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Extended Prediction Error Approach for MPC Performance Monitoring and Industrial Applications ⋆

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Abstract: Performance monitoring and diagnosis of model predictive control systems (MPC) has been a great interest for both academia and industry. In recent years some novel approaches for multivariate control performance monitoring have been developed without the requirement of process models or interactor matrices. Among them the prediction error approach has shown to be a promising one, but it has certain limitations in applications. This paper further develops the prediction error approach for performance monitoring of model predictive control systems, and demonstrates its applications in two industrial MPC performance monitoring and diagnosis problems.

Keywords: Multivariable systems; Performance evaluation; Performance monitoring; Prediction error methods.

1. INTRODUCTION

Since notable work of (Harris, 1989), research on the control performance assessment (CPA) or control performance monitoring (CPM) has achieved a great progress and remains to be a very active area. There is a great demand from industry for this research to produce practical solutions, particularly for MPC monitoring. "It is an important asset-management technology to maintain highly efficient operation performance of automation systems in production plants." (Jelali, 2006). Over the last decade, CPA/CPM has considerable achievements in industrial applications especially with the univariate CPA or CPM. Many algorithms including commercial software are available. There are several interesting reviews addressing related research achievements in different stages (Harris et al., 1999; Huang et al., 1999; Jelali, 2006; Qin, 2007).

Although many publications are available, multivariable CPA or CPM still has many stumbling blocks in practical applications. Recently some progress has been made towards this direction (Jelali, 2006; Huang et al., 2006). In particular, performance assessment of model predictive control (MPC) has been the current interest since MPC is the most effective and widely used advanced multivariate control strategies in modern industries. The existence of the constraints and economic optimization makes the MPC controller with certain specialities, so that the existing CPA/CPM may not be directly applicable for it (Xu et al., 2007).

For multivariable CPA/CPM to be practical, it must reduce *a priori* knowledge requirement. Traditional approaches for the multivariable CPA with minimum variance control as benchmark need to estimate the interactor matrices, which is equivalent to knowing the process model (Huang et al., 1999) or at least the first few Markov parameter matrices. Most recently, some new methods have been developed to address the multivariable CPA problems with only the input/output data (Jelali, 2006; Huang et al., 2006). Obviously, simpler methods require less *a prior* process knowledge but also provide less diagnostic information. There is a tradeoff between simplicity and diagnostic capability of the CPA/CPM methods. Thus, how to extract the most information out of the data is of considerable interest.

What simple index may be considered as a measure or one of MPC performance measures? Consider that, if a closed-loop output is highly predictable, one should be able to do better, i.e. to compensate the predictable content by a well designed controller. This is the principle of predictive control. Should a better controller be implemented, the closed-loop output would have been less predictable. Therefore, high predictability of a closed-loop output implies high potential to improve its performance by controller re-tuning and/or re-design, or in other words, the existing controller may not have been satisfactory in terms of exploring its potential. Motivated by the prediction-error approach of (Huang et al., 2006), this paper further develops closed-loop prediction-error measures that are more relevant to practical problems. Furthermore, applications of prediction-error measures for two industrial model predictive control systems are reported in this paper. The remainder of this paper is organized as follows: Section 2

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revisits the concept of prediction-error and closed-loop potentials; Section 3 further develops the prediction-error approach; Section 4 provides two industrial case studies and illustrates the utility of the new performance measures; finally the conclusion is drawn in section 5.

2. FORMULATION OF CLOSED-LOOP POTENTIAL FOR MULTIVARIATE CPM/CPA

As argued in the last section, if a closed-loop output is highly predictable, then a well designed predictive controller should be able to compensate the predictable content. Therefore, a higher predictability of a closed-loop output implies a higher potential to improve its performance by controller re-tuning or re-design; that is to say, the existing controller may not have been satisfactory in terms of performance. So the multi-step prediction error for multivariate processes potentially provides useful information for performance assessment. In this section, we shall revisit the concepts of multi-step prediction error and closed-loop potentials as defined in (Huang et al., 2006).

For multivariable process, the closed-loop output with zero setpoint driven by white noise can be described by a time series model:

$$Y_t = G_{cl} a_t \tag{1}$$

where G_{cl} is closed-loop time series model and a_t is white noise with mean zero and covariance Σ_a. Transfer the above model to a moving average (MA) form:

$$Y_t = \sum_{k=0}^{\infty} F_k a(t-k) = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} + F_i a_{(t-i)} + \cdots \tag{2}$$

Clearly, the above time series model can be estimated without any a priori knowledge about the process.

