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Agent and Link Redundancy for Autonomous Formations ⋆

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Abstract:
More and more often the multiagent formations found in high-risk military and civilian missions are designed to sustain the loss of some agents or control/communication links. In a military context, the agent loss could be a result of enemy attack and/or mechanical breakdown, while the link loss is usually due to enemy jamming, reduced transmission power, obstacles and/or hardware failure. If the agents are tasked to tightly maintain their inter-agent distances in order to maintain a desirable shape of the formation, loss of certain agents or links could lead to catastrophic consequences. This preliminary work proposes to address this potential problem by enforcing redundancy in the formation design, by addition of extra agents and/or links beyond the minimum necessary. Following an existing graphical model of the formations, we extend the notions and definitions for simple rigidity to ones including the redundancy concept. We remark that this engineered redundancy naturally reflects a level of operational robustness/health of the formation. We differentiate the measure into a deterministic metric and a statistical metric, and provide some general results on *redundant rigidity*.

1. INTRODUCTION

It has not been long ago that UAVs became recognized as one of the enabling technologies for operational success in modern military operations (see for example an AFSAB (1996) report). And more often than ever, it is desirable for them to fly in formations (see for example a NASA (2004) report). More recently, UAV formations are also found in civil applications such as environmental monitoring, and bush fire surveillance.

We are investigating multiagent UAV formations operating in a hazard operational environment, including for use in electronic warfare, where robustness and health of the formation is of prime concern. By formation health we are concerned with the degree of functionality of the formation in different configurations, measured against different levels of operational requirement. The word robustness is intended to suggest a focus on ensuring satisfactory performance under different operational scenarios, when both agent and communication/control links could be lost due to equipment malfunction, or enemy attack.

Vulnerability of UAVs in combat operations has been studied, see for example Haulman (2003), who concluded from a study of thirteen years' crash records, that:

- UAV flights should be carefully synchronized with each other and with the flights of other systems.
- UAVs should be specialized and used for a greater variety of missions.
- The Air Force should develop countermeasures to enemy UAVs.
- UAVs should be improved to reduce their vulnerability to weather, enemy air defenses, and mechanical and communication failures.

While it is not the focus of this paper for us to investigate how these vulnerabilities could be avoided, we still wish to elaborate the last bullet point, where the potential vulnerabilities include:

- communication link loss, perhaps as a result of enemy jamming or occlusion.
- Selective or random attack of one or more agents in a formation.
- Loss of an agent as a result of mechanical and/or communication failure, even without enemy attack, but perhaps due to environmental changes (such as smoke, heat, ice, etc).

While there are many other examples, we collect these scenarios, and using an abstracted graphical model of a formation, categorize them into two large types, *link loss* and *agent loss*. In the literature, for example in Gross and Yellen (2006), these have been considered in the

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network survivability problem, where the desired property to preserve is connectivity.

The focus of this paper is then to address this practical problem of mitigating the effect of losses of links or agents in a UAV formation, by ways of using redundant agents and/or links in the formation. We measure the robustness of the formation, determined by how many agents or links it can afford to lose while still preserving the cohesiveness of the formation. We characterize the redundancy in such robust formations, and illustrate the concept by studying a number of special classes of formations.

In Section 2, we provide some background on the two key tools for this study, graph rigidity theory and multiagent formations. In Section 3, we define the appropriate measure for redundancy in graph theoretical terms, discuss in detail some general results. In Section 4, we investigate the redundancy of a number of special classes of rigid formations, and distinguish the deterministic and statistical ways of measuring redundant rigidity. Finally, we provide some concluding remarks.

