Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Sliding Mode Control and Flatness-Based Concept for Real-Time Ramp Metering

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Abstract: The aim of this paper is to present an application of a sliding mode control (SMC) and flatness-based concept to real-time ramp metering. Such application is a novel attempt in the field of traffic control. Differentially flat concept provides simple algorithms to generate optimal trajectories, without integration of any differential equation. On the other hand, SMC is known to be a robust control method appropriate for uncertain systems such that the traffic ones. The proposed approach is based on the well-known space discrete first order macroscopic model. A simple case study shows very promising results for further works including traffic control for more complex motorway network. *Copyright 2008 IFAC.*

Keywords: Traffic Control ; Sliding Mode Control ; Flatness-Based Concept.

1. INTRODUCTION

The urban and inter-urban traffic congestion has been identified as a crucial socio-economic problem in most country around the world. Indeed, with the steadily increasing of the number of vehicles and the need of transportation of goods and peoples, the capacity of the road infrastructures is reached. This lead to the frequent appearance of the congestion phenomena with dramatic consequences as the safety reduction, and the pollution increase, (Papageorgiou [2001]). Moreover, congestion represents a source of continuously increase of the direct and indirect costs (increased fuel consumption, air pollution, health problems, etc.).

Several studies have shown that the continuous expanding and constructing of a new infrastructures cannot solve the congestion problems. Given the huge costs of congestion to the society and the urgent need for solutions, traffic control and dynamic management systems seem to be the only viable and efficient solutions to improve traffic flow networks and to ensure a safe displacement of goods and people. They also contribute in pollution reduction. Traffic control in motorway networks consists on the use of several actions and measurements such as; dynamic speed limits, route guidance, ramp metering, ..., [1].

Ramp metering is the most direct and efficient measurement which makes it possible to improve the traffic on freeways and maximize the use of traffic capacity in such infrastructures. The principle of such measurement is to act on the on-ramp flow by adjusting the metering rate such that the mainstream freeway density remains below the critical value. This allows to prevent congestion formation and traffic breakdown (Hegyi et al. [2005]). The ramp metering problem has been studied since the Sixties. The early work in this topic was the demand-capacity (DC) strategy (Wattleworth [1964]). This strategy represents an open-loop (feedforward) disturbance-rejection policy which is known to be quite sensitive to various non-measurable disturbances. Masher et al. [1975] have proposed a "occupancy strategy" that pursue the same philosophy of (DC) strategy but it's based on the occupancy estimation of the input flow. A range of other ramp metering strategies has been developed. The Linear Quadratic feedback control algorithm has been proposed by Isaksen and Payne [1973] in 1973. Papageorgiou et al. have proposed a variety of ramp metering strategies, as ALINEA (*Asservissement Linaire d'Entre Autoroutière*) considered as a closed-loop (feedback) ramp metering strategy (Papageorgiou et al. [1991]), METALINE, ... Other strategies rest on the optimization techniques to solve optimal ramp control problems (Yuan and Kreer [1971], Zhang et al. [1994]). Ramp metering problems was also solved using expert systems, neural and/or fuzzy control, ... (Ho and Ioannou [1996]). Some researchers have proposed ramp metering laws based on feedback control (Pera and Nenzi [1973], Owens and J. [1988], Kachroo and Krishen [2000]).

Another nonlinear technique recently developed in the industrial field is that based on the flatness concept. The advantage of such approach is its simplicity and its power for the trajectories parametrization and tracking. Nevertheless, although it is widespread in the industrial field, to the authors best knowledge, few methods based on this concept was employed in the traffic area. The first developments was proposed by (Abouaissa et al. [2007, 2006]), (Iordanova [2006]). On the other hand, (SMC) is known to be a robust control method appropriate for uncertain systems such that the traffic one. Theoretical results but also practical design examples shown that high robustness is maintained against various kinds of uncertainties such as exogenous signal and measurement errors (Mammar et al. [2006]). The combination of the two concepts (i.e. SMC and Flatness-Based Control) seem to be a novel attempt (Manish [2004]) mainly in the traffic flow area.

[1] The remarkable paper proposed by Papageorgiou et al. [2003] provides a more detailed survey of the traffic control strategies

The objective of this paper then is to apply the SMC and differential flatness to real-time ramp metering.
The paper is organized as follow: Section 2 recalls the flatness-based concept and the sliding mode control principle. The first order sliding mode control of differentially flat systems is presented in Section 3 and successfully applied to local ramp metering. Section 4 provides some numerical simulations and demonstrates the relevance of the proposed approach. The conclusion 5 summarizes the main results and lists some perspectives for future research.

