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Normalised expression and evaluation of PI tuning rules

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Abstract: This manuscript addresses the problem of casting the heterogeneous panorama of model-based PI tuning rules into some uniform framework. By adopting convenient normalisations, different rules can be expressed in such a way to allow objectively grounded comparisons. The presented work is part of a larger research project, the ultimate goal of which is to allow qualifying the contribution of newly introduced rules with respect to existing ones, and possibly to design PI autotuners encompassing more than one rule, and capable of selecting the 'best' one for each particular tuning problem. The extension of the idea to the PID case will be treated in future works.

Keywords: PID autotuning; process control; adaptive systems.

1. INTRODUCTION AND MOTIVATION

The industrial importance of autotuning, particularly with reference to PI/PID regulators, is witnessed by a huge literature, a few examples of which are Åström and Hägglund (1995); Yu (1999); Lelic and Gajic (2000); Leva et al. (2002); Ang et al. (2005). Among the reasons for such a great interest are the value added to control systems by accurate tuning bib (September 2004), and the possible use of (auto)tuning techniques to *assess* a control system, see e.g.Thyagarajan et al. (2003) or, more in general, the issues evidenced in works such as, Shinskey (2002) or Li et al. (february 2006a).

A particularly important subject within the PI/PID autotuning domain is *model-based* autotuning. That subject has been extensively investigated over the last years Leva (2001), and a huge number of methods have been proposed in the literature, see e.g. Lelic and Gajic (2000) or, synthetically, the extensive review O'Dwyer (2003). However, as pointed out in some recent works such as Leva (2007a), it is difficult to compare such methods on an objective basis, so as to help potential users decide which one is 'best suited' for a particular tuning problem.

The aim of this manuscript is to formulate a proposal to tackle the mentioned problem, by conveniently normalising the quantities pertaining to the tuning problem, and by employing the results of that normalisation to establish some comparative relationships among tuning methods. The scope is here restricted to a single type of methods for space reasons, but the underlying ideas are apparently general.

To address the matter, the first (and less obvious than it may appear) point is to specify the boundaries of the treatise precisely. The literature, for physiological reasons, is of very little help in this respect, since the various published methods are in general conceived with very different goals, and the derivation of each method is coupled to its particular goal right from the beginning.

Therefore, we first state that this research deals with explicit model-based methods, i.e., tuning rules in the form $\theta_R = f_T(\theta_M, \tau_D)$, where θ_R is the vector of regulator parameters, θ_M that of parameters of the process model used for the tuning (of fixed structure), and τ_D a possible vector of method's design variables. Then, we decide to disregard - for the purpose of the normalisations and possible comparisons addressed in this research - the way the process model was parametrised. This is a critical decision, despite the fact that the particular procedure used to parametrise the process model is seldom accounted for in the model-based tuning literature.

We omit for space reasons a discussion on that very important matter, which the interested reader can however find e.g. in Leva and Piroddi (2007). Suffice here to recall the conclusions of that discussion, i.e., that a viable way to obtain a common basis for meaningful normalisations and comparisons is to define the concepts of Nominal Tuning Model (NTM), Nominal Control Problem (NCP) and Nominal Control System (NCS): the first is the model used for the tuning, the second has the aim of fulfilling some 'tuning desires' under the hypothesis that the NTM is an exact copy of the process, and the third is the result of the second, i.e., the control system containing the tuned regulator and the NTM. Of course 'nominal' entities differ from their 'real' counterparts owing to the fact that the NTM is not the process, but in many autotuning applications the NTM more or less exhausts the available information. It makes therefore sense to provide, beside the NCS, some *a priori* quantification, e.g., some overbound on the admissible model error committed by the NTM. Such overbounds can be computed based on the NCS, as discussed e.g. in Leva and Colombo (2001).

Finally, independently if the employed tuning method, we reasonably assume that the 'tuning quality' achieved in the NCS can be assessed by convenient indices, termed here Tuning Quality Indices (TQIs). The reason for normalising tuning methods is that doing so eases the use and the interpretation of such quality indices (that, it is worth stressing, are conceived as tuning evaluation quantities independently of the used tuning method). This is the scope of the presented research, its motivation being the need for shared and agreed methodologies to assess the additional benefits of newly introduced tuning rules and their differences with respect to existing ones, and to help decide which tuning method is best suited to a particular problem.