2. BACKGROUND

Rigid graph theory (Tay and Whiteley (1985); Jackson and Jordan (2005)) is a tool that has been used to analyse the property of formation rigidity, see for example Olfati-Saber and Murray (2002); Yu et al. (2007). Agents are modeled as points. Agent pairs with the inter-agent distance being actively constrained to be constant can be thought of as being joined by bars with lengths enforcing the inter-agent distance constraints. The system can be therefore modeled by a graph where vertices represent point-like agents and inter-agent distance constraints are abstracted as edges. Naturally, we can contemplate other constraints than distance, for example, those involving angle, or angle and distance. However, the theory begins with distance constraints, and we restrict discussion to this case. Rigid graph theory is concerned with stating properties of graphs that ensure that the formation being modeled by the graph is rigid. By keeping the formation rigid, one ensures that a higher level command can be given to the entire formation without having to explicitly consider many low-level issues, such as inter-agent collision, relative position, maintaining the range of communication etc. Moreover, certain operations, such as cooperative emitter localization, do require specific formation geometry to maximize the accuracy.

Figure 1 shows several examples of two-dimensional graphs, two of which are rigid and one of which is not rigid. In a nonrigid graph, part of the graph can flex or move, while the rest of the graph stays still. The notion of rigidity conforms to our normal intuition and corresponds to the rigidity of frameworks in civil/mechanical engineering.

A rigid graph is one for which for almost all choices of edge lengths and vertex positions for which a corresponding formation exists, the corresponding formation is rigid, Tay and Whiteley (1985). Intuitively, if enough of the distances between certain pairs of agents are maintained, such that all the inter-agent distances are preserved as a consequence, then the formation is said to be rigid.

It proves possible in two dimensions to characterize rigidity in purely combinatorial terms, i.e. counting-type con-

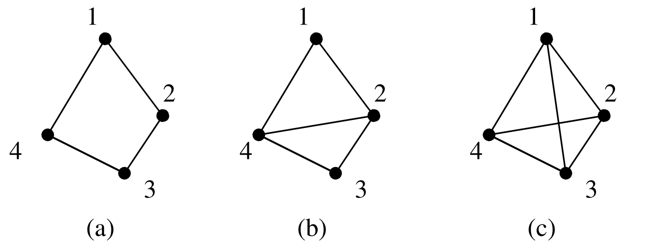

Fig. 1. Illustration of (a) non-rigid formation, (b) minimally rigid formation, and (c) non-minimally rigid formation, moreover any edge may be removed without losing rigidity.

ditions related to the graph (discarding therefore the agent coordinates) can be used to conclude the rigidity or otherwise of a generic formation corresponding to the graph. This is the celebrated Laman's Theorem, see Laman (1970), for which no three-dimensional equivalent exists. In three dimensions, differing necessity and sufficiency conditions are known for a graph to correspond to a formation which will be rigid for generic values of the constrained inter-agent distances, see Tay and Whiteley (1985).

Commonly in the literature, an information structure with a minimum number of communication links (or distance constraints) is to be exploited while preserving the rigidity of the formation. This leads to a widely used notion of *minimal rigidity.*

A graph is called *minimally rigid* if it is rigid and if there exists no rigid graph with the same number of vertices and a smaller number of edges, i.e., a graph is *minimally rigid* if it is rigid and if no single edge can be removed without losing rigidity.

It is interesting to note that *minimal* rigidity is not desirable in the context of this paper, since it, by our later definition, has no redundancy (robustness) at all. We re-state the following theorem (in, for example, Yu et al. (2007)) which provides the minimum number of links required to ensure rigidity for a formation with given number of agents.

Theorem 1. If a graph $G = (V, E)$ in $\Re^d$ ($d \in \{2, 3\}$) with at least d vertices is rigid, then there exists a subset E' of edges such that $G' = (V, E')$ is minimally rigid. This also implies the following:

- $|E'| = d|V| - d(d + 1)/2$.
- Any subgraph $G'' = (V'', E'')$ of G' with at least d vertices satisfies $|E''| \leq d|V''| - d(d + 1)/2$.

Though rigidity in three (and higher) dimensions can be defined, in this paper, we limit ourselves to two dimensions. Unless otherwise mentioned, we assume all the formations can be modeled using simple graphs, i.e. there is only a single constraint between each pair of agents.

