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An adaptive $\mathcal{H}_\infty$ control for robotic manipulator with compensation of input torque uncertainty

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Abstract: This paper examines the problem of link position tracking control for robot manipulators with input toque uncertainty. It is assumed that the input torque uncertainty can be regarded as dead-zone phenomena at each link of manipulator and all the system parameters for robotic manipulator and dead-zone model are unknown. The proposed method ensures that the unknown parameters are estimated adaptively, besides, an approximated errors of input nonlinearities and external disturbances are attenuated by means of $\mathcal{H}_\infty$ control performance. In spite of considering the nonlinear adaptive $\mathcal{H}_\infty$ control problems, based on our proposed method, the compensator can design without solving the Hamilton-Jacobi-Isaacs equation. Numerical simulation results are given to illustrate the effectiveness of our proposed method.

1. INTRODUCTION

This paper examines the problem of link position tracking control for robot manipulators with input toque uncertainty.

It is well known that a rigid n–link, serially connected, direct–drive robot manipulators can be described as a following form

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = F(\tau) \tag{1}$$

where $\theta \in \mathbb{R}^n$ is the rotational angle of robotic arm, $\tau \in \mathbb{R}^n$ is control input torques, $M(\theta) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(\theta,\dot{\theta}) \in \mathbb{R}^{n\times n}$ is the matrix composed of centrifugal and Coriolis term, $g(\theta) \in \mathbb{R}^n$ is the gravity vector, $F(*) \in \mathbb{R}^n$ is an unknown input toque uncertain functional vector. In general, it is very difficult to know the physical parameters of robotic manipulators, precisely. Then the system matrix and vectors $M(\theta)$, $C(\theta,\dot{\theta})$ and $g(\theta)$ have some parametric uncertainties. Moreover, in practice, the actuator model is not known exactly, then the input term of the robotic manipulator should be treat unknown function. Hence, in the presence of the parametric uncertainties and input torque uncertainty, the convergence of the tracking error may not be guaranteed. Therefore, to achieve a good tracking control performance in robotic manipulators, it is necessary to compensate the effects due to parametric uncertainties, input torque uncertainty, external disturbances, and so forth.

A common assumption in most of the previously designed controllers is that the right hand side of (1) can be described as follows:

$$F(\tau) = \tau + d \tag{2}$$

where $d \in \mathbb{R}^n, \in \mathcal{L}^2$ is unknown external torque disturbances vector. It seems that the input torque uncertainty is considered as input torque disturbance. Based on this assumption, there have been researched about the robust control method for robotic manipulators. In order to compensate for such parametric uncertainties, passivity based adaptive control methods have been proposed [Slotine and Li [1988], Ortega and Spong. [1989], Spong [1992]]. It was also shown that the external torque disturbance can be compensated by the notion of a nonlinear $\mathcal{H}_\infty$ control method[Chen, Chang, and Lee [1997]]. In this method, we have to solve a Hamilton–Jacobi–Isaacs (HJI) equation for a controller. However, it is well known that we have a great difficulty to solve a HJI equation. Based on $\mathcal{L}_2$ and $\mathcal{L}_\infty$ gain analysis, adaptive control for robotic system with disturbance attenuation method was also proposed[Tomei [1999]]. Dissipative based adaptive control method which can compensate the parametric uncertainties and attenuate the external torque disturbance in the sense of $\mathcal{H}_\infty$ optimality have been proposed for robotic manipulators [Shen and Tamura [1999]]. The most advantage point of this dissipative based adaptive control method can design the disturbance attenuation control system without solving the HJI equation.

For high or ultra precision link position tracking control for robotic manipulators, not only the effects of disturbances but also explicitly considering the input torque uncertainty is very important. To capture the input torque uncertainty, it is assumed that the uncertainty can be written by the friction phenomena which is described as

$$F(\tau) = \tau + F_f + d \tag{3}$$

where $F_f \in \mathbb{R}^n$ are friction forces vector and $d \in \mathbb{R}^n, \in \mathcal{L}^2$ are unknown external torque disturbances vector. It is assumed that the friction forces can be given by LuGre model, an adaptive friction compensator for robotic manipulator is also proposed which is based on the passivity based control [Panteley, Ortega, and Gäfvert [1998]]. In this method, it is assumed that the all the parameters of the LuGre friction model and robotic manipulators are unknown, and position tracking and friction compensation are achieved with a very simple adaptive law. But, unfortunately, this method included the discontinuous function and it is not enough to compensate the approximation errors of the friction model. Another adaptive friction compensation method was also proposed [Tomei [2000]]. This method is required that many nominal parameters for friction model should be known. Recently, an adaptive $\mathcal{H}_\infty$ controller with friction compensation and disturbance attenuation for position tracking for a robotic manipulator was given [Sato and Tsuruta [2006]]. This method is also assumed that the friction model can be described as LuGre model, but it does not include the discontinuous function. Some experimental results are given to show the effectiveness of this method.