2. FLAT SYSTEMS AND SLIDING MODE CONTROL

The concept of Flat systems was introduced by Fliess and coworkers (Fliess et al. [1992, 1995]) more than 10 years ago. This special class of non-linear control systems described by ordinary equations: differentially flat systems form a special class of nonlinear control systems for which systematic control methods are available once a flat-output is explicitly known. The flatness-based concept was developed in a differentially algebraic context and were later expressed using Lie- Bcklund transformation (Fliess et al. [1999]).
The flatness-based control methods may be expected to play a very significant role in high technology applications in the next few years, similar to what happened for nonlinear control in the last decade (Ramirez and Agrawal [2004], Rudolph [2003]). The main property of the flat systems is that all the state and input variables can be expressed directly, without integration of any differential equations, in term of the set of so-called "flat output" and a number of its time derivatives. More precisely, the entire system behavior is determined by the trajectory of a finite collection of quantities: flat outputs. This leads to a simple and elegant trajectories design. For a given system, the number of flat outputs is equal to the number of the system inputs. The flatness concept is closely related to the state feedback linearization.

2.1 Flat Systems Definition

In this section, We just sketch a tutorial definition of flatness for state-space control system. Consider the smooth system defined using the following equation:

$$\dot{x} = f(x, u); \quad x \in R^n, \quad u \in R^m \tag{1}$$

where $x = (x_1, x_2, ..., x_n)$ vector of state variables and $u = (u_1, u_2, ..., u_m)$, m scalar control. The system (1) is flat if and only if, there exist m real smooth functions $h = (h_1, h_2, ..., h_m)$ depending on x and a finite number of u derivatives, says β, such that, generically, the solution (x, u) of the square differential-algebraic system ($t \mapsto y(t)$ is given)

$$\dot{x} = f(x, u), \quad y(t) = h(x, u, \dot{u}, ..., u^{(\beta)}) \tag{2}$$

does not involve any differential equation and thus is of the form:

$$x = \Phi(y, \dot{y}, ..., y^{(\beta)}), \quad u = \Psi(y, \dot{y}, ..., y^{(\beta+1)}) \tag{3}$$

where, Φ and Ψ are smooth functions, and β is some finite number (Rouchon [2005]). The quantity y is of fundamental importance: it is called "*flat output*" or linearizing output. In the control language, the flat output y is such that, the inverse of $\dot{x} = f(x, u), y = h(x, u, \dot{u}, ..., u^{(\beta)})$ has no dynamics (Isidori et al. [1986]). Differentially flat systems are very useful in the situations where the explicit generation of trajectories is required. Since the behavior of the flat systems is determined by the flat outputs, one can plan the trajectories in the outputs space and then connect these to appropriate inputs. More precisely, from the trajectories of the flat outputs y , we can deduce immediately the trajectories of the state x and the input u variables. Applications of the flatness concept to problems of engineering field have grown steadily in recent years and a variety of case studies have been shown to be flat and flatness based controllers based on trajectories generation by polynomial interpolation and then closing the loop on the obtained trajectories have been developed. Generally, the problem of flatness characterization is fully open for multi-input systems $dim(u) > 1$. Indeed, there is no algorithm to decide once the equations $\dot{x} = f(x, u)$ are given, if there exists such map h, called flat output map (Rouchon [2005]). The situation is somehow comparable to integrable Hamiltonian systems: there is no algorithm to decide whether a given Hamiltonian $H(q, p)$ yields an integrable system; many examples of physical interest are integrable and for these systems we have the form of their general solution in terms of the initial conditions; only necessary conditions are available.

For flat systems, the situation is very similar: no algorithm to decide whether a system is flat or not; many examples of engineering interest are flat and their general solution reads in term of the derivatives of a flat-output y that has a clear physical interpretation. few necessary conditions are available (see, e.g., the ruled-manifold criterion (Rouchon [1995]). To summarize: the role of flat-systems within the set of under-determined ordinary differential systems is very similar to the role of integrable systems within the set of determined ordinary-differential systems.