3. REDUNDANCY IN UNDIRECTED FORMATION DESIGN

In this section we use a purely topological design perspective, i.e. we deal with the graph model only; however, there is sometimes a need to evaluate the robustness of a realization of a graph (when each agent is assigned coordinates), which we will not discuss in this paper.

The graph model itself is a combination of sensing, communication and control graphs, that is, the presence of an edge representing a distance constraint also implies that sensing is required and a communication link is to be maintained. For example, loss of a communication link due to mechanical failure will probably result in a formation being unable to maintain the corresponding distance constraint since the associated agents cannot communicate with each other; or occlusion of the sensor means one agent will not be able to sense the other agent's position and thus be unable to maintain the distance to that agent.

In the following, we will use the terms "node/vertex of a graph" and "agent of a formation" interchangeably, likewise for "link" and "edge/arc of a graph". Following convention and for clarity, most discussion uses only the terminology of rigid graph theory.

3.1 Redundancy concepts

There have been studies which directly or indirectly embed the concept of redundancy into the rigid graph theory. We give a number of examples here.

A *circuit* in rigid graph theory is a rigid graph that, after deletion of any single edge, becomes a minimally rigid graph. See Graver et al. (1993)

A graph is *redundantly rigid* if, after deletion of any single edge, it becomes a rigid graph. Thus it is trivial to see that a circuit is necessarily redundantly rigid. See Graver et al. (1993).

In Jackson and Jordan (2005), a graph is termed *2-rigid* if it remains rigid after deletion of any one edge; note this is the same as the definition of a redundantly rigid graph.

In Graver et al. (1993), the notion of *birigid* is used interchangeably with *vertex birigid*. A birigid graph remains rigid after deletion of any single vertex and the edges incident to this vertex.

Another type or redundancy becomes evident in considering a class of graphs termed *globally rigid*. These are graphs with the property that in an associated formation, specification of the lengths corresponding to edges determines the formation uniquely up to congruence. An important and recent result is that a graph is globally rigid in $\Re^2$ if and only if it is 3-connected and redundantly rigid, Jackson and Jordan (2005). In sensor network localization problems, a network is localizable if and only if it is globally rigid and there are three or more noncollinear anchor nodes, see Aspnes et al. (2006).

In fact, 3-connectedness itself is a characterization of a type of redundancy in graphs, which says that the graph can lose any two vertices but remain connected. (Connectivity is of course a critical property for many multiagent systems, for example, achieving consensus requires the network to be connected.)

More formally, a graph G is said to be k-connected (sometimes termed k vertex-connected) if there does not exist a set of $k-1$ vertices whose removal disconnects the graph. The vertex connectivity $K_v(G)$ is defined as the minimum number of nodes whose removal will disconnect the graph. Hence if G is k-connected, $K_v(G) \geq k$. Refer to for example Gross and Yellen (2006).

A similar characterization exists for connectedness redundancy involving edges. A graph G is k-edge-connected if there does not exist a set of $k-1$ edges whose removal disconnects the graph. The edge connectivity $K_e(G)$ is defined as the minimum number of edges whose removal will disconnect the graph. Hence if G is k-edge-connected, $K_e(G) \geq k$ (Gross and Yellen (2006)).

A nontrivial result is that a k-(vertex)-connected graph is necessarily k-edge-connected, but not vice versa. More generally, the vertex connectivity of a graph is always smaller than or equal to the edge connectivity of the same graph, in fact for any connected graph G, $K_v(G) \leq K_e(G) \leq \delta(G)$, where $\delta(G)$ denotes minimum vertex degree (Gross and Yellen (2006)).

3.2 Characterization of redundancy in rigid graphs

We proceed by analogy with the extended connectivity definition of a graph. To avoid trivialities, we will require that all graphs have at least 3 vertices to be considered rigid. Some literature uses the term "trivially rigid" in relation to graphs with 2 vertices and 1 edge, or even a single vertex graph. With mild departure from conventions, we for convenience are labelling such graphs here as non-rigid.

