar{q}_4(0) = (0, 0, sin(-\pi/4), \cos(-\pi/4))$, $\bar{p}_j = (1, 0, 0, 0)$ and $\bar{p}_{jk} = (1, 0, 0, 0)$, for $j, k = 1, .., 4$. The controller gains used in our simulations are

$$\Gamma_j = diag(5, 5, 5), \quad \alpha_{1j} = 70, \quad \alpha_{2j} = 90$$
$$\Gamma_{jk} = diag(5, 5, 5), \quad k^p_{jk} = 15, \quad k^d_{jk} = 25$$

for $j, k = 1, .., 4$.

The obtained results are illustrated in figures (1)-(2). Figure (1) shows the components of the unit quaternion, $\bar{q}^i_j$, $i = 1, ...4$, representing the attitude of the four spacecraft in the formation (we use the superscript (i) to denote the i^{th} component of a vector). Note that each spacecraft converges to the same desired attitude. Figure (2) illustrates the components of the angular velocity error vectors $\delta\omega_j$ for the four spacecraft. It is clear that the angular velocity of the four spacecraft converge to the same specified desired angular velocity.

7. CONCLUSION

We have considered the decentralized attitude alignment problem among a team of spacecraft within a formation without velocity measurements. The main contribution of this paper is the extension of the work proposed in Tayebi (2007) to formation flying control without velocity measurement. The presented approach is based on the introduction of an auxiliary system for each spacecraft and for each pair of spacecraft with a communication link. The vector parts of the unit quaternion describing the discrepancy between the output of these auxiliary systems and the attitude tracking error as well as the relative attitude errors between spacecraft, are used in the control law to generate the necessary damping that would have been directly generated by the angular velocities and the relative angular velocities of the spacecraft. Almost global asymptotic stability results are obtained in the sense that the closed-loop system has several equilibria that represent

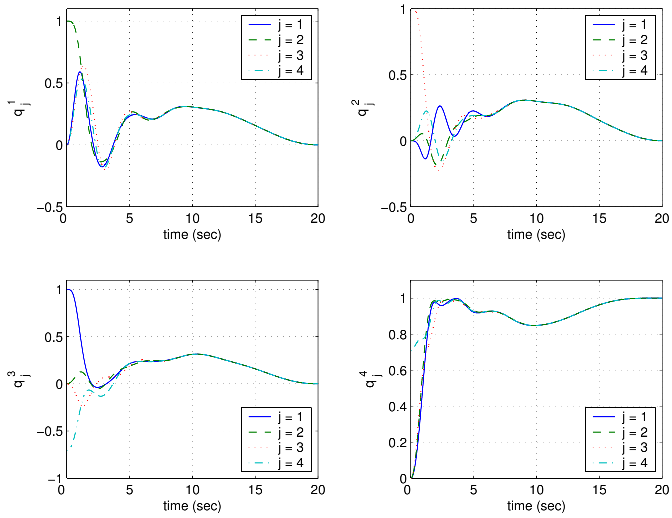

Fig. 1. Spacecraft attitudes

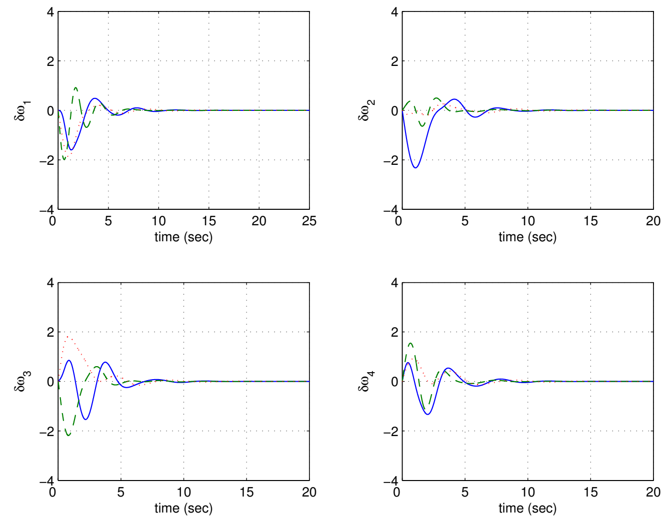

Fig. 2. The three components of the angular velocity error vectors for the four spacecraft

the same physical configuration, but only one of them is an attractor. Simulation results have shown a scenario of four spacecraft align and track a desired trajectory.