Before entering the matter, an important remark (and consequently a possible *caveat*) is in order. Neglecting the way the NTM was parametrised is necessary, since if tuning method evaluations and comparisons started not from the NTM but from some set of process data, and if correspondingly the 'tuning method' included the NTM parametrisation phase, then many evaluations could be impaired, and also many comparisons reversed, by simply changing the way the NTM is parametrised. In other words, to 'compare tuning methods', one can take basically two routes, that we briefly sketch out considering, for simplicity, methods that share the NTM structure.

- One can fix the NTM parametrisation method, then consider several classes of processes *not* having the same structure as the NTM, apply the (single) parametrisation method and the (various) tuning methods, and so compare them—we disregard the case in which the process and the NTM have the same structure because it is simply unrealistic in virtually the totality of the autotuning domain. In any case, such an analysis may be of interest to some extent, but exhausts its generality at the applicability boundaries of the parametrisation method considered. Moreover, there is no warranty that *another* parametrisation method gives different results. Note, incidentally, that we are neglecting the number of arbitrary choices that one has to take to turn a formalised parametrisation method into a parametrisation *procedure*. Consider for example the well known method of areas: how many samples in the step transient? Which approximation to compute the integrals? Which type of noise filtering? Which type of outlier removal? As can easily be seen, this route is not that likely to produce results with some general value.
- Alternatively, one can take a 'nominal' point of view and evaluate the way the controller controls the NTM. This is certainly a reduced objective, but it is clear how to attain it. The NTM parametrisation method has no role here, except as the source of some modelling error, which is unavoidable since the process/NTM structural correspondence is practically utopia. Instead of considering the NTM parametrisation *a priori*, the idea is then to quantify the admissible model error for a given tuning method *a posteriori*, including such a quantification in the evaluation indices—a subject that however we do not treat here for space reasons.

In this research, the second route is followed, for the reasons given above. Further discussions on the role of the NTM, and its parametrisation, can be found in Leva and Schiavo (2005); Leva (2007a); Leva and Piroddi (2007); we therefore avoid delving here in more detail on the matter. An important remark, for the safe of clarity, is however in order. The results obtained with the neglection of the NTM parametrisation phase can be interpreted correctly only if one takes into account that there are basically two possible scenarios.

- In the first one, the NTM is a good representative of the process 'dominant' (i.e., typically, low-frequency) dynamics. Parametrising a NTM in that way in general is far from ideal for the tuning, but is done very frequently for practical reasons. In such cases, the 'usual' TQIs like the cutoff frequency and the phase margin make sense, but not always really indicate the 'quality' of the obtained closed-loop transients, nor the NTM can in general be used, by simulating the NCS, to forecast the tuning results.
- In the second one, the NTM is parametrised so as to be precise around the cutoff frequency, see again Leva and Piroddi (2007) for a discussion on the matter. In this case more informative TQIs can be usefully computed, and the NTM can be effectively employed to forecast the tuning results.

In both scenarios, as anticipated, it is possible to give an *a priori* quantification of the obtained robustness by means of the nominal control sensitivity function, the inverse of its frequency response magnitude providing an overbound for the acceptable additive model error frequency response magnitude. The corresponding criteria tend to be excessively conservative, particularly in the second scenario, but if correctly interpreted (see e.g. Leva and Bascetta (2007)) such overbounds are informative anyway, and at least permit method comparisons also from the standpoint of the admissible NTM error.

Given all this, in this manuscript the tuning problem is viewed essentially from the side of the NCS, i.e., as much information as possible is sought based on the sole NTM. The work presented herein relates to others like Leva (2007a,b), that are part of the same research. The peculiarity of this manuscript, and therefore its particular contribution within the wider research path it belongs to, is to focus the attention on the usefulness of 'normalising the tuning rules' in the model-based context (what is meant for 'normalising' will be clarified in the following).

2. PI TUNING QUALITY INDICES

In this study we refer to the one-degree-of-freedom (1-d.o.f.) PI regulator

$$R(s) = K\left(1 + \frac{1}{sT_i}\right) \tag{1}$$

and, since we consider model-based tuning rules, we denote by $M(s)$ the transfer function of the NTM.

Five TQIs are considered herein.

The first and second TQIs are the cutoff frequency ω_c and the phase margin φ_m.

The third TQI is the regulator high-frequency magnitude

$$R_\infty := \lim_{\omega \to \infty} |R(j\omega)| \tag{2}$$

that equals the limit for $\omega \to \infty$ of the control sensitivity function frequency response magnitude, therefore quantifying the sensitivity of the control signal to measurement noise.