A graph G is n-edge-rigid if it remains rigid after deletion of any $(n-1)$ edge(s). The unique value of *edge-rigidity* $R_e(G)$ of a graph G is then defined as the minimum number of edges whose removal will result in a non-rigid graph. Note that $R_e(G) = 0$ if G is not rigid, including when G has fewer than 3 vertices.

Remark 2. Some other terms have been used (for example, 2-rigid by Jackson and Jordan (2005), redundantly rigid by Aspnes et al. (2006), and edge birigid as in Graver et al. (1993)) for the definition of 2-edge-rigidity.

Similarly, a graph G is n-vertex-rigid if it remains rigid after deletion of any set of $(n-1)$ vertices, and at least 3 vertices remain. Therefore the unique value of *vertex-rigidity* $R_v(G)$ of a graph G is defined as the minimum number of vertices whose removal will result in a graph with at least 3 vertices that is not rigid. If G has $|V|$ vertices and the removal of any set of $|V|-3$ vertices result in a rigid graph, the vertex rigidity is defined as $|V|-2$. Note that $R_v(G) = 0$ if G is not rigid, including when G has fewer than 3 vertices.

Trivially, for a minimally rigid graph $G_m = (V_m, E_m)$ where $|V_m| \geq 3$, $R_v(G_m) = R_e(G_m) = 1$. In the sequel we will present three basic yet powerful lemmas, and use them to prove a main theorem of this section, which shows that vertex-rigidity is a more demanding concept than edge-rigidity. First let us briefly explore the connection of these two new definitions to the minimum vertex degree $\delta(G)$ of a (non-minimally) rigid graph G.

Lemma 3. For any graph G, there holds $R_e(G) \leq \delta(G)-1$ and $R_v(G) \leq \delta(G) - 1$.

Proof: Consider, to obtain a contradiction, a graph $G = (V, E)$, with $R_e(G) = k > \delta(G) - 1$; then the graph obtained after removal of any set of $k-1$ edges will be rigid. Now consider the vertex v with degree equal to $\delta(G)$; removal of $\min(k-1, \delta(G))$ of the edges incident to v will leave v connected to at most one other agent, and therefore the resulting graph is not rigid. We obtained a contradiction. We can use a similar argument for the second part of the Lemma.

Next, let us consider the following Lemma useful in construction of a (redundant) rigid graph.

Lemma 4. Consider a graph G' obtained by adding a vertex v and at least two edges incident on v to $G = (V, E)$; then G' is rigid if G is rigid.

Proof: When the number of new edges is exactly 2, this operation is referred to as 0-extension of a Henneberg Sequence (see for example Tay and Whiteley (1985)), and the Lemma holds. Obviously, when the number of new edges exceeds 2, the addition of extra edges (beyond 2) does not destroy the rigidity.

Using Lemma 3 and 4 above, we can obtain the following result, stated as:

Lemma 5. For a graph $G = (V, E)$ with $|V| \geq 4$, $R_v(G) = 2$ implies $R_e(G) \geq 2$.

Proof: To obtain a contradiction, let us suppose $G = (V, E)$ is 2-vertex-rigid but not 2-edge-rigid. Assume $G' = (V, E \backslash \{e\})$ is not rigid, and e is incident to vertices w and v. First, both w and v have to have degree at least equal to 3 by Lemma 3. Next, let us consider the graph G'' obtained from G by removing v and the set of edges incident to v, denoted as $cover(v)$. Note that $e \in cover(v)$, and by assumption G'' is rigid. Now, observe that by adding v and the edge set $cover(v) \backslash \{e\}$ to G'', by Lemma 4, the obtained graph (in this case G') is rigid. This contradicts our assumption that $G' = (V, E \backslash \{e\})$ is not rigid. Therefore, G has to be at least 2-edge-rigid.

In fact Lemma 5 can be generalized, and serves as the basis step of an inductive proof of the generalization. The main result of this paper, as stated below can be proved in a similar spirit.

