

# A Construction of Disturbance Observer to Cope with Frequency Variation and Its Application to Vibration Suppression Control System<sup>\*</sup>

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**Abstract:** In this paper, a synthesis method of new disturbance observer, which can cope with frequency variation, is proposed. The main idea is to synthesize the disturbance observer as a gain-scheduled controller. The proposed disturbance observer gives the same low sensitivity property for disturbances with frequency variation as the conventional disturbance observer does. As an application, we consider vibration suppression control system for a flexible beam with multiple vibration modes. By numerical simulations and experiments, the proposed control scheme is shown superior to the conventional disturbance observer with respect to vibration suppression as compared.

# 1. INTRODUCTION

Disturbance observer is effective controller to reduce adverse effects of disturbances. There are many articles which demonstrate the usefulness of disturbance observers(Godler al. (1994) and Ohnishi (1995)). In these articles, disturbances are assumed to have constant and single frequency. On the other hand there are many industrial situations where the disturbance frequency varies according to the interaction with the environment. For these disturbances the conventional disturbance observer cannot be applied because it is effective only to the disturbance with fixed frequency.

Also, in order to asymptotically cancel the adverse effects of periodic disturbance, a new synthesis method of repetitive control has been proposed (Yu et al. (2001)). Based on the observation, digital repetitive controller can be synthesized to reduce periodic disturbance whose period is not exactly an integer multiple of the sampling interval. However, when the disturbance is periodic with the period different from an integer multiple of the sampling interval, it may be difficult to attain perfect asymptotic disturbance reduction. Thomas et al. (1997) have proposed the tip positioning controller of a single flexible link using variable structure model following controller. Since the controller can force the error between the model and the plant variables to zero as time tends to infinity, responses of the tip positioning can be improved by a suitable choice of the damping factor and natural frequency of the desired

\* This work was supported in part by Hitech Research Center, Project for Private University from the Ministry of Education, Culture, Sports, Science and Technology. model. Choi et al. (1999) have proposed vibration control and position tracking control of a flexible smart structure using the quantitative feedback theory(QFT). The QFT is a frequency domain design technique utilizing the Nichols chart. Based on the technique of loop shaping, controller is synthesized so that specifications of robust stability, tracking and disturbance reduction are satisfied. However, in these articles, the variation caused by disturbance frequency has not been considered.

Meanwhile, the vibration suppression control using  $H_{\infty}$ controller has been studied. Sivriogle et al. (1997) have proposed an active vibration control method by means of LMI-based mixed  $H_2/H_{\infty}$  state feedback control. Attainable specifications of both time-domain and frequencydomain are shown by numerical simulations. Tchernychev et al. (1997) have proposed a synthesis method of robust controllers for an experimental flexible beam with noncolocated sensors-actuator. By using the constrained  $H_{\infty}$  control approach (Sideris et al. (1993)), it is shown that time-domain constraints are directly treated without conversion into the frequency-domain. By these control methods, the vibration suppression performance in the wide range of frequency can be carried out. However, since the control performance is decided by minimizing the  $H_{\infty}$ norm of the transfer function matrix from disturbance input to the control output, the obtained performance of control system may be conservative.

To improve the conservative performance, Yang et al. (1997) have proposed vibration suppression control of smart structures using backpropagation neural network. The robustness of the controller against the model uncertainty of the first bending mode in the frequency domain



Fig. 1. Disturbance observer to cope with frequency variation (DOFV)

is demonstrated. Zhou et al. (2001) have proposed the approach to estimate the imbalance force and the imbalance itself of a rigid rotor system during acceleration mode using a time-varying observer. However, the control strategy for time-variant system has not been considered.

In this paper, a new synthesis method of disturbance observer which can cope with the frequency variation (DOFV) is proposed. The main idea is to synthesize disturbance observer as a gain-scheduled controller based on an LMI-based approach. The proposed disturbance observer has following properties:

- (1) Same low sensitivity property as traditional disturbance observer can be obtained for wide frequency area.
- (2) Many design constrains are expressed as matrix inequalities and they can be solved efficiently.

To demonstrate the usefulness of the proposed DOFV, we apply to the vibration suppression control system of flexible beam. Both simulations and experiments show that vibration of the beam excited by disturbance is remarkably suppressed in the wide range of the frequency.