In one point of view, the input torque uncertainty description (3) is effective, but it is not enough to caputure the input torque uncertainty. As for the practical applications, it should be taken into account that the input torque uncertainty exists due to dead–zone or hysteresis characteristics of actuators. For such uncertainties, if we give some appropriate assumptions [Su, Stepanenko, Svobada, and Leungć [2000], Wang, Su and Hong [2004]], then the right hand side of (1) can be written as

$$F(\tau) = \tau + d = Ku + d \tag{4}$$

where $u \in \mathbb{R}^n$ is actuator input, $K \in \mathbb{R}^{n\times n}$ is transmission matrix, and $d \in \mathbb{R}^n, \in \mathcal{L}^2$ are unknown external torque disturbances vector. In the presence of the input torque uncertainty, it is natural to assume that the transmission matrix K is unknown.

In this paper, we proposed a novel adaptive $\mathcal{H}_\infty$ control method for robotic manipulator with input torque uncertainty. It is assumed that the input torque uncertainty can be divided into unknown input vector and input torque disturbance, besides, unknown input vector can be described as unknown transmission matrix multiply the input torque, as shown in (4). Especially, our proposed method can compensate a parametric uncertainty of robotic manipulators and an uncertainty of the transmission matrix. In addition, our proposed method is also based on inverse optimal control strategy, an adaptive $\mathcal{H}_\infty$ controller can be designed without solving the HJI equation. We demonstrate the practical advantages of our proposed controller through comparative numerical simulations. As a result, our proposed method suggested that the compensation of the input torque uncertainty should not only be treated as a input disturbance, but also divided into uncertain transmission matrix and input disturbance torques.

2. ROBOTIC SYSTEM AND ERROR DYNAMICS

We consider an n–revolute joints rigid manipulator with input torque uncertainty which can be expressed as

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = D(\tau) + w, \tag{5}$$

where $\theta \in \mathbb{R}^n$ is the rotational angle of robotic arm, $\tau \in \mathbb{R}^n$ is control input torques, $M(\theta) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(\theta,\dot{\theta}) \in \mathbb{R}^{n\times n}$ is the matrix composed of centrifugal and Coriolis term, $g(\theta) \in \mathbb{R}^n$ is the gravity vector, $w \in \mathbb{R}^n, \in \mathcal{L}^2$ is unknown external torque disturbances vector, and $D(*) \in \mathbb{R}^n$ is unknown input toque uncertain functional vector. In general, a revolute joints robotic system has following properties [Ortega and Spong. [1989]],

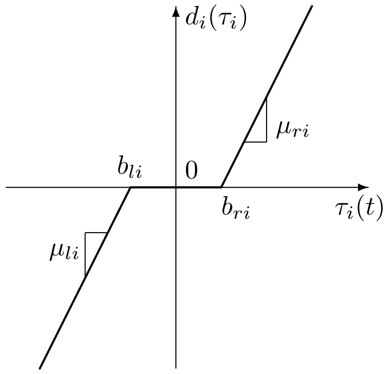

Fig. 1. Dead–zone model

C–1: $M(\theta)$ is bounded and positive symmetric matrix.

C–2: Matrix $\dot{M}(\theta) - 2C(\theta,\dot{\theta})$ is skew–symmetric.

C–3: We can parametrize the dynamic equations (5) as,

$$M(\theta)\dot{\xi} + C(\theta,\dot{\theta})\xi + g(\theta) = Y_d(\theta,\dot{\theta},\xi,\dot{\xi})\beta, \tag{6}$$

where $Y_d(\theta,\dot{\theta},\xi,\dot{\xi})$ is an $n \times r$ matrix of known functions, known as regressor, and β is an r–dimensional vector of parameters which are constructed by the physical parameters of $M(\theta), C(\theta,\dot{\theta}), g(\theta)$. In general, β is an unknown parameter vector because of the physical parameters of $M(\theta), C(\theta,\dot{\theta}), g(\theta)$ are unknown. But if we can know the nominal physical parameters $M(\theta)$, $C(\theta,\dot{\theta})$, $g(\theta)$ as $M_0(\theta)$, $C_0(\theta,\dot{\theta})$, $g_0(\theta)$, then we can describe (6) as follows:

$$M_0(\theta)\dot{\xi} + C_0(\theta,\dot{\theta})\xi + g_0(\theta) = Y_d(\theta,\dot{\theta},\xi,\dot{\xi})\beta_0. \tag{7}$$

