Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Implementation of Disturbance Attenuation System Based on Frequency Estimation ⋆

Tatsuo Narikiyo * **Katsuhiko Fuwa** ** **Takeshi Murano** ***

* *Toyota Technological Institute, RIKEN BMC Research Center, Nagoya, Japan (Tel: +81-52-809-1816; e-mail: n-tatsuo@toyota-ti.ac.jp).*
** *Nagoya Institute of Technology, Nagoya, Japan (Tel: +81-52-735-5444; e-mail: fuwa@elcom.nitech.ac.jp)*
*** *Toyota Technological Institute, Nagoya, Japan*

Abstract: A well-known control system that can reduce the adverse effects of disturbances is a disturbance observer. Whenever we apply disturbance observer, disturbance frequency should be known and constant. However, in many cases of industrial systems disturbance frequency is varied for some frequency range. Therefore, it may be difficult to reduce the adverse effect of such disturbance by use of the traditional disturbance observer. In this paper, a design method of disturbance attenuation system that can cope with the frequency variation (DOFV) is proposed. The main idea of this design method is to combine DOFV with frequency estimator that can estimate disturbance frequency in real time. Even though the proposed disturbance attenuation system is low degree and low gain controller, it has superior steady state characteristic. Numerical simulations and experiments show the usefulness of the proposed disturbance attenuation system.

1. INTRODUCTION

To reduce the adverse effects of disturbances is one of the most important control problems. Therefore disturbance attenuation control problems have been intensively studied. For example, high gain H_∞ observer(Fujita et al. (1991)) and high gain observer to cope with unknown inputs(Sagara et al. (1997)) have been proposed. These observers are useful for disturbance attenuation problems caused by unknown disturbances whose frequencies are in some range of spectrum. However, these high gain approaches cause the problem of noise sensitivity. Repetitive controller(She et al. (1997)) and disturbance observer(Ohnishi et al. (1987)) have been applied to the disturbance attenuation problem caused by pure sinusoidal disturbance. To apply these disturbance attenuation systems a frequency of the disturbance has to be known and to be constant. Therefore it may be difficult to reduce the effects of disturbance whose frequency is unknown and varied. Sensitivity reduction approach with H_∞ controller(Mita et al. (1995)) has been shown to be useful for disturbance attenuation problem caused by these disturbances by means of the suitable design of weighting function in the neighbourhood of resonant frequency. However, since design of the weighting function is extremly sophisticated in this approach, the obtained controller may be resulted too conservative. On the other hand, we have proposed a disturbance observer which can cope with frequency variation(DOFV)(Fuwa et al. (1998, 2002)). The excellent disturbance attenuation properties of DOFV for the pure sinusoidal disturbance have been shown by experiments. The main advantages of DOFV are as follows.
1. Same low sensitivity property as traditional disturbance observer can be obtained for wide frequency area.
2. Many design constraints are expressed as matrix inequalities and they can be solved efficiently.
However in order to apply DOFV we have to measure the disturbance frequency in real time. Therefore frequency estimation scheme is required to apply DOFV.

In this paper we propose a design method of disturbance attenuation system to apply to the real industrial system. The main idea of this design method is to combine DOFV with frequency estimator(Hus et al. (1999)) that can estimate disturbance frequency in real time. To demonstrate the usefulness of the proposed disturbance attenuation system, it is applied to experimental system. Throughout this paper the following assumptions for the disturbance are required.

(1)Matching condition is satisfied
(2)Disturbance is a pure sinusoidal signal
(3)Disturbance frequency is varied stepwise in some range of spectrum
(4)Duration time of constant frequency is sufficiently long

This paper is organized as follows. In Section 2 we discuss about frequency estimation. In Section 3 we propose a new disturbance attenuation system by means of DOFV. In Section 4 the proposed system is numerically applied to both the stable and unstable plant. Finally we apply to the experimental system to demonstrate the usefulness of the proposed disturbance attenuation system.

⋆ This work was supported in part by both the Scientific Research Fund (B)19360110 and Hitech Research Center, Project for Private University from the Ministry of Education, Culture, Sports, Science and Technology.

2. FREQUENCY ESTIMATOR

In this section we briefly introduce frequency estimator proposed by Hus et al. (1999).