Theorem 6. For any graph $G = (V, E)$, $R_v(G) \leq R_e(G)$.

Proof: If G is not a rigid graph, including when G has fewer than 3 vertices, then by definition we have $R_v(G) = 0$ and $R_e(G) = 0$, and therefore the theorem is trivially true.

Now consider the cases only when G is rigid and has at least 3 vertices, so that both $R_e(G)$ and $R_v(G)$ are necessarily at least 1. If $R_v(G) = 1$, which implies G is rigid, then $R_e(G) \geq 1$; therefore the theorem holds. It is also true for $R_v(G) = 2$ by Lemma 5.

Now let us consider the general case using Lemma 5 as the basis of an inductive proof. To form an induction, suppose the claim is true for $R_v(G) = k-1$, where $k \geq 3$. To obtain a contradiction, let us first suppose for a graph G, $R_v(G) = k$ and $R_e(G) = k-1$. We will comment below on the sub-cases of $R_v(G) = k$ and $R_e(G) \in \{1, 2, \ldots, k-2\}$.

Consider the graph G' obtained by deletion of any edge e from G, and let $W = \{w_1, w_2, \ldots, w_{k-2}\}$ be any set of $k-2$ vertices in G'. We consider two possibilities.

- e is incident to at least one vertex in W. This means that $e \in cover(w_i)$ where $i \in \{1, 2, \ldots, k-2\}$. The graph G^* obtained by deleting W from G' is the same as the one obtained by deleting W from G; therefore G^* is rigid.
- e is not incident to any vertex in W. Consider the graph G^* obtained by deletion of W from G, and by assumption, $R_v(G^*) = 2$ since $R_v(G) = k$. This, by Lemma 5, implies that the graph $G'' = G^* \backslash e$ is rigid. Observing that if we exchange the two steps, that is, we first delete e from G then remove W, the same G'' is obtained and is rigid.

The fact that in both cases, the obtained graph (G^* or G'') is rigid, implies that $R_v(G') \geq k-1$; by the induction step $R_e(G') \geq R_v(G')$, so we have $R_e(G') \geq k-1$ which implies $R_e(G) \geq k$ since e is arbitrarily selected. We obtain a contradiction to $R_e(G) = k-1$. This completes our proof for this sub-case. It is not hard to follow similar arguments for all the other sub-cases linked with $R_v(G) = k$. This completes the proof.

Next, we could also contemplate the mixture of *independent* agent and link losses; the word independent indicates that some links are not incident to the lost agents.

Lemma 7. For any graph $G = (V, E)$ with $R_v(G) = k$, there holds $R_v(G \backslash M) \geq k - m$, where M is any set of m ($m < k$) edges in G.

Proof: Let us first prove the case for $m = 1$. Suppose $M = \{e\}$ and let G' be the graph obtained by deleting an edge e from G. Let W be any set of $k-2$ vertices in G'. We consider two possibilities:

- e is incident to at least one vertex in W. This means that $e \in cover(w_i)$ where $i \in \{1, 2, \ldots, k-2\}$ and $cover(w_i)$ denotes the set of edges incident to w_i. The graph G^* obtained by deleting W from G' is the same as the one obtained by deleting W from G; therefore G^* is rigid.
- e is not incident to any vertex in W. Consider the graph G^* obtained by deletion of W from G, and by assumption, $R_v(G^*) = 2$ since $R_v(G) = k$. This, by Lemma 5, implies that the graph $G'' = G^* \backslash e$ is rigid. Observing that if we exchange the two steps, that is, we first delete e from G then remove W, the same G'' is obtained and is rigid.

The fact that in both subcases, the obtained graph (G^* or G'') is rigid, implies that $R_v(G') \geq k-1$ and therefore we prove the result when $m = 1$. By iteratively applying these arguments m times, we obtain a complete proof to this Lemma.