As we can see the aforementioned arguments, β_0 is known parameter vector. Moreover, if we do not consider the gravity term $g(\theta)$, then the regressor form can be written as follows:

$$M(\theta)\dot{\xi} + C(\theta,\dot{\theta})\xi = Y_g(\theta,\dot{\theta},\dot{\xi},\xi)\beta. \tag{8}$$

Moreover, if we assume that the input torque uncertainty on each link does not affect to the other links, then, we can describe input uncertainty as

$$D(\tau) = \begin{bmatrix} d_1(\tau_1) \\ d_2(\tau_2) \\ \vdots \\ d_n(\tau_n) \end{bmatrix} \tag{9}$$

To model the effect of the input torque uncertainty, we employ the dead–zone model as follows:

$$d_i(\tau_i) = \begin{cases} \mu_{ri}(\tau_i(t) - b_{ri}) & \text{for } \tau_i(t) \geq b_{ri} \\ 0 & \text{for } b_{li} < \tau_i(t) < b_{ri} \\ \mu_{li}(\tau_i(t) - b_{li}) & \text{for } \tau_i(t) \leq b_{li} \end{cases} \tag{10}$$

where $i = 1, 2, \cdots, n$ [Su, Stepanenko, Svobada, and Leungć [2000], Wang, Su and Hong [2004]]. We make the assumptions that the dead–zone has the following properties:

(A1) The dead–zone output $d_i(\tau_i)$ is not available for measurement.
(A2) The dead–zone slopes in positive and negative region are same, i. e., $\mu_{ri} = \mu_{li} = \mu_i$.
(A3) The dead–zone parameters b_{ri}, b_{li}, and μ_i are unknown, but their signs are known: $b_{ri} > 0$, $b_{li} < 0$, $\mu_i > 0$.
(A4) The dead–zone parameters b_{ri}, b_{li}, and μ_i are bounded, i. e., each lower and upper bounds are known and it can be described as follows:

$$b_{ri} \in [b_{ri\min}, b_{ri\max}], \quad b_{li} \in [b_{li\min}, b_{li\max}],$$
$$\mu_i \in [\mu_{i\min}, \mu_{i\max}].$$

(A1) $\sim$ (A4) are satisfied in practical applications. Then, dead–zone model (10) can be rewritten as follows:

$$d_i(\tau_i) = \mu_i \tau_i(t) + d'_i(\tau_i) \tag{11}$$

where $d'_i(\tau_i)$ can be described from (10) and (11) as

$$d'_i(\tau_i) = \begin{cases} -\mu_i b_{ri} & \text{for } \tau_i(t) \geq b_{ri} \\ -\mu_i \tau_i(t) & \text{for } b_{li} < \tau_i(t) < b_{ri} \\ -\mu_i b_{li} & \text{for } \tau_i(t) \leq b_{li} \end{cases} \tag{12}$$

From (A2) and (A4), we can evaluate $d(u(t))$ as follows:

$$|d'_i(\tau_i)| \leq \rho_i \tag{13}$$

where ρ_i is upper–bound of $d'_i(\tau_i)$, which can be chosen as

$$\rho_i = \max\{\mu_{i\max} b_{ri\max}, -\mu_{i\max} b_{li\min}\} \tag{14}$$

where $b_{li\min}$ is negative values.

Based on the aforementioned assumptions, the control objective is to design an adaptive controller which can attenuate the disturbances and the effects of input uncertainties to the control performance by means of $\mathcal{H}_\infty$ control, and to let the joint angles θ track a specified desired trajectory, $\theta_d(t)$, i. e., which satisfies as follows:

$$\lim_{t \to \infty} \theta(t) = \theta_d(t).$$

Let $\theta_d \in \mathbb{R}^n$ be bounded and differentiable up to second order.

From (11), the robot dynamics (5) can be rewritten as follows:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = K\tau + w \tag{15}$$

where

$$K = \mathrm{diag}\{\mu_1, \cdots, \mu_n\} \tag{16}$$
$$\tau = \mathrm{col}(\tau_1, \cdots, \tau_n) \tag{17}$$
$$w = \mathrm{col}(d'_1(\tau_1), \cdots, d'_n(\tau_n)) + d \tag{18}$$

and $K \in \mathbb{R}^{n \times n}$ is unknown diagonal transmission matrix.

