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Adaptive Neural Control of SISO Time-Delay Nonlinear Systems with Unknown Hysteresis Input ⋆

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Abstract: In this paper, adaptive variable structure neural control is investigated for a class of SISO nonlinear systems in a Brunovsky form with state time-varying delays and unknown hysteresis input. The unknown time-varying delay uncertainties are compensated for using appropriate Lyapunov-Krasovskii functionals in the design. The effect of the unknown hysteresis with the Prandtl-Ishlinskii model is mitigated using the proposed adaptive control. By utilizing the integral-type Lyapunov function, the closed-loop control system is proved to be semi-globally uniformly ultimately bounded. Simulation results demonstrate the effectiveness of the approach.

Keywords: Time-vary delays; hysteresis; neural networks.

1. INTRODUCTION

In recent years, control of nonlinear systems preceded by unknown hysteresis nonlinearities has received a great deal of attention, since the hysteresis nonlinearities are common in many industrial processes. Control of a system with hysteresis nonlinearities is challenging, because they are non-differentiable nonlinearities and severely limit system performance such as giving rise to undesirable inaccuracy or oscillations, even leading to instability (Tao and Kokotovic, 1995). Due to the nonsmooth characteristics of hysteresis nonlinearities, traditional control methods are insufficient in dealing with the effects of unknown hysteresis. Therefore, the advanced control techniques to mitigate the effects of hysteresis has been called upon and has been studied for decades.

The most common approach is to construct an inverse operator to cancel the effects of the hysteresis in (Tao and Kokotovic, 1995) and (Tan and Baras, 2004). However, it is a challenging work to construct the inverse operator for the hysteresis, due to the complexity and uncertainty of hysteresis. As an alternative, approaches combining the hysteresis model with the control technique without constructing an inverse model have also been developed. In (Su *et al.*, 2000), robust adaptive control was investigated for a class of nonlinear system with unknown backlash-like hysteresis, for which, adaptive backstepping control was designed in (Zhou *et al.*, 2004). Most of the above works are discussing about backlash-like hysteresis. Actually for the backlash-like hysteresis, it can be written as a linear-in-input term plus a bounded term, which can be compensated for using the standard robust adaptive control design. However, backlash-like hysteresis only can represents certain type of hysteresis nonlinearity. Hysteresis is a very complex phenomenon and there exist many hysteresis models in the literature. Interested readers can refer to (Macki *et al.*, 1993) for a review of the hysteresis models. Some of them are more complex than the backlash-like one, such as Prandtl-Ishlinskii model, but they can capture the hysteresis phenomenon more accurately. In (Su *et al.*, 2005) and (Wang and Su, 2006), adaptive variable structure control and adaptive backstepping methods were proposed for a class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity with the Prandtl-Ishlinskii model representation respectively. However, the nonlinear functions in these works were assumed to be known, which limited the applications of the proposed control. In this paper, to deal with the presence of function uncertainties, approximation based techniques using neural networks is proposed, since the neural networks has the universal approximation capabilities, learning and adaptation, parallel distributed structures (Narendra and Parthasarathy, 1990), (Lewis *et al.*, 1999), and (Ge *et al.*, 2002).

Time-delay is frequently encountered in the models of engineering systems, natural phenomena, and biological systems. The existence of time-delays in a system frequently becomes a source of instability, and may degrade the control performance. Therefore, a number of different approaches have been proposed in order to stabilize such systems with time-delays (Nguang, 2000), (Niculescu, 2001), (Richard, 2003),(Ge *et al.*, 2003) and (Ge and Tee, 2007). Motivated by (Ge *et al.*, 2003), where adaptive neural controls were firstly presented for classes of nonlinear systems with unknown constant time delays, we consider a class of uncertain SISO nonlinear systems with unknown time-varying delays and hysteresis nonlinearities in this paper. The main contributions of the paper lie in:

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(i) the removal of the need for prior knowledge of functional uncertainties with the use of neural network approximation, compared with the works (Su *et al.*, 2005) and (Wang and Su, 2006);
(ii) the introduction of a continuous function in $h(z)$, through which the controller singularity problem is avoided without designing practical controller like in (Zhang and Ge, 2007) and (Ge *et al.*, 2003), so that the proof of Theorem is simplified as will be seen later;
(iii) the combination of Lyapunov-Krasovskii functional and Young's inequality in eliminating the unknown time-varying delay $\tau_i(t)$ in the upper bounding function of the Lyapunov functional derivative, which makes NN parametrization with known inputs possible.