With the MA model, one can obtain the optimal ith step prediction:

$$Y_{t|t-i} = F_i a_{(t-i)} + F_{i+1} a_{(t-i-1)} + \cdots \tag{3}$$

and the prediction error:

$$e_{t|t-i} = Y_t - Y_{t|t-i} = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} \tag{4}$$

The covariance of the prediction error can be calculated as:

$$cov(e_{t|t-i}) = F_0 \Sigma_a F_0^T + F_1 \Sigma_a F_1^T + \cdots + F_{i-1} \Sigma_a F_{i-1}^T \tag{5}$$

Define its scalar measure:

$$s_i = tr(cov(e_{t|t-i})) = tr(F_0 \Sigma_a F_0^T + \cdots + F_{i-1} \Sigma_a F_{i-1}^T) \tag{6}$$

s_i is monotonically increasing with i, as $i \to \infty$, $e_{t|t-i} \to Y_t$, and $s_\infty = tr(cov(Y_t))$. If we plot s_i versus i, the plot reflects how the prediction error increases with the prediction horizon.

Huang et al. (2006) defined a closed-loop potential as

$$p_i = \frac{s_\infty - s_i}{s_\infty} \tag{7}$$

The closed-loop potential can be interpreted as: "If a deadbeat control action can be applied from time i, then the sum of squared error (SSE) can be reduced by $100 \times p_i$ percent. From stochastic view point, if i is greater than the interactor order d, it is possible that the variance of the multivariate output can be reduced by $100 \times p_i$ percent of the current variance. Since the order of the actual interactor matrix may not be known, one can check the trajectory of the closed-loop potential versus a range of possible time lag d". As s_i is monotonically increasing with i, p_i is monotonically decreasing. When $i \to 0$, $s_0 = tr(cov(Y_t - Y_{t|t})) = 0$, $p_0 = 1$. Therefore, the index p_i starts from 1 at $i = 0$ and monotonically decreases to 0 at $i \to \infty$. Larger the closed-loop potential is, more potential the control performance can be improved.

3. THE EXTENDED CLOSED-LOOP POTENTIAL

3.1 Closed-loop potential based on mean square error

The above closed-loop potential has the following advantages:

1. It has a simple expression and easy to implement.
2. The calculation of the index needs only the output data. It does not require any a priori knowledge about the interactor matrix or the process model.
3. By plotting the trajectory of the closed-loop potential versus the time lag d, we can get the potential plot, which is useful in performance assessment of model predictive control whose control strategy is based on predictions.

From the potential plot we can draw the conclusion whether or how much the present closed-loop has potential to improve. Furthermore, with the plot, we can compare performance of same controller with different tuning parameters. However, the closed-loop potential is variance based performance measure, which may not be general enough to consider, for example, the tracking performance such as offset. To generalize it, we need to analyze the closed-loop potential as it is originally defined. Substituting Eqn. (6) into (7) yields

$$p_i = \frac{tr(F_d \Sigma_a F_d^T + F_{d+1} \Sigma_a F_{d+1}^T + \ldots)}{tr(F_0 \Sigma_a F_0^T + F_1 \Sigma_a F_1^T + \ldots)}$$

If the process has a simple interactor matrix dI (Huang et al., 1999), then the numerator represents the variance that can be eliminated through minimum variance control, denoted here as $tr(\Sigma_R)$. The denominator represents the actual variance of the output, denoted as $tr(\Sigma_Y)$. Thus

$$p_i = \frac{tr(\Sigma_R)}{tr(\Sigma_Y)}$$

Namely p_i is the percentage of output variance that could be reduced by minimum variance control for a process with a simple interactor matrix of order d. Obviously, most of MIMO processes do not have a simple interactor matrix. Therefore, instead of calculating a single closed-loop potential, one should calculate a set of closed-loop potentials over a horizon of time lags. One can then determine the control performance according to the trajectory of the closed-loop potentials.