2.2 Sliding Mode Control

Sliding mode control is known to be a robust control method appropriate for uncertain systems such that the traffic one. Theoretical results but also practical design examples shown that high robustness is maintained against various kinds of uncertainties such as exogenous signal and measurement errors (Mammar et al. [2006]). This control scheme is based on the concept of changing of the structure of the controller in response to the changing state of the system in order to obtain a desired value. A high speed switching control action is used to switch between different structures of the controller and the trajectory of the system is forced to move along a chosen switching manifold in the state space. The behavior of the closed loop system is thus determined by the sliding surface (Manish [2004]). SMC is characterized by the following advantages:

- SMC is insensitive to systems' parameters variation and to external perturbations as well as modelling errors,
- the dynamic behavior of the system may be tailored by the particular choice of switching function.

Since the first order SMC that may be implemented only if the relative degree of the sliding surface s is equal to 1, several SMC extension have been developed (see e.g.

Manish [2004]). In this paper one focused on the standard or first SMC.

3. REAL-TIME RAMP METERING

Generally, traffic ramp metering can be either local (isolated) or coordinated. Isolated ramp metering can be implemented locally in the vicinity of each ramp to calculate the corresponding ramp metering values. Coordinated ramp metering aims to use available traffic measurements from larger freeway sections simultaneously. According to a recent results (Papageorgiou et al. [1997], Smaragdis et al. [2004]) isolated ramp metering are more easy to design and implement. The first necessary step in the design of feedback controller for ramp metering, is to describe the system dynamics via a traffic model. Several works (see e.g. Papageorgiou et al. [1991]) have shown that macroscopic models are well adapted in this context.

In this paper, one assume that the traffic dynamics are governed by a first order model like LWR one (see e.g., Lighthill and Whitham [1955], Richards [1956]). For the space discrete representation of the first order model, one consider the simple section depicted in (fig. 1):

Define, $\rho(t)$, the traffic density as the number of vehicles in

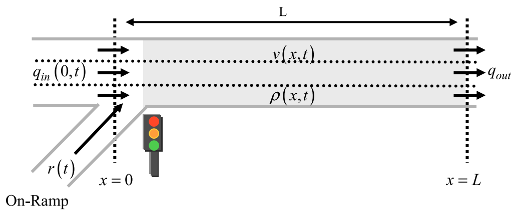


Fig. 1. Freeway section with on-ramp

this section at time t divided by the section length L. $q_{in}(t)$ is the traffic volume defined as the number of vehicles entering the section. $q_{out}(t) = \rho(t)v(t)$ is the number of vehicles leaving the section. $r(t)$ is the on-ramp flow. The space discrete form of the first order model reads [2] :

$$\dot{\rho}(t) = \frac{1}{L\lambda}(q_{in}(t) - q_{out}(t) + r(t)) \tag{4}$$

here, λ is the number of lanes in the section. For simplicity, one consider that $\lambda = 1$. The mean speed $v(t)$ of vehicles present in the section is defined by the expression Greenshields [1935]:

$$v(t) = v_f(1 - \frac{\rho(t)}{\rho_m}) \tag{5}$$

v_f is the free flow speed and ρ_m, the jam density. Notice that other types of fundamental diagrams, like that proposed by (May [1990]), may be considered.

3.1 Flatness of the traffic model

The studied system, characterized by one state variable (traffic density) is flat with $y = \rho$ its flat output. Indeed, taking the first time derivative of this flat output, one obtain:

$$\dot{y} = \frac{1}{L}(q_{in}(t) - y(t)v(t) + r(t)) \tag{6}$$

[2] one can use more complex space discrete model as proposed in (see, e.g. Mammar et al. [2006]). We didn't do it in order to illustrate the principle of the approach and because the fact that SMC deals with uncertain and incomplete models.

It's follow that all systems' variables can be expressed in term of the flat output and its first time derivative:

$$\begin{cases} \rho = y \\ r(t) = L\dot{y}(t) + y(t)v(t) - q_{in}(t) \end{cases} \tag{7}$$

The equation of the state variable allows choosing a suitable trajectory of the density. The expression of the control (input) variable allows adding additional constraints to this density trajectory. This means that all important properties of the system (4) are contained in such a differential parametrization.