The organization of this paper is as follows. The problem formulation and preliminaries ar given in Section 2. In Section 3, adaptive variable structure neural control is developed for a class of SISO time-varying delay systems with hysteresis. Simulation studies are shown to demonstrate the effectiveness of the approach in Section 4, followed by conclusion in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of uncertain SISO state time-varying delay nonlinear systems in Brunovsky form with unknown hysteresis in the following

$$\begin{aligned} \dot{x}_j &= x_{j+1}, \quad j = 1, ..., n-1 \\ \dot{x}_n &= f(x) + g_\tau(x_\tau) + b(x)u + \Delta(x,t) \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau_{max}, 0], \quad i = 1, ..., n \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, ..., x_n]^T \in R^n$ is the state, $x_\tau = [x_1(t-\tau_1(t)), ..., x_n(t-\tau_n(t))]^T \in R^n$ is the delay state, $\tau_1(t), ..., \tau_n(t)$ are unknown time-varying state delays, $\phi_1(t), ..., \phi_n(t)$ are known continuous initial state vector functions, τ_{max} as will be defined later is a known constant, $y \in R$ denotes the output; $f(x), g_\tau(x_\tau)$ are unknown continuous functions, $b(x)$ is the unknown differentiable control gains, $\Delta(x,t)$ is uncertain disturbance, $u \in R$ is the output of the hysteresis nonlinearity, which is represented by the Prandtl-Ishlinskii model as follows

$$u(t) = p_0 v(t) - d[v](t) \tag{2}$$

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr \tag{3}$$

$$\begin{aligned} F_r[v](0) &= f_r(v(0), 0) \\ F_r[v](t) &= f_r(v(t), F_r[v](t_j)), \text{ for } t_j < t \le t_{j+1}, \\ &\qquad 0 \le j \le N-1 \\ f_r(v, w) &= \max(v - r, \min(v + r, w)) \end{aligned}$$

with $p_0 = \int_0^R p(r)dr$, $p(r)$ is a given density function, satisfying $p(r) \ge 0$ with $\int_0^\infty rp(r)dr < \infty$, and F_r is known as the play operator. Since $p(r)$ vanishes for large values of r, the choice of $R = \infty$ as the upper limit of integration in the literature is just a matter of convenience. In addition, the function v is monotone on each of the subintervals $(t_j, t_{j+1}]$, $0 \le j \le N-1$. See (Su *et al.*, 2005) and (Wang and Su, 2006) for the details.

Substituting (2) into (1), we obtain

$$\begin{aligned} \dot{x}_j &= x_{j+1}, \quad j = 1, ..., n-1 \\ \dot{x}_n &= f(x) + g_\tau(x_\tau) + b(x)p_0 v - b(x)d[v](t) + \Delta(x,t) \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau_{max}, 0], \quad i = 1, ..., n \\ y &= x_1 \end{aligned} \tag{4}$$

The control objective is to design an adaptive controller v for system (4) such that the output y follows the specified desired trajectory y_d.

Define x_d and e as

$$\begin{aligned} x_d &= [y_d, \dot{y}_d, ..., y_d^{(n-1)}]^T \\ e &= x - x_d = [e_1, e_2, ..., e_n]^T \end{aligned}$$

and the filtered tracking error s as

$$s = (\frac{d}{dt} + \lambda)^{n-1} e_1 = \sum_{j=1}^{n-1} \lambda_j e_j + e_n \tag{5}$$

where $\lambda_j = C_{n-1}^{j-1} \lambda^{n-j}$, $j = 1, ..., n-1$, $\lambda > 0$, $i = 1, ..., n$ are positive constants, specified by the designer.

To facilitate control design later in Section 3, we need make the following assumptions.

Assumption 1. The sign of $b(x)$ is known, and there exist constants b_0 and b_1 such that $0 \le b_0 \le |b(x)| \le b_1$, $\forall x \in R^n$. Without loss of generality, we shall assume that $b(x) > 0$, $\forall x \in R^n$. In addition, the constants b_0 and b_1 are used to handle the stability analysis only.