The fourth TQI is the ratio between the PI integral time and the dominant time constant of the $M(s)$. Denoting by T that time constant the TQI is then defined as

$$\tau_D := \frac{T_i}{T} \tag{3}$$

and measures how much the regulator zeroes are 'at low frequency' with respect to the control bandwidth, thus the keenness of the control system to result in poor load disturbance recovery, and in 'control kicks' in response to set point steps.

The fifth TQI is the regulator magnitude at the cutoff frequency

$$R_c := |R(j\omega_c)| \tag{4}$$

and quantifies the inverse of the ∞-norm of the transfer function from load disturbance to controlled variable.

If some robustness quantification is desired, a sixth TQI can be introduced based on the nominal control sensitivity function

$$C(s) := \frac{M(s)}{1+R(s)M(s)}. \tag{5}$$

A very conservative choice is the inverse of the ∞-norm of that function, since a well known robust stability result states that the stability of the nominal control system (that containing the NTM) carries over to all the systems for which, denoting by $\mathscr{E}_a(s)$ the additive model error relative to the NTM $M(s)$, the relationship

$$\|C(s)\mathscr{E}_a(s)\|_\infty < 1 \tag{6}$$

holds true. More informative a TQI, particularly if the NTM is parametrised so as to be precise around the cutoff, is the same norm computed over a band around the cutoff itself. A possible TQI is then for example

$$D_{min} := \min_{\omega \in \left(\frac{\omega_c}{10}, 10\omega_c\right)} \left|\frac{1}{C(j\omega)}\right| \tag{7}$$

3. NORMALISATION QUANTITIES

For space limitations, in this manuscript we consider only PI tuning rules based on the First Order Plus Dead Time (FOPDT) model structure, i.e.,

$$M(s) = \mu \frac{e^{-sL}}{1+sT}. \tag{8}$$

Extension to more complex regulators, starting e.g. from the PID, are possible but lengthy to present. The normalisation adopted here (that is almost trivial given the simple regulator addressed) consists of defining the normalised complex variable

$$\sigma := sT \tag{9}$$

and consequently the normalised PI

$$r(\sigma) := \mu R\left(\frac{\sigma}{T}\right) \tag{10}$$

and the normalised FOPDT model

$$m(\sigma) := \frac{1}{\mu} M\left(\frac{\sigma}{T}\right). \tag{11}$$

It is now possible to express the normalised open-loop nominal transfer function

$$\begin{aligned} \ell(\sigma) &:= r(\sigma)m(\sigma) = \mu R\left(\frac{\sigma}{T}\right)\frac{1}{\mu}M\left(\frac{\sigma}{T}\right) \\ &= R(s)M(s) = L(s) \end{aligned} \tag{12}$$

and the normalised nominal control sensitivity function

$$c(\sigma) := \frac{r(\sigma)}{1+\ell(\sigma)} = \frac{\mu R\left(\frac{\sigma}{T}\right)}{1+L(s)} = \mu C(s). \tag{13}$$

Observing now that clearly

$$r(\sigma) = \mu K \frac{1+\sigma T_i/T}{\sigma T_i/T} \tag{14}$$

it turns out that, for any tuning evaluation based on $L(s)$ and $C(s)$, any tuning rule of the type considered in this section is characterised by the two quantities

$$\alpha_\mu := \mu K, \quad \alpha_T := \frac{T_i}{T}, \tag{15}$$

together with the L/T ratio exhibited by the NTM, since defining

$$\theta := \frac{L}{T} \tag{16}$$

immediately leads to

$$r(\sigma) = \frac{\alpha_\mu}{\alpha_T}\frac{1+\sigma\alpha_T}{\sigma}, \tag{17}$$

$$\ell(\sigma) = \frac{\alpha_\mu}{\alpha_T}\frac{e^{-\sigma\theta}}{\sigma}\frac{1+\sigma\alpha_T}{1+\sigma}, \tag{18}$$

$$c(\sigma) = \frac{\alpha_\mu(1+\sigma\alpha_T)(1+\sigma)}{\sigma\alpha_T(1+\sigma)+\alpha_\mu(1+\sigma\alpha_T)e^{-\sigma\theta}}. \tag{19}$$