3.2 trajectory planning: open loop control

In order to define the trajectory planning, an open loop control law must be determined. For this, a suitable desired trajectory y^* has to be defined. According the expression of the control variable in (7) this trajectory must have smooth derivatives up to order two. From the initials and finales conditions of the density $(y(t_i) = y_i,\ \dot{y}(t_i) = 0)$ and $(y(t_f) = y_f,\ \dot{y}(t_f)) = 0)$, one can build this reference trajectory for the density (flat output) using a polynomial interpolation because of the reduced computational effort in the real time environment (Ramirez and Agrawal [2004]). Indeed, since, at the system equilibrium points the constant values of the output variable $(\rho(t))$ and the flat output, $y(t)$, perfectly coincide, one pose ourselves, instead of the original problem, the equivalent problem of controlling, or transferring along a desired trajectory $y^*(t)$, the flat output y, between the given initial and final equilibria. One desire, then to transfer the flat output y between the values $y^*(t_i) = y_i$ and $y^*(t_f) = y_f$. Using the polynomial interpolation, this is accomplished by prescribing the following desired trajectory for the flat output y:

$$y^*(t) = \begin{cases} y_i & \text{for } t < t_i \\ y_i + (y_f - y_i)\sigma(t, t_i, t_f) & \text{for } t_i \le t \le t_f \\ y_f & \text{for } t > t_f \end{cases} \tag{8}$$

where $\sigma(t, t_i, t_f)$ is a polynomial function of time, exhibiting a sufficient number of zero derivatives at times, t_i and t_f, while also satisfying: $\sigma(t_i, t_i, t_f) = 0$ and $\sigma(t_f, t_i, t_f) = 1$. Because for the studied system, we have four conditions, we need then a degree 3 polynomial [3] (fig 2):

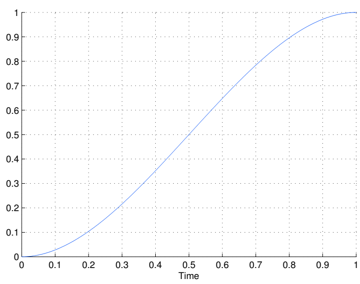


Fig. 2. $\sigma(t)$ interpolation function

$$\sigma(t) = \begin{cases} 0 & \text{for } t < t_i \\ 3t^2 - 2t^3 & \text{for } t_i \le t \le t_f \\ 1 & \text{for } t > t_f \end{cases} \tag{9}$$

[3] For the polynomial calculation see e.g. Rudolph [2003], Ramirez and Agrawal [2004]

In this way with the expression of the control variable in (7) and the relation (8), the nominal open loop control can be calculated:

$$u^*(t) = L\dot{y}^* + v_{out}(t)y^*(t) - q_{in} \quad (10)$$

3.3 Sliding mode control of the flat system

In the following, a sliding mode controller which asymptotically regulates the output y towards the desired equilibrium position is proposed. Since the control input to the system, r, is a differential function (7) of the flat output y , we can impose on the highest derivative of y, a linear relation involving only small order derivatives of the same output component. This gives the required linearizing controller expression in terms of the flat outputs. From the expression for r it follows that the linearized equation for the system is simply given by:

$$\dot{y} = w \quad (11)$$

The system's dynamic behavior is determined by the following condition:

$$s(y) = 0 \quad (12)$$

The control objective is to ensure a traffic density equal to a desired value (typically, this value is sensibly inferior to the critical density). A sliding surface expression is proposed which depicts a desired first order dynamic response for the controlled flat output y towards its desired equilibrium value y^*. In this context, the sliding surface can be:

$$s(y) = \rho(t) - \rho^*(t) = y(t) - y^*(t) \quad (13)$$

To ensure the convergence of the flat output y toward the desired value y^* in a finite time, one can impose the following sliding mode controlled dynamics (the constant plus proportional rate reaching law Manish [2004]) on the evolution of the sliding surface function s.

$$\dot{s}(y) = -k_1 sign(s) - k_2 s \quad (14)$$

k_1 and k_2 are positif parameters. There a good choice must ensure the reduction of both the convergence time and the chattering phenomena near the sliding manifold (Mammar et al. [2006]). Thus, the sliding mode dynamics yield the following required dynamics of the flat output y:

$$\dot{y} = -k_1 sign(s) - k_2 s \quad (15)$$

The auxillaire input w of the control variable is then given by:

$$w = -k_1 sign(s) - k_2 s \quad (16)$$

Substituting equations (16) and (12) into (7) gives the control law:

$$r(t) = L(-k_1 sign(s) - k_2 s) + yv - q_{in} \quad (17)$$

The control variable r can now be expressed in term of the original state variable ρ, by a simple substitution of the flat output y by ρ.

$$r(t) = L(-k_1 sign(\rho - \rho^*) - k_2(\rho - \rho^*)) + \rho v - q_{in} \quad (18)$$

The equation (17) is the 'state feedback' sliding mode control based on differential flatness for local ramp metering.