Assumption 2. The desired trajectory vector is continuous and available, and $[x_d^T, y_d^{(n)}]^T \in \Omega_d \subset R^{n+1}$ with Ω_d known compact set.

Assumption 3. There exist known nonnegative function $\phi(x)$ and unknown positive constant ρ^* such that $|\Delta(x,t)| \le \rho^* \phi(x)$, $\forall x \in R^n$ and $\forall t \ge 0$.

Assumption 4. The unknown continuous function $g_\tau(x_1(t-\tau_1(t)), ..., x_n(t-\tau_n(t)))$ satisfies the inequality

$$|g_\tau(x_1(t-\tau_1(t)), ..., x_n(t-\tau_n(t)))| \le \sum_{k=1}^n \varrho_k(x_k(t-\tau_k(t)))$$

with $\varrho_k(x_k(t))$ being known positive continuous functions, $i = 1, ..., n$.

Assumption 5. The unknown state time-varying state delays $\tau_i(t)$ satisfy the inequality

$$0 \le \tau_i(t) \le \tau_{max}, \ \dot{\tau}_i(t) \le \bar{\tau}_{max} < 1, \ i = 1, ..., n$$

with the known constants τ_{max} and $\bar{\tau}_{max}$.

Assumption 6. There exist known constants p_{0min} and p_{max}, such that $p_0 > p_{0min}$, and $p(r) \le p_{max}$ for all $r \in [0, R]$.

3. CONTROL DESIGN AND STABILITY ANALYSIS

From (4) and (5), we obtain

$$\dot{s} = f(x) + g_\tau(x_\tau) + b(x)p_0 v - b(x)d[v](t) + \Delta(x,t) + \nu \tag{6}$$

where $\nu = \sum_{j=1}^{n-1} \lambda_j e_{j+1} - y_d^{(n)}$.

Define the following integral Lyapunov function candidate, which was firstly proposed in (Ge *et al.*, 1999) to avoid control singularity:

$$V_s = \int_0^s \frac{\sigma}{b(\psi, \sigma + \nu_1)} d\sigma \tag{7}$$

where $\nu_1 = y_d^{(n-1)} - \sum_{j=1}^{n-1} \lambda_j e_j$, and $\psi = [x_1, ..., x_{n-1}]^T$. Then, V_s can be rewritten as the following form by using Second Mean Value Theorem for Integrals in (Zhang and Ge, 2007)

$$V_s = \frac{s^2}{2b(\psi, \lambda_s s + \nu_1)}, \quad \lambda_s \in (0,1)$$

Since $0 < b_0 < b(x)$, it is shown that V_s is positive definitive with respect to s. Differentiating V_s with respect to time t, we obtain

$$\dot{V}_s = \frac{s}{b(x)}\dot{s} + \int_0^s \sigma\Big[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \psi}\dot{\psi}\Big] d\sigma + \dot{\nu}_1 \int_0^s \sigma\Big[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \nu_1}\Big] d\sigma \tag{8}$$

Because $\partial b^{-1}(\psi, \sigma + \nu_1)/\partial \nu_1 = \partial b^{-1}(\psi, \sigma + \nu_1)/\partial \sigma$ and $\dot{\nu}_1 = -\nu$, it is shown that

$$\dot{\nu}_1 \int_0^s \sigma\Big[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \nu_1}\Big] d\sigma = -\frac{\nu s}{b(x)} + \int_0^s \frac{\nu}{b(\psi, \sigma + \nu_1)} d\sigma \tag{9}$$

Substituting (6) and (9) into (8), we have

$$\begin{aligned}\dot{V}_s &= \frac{s}{b(x)}\Big[f(x) + g_\tau(x_\tau) + b(x)p_0 v - b(x)d[v](t) + \Delta(x,t) \\ &\quad + \nu\Big] + \int_0^s \sigma\Big[\sum_{k=1}^{n-1} \frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial x_k} x_{k+1}\Big] d\sigma - \frac{\nu s}{b(x)} \\ &\quad + \int_0^s \frac{\nu}{b(\psi, \sigma + \nu_1)} d\sigma \\ &\le sQ(z) + s\Big[p_0 v - d[v](t)\Big] + \frac{(n + \phi^2(x))s^2}{2b^2(x)} \\ &\quad + \frac{1}{2}\sum_{i=1}^{n} \varrho_i^2(x_i(t - \tau_i(t))) + \frac{\rho^{*2}}{2}\end{aligned} \tag{10}$$

where

$$Q(z) = \frac{f(x)}{b(x)} + s\int_0^1 \Big[\theta \sum_{k=1}^{n-1} \frac{\partial b^{-1}(\psi, \theta s + \nu_1)}{\partial x_k} x_{k+1} + \frac{\nu}{b(\psi, \theta s + \nu_1)}\Big] d\theta$$

with $z = [x^T, s, \nu, \nu_1]^T \in R^{n+3}$.