With the above interpretation, we are ready to extend it to a more general solution, namely extend it from variance measure to mean square error measure. In this way, the tracking performance such as offset will be considered automatically in the closed-loop potential calculation. The scalar measure of mean square error of output MSE_Y is defined as $tr(MSE_Y) = tr(E[y_t - y_t^{sp}][y_t - y_t^{sp}]^T)$, which can be written as $tr(\delta\delta^T) + s_\infty$ where δ stands for the offset.

With the consideration of mean square error, the new closed-loop potential p_{mi}, analogous to the definition of p_i, can be defined as

$$p_{mi} = \frac{tr(MSE_R)}{tr(MSE_Y)} \tag{8}$$

where MSE_R represents the mean square error that can be eliminated through optimal control (minimum variance control plus offset compensation), which yields

$$MSE_R = \delta\delta^T + \Sigma_R \tag{9}$$

and MSE_Y can be written as

$$MSE_Y = \delta\delta^T + \Sigma_Y \tag{10}$$

Substituting Eqns (9) and (10) into Eqn.(8) yields

$$\begin{aligned} p_{mi} &= \frac{tr(\delta\delta^T + \Sigma_R)}{tr(\delta\delta^T + \Sigma_Y)} \\ &= \frac{tr(\delta\delta^T) + s_\infty - s_i}{tr(\delta\delta^T) + s_\infty} \end{aligned} \tag{11}$$

With $s_0 = tr(E(e^2_{t|t})) = 0$,

$$p_{m0} = \frac{tr(\delta\delta^T) + s_\infty}{tr(\delta\delta^T) + s_\infty} = 1$$

From (11), we can see that p_{mi} is monotonically decreasing starting from 1 as well, but as $i \to \infty$, $p_{mi} \to \frac{tr(\delta\delta^T)}{tr(\delta\delta^T)+s_\infty}$ which is not necessarily to be zero, the value of which depends on ratio of offset to the mean square error. In this way, the closed-loop potential will not be zero when there is offset even if the time lag goes to infinity. This is more reasonable since if the time lag goes to infinity, there is no potential to predict the dynamic variability but the offset is always practicable and controllable. Therefore, as time lag goes to infinity, the potential depends on the offset only.

Furthermore, when there exists offset, $tr(\delta\delta^T)$ is positive,

$$\begin{aligned} p_{mi} - p_i &= \frac{tr(\delta\delta^T) + s_\infty - s_i}{tr(\delta\delta^T) + s_\infty} - \frac{s_\infty - s_i}{s_\infty} \\ &= \frac{s_i tr(\delta\delta^T)}{s_\infty(tr(\delta\delta^T) + s_\infty)} > 0 \end{aligned} \tag{12}$$

The result above shows that the closed-loop with offset has larger potential than the one without offset as expected. Using an alternative expression:

$$p_{mi} - p_i = \frac{s_i tr(\delta\delta^T)}{s_\infty(tr(\delta\delta^T) + s_\infty)} = \frac{s_i}{s_\infty} \times \frac{1}{1 + \frac{s_\infty}{tr(\delta\delta^T)}} \tag{13}$$

We can draw the conclusions:

$$\begin{cases} \lim\limits_{tr(\delta\delta^T)\to 0} (p_{mi} - p_i) = 0 \\ \lim\limits_{tr(\delta\delta^T)\to \infty} (p_{mi} - p_i) = \frac{s_i}{s_\infty} \end{cases} \tag{14}$$

For comparison, in the sequel, we call the new developed closed-loop potential as the extended closed-loop potential (ECP), while the closed-loop potential defined in Huang et al. (2006) as the original closed-loop potential (OCP). When the offset is close to zero, the ECP is close to OCP. When the offset is much larger than s_∞, the ECP will approach to the constant 1, which implies that there is always maximum potential to improve for the closed-loop no matter what is the time delay.

The procedures of implementing the ECP are summarized as follows:

(1) Calculate the offset
(2) Estimate a time-series model using closed-loop data
(3) Transfer the time series model to a moving average form as in (2)
(4) Calculate the optimal ith prediction and the prediction error respectively according to (3) and (4)
(5) Calculate the covariance of the prediction error as in (5)
(6) Calculate the ith ECP respectively according to (11).
(7) Plot the potential trajectory and analyze the performance of the closed-loop.

From the procedures discussed above, we can see that the proposed extension has all features that the previous one has, and moreover, it can evaluate the tracking performance of loops with offset through a natural extension.