4. SIMULATION RESULTS

The proposed new algorithms are validated by carried-out a set of simulations. To perform a comparative studies mainly with the results from a feedback control law proposed by Kachroo et al., (Kachroo and Krishen [2000]), one take the same data used by these authors. Thus, we used the Greenshields's fundamental diagram with the free flow speed $v_f = 60km/h$ and the maximal density $\rho_m = 120veh/km$. The critical density in this case is then: $\rho_{cr} = 60veh/km$ with a maximum flow (corresponding to the section capacity) of $q_m = 1800veh/h$. We choose the target signal to be $\rho_{cr} = 55veh/km$. The controller is synthesized using parameters $k_1 = 1$ and $k_2 = 0.1$.

4.1 Traffic ramp metering simulations in uncongested mode

We start with the case when the initial density is $\rho = 40veh/km$. The inflow to the mainstream section was kept at $1500veh/h$. The figures (fig. 3) show that the designed controller allows to maintain the desired parameter values.

4.2 Traffic ramp metering simulations in congested mode

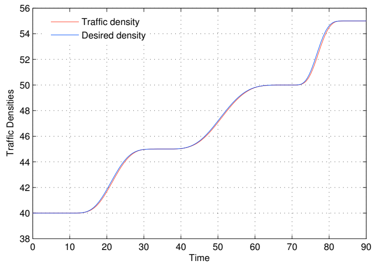
The seconde case study sets the initial density is $\rho = 65veh/km$. The inflow to the mainstream section was kept at $1500veh/h$. The figures (fig. 4) show that, in this case also, the designed controller allows to maintain the desired density at the prescribed values.

4.3 Traffic ramp metering simulations in congested mode with a random inflow

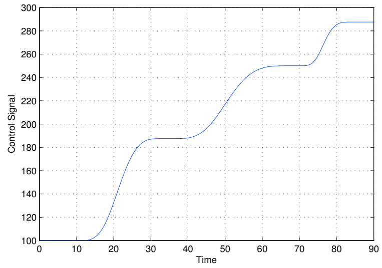
Let us consider that the input to mainstream flow is a random variables between 1400 and 1600 veh/h. One consider the case when the traffic is congested ($\rho(0) = 65veh/km$). In this case also, the figures (fig. 5) show that the controller is able to maintain the traffic variables at the prescribed values. Notice that for high level of congestion the proposed approach must be enriched to take into account the queue length that will be formed in the on-ramp.

5. CONCLUSION

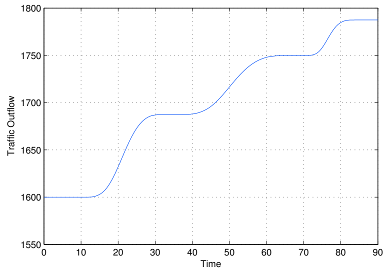
In this paper, we presented the SMC of differentially flat systems in the framework of local ramp metering control problems setting in space discretized lumped parameter. The suggested control algorithms are based on the system inversion concept carried-out using the flatness-based notion that do not requires integration of any differential equation. In this context, the analysis and design of a controller is greatly simplified by means of differential flatness. Whereas the open loop flatness controller is not desirable for the traffic density control in the studied section, and in order to treat the various fluctuations of this control schema, we proposed an additional first order sliding mode control. Indeed, SMC is a robust control scheme based on the concept of changing the structure of the controller in response to the changing state of the system in order to obtain a desired response. The main advantage of SMC is its insensitivity to variation in system parameters, external disturbances and modelling errors. The combination of differential flatness with such SMC techniques qualifies as a valuable control scheme. Simulations results demonstrate the relevance of the proposed approach and the authors will further exploit the principle of the proposed method and the possibility to extend it to more large and complex networks using the high order SMC.