Let us now consider the five proposed TQIs. Defining, in accordance with (9), a normalised frequency $\psi = \omega T$, one obtains for the normalised cutoff frequency ψ_c the simple relationship

$$\frac{\alpha_\mu}{\alpha_T}\frac{1}{\psi_c}\sqrt{\frac{1+(\psi_c\alpha_T)^2}{1+\psi_c^2}} = 1. \tag{20}$$

Bringing (18) in, for φ_m one similarly has

$$\varphi_m = 180^o - \arg^o(\ell(j\psi_c)) \tag{21}$$

As for the remaining TQIs, finally,

$$R_\infty = \lim_{\psi\to\infty}\left|\frac{1}{\mu}r(j\psi)\right|, \tag{22}$$

$$R_c = |r(j\psi_c)|, \tag{23}$$

$$\tau_D = \alpha_T. \tag{24}$$

Solving now for the TQIs, lengthy but trivial computations yield

$$\begin{aligned}
\psi_c &= \psi_c(\alpha_\mu, \alpha_T) \\
&= \frac{\sqrt{\alpha_\mu^4\alpha_T^4 - 2\alpha_T^2(2+\alpha_\mu^2) + 4\alpha_\mu^2 + 1} + \alpha_\mu^2\alpha_T^2 - 1}{2}, \\
\varphi_m &= \varphi_m(\alpha_\mu, \alpha_T, \theta) \\
&= 90^\circ - \psi_c\theta\frac{180^\circ}{\pi} + \arctan^\circ(\alpha_T\psi_c) - \arctan^\circ(\psi_c), \\
R_\infty &= R_\infty(\alpha_\mu, \mu) \\
&= \left|\frac{\alpha_\mu}{\mu}\right|, \\
R_c &= R_c(\alpha_\mu, \alpha_T) \\
&= \frac{\alpha_\mu}{\alpha_T\psi_c}\sqrt{1+\alpha_T^2\psi_c^2}, \\
\tau_D &= \tau_D(\alpha_T) \\
&= \alpha_T.
\end{aligned} \tag{25}$$

Given the meaning of ψ_c, the relationships (25) are valid subject to

$$\begin{aligned} &\alpha_\mu^4\alpha_T^4 - 2\alpha_T^2(2+\alpha_\mu^2) + 4\alpha_\mu^2 + 1 \geq 0 \\ &\sqrt{\alpha_\mu^4\alpha_T^4 - 2\alpha_T^2(2+\alpha_\mu^2) + 4\alpha_\mu^2 + 1} + \alpha_\mu^2\alpha_T^2 - 1 > 0 \end{aligned} \tag{26}$$

4. NORMALISED TUNING RULES

The reader may wonder why one should introduce the two quantities α_μ and α_T instead of basing methods' evaluations and comparisons directly on the above (or possibly other) TQIs. To sketch out an answer, in this section we first write the normalised version of some well established model-based PI tuning rules, namely

- the IMC-PI formulæ Morari and Zafiriou (1989); Braatz (1996); Leva and Colombo (2004), indicated here with 'IMC',
- the IMC improved PI or 'Rivera PI' Rivera et al. (1986), termed here 'Riv',
- the SIMC rules by Skogestad (2005, 2006), here 'Sko',
- the 'Direct Synthesis for disturbance' method by Chen and Seborg Chen and Seborg (2002), indicated with 'DSd',
- the formulæ used in the ABB Easy-Tune, as reported in Li et al. (february 2006b), and termed here the 'ABB' PI,
- the rules based on the minimisation of the IAE (Integral of the Absolute Error) given in Lopez et al. (1967), indicated with 'LSM',
- and the formulæ by Daniel and Cox (termed there 'D&C') reported in Cox et al. (1997), a specialisation of Vrančić et al. (1996) to the FOPDT model case,

that, for the reader's convenience, are summarised in their native form in table 1. Notice that some methods have as design parameter a 'desired closed-loop time constant' (λ), while others have no design parameters at all.