Let define the tracking error as

$$e = \theta - \theta_d. \tag{19}$$

Besides, we also define the error signal as

$$s = \dot{e} + \lambda e. \tag{20}$$

Taking time derivative of each side of (20) and substitute it into (15), then error equation for robotic system (15) can be written as

$$M(\theta)\dot{s} - \lambda M(\theta)\dot{e} + C(\theta, \dot{\theta})\dot{e} + Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\beta = K\tau + w \tag{21}$$

where $Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\beta$ is the regressor description which is described as

$$M(\theta)\ddot{\theta}_d + C(\theta, \dot{\theta})\dot{\theta}_d + g(\theta) = Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\beta. \tag{22}$$

Moreover, we can derive the following equation:

$$(\lambda M(\theta) - C(\theta, \dot{\theta}))\dot{e} = \lambda Y_g(\theta, \dot{\theta}, e, \dot{e})\beta, \tag{23}$$

and substitute (23) into (21), then the error equation for robotic system (15) can be further written as

$$\begin{aligned} M(\theta)\dot{s} &= -C(\theta, \dot{\theta})s - Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\beta + \lambda Y_g(\theta, \dot{\theta}, e, \dot{e})\beta \\ &\quad + K\tau + w \\ &= -C(\theta, \dot{\theta})s - Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)e_\beta - Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\beta_0 \\ &\quad + \lambda Y_g(\theta, \dot{\theta}, e, \dot{e})e_\beta + \lambda Y_g(\theta, \dot{\theta}, e, \dot{e})\beta_0 + K\tau + w \\ &= -C(\theta, \dot{\theta})s - Y_e(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d, e, \dot{e})e_\beta \\ &\quad - Y_e(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d, e, \dot{e})\beta_0 + K\tau + w \end{aligned} \tag{24}$$

where $e_\beta = \beta - \beta_0$ and

$$Y_e(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d, e, \dot{e}) = Y_d(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d) - \lambda Y_g(\theta, \dot{\theta}, e, \dot{e}). \tag{25}$$

For simplicity of notation, we will omit the parameter description of regressor matrices, appropriately, then we only describe as Y_d, Y_e, and Y_g.

3. ADAPTIVE $\mathcal{H}^\infty$ CONTROL METHOD

Based on the control objective and the subsequent stability analysis, the following adaptive controller is developed:

$$\tau = \bar{K}^{-1}\left(-e - Bs + Y_e\beta_0 + Y_e\hat{e}_\beta + Y_a(\tau_0)\hat{\beta}_a + v\right). \tag{26}$$

$\bar{K} \in \mathbb{R}^{n \times n}$ is an approximate diagonal transmission matrix as follows:

$$\bar{K} = \mathrm{diag}\{\bar{k}_1, \cdots, \bar{k}_n\}, \tag{27}$$

where $\bar{k}_i$ are constants which are the design parameters. $\hat{e}_\beta \in \mathbb{R}^{r \times 1}$, $\hat{\beta}_a \in \mathbb{R}^{n \times 1}$ denote a subsequently designed parameter estimates, and v is a new control input signal which will be given later. Moreover, $Y_a(\tau_0)$ denotes as

$$Y_a(\tau_0) = \mathrm{diag}\{-\tau_{01}, \cdots, -\tau_{0n}\} \tag{28}$$

where τ_{0i} is the i–th element of the vector τ_0 which is defined as

$$\tau_0 = e + Bs - Y_e\beta_0 - Y_e\hat{e}_\beta - v \tag{29}$$

Based on the subsequent stability analysis, the parameter estimates $\hat{e}_\beta$, $\hat{\beta}_a$ are generated from the following adaptation laws:

$$\dot{\hat{e}}_\beta = -\Gamma_\beta Y_e^{\mathsf{T}}(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d, e, \dot{e})s \tag{30}$$
$$\dot{\hat{\beta}}_a = -\Gamma_a Y_a(\tau_0)s \tag{31}$$

where $\Gamma_\beta \in \mathbb{R}^{r \times r}$ and $\Gamma_a \in \mathbb{R}^{n \times n}$ denote constant, diagonal positive definite adaptation gain matrices.