3.2 The individual closed-loop potential

In the previous algorithm, to calculate potential of individual variable, the trace operator $tr[\cdot]$ is replaced by diagonalization operator $diag[\cdot]$ (Huang et al., 2006):

$$pind_i \triangleq \frac{\tilde{s}_{\infty i} - \tilde{s}_i}{\tilde{s}_{\infty i}} \tag{15}$$

where $\tilde{s}_i$, $\tilde{s}_{\infty i}$ are defined as:

$$\begin{aligned} \tilde{s}_i &\triangleq [diag(F_0\Sigma_a F_0^T + F_1\Sigma_a F_1^T + \cdots + F_{i-1}\Sigma_a F_{i-1}^T)](i) \\ \tilde{s}_{\infty i} &= [diag(F_0\Sigma_a F_0^T + F_1\Sigma_a F_1^T + \cdots)](i) \end{aligned} \tag{16}$$

where, following Matlab notation, i stands for the ith element of the vector. Such individual potential is convenient for assessing potential of each controlled variable in a multivariate system. However, the contribution of each variable to the potential of total system is not explicit. Namely, sum of individual closed-loop potentials is not the same as the (overall) multivariate closed-loop potential. In view of this problem, a new individual potential is defined as:

$$pmind_i \triangleq \frac{\tilde{s}_{\infty i} - \tilde{s}_i}{s_\infty} \tag{17}$$

where the $\tilde{s}_i$, $\tilde{s}_{\infty i}$ have been defined in (16).

At each i, making the summation over each individual potential, the result is the multivariate potential index defined in the last section:

$$\sum pmind_i = \frac{\sum(\tilde{s}_{\infty i} - \tilde{s}_i)}{s_\infty} = \frac{s_\infty - s_i}{s_\infty} = p_i \tag{18}$$

When there is offset, the new individual potential index is:

$$p\tilde{m}ind_i \triangleq \frac{\widetilde{\Gamma}_i + \tilde{s}_{\infty i} - \tilde{s}_i}{\delta\delta^T + s_\infty} \tag{19}$$

where $\widetilde{\Gamma}_i = [diag(\delta\delta^T)](i)$. The summation is the same as the multivariate (overall) potential (ECP) as shown below:

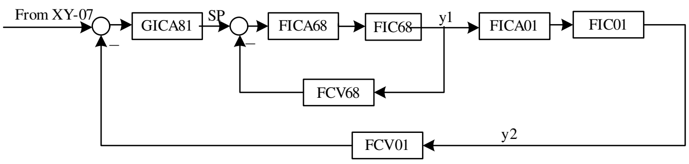

Fig. 1. Block diagram of a cascade control loop.

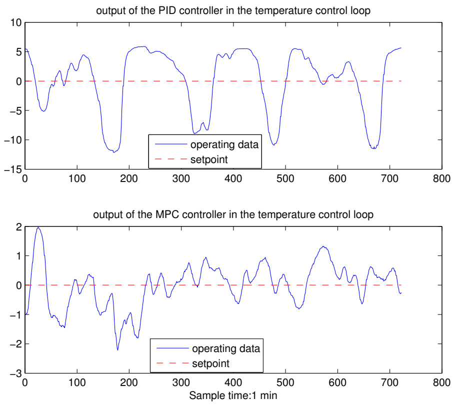

Fig. 2. Output of the two controllers and their setpoints.

$$\sum \tilde{pmind}_i = \frac{\widetilde{\Gamma}_i + \tilde{s}_{\infty i} - \tilde{s}_i}{s_\infty} = \frac{tr(\delta\delta^T) + s_\infty - s_i}{tr(\delta\delta^T) + s_\infty} = p_{mi} \qquad (20)$$

The new individual potential can reflect contribution of each variable. It provides us with a guideline on how to improve the performance of closed-loop via balancing the tuning of each variable.

4. INDUSTRIAL APPLICATIONS

In this section both the previous and the proposed closed-loop potential measures will be applied to evaluate the performance of two industrial control systems, one with offset and the other without.

4.1 Case study with offset

Process description Fig. 1 shows a block diagram of a cascade temperature control loop in a distillation column. To control the components in the column top and the column bottom, the column's temperature gradient (the setpoint of the loop calculated from XY-07) must be maintained appropriately. The basic method to control the temperature gradient is to regulate the reflux flow. Consider the main controller GICA81 (temperature controller) of the loop in this study. Two control strategies: PID and MPC have been used for the process. The output of GICA81 is the setpoint of the inner controller FICA68 (flow controller). The setpoint is calculated by a formula with the parameter of temperature measured in the column. The closed-loop output under the two controllers are shown in the top panel and bottom panel of Fig. 2, respectively. Performance between these two control strategies is compared and analyzed using the OCP and ECP, respectively, elaborated next.