The following notations are used throughout in this paper. Re $\{\lambda(M)\}$  denotes real part of eigenvalues of matrix M.  $\{A, B, C, D\}$  denotes the transfer function matrix with order n, i.e.

$$\{A, B, C, D\} := C (sI_n - A)^{-1} B + D$$

where  $I_n$  denotes the  $n \times n$  identity matrix.  $O_{l \times m}$  denotes the  $l \times m$  zero matrix.

# 2. CONSTRUCTION OF DOFV

#### 2.1 Problem Statement

Fig.1 depicts the overall structure of the control system for disturbance reduction.  $G_p(s)$  represents a given plant,  $r(s), u(s), d(s), y(s), n_o(s)$ , and e(s) are reference input, control input, disturbance input, measurement output, sensor noise input and error signal, respectively. K is the state feedback gain matrix to assign desired poles to the closed loop system.  $C_y(s)$  and  $C_e(s)$  are observer transfer functions for disturbance reduction. When a disturbance  $\hat{d}$ with frequency variation is impressed in plant, we change properties of  $C_y(s)$  and  $C_e(s)$  according to its frequency. By measuring of the channel output y and error signal e, we can derive the estimation  $\hat{d}$  for d. By the feedback of  $\hat{d}$  to the input, disturbance d can be reduced. Thus, by changing the properties of observer according to the disturbance frequency, disturbance can be reduced for wide range of frequency.

The plant dynamics is described by equations

$$\dot{x}(t) = Ax(t) + B\{e(t) + d(t)\} 
y(t) = Cx(t)$$
(1)

where  $x(t) \in \mathbb{R}^n$ ,  $u(t), d(t) \in \mathbb{R}^m$ ,  $y(t) \in \mathbb{R}^l$  are state, control input, disturbance, measurement output, respectively. It is assumed that the pair (A, B) is controllable and the pair (C, A) is observable.

We consider the case when disturbance d(t) is described by the following state equations:

$$\dot{x}_d(t) = A_d x_d(t), \quad x_d(t) \in R^{\xi} 
d(t) = C_d x_d(t).$$
(2)

We assume  $\operatorname{Re}\{\lambda(A_d)\} \geq 0$  and characteristic polynomial of  $A_d$  is minimal, and also the pair  $(C_d, A_d)$  is observable. Expressing the eigenvalues of  $A_d$  as

$$\lambda_s = \sigma_s + j\omega_s \text{(algebraic multiplicity } \frac{m_s}{2}\text{)}$$
  
$$\bar{\lambda}_s = \sigma_s - j\omega_s \text{(algebraic multiplicity } \frac{m_s}{2}\text{)}.$$

where  $\omega_s$  is assumed to be measured and satisfies quasistatic process (Shamma et al. (1990)). Without loss of generality, we can assume that  $A_d$  admits the following canonical form.

$$A_d = \text{blockdiag}\left\{A_{d1}, \cdots, A_{dh}\right\} \tag{3}$$

with

$$A_{ds} := \begin{bmatrix} \Lambda_s & I_2 \\ & \ddots & \ddots \\ & & \Lambda_s & I_2 \end{bmatrix} (m_s \times m_s), \ \Lambda_s := \begin{bmatrix} \sigma_s & \omega_s \\ -\omega_s & \sigma_s \end{bmatrix}$$
$$s = 1, \cdots, h.$$

For 
$$J_2 := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
, we can write  
$$A_{ds} = A_d^{\sigma_s} + \omega_s A_d^{\omega_s}$$

where

$$A_d^{\sigma_s} := \begin{bmatrix} \sigma_s I_2 & I_2 \\ & \ddots & \ddots \\ & & \sigma_s I_2 & I_2 \end{bmatrix}, A_d^{\omega_s} := \begin{bmatrix} J_2 & O_{2 \times 2} \\ & \ddots & \ddots \\ & & J_2 & O_{2 \times 2} \end{bmatrix}.$$

So,

$$\begin{split} A_{d} &= \operatorname{blockdiag} \left\{ A_{d}^{\sigma_{1}}, \cdots, A_{d}^{\sigma_{h}} \right\} \\ &+ \operatorname{blockdiag} \left\{ \omega_{1} A_{d}^{\omega_{1}}, \cdots, \omega_{h} A_{d}^{\omega_{h}} \right\} \\ &= \operatorname{blockdiag} \left\{ A_{d}^{\sigma_{1}}, \cdots, A_{d}^{\sigma_{h}} \right\} \\ &+ \sum_{s=1}^{h} \omega_{s} \operatorname{blockdiag} \left\{ O_{\sum_{i=1}^{s-1} m_{i} \times \sum_{i=1}^{s-1} m_{i}}, \ A_{d}^{\omega_{s}}, \right. \\ &\left. O_{\sum_{i=s+1}^{h} m_{i} \times \sum_{i=s+1}^{h} m_{i}} \right\} \end{split}$$