The closed–loop error system can be determined after substituting (26) into (24) as follows:

$$\begin{aligned} &M(\theta)\dot{s} + C(\theta, \dot{\theta})s + Y_e\beta_e + Y_e\beta_0 \\ &- K\bar{K}^{-1}\left\{-e - Bs + Y_e\beta_0 + Y_e\hat{e}_\beta + Y_a(\tau_0)\hat{\beta}_a + v\right\} \\ &- w = 0. \end{aligned} \tag{32}$$

The parameter estimation error $\tilde{e}_\beta(t) \in \mathbb{R}^{r \times 1}$ is defined as

$$\tilde{e}_\beta = e_\beta - \hat{e}_\beta, \tag{33}$$

then the closed–loop error system (32) can be further written as

$$\begin{aligned} &M(\theta)\dot{s} + C(\theta, \dot{\theta})s + Y_e\tilde{e}_\beta + Y_e\hat{e}_\beta + Y_e\beta_0 \\ &+ K\bar{K}^{-1}\left\{e + Bs - Y_e\beta_0 - Y_e\hat{e}_\beta - Y_a(\tau_a)\hat{\beta}_a - v\right\} \\ &- w - e + e - Bs + Bs - v + v = 0 \end{aligned} \tag{34}$$

Besides, we can rewrite (34) as

$$M(\theta)\dot{s} + C(\theta,\dot{\theta})s + e + Bs + Y_e\tilde{e}_\beta - v - w + (K\bar{K}^{-1} - I)\tau_0 - K\bar{K}^{-1}Y_a(\tau_0)\hat{\beta}_a = 0 \quad (35)$$

Note that K, $\bar{K}$, and $Y_a(\tau_0)$ are diagonal matrices, then the right hand side of 8–th and 9–th term of (35) can be written as follows:

$$\begin{aligned}&(K\bar{K}^{-1} - I)\tau_0 - K\bar{K}^{-1}Y_a(\tau_0)\hat{\beta}_a \\ &= \begin{bmatrix} \frac{k_1}{\bar{k}_1} - 1 & & \\ & \ddots & \\ & & \frac{k_n}{\bar{k}_n} - 1 \end{bmatrix}\tau_0 \\ &- \begin{bmatrix} -\frac{k_1}{\bar{k}_1}\tau_{01} & & \\ & \ddots & \\ & & -\frac{k_n}{\bar{k}_n}\tau_{0n} \end{bmatrix}\hat{\beta}_a \\ &= Y_a(\tau_0)(\bar{\beta}_a - K\bar{K}^{-1}\hat{\beta}_a) \end{aligned} \quad (36)$$

where $\bar{\beta}_{ai} = 1 - \left(k_i/\bar{k}_i\right)$. k_i and $\bar{k}_i$ are the i–th diagonal elements of K and $\bar{K}$, respectively.

From (36), we can rewrite (35) as

$$M(\theta)\dot{s} + C(\theta,\dot{\theta})s + e + Bs + Y_e\tilde{e}_\beta - v - w + Y_a(\tau_0)\tilde{\beta}_a = 0 \quad (37)$$

where $\tilde{\beta}_a = \bar{\beta}_a - K\bar{K}^{-1}\hat{\beta}_a$ and, hence,

$$\dot{\tilde{\beta}}_a = -K\bar{K}^{-1}\dot{\hat{\beta}}_a \quad (38)$$

4. STABILITY ANALYSIS

Theorem 1. Given robotic system defined by (15), the control torque input given in (26) with (28) and (29), along with the adaptation law given in (30) and (31). If a new control input v is given as

$$v = -\frac{1}{2}R^{-1}s, \quad (39)$$

then, the closed–loop system is sub–optimal in the sense that it minimizes the upper bound on the quadratic cost functional J defined by

$$J = \sup_{w\in\mathcal{L}_2}\left\{\int_0^t \left(x^\top Qx + v^\top Rv - \gamma^2 w^\top w\right)\mathsf{d}\xi + V\right\} \quad (40)$$

for any $t \le \infty$, where $x = [e^\top \;\; s^\top]^\top$, $\gamma \in \mathbb{R}$ is positive constant, $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{n\times n}$ are positive definite matrixes, respectively. Besides, $V(t) \in \mathbb{R}$ is the following nonnegative function :

$$V = \frac{1}{2}e^\top e + \frac{1}{2}s^\top M(\theta)s + \frac{1}{2}\tilde{e}_\beta^\top\Gamma_\beta^{-1}\tilde{e}_\beta + \frac{1}{2}\tilde{\beta}_a^\top\Gamma_a^{-1}\bar{K}K^{-1}\tilde{\beta}_a \quad (41)$$