To overcome the design difficulties from the unknown time-varying delays $\tau_1(t), ..., \tau_n(t)$, the following Lyapunov-Krasovskii functional can be considered

$$V_U(t) = \frac{1}{2(1 - \bar{\tau}_{max})} \sum_{i=1}^{n} \int_{t-\tau_i(t)}^{t} \varrho_i^2(x_i(\tau)) d\tau \tag{11}$$

Its time derivative is

$$\dot{V}_U(t) = \frac{1}{2(1 - \bar{\tau}_{max})} \sum_{i=1}^{n} \Big[\varrho_i^2(x_i(t)) - \varrho_i^2(x_i(t - \tau_i(t))) (1 - \dot{\tau}_i(t))\Big] \tag{12}$$

Combining (10) and (12), we obtain

$$\dot{V}_s + \dot{V}_U \le sh(z) + s\Big[p_0 v - d[v](t)\Big] + \frac{\rho^{*2}}{2} + \frac{1}{2(1 - \bar{\tau}_{max})}\Big[1 - \frac{s^2}{c_s^2}\Big] \sum_{i=1}^{n} \varrho_i^2(x_i) \tag{13}$$

where

$$h(z) = Q(z) + \frac{(n + \phi^2(x))s}{2b^2(x)} + \frac{s}{2(1 - \bar{\tau}_{max})c_s^2} \sum_{i=1}^{n} \varrho_i^2(x_i) \tag{14}$$

and c_s is a positive design constant that can be chosen arbitrarily small.

Remark 1. Due to the appearance of $\frac{1}{z}$ in $Q(z)$ or α^* in (Ge *et al.*, 2003), and the appearance of $\frac{1}{s_1}$ in $h_1(z_1)$ in (Zhang and Ge, 2007), practical robust controller was constructed. However, in this paper, $h(z)$ in (14) does not contain $\frac{1}{s}$ by introducing continuous function $\frac{s}{2(1-\bar{\tau}_{max})c_s^2} \sum_{i=1}^{n} \varrho_i^2(x_i)$. It is easy to know that if $|s| > c_s$, the final term of right hand side of (13) is less than zero; if $|s| \le c_s$, then it is bounded. Accordingly, (13) makes the control design and stability analysis possible while the controller cannot occur the singularity problem. This approach can effectively simplify the proof of Theorem 1 without discussing many cases like (Ge *et al.*, 2003) and (Zhang and Ge, 2007) as will be seen later.

If we use RBFNN in (Ge *et al.*, 2002), $\hat{W}^T S(z)$, as the approximation of the function $h(z)$ (14) on the compact set Ω, which will be defined later in Theorem 1, then we have

$$h(z) = \hat{W}^T S(z) - \tilde{W}^T S(z) + \mu_l \tag{15}$$

where the approximation error μ_l satisfies $|\mu_l| \le \mu_l^*$ with positive constant μ_l^*.

Consider the following control laws

$$v = -\frac{sgn(s)}{p_{0min}}[k_0(t)|s| + |\hat{W}^T S(z)|] + v_h \tag{16}$$

$$v_h(t) = -\frac{sgn(s)}{p_{0min}} \int_0^R \hat{p}(t,r)|F_r[v](t)| dr \tag{17}$$

$$k_0(t) = k_1 + k_2(t) + \frac{1}{2} \tag{18}$$

where k_1 is any positive constant and $k_2(t)$ can be chosen as

$$k_2(t) = \frac{k_3}{2(1-\bar{\tau}_{max})s^2}\sum_{i=1}^{n}\int_{t-\tau_{max}}^{t}\varrho_i^2(x_i(\tau))d\tau \quad (19)$$

with k_3 a positive constant specified by the designer.