$$=: A_d^{\sigma} + \sum_{s=1}^h \omega_s A_d^s .$$

$$\tag{4}$$

By equations (1) and (3), the state equation of the augmented system including the disturbance is

$$\dot{x}_L(t) = A_L x_L(t) + B_L e(t) 
y(t) = C_L x_L(t) 
d(t) = C_d x_L(t)$$
(5)

where e(t) := u(t) - d(t) and

$$x_L(t) := \begin{bmatrix} x(t) \\ x_d(t) \end{bmatrix}, \ A_L := \begin{bmatrix} A & BC_d \\ O_{\xi \times n} & A_d \end{bmatrix}, \ B_L := \begin{bmatrix} B \\ O_{\xi \times m} \end{bmatrix},$$
$$C_L := \begin{bmatrix} C & O_{l \times \xi} \end{bmatrix}, \ C_{dL} := \begin{bmatrix} O_{m \times n} & C_d \end{bmatrix}.$$

For system (5), note that the observability of the pair  $(C_L, A_L)$  is guaranteed by the following conditions.

(1) The pair (C, A) is observable. (2) For  $\lambda_j \in \lambda(A_d)$ C

$$\operatorname{rank} \begin{bmatrix} A_L - \lambda_j I_{n+\xi} \\ C_L \end{bmatrix} = \operatorname{rank} \begin{bmatrix} A - \lambda_j I_n & BC_d \\ O_{\xi \times n} & A_d - \lambda_j I_\xi \\ C & O_{l \times \xi} \end{bmatrix}$$
$$= n + \xi, \ j = 1, \cdots, \xi.$$

Under these conditons, which are assumed here and after, the state observer for the augmented system (5) can be constructed. Next, we show a synthesis method of DOFV using minimal order observer.

#### 2.2 DOFV using Minimal Order Observer

In this sub-section, we synthesize DOFV by Gopinath's method. Assume that C has a full row rank, that is, rank C = l. Define  $q := n + \xi - l$  and

$$T := \begin{bmatrix} C & O_{l \times \xi} \\ M_1 & M_2 \end{bmatrix}$$
(6)

where  $\begin{bmatrix} M_1 & M_2 \end{bmatrix}$  is  $q \times (n + \xi)$  real constant matrix and is assigned so that T is nonsingular. We express the inverse matrix of T with  $n \times l$  matrix  $U_{11}$  as

$$T^{-1} := \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}.$$
 (7)

Now, we consider state transformation  $\bar{x}_L(t) = T x_L(t)$ , where the matrix  $T A_L T^{-1}$  can be partitioned as

$$TA_{L}T^{-1} =: \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$$
(8)

with

$$\begin{split} \bar{A}_{11} &= CAU_{11} + CBC_dU_{21}, \\ \bar{A}_{12} &= CAU_{12} + CBC_dU_{22}, \\ \bar{A}_{21} &= M_1AU_{11} + M_1CBC_dU_{21} + M_2A_dU_{21}, \\ \bar{A}_{22} &= M_1AU_{12} + M_1CBC_dU_{22} + M_2A_dU_{22}. \end{split}$$

Here, only  $\bar{A}_{21}$  and  $\bar{A}_{22}$  depend on the parameters  $\omega_s$ . It can be easily seen that  $\bar{A}_{22}$  admits the following affine dependence on  $\omega_s$ :

$$\bar{A}_{22} = \bar{A}_{22}^o + \sum_{s=1}^h \omega_s \bar{A}_{22}^s \tag{9}$$

with

$$\begin{split} \bar{A}_{22}^o &:= M_1 A U_{12} + M_1 C B C_d U_{22} + M_2 A_d^\sigma U_{22}, \\ \bar{A}_{22}^s &:= M_2 A_d^s U_{22}. \end{split}$$

On the other hand, if the pair  $(C_L, A_L)$  is observable, then the pair  $(\bar{A}_{12}, \bar{A}_{22})$  is observable too. Then eigenvalues of matrix

$$\hat{A} := \bar{A}_{22} - L\bar{A}_{12} \tag{10}$$

can be arbitrarily assigned by a suitable choice of matrix  $L. \label{eq:L}$ 

Now, we consider  $\omega_s$  to be restricted on the box

$$\omega_s^1 \le \omega_s \le \omega_s^2, \ s = 1, \cdots, h \tag{11}$$

with vertices

$$Q_1 = \left(\omega_1^1, \ \omega_2^1, \cdots, \omega_h^1\right), \ Q_2 = \left(\omega_1^1, \ \omega_2^1, \cdots, \omega_h^2\right), \ \cdots,$$
$$Q_{2^h} = \left(\omega_1^2, \ \omega_2^2, \cdots, \omega_h^2\right).$$