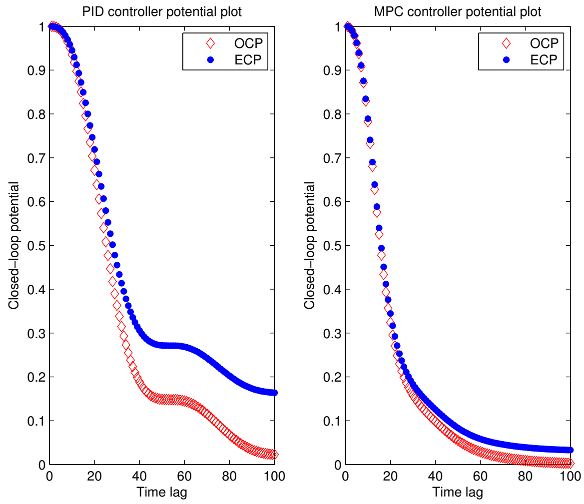

Fig. 3. OCP and ECP plot of the two controllers.

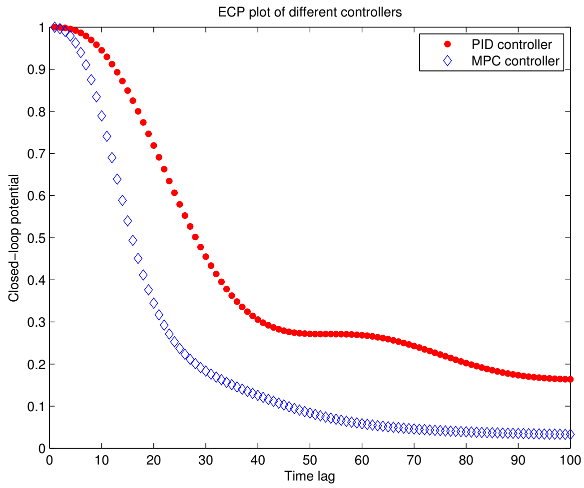

Fig. 4. ECP trajectory of the two controllers.

Analysis Following the steps described in Section 3, we can calculate the potential of each controller with and without considering offset, namely using ECP and OCP, respectively. The results are shown in Fig. 3.

From Fig. 3, we can see that there exist potentials for both controllers. Within each sub-figure, one can see the potential with consideration of offset (ECP) is always higher than the one without considering offset (OCP). The ECP never goes to zero if there is offset. The smaller the difference between the two potentials within each sub-figure is, the less the offset is. The MPC controller has less offset than the PID one.

Fig. 4 shows comparison of ECPs between the two controllers, PID and MPC. There exists more potential for the PID controller to improve, which implies that the performance of the MPC controller is better. Fig. 5 shows the potential plots of two controllers without considering the offset (namely OCP). Less difference of the potentials between the two controllers, particularly when time lag is large, can be seen from Fig. 5, indicating limitation

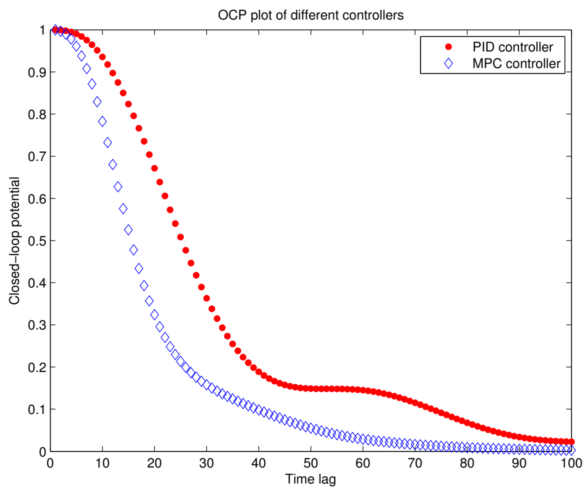

Fig. 5. OCP trajectory of the two controllers.