In fact, Lemma 7 immediately leads to an alternative proof to Theorem 6, as shown below:

Alternative Proof to Theorem 6: Consider a graph $G = (V, E)$ with $|V| \geq 4$ and $R_v(G) = k$; and let $M \subset E$ be any set of $k-1$ edges, by Lemma 7 there holds $R_v(G \backslash M) \geq 1$ which means that $G \backslash M$ is rigid. By definition of edge-rigidity, there holds $R_e(G) \geq k$. Therefore we proved that $R_e(G) \geq R_v(G)$.

In the previous proofs, we often relied on arguments that remove edges first, or a vertex first. In fact, Lemma 7 and Theorem 6 lead to a corollary that says that for a vertex rigid graph, one could perform a combination of independent agent and link losses in *any order*, as stated below:

Corollary 8. For any graph $G = (V, E)$ with $|V| \geq 4$ and $R_v(G) \geq k$; let $N \subset V$ be any set of n vertices and the union of their covers be $U = \bigcup_{v_i \in N} cover(v_i)$; denote $M \subset E \backslash U$ as any set of m edges. Consider the graph $G' = (V \backslash N, E \backslash M \backslash U)$ obtained after the losses; then G' is rigid if $n + m < k$.

4. REDUNDANCY OF SPECIFIC FORMATIONS

In this section, we comment on the redundancy properties of formations with special types of graphs. We also discuss how one might define, in contrast to a "restrictive" deterministic definition, a statistical measure of redundancy that is tailored to some application scenarios. Proofs are omitted due to space limitations but are made available in an extended paper (Yu and Anderson (2008)).

4.1 Case study: $n = 2$

We focus on the case of $n = 2$ in the sequel; this is equivalent to restricting to a graph G with $R_e(G) = 2$ or $R_v(G) = 2$. Since the redundant rigidity concept is defined by analogy to connectivity of a graph, let us try to relate the two. A more complete work of Summers et al. (2008) on 2-vertex-rigidity has recently become available.

Lemma 9. For any rigid graph G, there holds $R_e(G) = 2$ only if $K_e(G) \geq 3$.

Lemma 10. For any rigid graph G, there holds $R_v(G) = 2$ only if $K_v(G) \geq 3$.

Actually, the preceding Lemmas can be generalized as follows:

Corollary 11. For any rigid graph G, there holds $R_v(G) < K_v(G)$, and $R_e(G) < K_e(G)$.

Remark 12. Corollary 11 could actually trivially lead to the Lemma 3, given the graph connectivity result that $K_v(G) \leq K_e(G) \leq \delta(G)$ stated in the end of subsection 3.1.

Remark 13. Comparing Corollary 11 and Theorem 6, and considering that $K_v(G) \leq K_e(G)$, we note an open question (to which we have no complete answer yet): is there a relationship between $R_e(G)$ and $K_v(G)$, and if so, what is the relationship?

Let us also compare our formal definition of redundancy with those reviewed in Subsection 3.1.

Lemma 14. A 2-vertex-rigid graph is necessarily globally rigid.

The converse of Lemma 14 does not hold. For example, the graph on 5 agents of Figure 3(a) is globally rigid but not 2 vertex-rigid (for removal of the vertex in the center will destroy rigidity of the remaining graph). Also, in contrast to the result of Lemma 14, a 2 edge-rigid graph is not necessarily globally rigid; Figure 2(b) gives a counterexample (the vertices 1 and 4 form a *cut-set* Gross and Yellen (2006), demonstrating that the graph is not 3-connected). Further, we have:

Lemma 15. A circuit is a 2 edge-rigid graph, but not necessarily 2 vertex-rigid, nor globally rigid.

In the next section, we will elaborate more on these special classes of formations with respect to the redundancy measure.

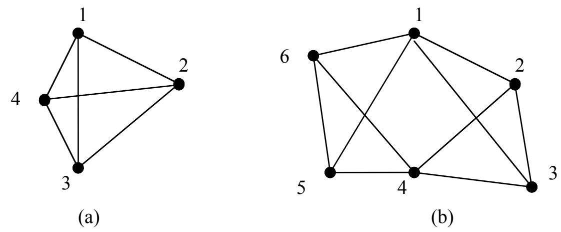

Fig. 2. Examples of circuits. The formation represented in (a) is 2-vertex-rigid and globally rigid; (b) is NOT globally rigid and NOT 2-vertex-rigid.