(a) Traffic density evolution

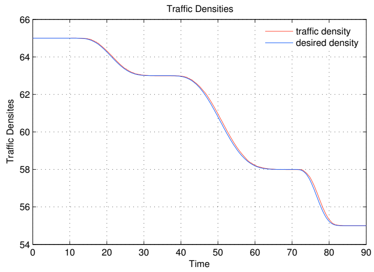


(b) ramp signal

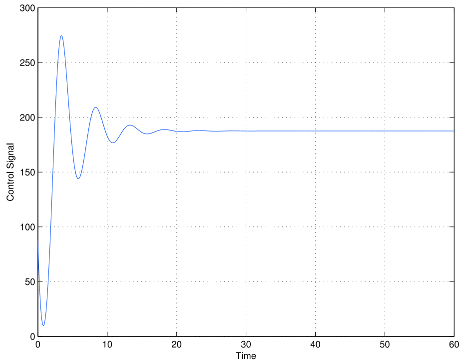


(c) Traffic Flow output

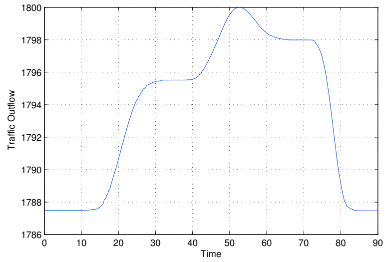
Fig. 3. Ramp metering in uncongested mode



(a) Traffic density evolution

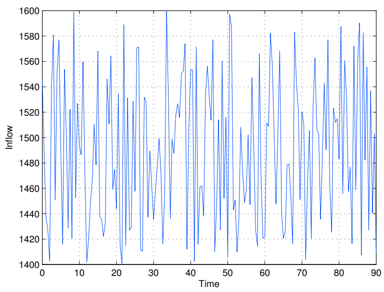


(b) ramp signal

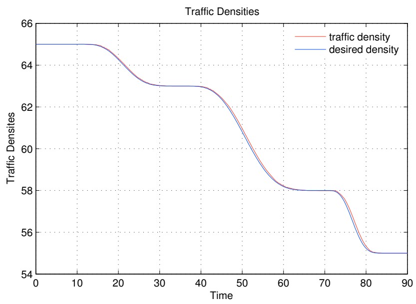


(c) Traffic Flow output

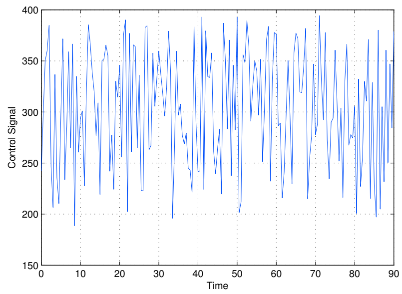
Fig. 4. Ramp metering in congested mode



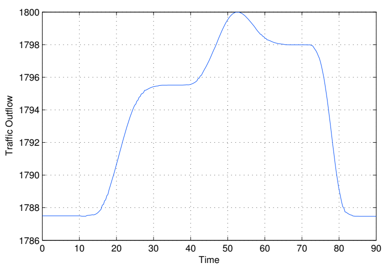
(a) Input Flow



(b) Traffic density evolution



(c) ramp signal



(d) Traffic Flow output

Fig. 5. Ramp metering in congested mode and random inflow

REFERENCES

H. Abouaissa, V. Iordanova, and D. Jolly. Flatness based control of traffic flow. In *9th International IEEE Conference on Intelligent Transportation Systems, Toronto, Canada*, pages 1060–1065, 17-20 September 2006.

H. Abouaissa, V. Iordanova, and D. Jolly. Integrated control of highway systems using flatness-based concept. In *SCS-IFAC, International Conference on Advances in Vehicle Control and Safety,*, Bueno-Aires Argentina, 8-10 February 2007.

M. Fliess, J. Levine, P. Martin, and P. Rouchon. Sur les systèmes non linéaires différentiellement plats. *C.R. Acad. Sci.*, I-315:619–624, 1992.

M. Fliess, J. Levine, P. Martin, and P. Rouchon. Flatness and defect of non-linear systems: introductory theory and examples. *International Journal of Control*, 61(6): 1327–1360, 1995.

M. Fliess, J. Levine, P. Martin, and P. Rouchon. A lie-backlund approach to equivalence and flatness of nonlinear systems. *IEEE Trans. Auto. Cont.*, 44(5):928–937, 1999.