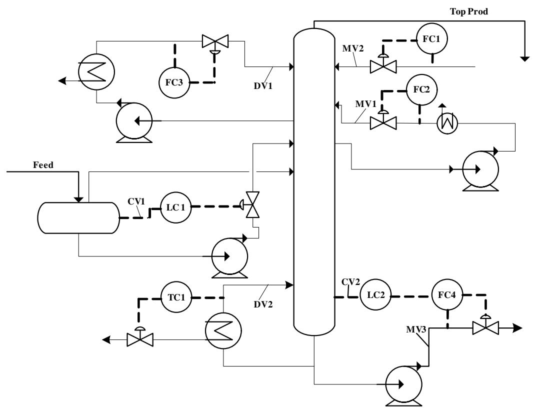

Fig. 6. Schematic diagram of the industrial process.

of closed-loop potential without considering the offset in measuring control performance.

4.2 Case study without offset

Process description Another application is the control performance assessment and diagnosis of the Residual Fluid Catalytic Cracking Unit (RFCCU) MPC controller in a petrochemical complex, where multiloop PID controller and MPC controller are applied respectively. Fig. 6 is a simplified process flow chart. With the absorbent of stable gasoline, the absorption & desorption column absorbs the fractions of C_3 and C_4 in the unstripped gas. The whole column is divided into absorption part on top and desorption part on bottom. The unstripped feed gas goes into the middle column and stable gasoline is compressed in from the top column. Two kinds of gas contact in the reverse direction and the fractions of C_3 and C_4 in unstripped gas are absorbed by stable gasoline. Besides the fractions of C_3 and C_4, gasoline in the desorption column contains the fraction of C_2. After contacting with the high temperature steam from the bottom column, C_2 in gasoline is desorbed. Finally, product flowing out of the top column is lean gas without C_3 and fractions have more C than C_3.

Table 1. List of process variables and their corresponding tag names.

CV1	CV2	MV1	MV2	MV3	DV1	DV2
L01	L02	F04	F10	F06	F08	T05

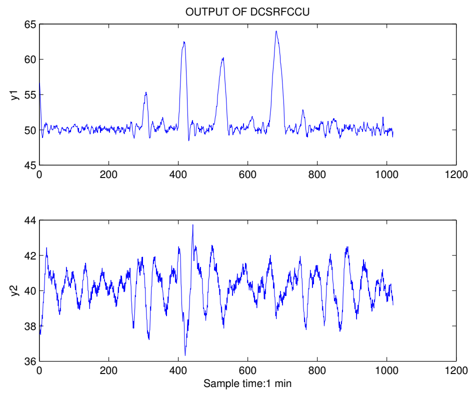

Fig. 7. Output data set under PID controller.

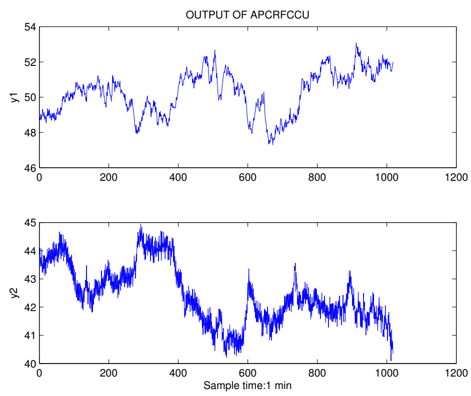

Fig. 8. Output data set under MPC controller.

The control system consists of three manipulated variables (MVs), two controlled variables (CVs) and two disturbance variables (DVs). A description of process variables and the corresponding tag names is shown in Table 1, where CV1 is the level of the feed's buffer tank and CV2 is the level of the bottom column.

For the PID controller, the configuration was setup in the distributed control system (DCS). Data sets collected under PID and MPC control are shown in Fig. 7 and Fig. 8 respectively.