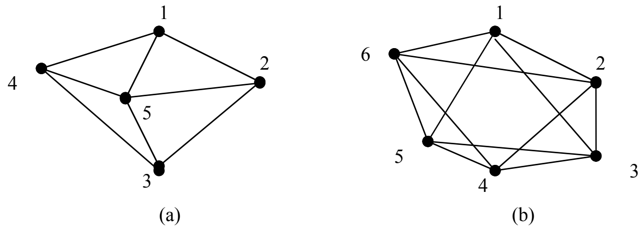

Fig. 3. Example of wheel and power of a cycle. The formation represented in (a) is a wheel formation with 5 vertices; (b) is a power graph of a cycle of length 6.

4.2 Wheel graph

A wheel graph W_n is a graph with n vertices, formed by connecting a single vertex to all vertices of an $(n-1)$-cycle. One might use wheel graphs to model a formation in which there is a central UAV , the commander, whose sensing region covers all other agents. Figure 2(a) and Figure 3(a) are two wheel graphs on 4 and 5 vertices, respectively.

Lemma 16. A wheel graph is 2 edge-rigid.

It is trivial to see a wheel graph with more than 4 vertices is not 2 vertex-rigid, since removal of the central node will leave only a cycle, which is not rigid. In contrast, the graph obtained after removal of any node other than the central node is always rigid.

It seems that applying the *deterministic* metric of 2 vertex-rigidity is too restrictive here, in that only a small fraction of the vertices (in this case, 1 out of n) is vulnerable. Further, consider a military scenario where the main hazard to the formation is enemy attack on a randomly selected agent (all agents having equal probability of selection). For a very large formation that can be modeled as a wheel graph (of course, the enemy would not know it is a wheel graph, or even should they know, would not know which agent is the central agent), we can say that it is deterministically 2 edge-rigid and statistically $\frac{n-1}{n}$ 2 vertex-rigid, or $R_v(W_n) = 2\frac{n-1}{n}$.

Further to this proposition, we have another interesting observation; for the same wheel graph W_n, there is the statistical measure for its 3 (vertex)-rigidity. Let us consider the case W_5; it is statistically $\frac{4}{5}$ 2-rigid, $\frac{2}{5}$ 3-rigid, and zero 4-rigid. For W_6, it is statistically $\frac{5}{6}$ 2-rigid, $\frac{1}{3}$ 3-rigid, and $\frac{1}{3}$ 4-rigid. We can define a statistical vertex rigidity factor, $svrf(W_n)$ to be the maximum of the products of the fraction times the level of redundancy. For example, $svrf(W_5) = \max\{\frac{8}{5}, \frac{6}{5}, 0, 0, \ldots\} = \frac{8}{5}$, and $svrf(W_6) = \frac{5}{3}$.

These observations motivate us to pose the following:

Conjecture 17. For a wheel graph W_n, there holds $svrf(W_n) < 2$.

Conjecture 18. For any graph G, if it is m-rigid but not $(m+1)$-rigid, we have $svrf(G) \leq m$.

4.3 Powers of a graph

In Anderson et al. (2007), powers of graphs are used in discussion of the property of global rigidity. Powers of graphs are also useful for formulating redundant rigidity concepts of the type appearing in this paper.

The k^{th} power G^k of a graph G is a graph obtained by adding an edge between all pairs of vertices of G with hop-distance at most k. Given a formation with graph G, and with an edge being present only when two agents are closer than some sensing radius R and no occlusion occurs, doubling of R will generate G^2, tripling will generate G^3, etc. Figure 3(b) gives an example of a second power graph of a cycle with length 6.

Proposition 19. A second power graph of a cycle C^2 is 2-vertex-rigid when $|V| \geq 4$.