Proof

After taking the time derivative of (41), substituting for the closed–loop error system (37), utilizing **C–2** and (38), the following expression is obtained

$$\begin{aligned}\dot{V} &= -\lambda e^\top e + e^\top s - s^\top C(\theta,\dot{\theta}) - s^\top e \\ &\quad - s^\top Bs - s^\top Y_e\tilde{e}_\beta + s^\top v + s^\top w - s^\top Y_a(\tau_0)\tilde{\beta}_a \\ &\quad + \frac{1}{2}s^\top M(\theta)s - -\tilde{e}_\beta^\top\Gamma_\beta^{-1}\dot{\tilde{e}}_\beta - \tilde{\beta}_a^\top\Gamma_a^{-1}\dot{\tilde{\beta}}_a \\ &= -\lambda e^\top e - s^\top Bs - \tilde{e}_\beta^\top\left(\Gamma_\beta^{-1}\dot{\tilde{e}}_\beta + Y_e^\top s\right) \\ &\quad - \tilde{\beta}_a^\top\left(\Gamma_a^{-1}\dot{\tilde{\beta}}_a + Y_a(\tau_0)s\right) + s^\top v + s^\top w \\ &= -s^\top Bs + s^\top v + s^\top w \end{aligned} \quad (42)$$

Then, we give the virtual system as

$$\dot{x} = f(x) + g(x)w + g(x)v \quad (43)$$

$$x = \begin{bmatrix} e \\ s \end{bmatrix}, f(x) = \begin{bmatrix} -\lambda e \\ \alpha Is \end{bmatrix}, g(x) = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (44)$$

For the virtual system, we give the following Hamilton–Jacobi–Isaacs (HJI) equation

$$\frac{\partial\tilde{V}}{\partial s}f(s(t)) + \frac{1}{4}\left\{\frac{\|\mathcal{L}_g\tilde{V}\|^2}{\gamma^2} - \left(\mathcal{L}_g\tilde{V}\right)^\top R^{-1}\left(\mathcal{L}_g\tilde{V}\right)\right\} + x^\top Qx \le 0 \quad (45)$$

where $\tilde{V}(t) \in \mathbb{R}$ is following nonnegative function :

$$\tilde{V}(t) = \frac{1}{2}e^\top e + \frac{1}{2}s^\top s = \frac{1}{2}x^\top x, \quad (46)$$

and positive definite matrix Q is given as follows:

$$Q = \begin{bmatrix} hI & 0 \\ 0 & B + (\alpha + h)I \end{bmatrix} \quad (47)$$

We can derive positive definite matrices Q and R satisfying HJI equation (45) with its solution $\tilde{V}(t)$ (46) and positive constant γ. Then, we can evaluate $\dot{V}$ in the final equation of (42) as follows:

$$\begin{aligned}\dot{V} &\le -s^\top(B + \alpha I)s + \frac{1}{4}s^\top R^{-1}s - \frac{1}{4\gamma}s^\top s - hx^\top x \\ &\quad + s^\top v + s^\top w \\ &= -s^\top(B + \alpha I)s + \left(v + \frac{1}{2}R^{-1}s\right)^\top R\left(v + \frac{1}{2}R^{-1}s\right) \\ &\quad - v^\top R^{-1}v - \gamma\left\|w - \frac{1}{2\gamma^2}s\right\|^2 + \gamma^2 w^\top w - hx^\top x \end{aligned} \quad (48)$$

Consequently, we select the new control input v as (39), then we can conclude that the all signals in the closed–loop system are bounded and the v is a sub–optimal control input which minimize the upper bound on the cost functional (40).

Remark

As we can see (40), the $\mathcal{L}_2$ gain from the unknown bounded dead–zone disturbance and the input torque disturbance to tracking error s is prescribed by given constant γ, that is, the $\mathcal{H}^\infty$ control performance is attained adaptively for generalized output $\sqrt{x^\top Qx + v^\top Rv}$.

5. NUMERICAL SIMULATIONS

The controller developed in (26) and the adaptation law given in (30) and (31) were simulated for a two–link robot