The adaptation laws are chosen as

$$\dot{\hat{W}} = \Gamma[S(z)s - \sigma\hat{W}] \quad (20)$$

$$\frac{\partial}{\partial t}\hat{p}(t,r) = \begin{cases} 0, & if\ \hat{p}(t,r) \geq p_{max} \\ \eta|s||F_r[u](t)|, & if\ 0 \leq \hat{p}(t,r) < p_{max} \end{cases} \quad (21)$$

with $\Gamma > 0$, σ and η are strictly positive constants.

Theorem 1. Consider the closed-loop system consisting of the plant (4), the control laws (16) (17) and adaptation laws (20) (21). Under Assumptions 1-6, for bounded initial conditions, the overall closed-loop neural control system is semi-globally stable in the sense that all of the signals in the closed-loop system are bounded, the parameter estimates

$$\hat{W} \in \Omega_w = \Big\{\hat{W} | \|\tilde{W}\| \leq \sqrt{\frac{2\mu}{\lambda_{min}(\Gamma^{-1})}}\Big\}$$

and $\forall x(0) \in \Omega_0$ (as will be defined later in the proof), the state vector

$$x \in \Omega_c = \Big\{x| \ \ \|x - x_d\| \leq c_0(1+\|\Lambda\|)\|\omega(0)\| + [1 + \frac{(1+\|\Lambda\|)c_0}{\lambda}]\sqrt{2b_1\mu}, x_d \in \Omega_d\Big\} \subset \Omega$$

proof: The proof includes two steps. We shall first assume that $x \in \Omega$, $\forall t \geq 0$, on which NN approximation (15) is valid, and construct adaptive NN control over Ω. Then, we shall show that there exists nonempty initial set Ω_0 such that the state x indeed remains in the compact set Ω for all $t \geq 0$ if initial state $x(0)$ initiates from Ω_0.

Step 1: Suppose that $x \in \Omega$, $\forall t \geq 0$, then NN approximation (15) is valid. Consider the following Lyapunov function candidate

$$V(t) = V_s(t) + V_U(t) + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr$$

Differentiating $V(t)$ with respect to time t and using (13), (15), (16) and (20) lead to

$$\begin{aligned}\dot{V}(t) \leq & s[\hat{W}^TS(z) - \tilde{W}^TS(z) + \mu_l] + \frac{\rho^{*2}}{2} \\ & + s\Big[p_0\Big(-\frac{sgn(s)}{p_{0min}}[k_0(t)|s| + |\hat{W}^TS(z)|] + v_h\Big) \\ & - d[v](t)\Big] + \frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{s^2}{c_s^2}\Big]\sum_{i=1}^{n}\varrho_i^2(x_i) \\ & + \tilde{W}^T[S(z)s - \sigma\hat{W}] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \\ \leq & -k_1s^2 - k_3V_U + \frac{\mu_l^{*2} + \rho^{*2}}{2} - \sigma\tilde{W}^T\hat{W} \\ & + \frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{s^2}{c_s^2}\Big]\sum_{i=1}^{n}\varrho_i^2(x_i) + s\Big[p_0v_h - \\ & \int_0^R p(r)F_r[v](t)dr\Big] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \quad (22)\end{aligned}$$

For the fourth term in (22), by completion of squares, we have

$$-\sigma\tilde{W}^T\hat{W} \leq -\frac{\sigma}{2}\|\tilde{W}\|^2 + \frac{\sigma}{2}\|W^*\|^2 \quad (23)$$

For the fifth term in (22), if $|s| > c_s$, then it is less than zero; if $|s| \leq c_s$, then it is bounded. Therefore, we have

$$\frac{1}{2(1-\bar{\tau}_{max})}\Big[1 - \frac{s^2}{c_s^2}\Big]\sum_{i=1}^{n}\varrho_i^2(x_i) \leq \frac{1}{2(1-\bar{\tau}_{max})}\varrho_{max} \quad (24)$$

where $\varrho_{max} = \max_{x\in\Omega_{c_s}}\sum_{i=1}^{n}\varrho_i^2(x_i)$.