Then, by equation (9),  $\bar{A}_{22}$  can be represented by

$$\bar{A}_{22} = \sum_{k=1}^{2^h} q_k A_k \tag{12}$$

with

$$q_{1} := \frac{\prod_{s=1}^{h} \left(\omega_{s}^{2} - \omega_{s}\right)}{\prod_{s=1}^{h} \left(\omega_{s}^{2} - \omega_{s}^{1}\right)}, \ q_{2} := \frac{\left(\omega_{h} - \omega_{h}^{1}\right) \prod_{s=1}^{h-1} \left(\omega_{s}^{2} - \omega_{s}\right)}{\prod_{s=1}^{h} \left(\omega_{s}^{2} - \omega_{s}^{1}\right)}, \\ , \cdots, q_{2^{h}} := \frac{\prod_{s=1}^{h} \left(\omega_{s} - \omega_{s}^{1}\right)}{\prod_{s=1}^{h} \left(\omega_{s}^{2} - \omega_{s}^{1}\right)}, \ \sum_{k=1}^{2^{h}} q_{k} = 1$$

and

$$egin{aligned} A_1 &:= ar{A}_{22}^o + \sum_{s=1}^h \omega_s^1 ar{A}_{22}^s, \ A_2 &:= ar{A}_{22}^o + \sum_{s=1}^{h-1} \omega_s^1 ar{A}_{22}^s \ &+ \omega_h^2 ar{A}_{22}^h, \cdots, \ &A_{2^p} &:= ar{A}_{22}^o + \sum_{s=1}^h \omega_s^2 ar{A}_{22}^s. \end{aligned}$$

To obtain DOFV, we consider LMI-based approach. The matrix  $\hat{A}$  is stable if and only if the following Lyapunov inequality

$$X\hat{A} + \hat{A}^T X < O \tag{13}$$

admits positive definite solution. Now, if L is made as

$$L = \sum_{k=1}^{2^{h}} q_k L_k,$$
 (14)

then by equations (12) and (14) the matrix  $\hat{A}$  becomes

$$\hat{A} = \sum_{k=1}^{2^{h}} q_k \left( A_k - L_k \bar{A}_{12} \right), \qquad (15)$$

and inequality (13) is guaranteed by

$$XA_k + A_k^T X - N_k \bar{A}_{12} - \bar{A}_{12}^T N_k^T < O, \ k = 1, \cdots, 2^h$$
 (16)

where

$$N_k = XL_k, \ k = 1, \cdots, 2^h, \tag{17}$$

that is, if there are matrices X > O and  $N_k$  satisfying LMIs (16), then  $\hat{A}$  is stable. This means that the matrix  $\hat{A}$  is stabilized in the box written by equation (11). From equations (14) and (17), the observer gain matrix L is written by

$$L = \sum_{k=1}^{2^{h}} q_k X^{-1} N_k.$$
(18)

*Remark.* The set  $\lambda(\hat{A})$  can be assigned by LMI-based approach (Chilali et al. (1996)). Consequently, the dynamical equation of minimal order observer to estimate  $x_L(t)$  is

$$\dot{\zeta}(t) = \hat{A}\zeta(t) + \hat{B}e(t) + Gy(t)$$
  

$$\dot{x}_L(t) = \hat{C}\zeta(t) + \hat{D}y(t),$$
(19)

where  $\zeta(t)$  is state of minimal order observer. Letting the matrix T be set as

$$TB_L := \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}, \ \bar{B}_1(l \times m)$$

parameter matrices of equation (19) found by the Gopinath's method are

$$\hat{B} := -L\bar{B}_{1} + \bar{B}_{2} (q \times m), \ G := \hat{A}L + \bar{A}_{21} - L\bar{A}_{11} (q \times l),$$
$$\hat{C} := T^{-1} \begin{bmatrix} O_{l \times q} \\ I_{q} \end{bmatrix}, \ \hat{D} := T^{-1} \begin{bmatrix} I_{l} \\ L \end{bmatrix}.$$