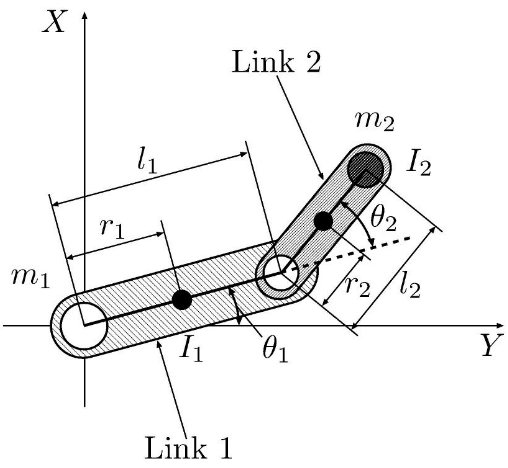

Fig. 2. 2–DOF robotic system

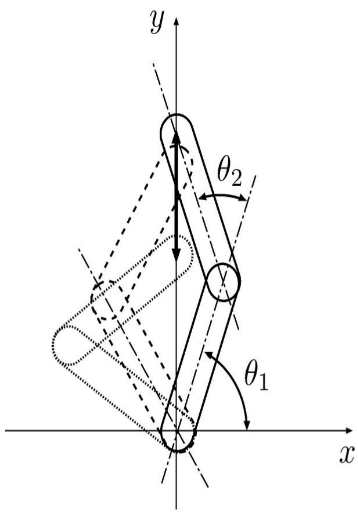

Fig. 3. Reference trajectories

planar manipulator. We consider a two–link robotic system moving in the horizontal plane shown in the Fig.2. Let us assume that the parameters of the unloaded manipulator are known and are given by $m_1 = 12.27$[kg], m_2=2.083[kg], l_1=0.2[m], l_2=0.2[m], r_1=0.063[m], r_2=0.080[m], $I_1 = 0.1149$[kgm^2], $I_2 = 0.0144$[kgm^2]. The properties C–1∼C–3 are satisfied in this manipulator and we can directly apply the proposed control method to the robotic system.

The control objective is to track the robot arm angle to the reference signals as

$$\tilde{\theta} = \begin{bmatrix} \theta_{1d} \\ \theta_{2d} \end{bmatrix} = \begin{bmatrix} 0.9 \sin 4t \\ -1.8 \sin 4t \end{bmatrix}, \qquad (49)$$

and its trajectories can be depicted as Fig. 3. The initial position is given as $[\theta_1(0)\ \theta_2(0)]^{\mathsf{T}}$=[0.0 0.0]$^{\mathsf{T}}$[rad]. Moreover, the physical parameters of dead–zone (10) are given as $\mu_1 = 1.0$, $\mu_2 = 1.0$, $b_{l1} = -2.0$, $b_{r1} = 2.0$, $b_{l2} = -0.5$, $b_{r2} = 0.5$. These parameters are identified by the experimental equipment.

To exhibit the effect of input uncertainty compensation, we also apply the controller as follows:

$$\tau = \bar{K}^{-1} \left(-e - Bs + Y_e \beta_0 + Y_e \hat{e}_\beta \right), \qquad (50)$$