For the last two terms in (22), using v_h in (17), we have

$$\begin{aligned}& s\Big[p_0v_h - \int_0^R p(r)F_r[v](t)dr\Big] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \\ & \leq -|s|\int_0^R \tilde{p}(t,r)|F_r[v](t)|dr + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \quad (25)\end{aligned}$$

Case(i): When $r \in R_{max} = \{r : \hat{p}(t,r) \geq p_{max}\}$, and $R_{max} \subset [0,R]$, according to (21), we have

$$\tilde{p}(t,r) \geq 0 \quad (26)$$

$$Proj(\hat{p}(t,r), \eta|s||F_r[u](t)|) = 0 \quad (27)$$

Substituting (26) and (27) into (25), we obtain

$$s\Big[p_0v_h - \int_0^R p(r)F_r[v](t)dr\Big] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \leq 0$$

Case (ii): When $r \in R_{max}^c$, which is the complement set of R_{max} in $[0,R]$, i.e., $0 \leq \hat{p}(t,r) < p_{max}$, according to (21), we have

$$Proj(\hat{p}(t,r), \eta|s||F_r[u](t)|) = \eta|s||F_r[u](t)| \quad (28)$$

Substituting (28) into (25), we obtain

$$\begin{aligned}& s\Big[p_0v_h - \int_0^R p(r)F_r[v](t)dr\Big] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \\ & \leq -|s|\int_0^R \tilde{p}(t,r)|F_r[v](t)|dr + |s|\int_0^R \tilde{p}(t,r)|F_r[u](t)|dr = 0\end{aligned}$$

Combining Case (i) with Case (ii), we have

$$\begin{aligned}& s\Big[p_0v_h - \int_0^R p(r)F_r[v](t)dr\Big] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\hat{p}(t,r)dr \\ & \leq 0 \quad (29)\end{aligned}$$

Substituting (23), (24) and (29) into (22), we have

$$\dot{V}(t) \leq -k_1 s^2 - k_3 V_U - \frac{\sigma}{2}\|\tilde{W}\|^2 + \frac{\sigma}{2}\|W^*\|^2 + \frac{\mu_0^2 + \rho^{*2}}{2} + \frac{1}{2(1-\bar{\tau}_{max})}\varrho_{max} \quad (30)$$

From Assumption 6 and the adaptation law (21), we know the boundedness of $|\tilde{p}(t,r)| \leq p_{max}$, which leads to the boundedness of $\frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr$. Adding and subtracting the term $\frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr$ on the right hand side of (30), we have

$$\begin{aligned}\dot{V}(t) &\leq -k_1 s^2 - k_3 V_U - \frac{\sigma}{2}\|\tilde{W}\|^2 - \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \\ &\quad + \frac{\sigma}{2}\|W^*\|^2 + \frac{\mu_0^2 + \rho^{*2}}{2} + \frac{1}{2(1-\bar{\tau}_{max})}\varrho_{max} \\ &\quad + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \\ &\leq -\lambda_{10} V(t) + \mu_{10} \end{aligned} \quad (31)$$

where constants λ_{10} and μ_{10} are defined as follows

$$\begin{aligned}\lambda_{10} &= \min\{2k_1 b_0, k_3, \frac{\sigma}{\lambda_{max}(\Gamma^{-1})}, 1\} \\ \mu_{10} &= \frac{\sigma}{2}\|W^*\|^2 + \frac{\mu_0^2 + \rho^{*2}}{2} + \frac{1}{2(1-\bar{\tau}_{max})}\varrho_{max} \\ &\quad + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \end{aligned} \quad (32)$$

Multiplying (31) by $e^{\lambda_{10} t}$ and integrating over $[0,t]$, we have

$$0 \leq V(t) \leq \frac{\mu_{10}}{\lambda_{10}} + [V(0) - \frac{\mu_{10}}{\lambda_{10}}]e^{-\lambda_{10} t} \leq \mu \quad (33)$$

where $\mu = \frac{\mu_{10}}{\lambda_{10}} + V(0)$. Therefore, $\|\tilde{W}\| \leq \sqrt{2\mu/\lambda_{min}(\Gamma^{-1})}$ and $|s| \leq \sqrt{2b_1 V(t)} \leq \sqrt{2b_1\mu}$.