Since the estimated disturbance d(t) can be obtained as

$$\hat{d}(t) = C_{dL}\hat{x}_L(t), \qquad (20)$$

DOFV  $C_e(s)$  and  $C_y(s)$  can be written as

$$C_{e}(s) = \left\{ \hat{A}, \ \hat{B}, \ \hat{C}, \ O_{n+\xi \times m} \right\}$$

$$C_{y}(s) = \left\{ \hat{A}, \ G, \ \hat{C}, \ \hat{D} \right\}.$$
(21)

#### 2.3 Stability Analysis of the Closed Loop System

In this subsection, we consider stability of the closed loop system shown in Fig.1. The transfer matrix U(s) from all exogenous inputs to all internal signals is

$$U(s) = \begin{bmatrix} U_{11}(s) & U_{12}(s) & U_{13}(s) \\ U_{21}(s) & U_{22}(s) & U_{23}(s) \\ U_{31}(s) & U_{32}(s) & U_{33}(s) \end{bmatrix},$$
(22)

where

$$\begin{split} U_{11}(s) &= U_{21}(s) := \alpha(s) = \left\{ A - BK, B, -K, I_m \right\}, \\ U_{12}(s) &= \alpha(s) \left\{ \chi C_e(s) + I_m \right\} - I_m, \\ U_{13}(s) &= U_{23}(s) = -\alpha(s)\chi C_y(s), \\ U_{22}(s) &= U_{12}(s) + I_m, \\ U_{31}(s) &:= \beta(s) = \left\{ A - BK, B, C, O_{l \times m} \right\}, \\ U_{32}(s) &= \beta(s) \left\{ \chi C_e(s) + I_m \right\}, \\ U_{33}(s) &= -\beta(s)\chi C_y(s) + I_l, \\ K_L &:= \left[ K \quad O_{m \times \xi} \right], \ \chi := C_{dL} + K_L. \end{split}$$



Fig. 2. Schematic diagram of experimental system

In equation (22), det  $U^{-1}(s) = \det \alpha^{-1}(s)$ . Since we choose a feedback gain matrix K such that  $\operatorname{Re}\lambda(A - BK) < 0$ , then  $\alpha(s), \beta(s) \in RH_{\infty}$ . Consequently, since  $U(s) \in RH_{\infty}$ , the closed loop system is internally stable.

# 3. VIBRATION SUPPRESSION CONTROL USING DOFV

#### 3.1 Experimental System

The schematic diagram for the experimental vibration suppression system is shown in Fig.2. The flexible beam is made of aluminum material with 1000 mm in length, 50 mm in width and 2 mm in thickness, and fixed by actuators at each side from each end. Two actuators of moving coil type are used as shakers to excite the flexible beam. One is used to be a disturbance source and the other is used to generate the control inputs. The flexible beam is vibrated with their largest amplitudes at the tip. To suppress the vibration at the tip of flexible beam, the acceleration is measured by using an accelerometer which is attached at the beam tip through the charge amplifier. By integrating the measured acceleration, the displacement and the velocity are obtained. The obtained displacement is sent to DSP via an A/D converter. The DSP which is linked by a personal computer is used to implement the synthesized controllers and to generate the disturbance signal. The control signal which is computed by DSP is sent to power amplifier to generate control force. The disturbance signal with frequency variation which is computed by DSP is also sent to power amplifier to generate exciting force. On the other hand, the information about disturbance frequency is directly given to DOFV.

The model of the flexible beam is given by identification technique of frequency response as follows.

$$G_p(s) = \frac{K_u(s^2 + 2\zeta_z\omega_z s + \omega_z^2)}{(s^2 + 2\zeta_{p1}\omega_{p1}s + \omega_{p1}^2)(s^2 + 2\zeta_{p2}\omega_{p2}s + \omega_{p2}^2)}$$
(23)

where  $K_u = 1525$ ,  $\zeta_z = 0.02$ ,  $\omega_z = 2\pi \times 20.3$ ,  $\zeta_{p1} = 0.015$ ,  $\omega_{p1} = 2\pi \times 16.1$ ,  $\zeta_{p2} = 0.015$ ,  $\omega_{p2} = 2\pi \times 24.6$ . As  $G_p(s)$  is stable, the closed loop system in Fig.1 is stabilized by  $K = O_{1\times 4}$ .