	K	T_i
IMC	$\dfrac{T}{\mu(L+\lambda)}$	T
Riv	$\dfrac{T+L/2}{\mu\lambda}$	$T+\dfrac{L}{2}$
Sko	$\dfrac{T}{\mu(L+\lambda)}$	$\min(T,4(L+\lambda))$
DSd	$\dfrac{T^2+TL-(\lambda-T)^2}{\mu(L+\lambda)^2}$	$\dfrac{T^2-TL-(\lambda-T)^2}{L+T}$
ABB	$\dfrac{1.164}{\mu}\left(\dfrac{L}{T}\right)^{0.977}$	$\dfrac{60}{40.44}T\left(\dfrac{L}{T}\right)^{0.68}$
LSM	$\dfrac{0.758}{\mu}\left(\dfrac{L}{T}\right)^{-0.861}$	$\dfrac{T}{1.02-0.323\frac{L}{T}}$
D&C	$\dfrac{1}{2\mu}\dfrac{6T^3+6LT^2+3L^2T+L^3}{3L^3T^2+3L^4T-3L^2T+L^3}$	$\dfrac{6T^3+6LT^2+3L^2T+L^3}{6T^2+3L^3T}$

Table 1. The PI tuning rules considered.

Applying the proposed normalisation and evidencing the L/T ratio as defined in (16), the rules of table 1 are expressed in normalised form as shown in table 2.

	α_μ	α_T
IMC	$\dfrac{1}{\theta+\lambda/T}$	1
Riv	$\dfrac{1+\theta/2}{\lambda/T}$	$1+1\dfrac{\theta}{2}$
Sko	$\dfrac{1}{\theta+\lambda/T}$	$\min(1,4(\theta+\lambda/T))$
DSd	$\dfrac{1+\theta-(1-\lambda/T)^2}{(\lambda+T)^2}$	$\dfrac{1+\theta-(1-\lambda/T)^2}{(1+\theta)^2}$
ABB	$1.164\theta^{0.977}$	$\dfrac{60}{40.44}\theta^{0.68}$
LSM	$0.758\theta^{-0.861}$	$\dfrac{1}{1.02-0.323\theta}$
D&C	$\dfrac{6+6\theta+3\theta^2+\theta^3}{6\theta L^2+6\theta^2L^2-6\theta^2+2\theta^3}$	$\dfrac{6+6\theta+3\theta^2+\theta^3}{3\theta^2+6/T^2}$

Table 2. Normalised tuning rules.

Apparently, the two quantities α_μ and α_T depend on the NTM parameters and the design variables of the tuning method (where present) in quite simple a way, as shown by table 2—certainly simpler a dependence than the dependence of the TQIs on the same quantities.

In fact, (25) evidence that, denoting the NTM parameters (μ, T and L in the FOPDT case) by θ_M, and the method's design variables (λ or the empty set in the considered rules) with θ_D, a mapping can be established in the form

$$\{TQIs\} = f\left(\alpha_\mu(\theta_M,\theta_D),\alpha_T(\theta_M,\theta_D),\theta_M\right) \tag{27}$$

where $\alpha_\mu(\cdot,\cdot)$ and $\alpha_T(\cdot,\cdot)$ are simpler and more tractable than $f(\cdot,\cdot,\cdot)$.

This simple remark constitutes a first reason for studying the behaviour of α_μ and α_T in the various situations the considered tuning rules may come across. A second reason is that (26), possibly coupled with a minimum phase margin constraint, provide a very simple way to find the validity limits of a given tuning rule. As a further remark, recall that the devised TQIs are to be used also (and preferably, in some sense) in cases where the NTM privileges fidelity around the (expected) cutoff, and therefore its parameters are not bound to represent the process in open-loop (e.g., the time constants of such NTMs may have hardly anything to do with the time scale of the process responses). In such cases the proposed TQIs do not replicate, as it may appear in simpler situations, well known similar normalisations and indices.

Having in any case established that α_μ and α_T can easily characterise a tuning rule, we now proceed to analyse - an interpret where possible - the behaviour of those two quantities. In doing so, further reasons in favour of the proposed normalisation will emerge.

5. INTERPRETATION OF α_μ AND α_T

We start this section with two further remarks. First, the NTM gain μ appears only as a scaling factor in R_∞, see (25), i.e., with the proposed normalisation it is only relevant to determine facts that are related to the high-frequency behaviour of the regulator frequency response, and consequently the control sensitivity; consistently, μ does not appear in table 2, and is not necessary to determine R_c (which is logical, but may not be completely intuitive at a first glance). Then, one of the TQIs (τ_D) coincides with one of the proposed quantities to characterise the regulator and the tuning rules intheir normalised form. This not only means that τ_D is important to quantify the load disturbance rejection capabilities of a NCS, but also confirms that the proposed normalisations make sense.