Define $\omega = [e_1, ..., e_{n-1}]^T \in R^{n-1}$. From (5), we know that (i) there is a state space representation for mapping $s = [\Lambda^T \;\; 1]e$, i.e., $\dot{\omega} = A_s\omega + b_s s$ with $\Lambda = [\lambda_1, ..., \lambda_{n-1}]^T$, $b_s = [0, ..., 0, 1]^T$, A_s being a stable matrix; (ii) there is a positive constant c_{10} such that $\|e^{A_s t}\| \leq c_{10}e^{-\lambda t}$, and (iii) the solution of ω is

$$\omega(t) = e^{A_s t}\omega(0) + \int_0^t e^{A_s(t-\tau)} b_s s(\tau)d\tau$$

Accordingly, it follows that

$$\begin{aligned}\|\omega(t)\| &\leq c_{10}\|\omega(0)\|e^{-\lambda t} + c_{10}\int_0^t e^{-\lambda(t-\tau)}|s(\tau)|d\tau \\ &\leq c_{10}\|\omega(0)\| + \frac{c_{10}\sqrt{2b_1\mu}}{\lambda}\end{aligned} \quad (34)$$

Noting $s = \Lambda^T\omega + e_n$ and $e = [\omega^T, e_n]^T$, we have

$$\|e\| \leq \|\omega\| + |e_n| \leq (1 + \|\Lambda\|)\|\omega\| + |s|$$

Substituting (34) into the above inequality leads to

$$\|e\| \leq c_{10}(1 + \|\Lambda\|)\|\omega(0)\| + [1 + \frac{(1+\|\Lambda\|)c_{10}}{\lambda}]\sqrt{2b_1\mu}$$

Since c_{10}, $\|\Lambda\|$ and λ are positive constants, and $\omega(0)$ and $s(0)$ depend on $x(0) - x_d(0)$, we conclude that there exists a positive constant $R(\tau_{max}, c, x(0), \tilde{W}(0))$ depending on $\tau_{max}, c, x(0)$ and $\tilde{W}(0)$ such that

$$\|e\| \leq R(\tau_{max}, c, x(0), \tilde{W}(0)), \quad \forall t \geq 0$$

where $c = \frac{\mu_{10}}{\lambda_{10}}$, and $R(\tau_{max}, c, x(0), \tilde{W}(0)) = c_{10}(1 + \|\Lambda\|)\|\omega(0)\| + [1 + \frac{(1+\|\Lambda\|)c_{10}}{\lambda}]\sqrt{2b_1\mu}$.

Noting $x = e + x_d$ and Assumption 2, we obtain

$$\begin{aligned}\|x\| &\leq \|e\| + \|x_d\| \leq c_{10}(1 + \|\Lambda\|)\|\omega(0)\| \\ &\quad + [1 + \frac{(1+\|\Lambda\|)c_{10}}{\lambda}]\sqrt{2b_1\mu} + \|x_d\| \in L_\infty\end{aligned}$$

Therefore, we can conclude that all the closed-loop signals are semi-globally uniformly ultimately bounded for bounded initial conditions.

Step 2: In the following, we shall find the conditions such that $x \in \Omega$, $\forall t \geq 0$. Firstly, define a set

$$\Omega_0 = \Big\{x(0) | \{x | \|x - x_d\| < R(0, 0, x(0), 0, 0)\} \subset \Omega, \; x_d \in \Omega_d\Big\}$$

which is not empty. It is easy to see that for all $x(0) \in \Omega_0$ and $x_d \in \Omega_d$, we have $x \in \Omega$, $\forall t \geq 0$. Then, for the system with $x(0) \in \Omega_0$, bounded $\hat{W}(0)$ and $x_d \in \Omega_d$, the following constants c^* and τ^*_{max} can be determined by

$$\begin{aligned}c^* &= \sup_{c \in R^+}\Big\{c | \{x | \|x - x_d\| < R(0, c, x(0), \tilde{W}(0))\} \subset \Omega, \; x_d \in \Omega_d\Big\} \\ \tau^*_{max} &= \sup_{\tau_{max} \in R^+}\Big\{\tau_{max} | \{x | \|x - x_d\| \leq R(\tau_{max}, c, x(0), \tilde{W}(0)), c \leq c^*\} \subset \Omega, x_d \in \Omega_d\Big\}\end{aligned}$$