### 3.2 Synthesis of DOFV

The control system is constructed by the observer-based controller. The convergent rate of  $\hat{x}_L(t)$  approaching  $x_L(t)$ 

is determined by the observer poles. So, the observer poles must be placed sufficiently left distant from imaginary axis. To find the observer gain such that  $\left|\operatorname{Re}\left\{\lambda(\hat{A})\right\}\right|$  is larger than  $\left|\operatorname{Re}\left\{\lambda(A)\right\}\right|$ , inequality (16) and LMIs for pole placement constraints(Chilali et al. (1996)) should be simultaneously solved.

Consequently, for instance,  $C_e(s)$  and  $C_y(s)$  at f = 16.1 Hz can be obtained as

$$C_e(s) = \frac{-1.090 \times 10^4 \left\{ (s + 2.551)^2 + (1.275 \times 10^2)^2 \right\}}{(s + 2.603)^2 + (1.274 \times 10^2)^2} \\ \times \frac{s - 6.622 \times 10}{\left\{ (s + 4.930)^2 + (1.091 \times 10^2)^2 \right\} (s + 6.968 \times 10)}$$
(24)

$$C_y(s) = \frac{7.148 \left\{ (s+1.517)^2 + (1.011 \times 10^2)^2 \right\}}{(s+2.603)^2 + (1.274 \times 10^2)^2} \\ \times \frac{(s+2.318)^2 + (1.545 \times 10^2)^2}{(s+4.930)^2 + (1.091 \times 10^2)^2} \\ \times \frac{s-6.622 \times 10}{s+6.968 \times 10}.$$
(25)

#### 4. SIMULATIONS

In this section, simulation results of vibration suppression control using DOFV are shown. Assume that the disturbance is a sinusoidal signal written by

$$A_d = 2\pi f \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad (26)$$

where f is disturbance frequency and the variation range is

$$10 [Hz] \le f \le 30 [Hz].$$
 (27)



Fig. 3. Output responses of plant without control



Fig. 4. Output responses of control system with disturbance observer (at f = 16.1 [Hz])



Fig. 5. Output responses of control system with disturbance observer (at f = 24.6 [Hz])



Fig. 6. Output responses of control system with DOFV

The frequency is started from 10 Hz and is increased at the rate of 1/3 Hz per second.

Fig.3 shows the deflection of the beam tip in case without control. From this result, it is confirmed that the natural frequency of the first bending mode is about 16.1 Hz and the natural frequency of the second bending mode is about 24.6 Hz, respectively. Fig.4 and Fig.5 show the control performances of the conventional disturbance observer synthesized at the fixed frequency 16.1 Hz and 24.6 Hz, respectively. From these results, it can be stated that the conventional disturbance observer suppresses the vibration at only single frequency specified in the synthesis. Fig.6 shows the control performance of the proposed DOFV. Compared to previous results, the control performance of the proposed DOFV is remarkably improved.

#### 5. EXPERIMENTS

In this section, experimental results of vibration suppression control using DOFV are shown. We implement the proposed DOFV into DSP and perform calculations in sampling time 2 msec. Experimental results are shown in figures from Fig.7 to Fig.10. Each condition of control parameters of these results corresponds to that of the simulation results from Fig.3 to Fig.6, respectively. Comparing the simulation results with the experimental results shows that the experimental results have almost the same excellent performance as the simulation results.

Also, in the neighborhood of time 30 sec, the result of performance depicted in Fig.10 is slightly worse than the simulation result in Fig.6. This may be caused by the existence of anti-resonance point of flexible beam. However, since the vibration is also suppressed naturally at the neighborhood of anti-resonance frequency, this will not be disadvantage in practical application.





Fig. 10. Output responses of control system with DOFV

From these results, it is shown that the proposed DOFV can be successfully applied to the vibration suppression control system.

# 6. CONCLUDING REMARK

In this paper, a design technique to reduce the adverse effects of disturbance with frequency variation is proposed. The proposed controller is called the Disturbance Observer to Cope with Frequency Variation (DOFV). DOFV is theoretically improved from conventional disturbance observer which gives excellent sensitivity property for a periodic disturbance with single frequency and it is synthesized as gain-scheduled controller using a LMI-based approach. Therefore, DOFV has the low sensitivity property against the disturbance with frequency variation. Moreover, in the LMI-based approach, many design constrains can be written by matrix inequalities and they can be solved numerically very efficiently.

In order to verify the effectiveness of disturbance reduction using DOFV, the numerical simulations and experiments are carried out. These results of both the simulation and experiment demonstrate almost the same excellent performance of vibration suppression. From these results, it can be stated that DOFV is useful and noble controller to reduce the disturbance with frequency variation.

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