Hence, to show that α_μ and α_T actually evidence (and make comparable, thanks to their normalised character) the different behaviours of the tuning rules considered (and also of possible others, of course), an example of their behaviour is now proposed. A NTM is considered with $\mu = 1$, $T = 1$, and the normalised delay

$$\tau := \frac{L}{L+T} \tag{28}$$

stepped in the range 0.1–0.7, i.e., from the dominance of rational dynamics to that of the delay, but remaining within the applicability limits of all the considered rules (except for a note on DSd later on).

To provide a fair and meaningful comparison, all the methods having as design parameter a 'desired closed-loop time constant' (λ) had that parameter selected so as to require a certain 'acceleration factor' with respect to the open-loop NTM dynamics, the mentioned factor being intuitively defined as

$$A_f := \frac{L+5T}{5\lambda}. \tag{29}$$

Figure 1 shows the behaviour of α_μ and α_T for A_f equal, from top to bottom, to 0.5, 1, 2, and 4 (reasonable values covering the majority of possible tuning desires).

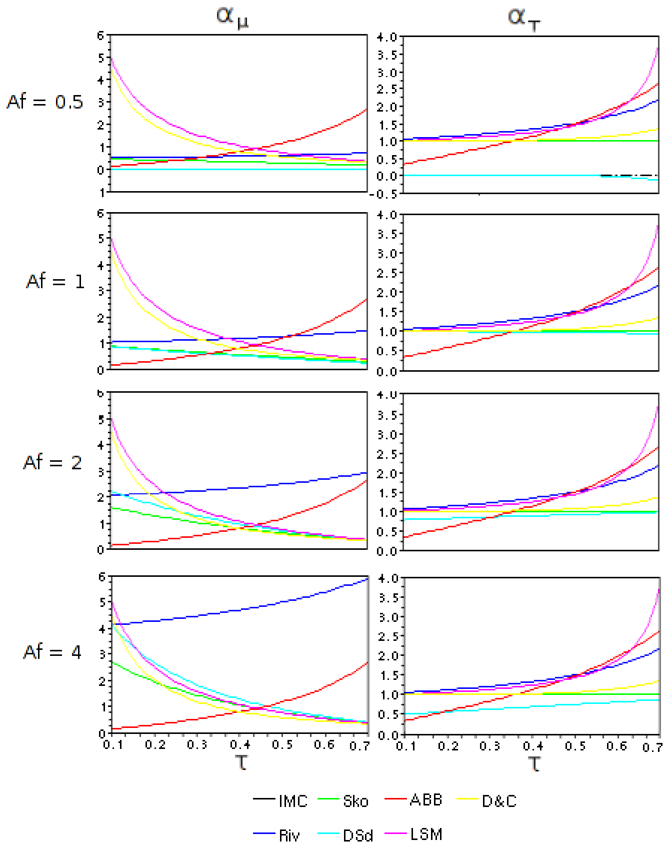

Fig. 1. Behaviour of α_μ and α_T in significant cases.

Note that for high values of τ and low A_f DSd may give negative values of α_μ and α_T: if only one is negative the tuning is apparently invalid, while if both are negative the NCS may still be stable, but the PI zero is positive. Both situations are undesired, so for simplicity having α_μ or α_T negative is considered out of the validity limits—a further usefulness of those two quantities.

From figure 1, two relevant facts emerge. First, for a given A_f, some methods tend to reduce, others to increase α_μ. Since the normalised delay is by common opinion a measure of 'how difficult it is' to control a NTM (that quantity is also sometimes called the 'controllability index'), methods of the first type are more keen to produce good robustness than methods of the second type. Methods of the first type are then advisable for high values of τ: for LSM and D&C this is true irrespective of A_f, while for DSd and Sko the fact is relevant for high values of A_f only. Then, for some methods (including those having λ as parameters, i.e., IMC, Riv, Sko and DSd, or the remark would be irrelevant) α_T is substantially independent of A_f, while for others this is not true. Methods of the first type aim at response speed essentially by increasing the PI gain, while methods of the second type (also) put the integral time into play. Methods of the second type are advisable if the tuning desire is to make the load disturbance response vanish quickly even at the cost of a larger peak deviation from the set point, while the reverse is true for methods of the first type.

We omit more detailed observations, that would require too much space to discuss, and are of less importance than the two above. It should however be clear that α_μ and α_T, and more in general the normalisation procedure here applied to the particular case of FOPDT-based PI tuning, is of help to characterise a tuning rule.