Performance assessment Following the procedure of section 3, the trajectories of the potentials of the closed-loop systems under two controllers can be calculated. In this example, all CVs are constraint CV, and the offset is not a concern as long as all CVs are within the constraints. Thus, the closed-loop potential does not need to consider the offset, namely OCP is sufficient for the application. However, we will demonstrate the utility of the new individual closed-loop potentials defined in Eqn. (17). Fig. 9 shows that there exist potentials for both controllers, especially when the time lag is small. Furthermore, the MPC controller appears to have more potential to improve,

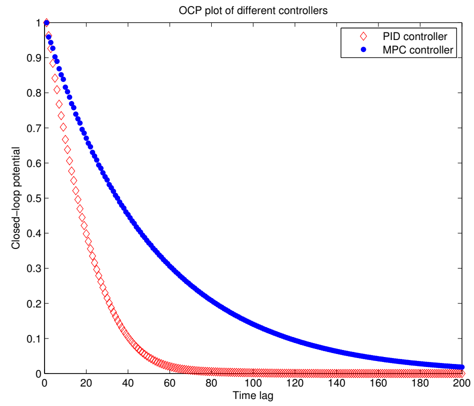

Fig. 9. OCP trajectory under different controllers.

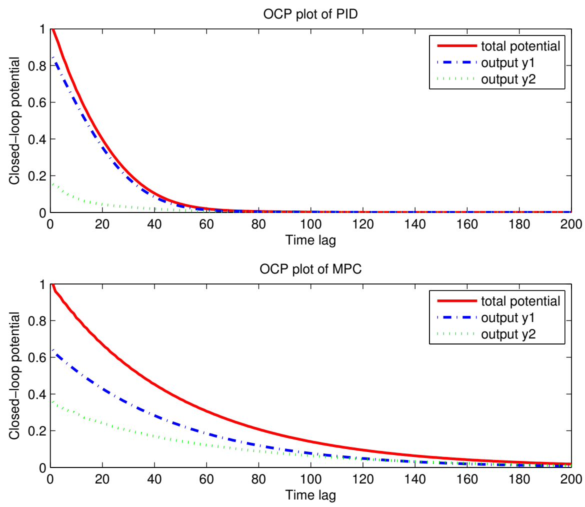

Fig. 10. Individual potential trajectory of different controllers.

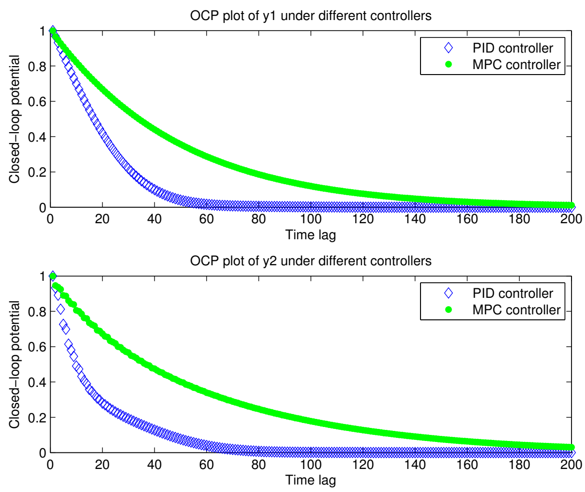

Fig. 11. Originally defined individual potential trajectory of different controllers.

which implies that the MPC controller may not be well tuned or not as good as the PID. For further analysis of the reason, we calculate the new individual potential for each controller shown in Fig. 10. For the PID controller, the output $y1$ is the key contributor to the closed-loop potential of the whole system (note a few spikes in $y1$ in Fig. 7 that can be an important contributor to the potential), while the output $y2$ has little potential, indicating a good tuning for it. For the MPC controller, although the output $y1$ has a larger contribution than that of $y2$, both of them have large potential, indicating that the controller can be re-tuned for both variables. Note that new defined individual potentials add up to the multivariate potential. Thus they can be used to assess the contributions of each individual variable on the overall potential. Although PID in this application appears better than MPC, i.e. less predictable, the existence of spikes in $y1$ worths further investigation.

To compare potential of individual CVs among different controllers, however, we need the individual potential plots originally defined by Huang et al. (2006). The original individual potential trajectories are plotted in Fig. 11. Obviously, both CVs under PID control have less potential to improve than the ones under MPC control, which reflects that the PID controller indeed has a better performance in this application (without considering offset).

5. CONCLUSION

Closed-loop potentials are promising measures of MPC performance. However, they have certain limitations as they are originally defined. In this paper, extensions of previous work are attempted and new closed-loop potentials are defined. The proposed performance measures have all the advantages of the previous ones in addition to new features that are lack in the previous results. Regardless of the dimension of plant, the closed-loop potential is a scalar, which facilitates the implementation, visualization, and interpretation. This paper is also the first report in actual industrial application of the closed-loop potential as a measure of MPC performance.

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