From (32), we know that if the adaptive control parameter σ is chosen to be sufficiently small and k_1, k_3, and $\lambda_{min}(\Gamma)$ are taken to be sufficiently large, then the constant $c = \frac{\mu_{10}}{\lambda_{10}}$ can be made arbitrary small. Therefore, for the initial condition $x(0) \in \Omega_0$, bounded $\hat{W}$ and $x_d \in \Omega_d$, if the adaptive control parameters are appropriately chosen such that $\frac{\mu_{10}}{\lambda_{10}} \leq c^*$, and the upper bound τ_{max} of time-varying delay $\tau_1(t), ..., \tau_n(t)$ satisfies $\tau_{max} \leq \tau^*_{max}$, then system state x indeed stays in Ω for all time. This completes the proof. ■

4. SIMULATION STUDIES

To demonstrate the effectiveness of the proposed approach, we consider the following first-order nonlinear system:

$$\begin{aligned}\dot{x} &= \frac{1 - e^{-x}}{1 + e^{-x}} + 0.1x(t - \tau(t)) + u + 0.2\sin(x) \\ y &= x\end{aligned}$$

where $u = p_0 v - \int_0^R p(r)F_r[v](t)dr$, $\tau(t) = 1 - 0.5\cos(t)$, $\tau_{max} = 2$, $\bar{\tau}_{max} = 0.6$; $p(r) = 0.35e^{-0.003(r-1)^2}$ for

$r \in [0, 100]$. Our objective is to design v such that the output y will follow the desired trajectory $y_d = \sin(2t) + 0.1cos(6.7t)$. For RBFNN, we consider the centers for $S(z)$ to be evenly spaced in a regular lattice in R^4. Employing four nodes for each input dimension, we end up with $4^4 = 256$ nodes for the network $\hat{W}^T S(z)$. The following initial conditions and controller design parameters are adopted in the simulation: $p_{\max} = 0.35$, $p_{0\min} = 0.35$, $\hat{p}(0, r) = 0$, $x(0) = 0.5$, $v(0) = 0$, $\Gamma = diag\{1.0\}$, $\sigma = 0.1$ and $\hat{W}(0) = 0$.

The simulation results are shown in Figs 1 and 2. We can observe the good tracking performance and the boundedness of the control signal.

5. CONCLUSION

Adaptive variable structure neural control has been proposed for a class of uncertain SISO nonlinear systems with unknown state time-varying delays and hysteresis nonlinearities. The uncertainties from unknown time-varying delays have been compensated for through the use of appropriate Lyapunov-Krasovskii functionals. The controller has been made to be free from singularity problem by utilizing integral Lyapunov function. Based on the principle of sliding mode control, the developed controller can guarantee that all signals involved are semi-globally uniformly ultimately bounded. Simulation results have verified the effectiveness of the proposed approach.

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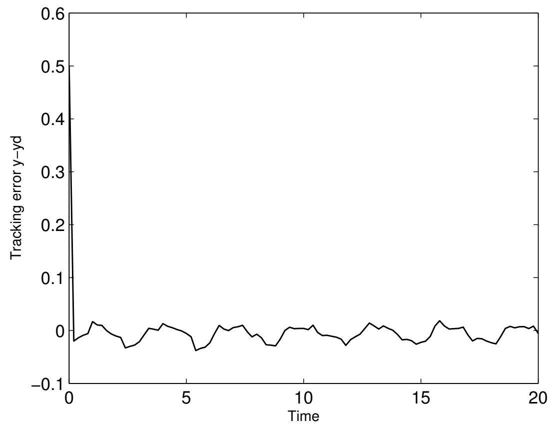

Fig. 1. Output tracking error $y - y_d$

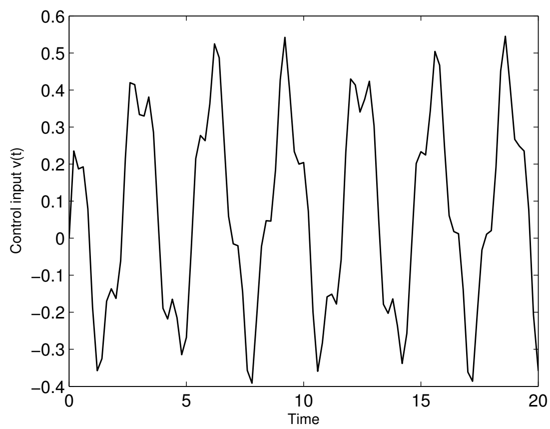

Fig. 2. Control input $v(t)$