6. A POSSIBLE APPLICATION

Besides being useful to evaluate a tuning method and understanding its behaviour in a synthetic way, α_μ and α_T can also be used as PI parameters directly. Consider for example, recalling (11) and (16), the normalised FOPDT NTM

$$m(\sigma) = \frac{e^{-\sigma\theta}}{1+\sigma} \tag{30}$$

and suppose to fix α_μ to a prescribed value $\overline{\alpha}_\mu$, which substantially means constraining the high-frequency control sensitivity in a relative way with respect to the NTM gain.

From (25) it is straightforward to compute ψ_c and φ_m, thus achieving a compromise between response speed and stability for the NCS. An example with $\theta = 0.25$ is briefly shown in figure 2, where $\overline{\alpha}_\mu = 0.5$ and α_T goes from 0.1 (lower values are rarely sensible) to 0.99.

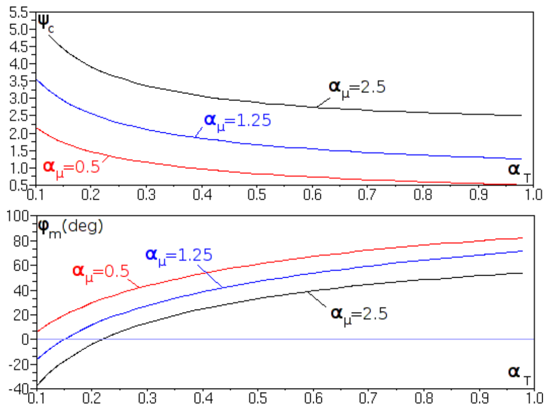

Fig. 2. Application example: ψ_c and φ_m.

Figure 2 can then be used to select a PI tuning. Some possible choices are compared in figure 3, where the line colours correspond to ($\overline{\alpha}_\mu$,φ_m) couples as listed in table 3.

$\overline{\alpha}_\mu$	φ_m	Line colour	$\overline{\alpha}_\mu$	φ_m	Line colour
0.5	40°	blue	1.25	50°	cyan
0.5	60°	red	2.5	25°	yellow
1.25	30°	green	2.5	40°	black

Table 3. Organisation of figure 3.

Figure 3 reports in the left column the responses of the controlled variable y and the control signal u to a unit load disturbance step applied when the (normalised) time equals 1; the right column reports the Bode magnitude plots of the regulators and the inverse of the control sensitivity functions.

The illustrated results should be self-explanatory. Suffice to note the consistent relationship between the speed/stability requests and the obtained robustness, indicated by $|1/C(j\psi)|$, that - recall (6) - gives a bound on the acceptable (normalised) additive model error relative to the NTM. By the way, that plot could also permit to evaluate the TQI D_{min}, which we omit here for brevity.

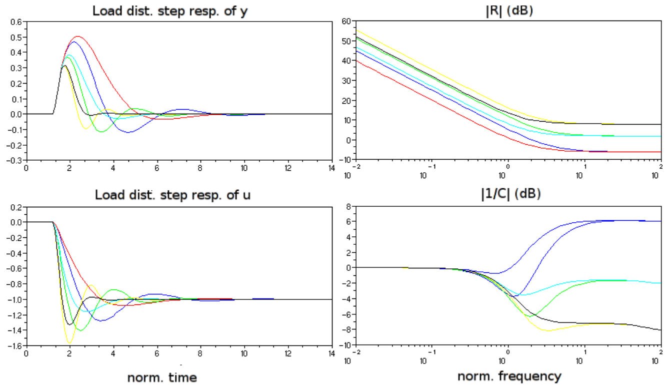

Fig. 3. Application example: closed-loop transients and relevant magnitude Bode diagrams.

7. CONCLUSIONS AND FUTURE WORK

By conveniently normalising PI tuning rules, meaningful comparisons can be established, and it is possible to understand the behaviour and characteristics of a rule in a synthetic way. In addition, the same normalisation approach can lead to particular tuning rules. Finally, normalising the required computations can help save time and memory space on low-end control devices.

As anticipated, the presented results were obtained within a larger research project, aimed both at qualifying newly introduced rules with respect to existing ones in a rigorous - or at least standardised - way, and at the realisation of autotuners with a *set* of tuning rules, and with rule selection capability among that set, based on the particular control problem at hand.

As such, future work will concern the extension of the presented idea to the PID case, and the inclusion in the framework of more, different tuning rules.

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