B. D. Greenshields. A study of traffic capacity. In *Proc. of the Highway Board*, volume 14, pages 448–477, 1935.

A. Hegyi, B. De Schutter, and H Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13:185 – 209, 2005.

F.-S. Ho and P. Ioannou. Traffic flow modeling and control using artificial neural networks. *IEEE Control Systems*, pages 16–26, 1996.

V. Iordanova. *Contribution à la modélisation et la commande du trafic: Approches par Bond-Graph et commande par platitude*. PhD thesis, Université d'Artois,, LGI2A, Béthune, France, Décembre 2006.

L. Isaksen and H. J. Payne. Suboptimal control of linear systems by augmentation with application to freeway traffic regulation. *IEEE Trans. Automat. Contr.*, 18: 210–219, 1973.

A. Isidori, C.H. Moog, and A. De Luca. A sufficient condition for full linearization via dynamic state feedback. In *In Proceedings 25th. IEEE Conf. Decision Control*, pages 203–208, 1986.

P. Kachroo and K. Krishen. System dynamics and feedback control design problem formulations for real time ramp metering. *Transactions of Society for Design and Process Science*, 4(1):37–54, 2000.

M. J. Lighthill and G. B. Whitham. On kinematic waves ii. a theory of traffic flow in long crowded roads. *Proceedings of the Royal Society*, A 229:317–345, 1955.

S. Mammar, S. Mammar, and M. Netto. Coordinated ramp metering via second order sliding mode control. In *IEEE ITSC 2006 IEEE Intelligent Transportation Systems Conference*, pages 261–266, Torronto, Canada, September 2006.

Vig Manish. Higher order sliding mode control of differentially flat systems. Master's thesis, INDIAN INSTITUTE OF TECHNOLOGY, BOMBAY, MUMBAI, India,, 2004.

D. P. Masher, D.W. Ross, P. J.Wong, P. L. Tuan, H. M. Zeidler, and S. Peracek. Guidelines for design and operating of ramp control systems. In *Stanford Research Institute*, Menlo Park California, 1975.

A. D. May. Traffic flow fundamentals. *Englewood Cliffs, NJ: Prentice-Hall*, 1990.

D. Owens and Schofield M. J. Access control on the m6 motorway: Evaluation of britain's first ramp metering scheme. *Traffic Engineering and Control*, 29:616–623, 1988.

M. Papageorgiou. *Traffic Control*, chapter 8. Handbook of Transportation Science, 2001.

M. Papageorgiou, H. Hadj-Salem, and J.-M. Blosseville. Alinea: A local feedback control law for on-ramp metering. *Transp. Res. Record*, 1320, 1991.

M. Papageorgiou, H. Haj-Salem, and F. Middleham. Alinea local ramp metering: summary of fields results. *Transportation Research Record*, 1603:90–98, 1997.

M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kostialos, and Y. Wang. Review of road traffic control strategies. *IEEE Transaction of Intelligent Transportation Systems*, 91(12):2043 – 2067, december 2003.

R. La. Pera and R. Nenzi. Tana - an operating surveillance system for highway traffic control. *Proceeding., Institute of Electrical and Electronics Engineers*, V. 61:542–556, 1973.

S. Ramirez and S. Agrawal. *Differentially Flat Systems*. Marcel Dekker, New-York, 2004.

P.I Richards. Shock waves on the highway. *Operations Research*, 4:42–51, 1956.

P. Rouchon. Flatness based control of oscillators. *ZAMM Z. Angew. Math. Mech*, 85(6):411–421, 2005.

P. Rouchon. Necessary condition and generecity of dynamic feedback linearization. *J. Math. Systems Estim. Control*, 5(3):345–358, 1995.

J. Rudolph. *Flatness Based Control of Distributed Parameter Systems*. Shaker Verlag, 2003.

E. Smaragdis, M. Papageorgiou, and E. Kosmatopoulos. A flow-maximizing adaptive local ramp metering strategy. *Transportation Research Part B*, 38:251–270, 2004.

J.A. Wattleworth. System demand capacity analysis on the inbound gulf freeway. In *Texas Transportation Institute Rept.*, pages 24–81, 1964.

L.S. Yuan and J.B. Kreer. Adjustment of freeway ramp metering rates to balance entrance ramp queues. *Transportation Research*, 5:127–133, 1971.

H. Zhang, Ritchie S.G., and Lo Z. A local neural network controller for freeway ramp metering. In *IFAC Transportation Systems*, pages 655–658, Tianjin, PRC,, 1994.