Proposition 20. A third power graph of a cycle C^3 is 3-vertex-rigid when $|V| \geq 5$.

This leads us to develop a more general result:

Theorem 21. A K^{th} power graph of a cycle C^k with vertex set V is K-vertex-rigid when $|V| \geq K+2$ and $K \geq 2$

5. CONCLUDING REMARKS

In this paper, we revisited a graph model of multiagent formations using rigid graphs, and have extended such models to incorporate redundancy in the design. We give perhaps the first set of definitions of both edge and vertex redundant rigidity in the field, with the aim of enhancing the robustness of a formation graph against agent and/or link loss(es). We discussed the perhaps simplest case of redundant rigidity, and stretched the results to more general cases along a few directions. We showed that in some sense, k-vertex-rigidity is a more demanding property than k-edge-rigidity. We evaluated a number of common topological models (special classes of rigid graphs) and measured their redundant rigidity both deterministically and statistically.

Our preliminary works on formally considering and utilizing redundancy concept in rigidity have led us to contemplate many problems with redundancy in mind. One could for example ask the following: Is there a complete characterization of n-rigid graphs for small n? What are the operations that preserve n-rigidity? What is the relationship between the connectivity and the edge or vertex redundant rigidity of a graph? Globally rigid formations are useful for sensor network localization problems; would it be useful to discuss the redundant globally rigid formations?

On top of everything that can be done for undirected formations, we can work with directed formations, such as occurred in Yu et al. (2007). Many results will have a different interpretation for directed formations and sometimes they are much more complicated.

REFERENCES

AFSAB. UAV technologies and combat operations. *Air Force Scientific Advisory Board (AFSAB), SAF/PA 96-1204*, I, 1996.

B. D. O. Anderson, P. N. Belhumeur, T. Eren, D. K. Goldenberg, A. S. Morse, W. Whiteley, and Y. R. Yang. Graphical properties of easily localizable sensor networks. *IEEE Transactions on Mobile Computing, Submitted*, 2007.

J. Aspnes, T. Eren, D. Goldenberg, A. S. Morse, W. Whiteley, Y. Yang, B. D. O. Anderson, and P. Belhumeur. A theory of network localization. *IEEE Transactions on Mobile Computing*, 5(12):1663–1678, December 2006.

J. E. Graver, B. Servatius, and H. Servatius. *Combinatorial Rigidity.* Graduate Studies in Math., 1993.

J.L. Gross and J. Yellen. *Graph Theory and Its Applications.* Chapman and Hall & CRC, 2006.

D. L. Haulman. U.S. unmanned aerial vehicles in combat, 1991-2003. *Internal Report, Air Force Historical Research Agency*, 2003.

B. Jackson and T. Jordan. Connected rigidity matroids and unique realizations of graphs. *Journal of Combinatorial Theory Series B*, 94:1–29, 2005.

G. Laman. On graphs and rigidity of plane skeletal structures. *J. Engrg. Math.*, 4:331–340, 1970.

NASA. J-UCAS X-45A represents an evolutionary step in technology and a look at future aviation. *The X-press, Dryden Flight Research Center, NASA*, 46(7), 2004.

R. Olfati-Saber and R. M. Murray. Distributed cooperative control of multiple vehicle formations using structural potential functions. In *Proc. of the 15th IFAC World Congress*, Barcelona, Spain, 2002.

T. H. Summers, C. Yu, and B. D. O. Anderson. Addressing agent loss in vehicle formations and sensor networks. *Preprint*, 2008.

T. Tay and W. Whiteley. Generating isostatic frameworks. *Structural Topology*, (11):21–69, 1985.

C. Yu and B. D. O. Anderson. Redundant rigidity theory and applications. *Preprint*, 2008.

C. Yu, J.M. Hendrickx, B. Fidan, B.D.O. Anderson, and V.D. Blondel. Three and higher dimensional autonomous formations: Rigidity, persistence and structural persistence. *Automatica*, pages 387–402, March 2007.