REFERENCES

T. Balch and R. C. Arkin. Behavior-Based Formation Control for Multirobot Teams. *IEEE Transactions on Robotics and Automation*, vol. 14, pp. 926939, December 1998.

R. W. Beard, J. Lawton, F. Y Hadaegh. A Coordination Architecture for Formation Control. *IEEE Transactions on Control Systems Technology*, Vol. 9, No. 6, pp. 777790, 2001.

F. Caccavale and L. Villani. Output feedback control for attitude tracking. *Systems & Control Letters*, Vol. 38, No. 2, pp. 91-98, 1999

B. T. Costic, D.M. Dawson, M.S. de Queiroz, and V. Kapila. A quaternion-based adaptive attitude tracking controller without velocity Measurements. *In proc. of the 39th IEEE Conference on Decision and Control*, Sydney, Australia, pp. 2424-2429, 2000.

J. Lawton, R. W. Beard, and F. Y. Hadaegh. Elementary Attitude Formation Maneuvers Via Leader-following and Behavior-based Control. in *Guidance, Naviagtion, and Control Conference and Exhibit*, (Denver, Colorado), AIAA 2000-4442, August 2000.

J. Lawton, B. Young and R. W. Beard. A Decentralized Approach to Elementary Formation Maneuvers. *Proc on the IEEE Int. Conf. on Robotics and Automation*, Vol. 3,IEEE, Piscataway, NJ, 2000, pp 2728-2733.

J. Lawton and R. W. Beard. Synchronized Multiple Spacecraft Rotations. Automatica, Vol. 38, No. 8, 2002, pp. 13591364.

N. E. Leonard and E. Fiorelli, Virtual Leaders, Artificial Potentials and Coordinated Control of Groups. *Proceedings of the IEEE Conference on Decision and Control*, IEEE Press, Piscataway, NJ, 2001, pp. 29682973.

F. Lizarralde and J. T. Wen. Attitude control without angular velocity measurement: A passivity approach. *IEEE Transactions on Automatic Control*, Vol. 41, No. 3, pp. 468- 472, 1996.

B. Young, R. W. Beard and J. Kelsey. A Control Scheme for Improving Multi-Vehicle Formation Maneuvers. *Proceedings of the American Control Conference (Arlington, VA)*, American Automatic Control Council, Evanston, IL, 2001.

W. Ren, and R. W. Beard. Virtual Structure Based Spacecraft Formation Control with Formation Feedback. *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Monterey, California, AIAA paper no: 2002-4963, August 2002

W. Ren and R. W. Beard. Decentralized scheme for spacecraft formation flying via the virtual structure approach. *AIAA Journal of Guidance, Control, and Dynamics*; 27(1), pp. 7382, 2004.

W. Ren. Distributed attitude alignment in spacecraft formation flying. *International journal of adaptive control and signal processing*, 21, pp. 95113, 2007.

D. P. Scharf, F. Y. Hadeagh and S. R. Ploen. A survey of spacecraft formation flying guidance and control (part II): control. *Proceeding of the 2004 American Control Conference*, Boston, Massachusetts, pp. 2976-2985, June 2004

M. D. Shuster. A survey of attitude representations. *Jour. Astronautical Sciences*, Vol. 41 , No. 4, pp. 439-517, 1993.

A. Tayebi A velocity-free attitude tracking controller for rigid spacecraft. *Proceedings of the* 46^{th} *IEEE Conference on Decision and Control*, New Orleans, LA, USA, 2007, pp. 6430-6434.

M. C. Vandyke and C. D. Hall. Decentralized coordinated attitude control within a formation of spacecraft. *journal of guidance, control and dynamics*, Vol. 29, No.5, pp.1101–1109, sept-oct 2006.

P. Wang and F. Hadaegh. Coordination and control of multiple microspacecraft moving in formation. *The Journal of the Astronautical Sciences*, vol. 44, pp. 315355, July- September 1996.

P. Wang, F. Hadaegh and K. Lau. Synchronized Formation Rotation and Attitude Control of Multiple Free-Flying Spacecraft. *Journal of Guidance, Control, and Dynamics*, vol. 22, pp. 2835, January-February 1999.