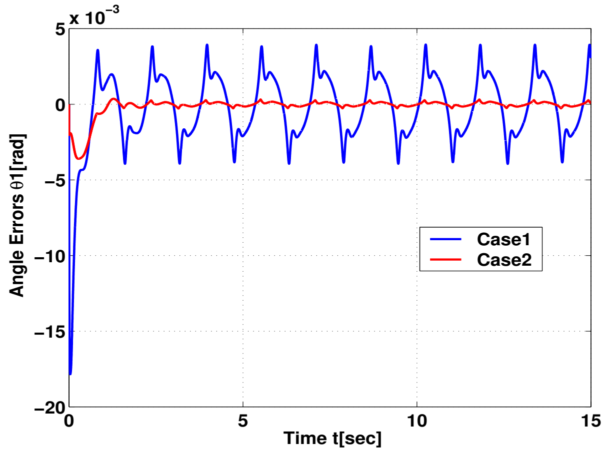

Fig. 4. Error Angles θ_1

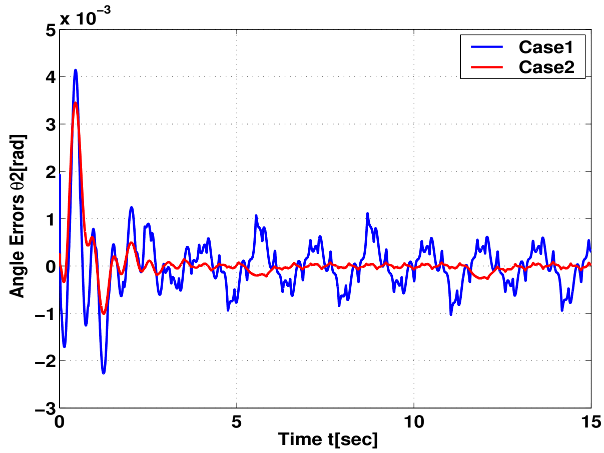

Fig. 5. Error Angles θ_2

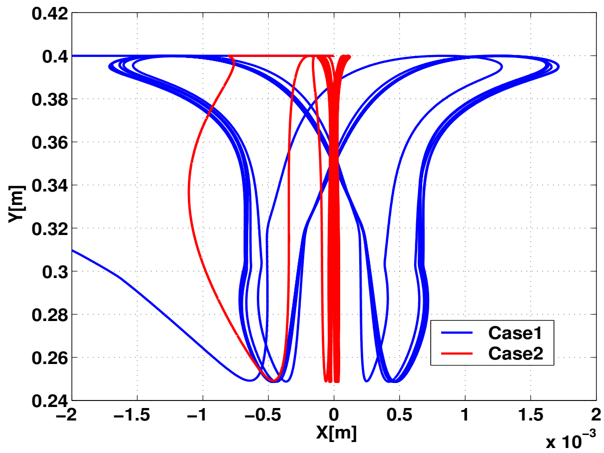

Fig. 6. The tip positions

This controller (50) ignores to compensate the input torque uncertainty, we call this case as 'case 1.' In all simulations we selected the design parameters as $\bar{K} = \mathrm{diag}\{1.0, 1.0\}$, $\lambda = 8.0$, $B = \mathrm{diag}\{30.0, 40.0\}$, $\Gamma_\beta = [3.0, 2.5, 1.5]^{\mathsf{T}}$, $\Gamma_a = \mathrm{diag}\{100.0, 180.0\}$, and $R = \mathrm{diag}\{0.05, 0.08\}$.

Figs. 4 and 5 show the error angles of link 1 and 2, respectively. As we can see these figures that our proposed

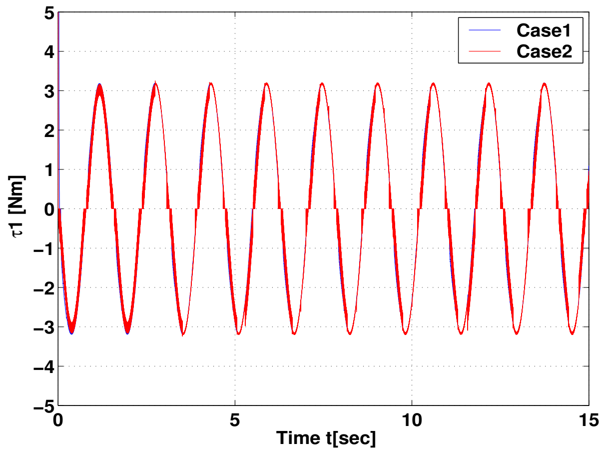

Fig. 7. input torques τ_1

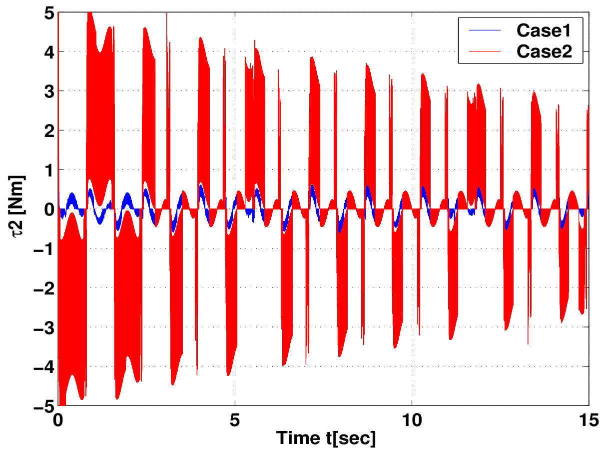

Fig. 8. input torques τ_2

method (case 2) shows a good tracking performance compared with case 1. Fig. 6 depicts the tip position of robotic manipulator. Using our proposed method, the tip position converges to the reference trajectory (i. e., 0[m] at x–axis, 0.25 $\sim$ 0.4 [m] at y–axis), but case 1 does not converge. Figs. 7 and 8 show the input torque for each link. It is found from these figures that our proposed method (case 2) exhibits a good compensation performance compared with case 1. As we can see these simulation results, we should treat the unknown input characteristics, especially, dead–zone phenomena, not only input torque disturbance but also input torque uncertainty.

6. CONCLUSIONS

This paper considered the problem of link position tracking control for robot manipulators with input toque uncertainty. It was assumed that the input torque uncertainty can be regarded as dead-zone effects at each link of manipulator and all the system parameters for dead–zone model and robot were unknown. The proposed method ensured that the unknown parameters were estimated adaptively, besides, an approximated errors of dead–zone model and external disturbances are attenuated by means of $\mathcal{H}_\infty$ control performance. Simulation results were given to illustrate the effectiveness of our proposed method. From the simulation results, we had shown that we should compensate the external disturbance to the robotic system and input uncertain characteristics, independently.

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