2.1 Adaptive Notch Filter

We consider a pure sinusoidal signal

$$d(t) = k \sin \omega_* t,\ \omega_* > 0, \tag{1}$$

where ω_* is unknown frequency and k is an amplitude, $k > 0$, is also unknown. To estimate the unknown frequency we introduce the following dynamical system.

$$\begin{cases} \ddot{z}(t) + 2\zeta\omega(t)\dot{z}(t) + \omega(t)^2 z(t) = \omega(t)^2 d(t) \\ \dot{\omega}(t) + \gamma z(t) \left\{ \omega(t)^2 d(t) - 2\zeta\omega(t)\dot{z}(t) \right\} = 0 \end{cases} \tag{2}$$

Where $\omega(t)$ represents the estimated frequency, $\zeta > 0$ and $\gamma > 0$ are tuning parameters to relieve the conflict between estimation speed and noise sensitivity. Choosing the initial condition as

$$\left[\, z(t_o) \quad \dot{z}(t_o) \quad \omega(t_o) \,\right]^T = \left[-\frac{\hat{k}}{2\zeta} \quad 0 \quad \omega_o \right]^T,$$

where $\hat{k}$ and ω_o are initial guesses for k and ω_*, solution to the dynamical system(2) converges to stable limit cycle with constant state $\omega(t) = \omega_*$.

$$\begin{aligned} &\left[\, z(t) \quad \dot{z}(t) \quad \omega(t) \,\right]^T \\ &= \left[-\frac{\hat{k}}{2\zeta} \cos \omega_* t \quad \frac{\hat{k}}{2\zeta}\omega_* \sin \omega_* t \quad \omega_* \right]^T \end{aligned}$$

This means that if the output of the dynamical system(2) excited by sinusoidal input $d(t)$ is set to $\omega(t)$, it is regulated to the frequency ω_* of disturbance $d(t)$. We call this dynamical system Adaptive Notch Filter(ANF).

To design ANF, we have to suitably choose ζ and γ and initial conditions. Unfortunately these parameters cannot be designed theoretically. Therefore we decide these parameters numerically by means of simulations.

2.2 Simulation for ANF

To illustrate the performances of given ANF, we present some simulations. Equation of disturbance $d(t)$ is shown in eqn.(1). Amplitude of this disturbance is constant $k = 1$ and frequency ω_* is varied ramdomly stepwise every 5 second in the range of frequency $[2\pi \times 14 \quad 2\pi \times 18]$. The initial conditions of ANF are $[\, z_1 \quad z_2 \quad z_3 \,]^T = [\, 0 \quad 0 \quad 2\pi \times 16 \,]^T$ and $\zeta = 0.1, \gamma = 0.1$. Fig.1 shows that estimated frequency can be tracked to the variation of frequency with small delay of estimation. Fig.2 shows a trajectory of solution to eqn.(2) in the state space. The trajectory is asymptotically converged to stable limit cycle with constant state $z_3 = \omega_*$.

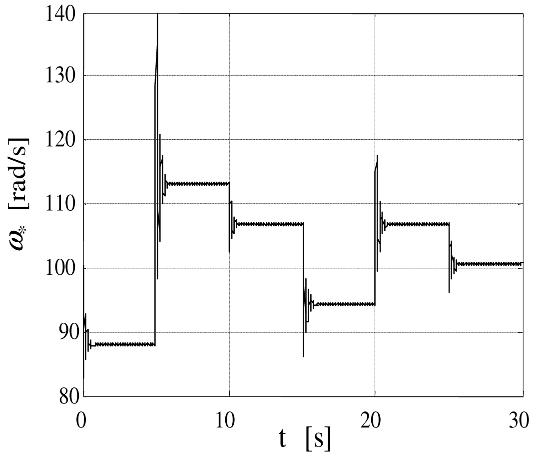

Fig. 1. Estimation of stepwise varied angular velocity

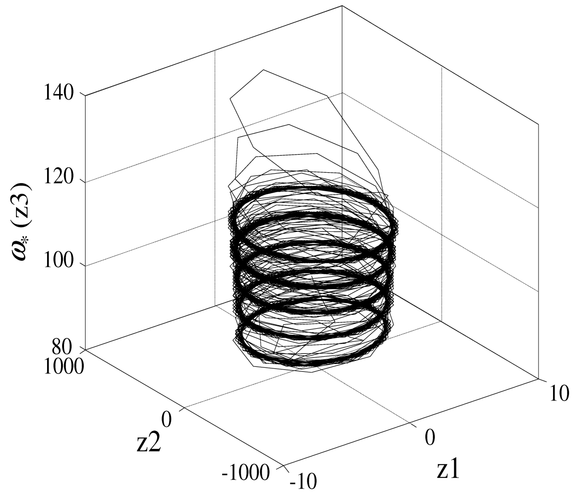

Fig. 2. Limit cycles of ANF

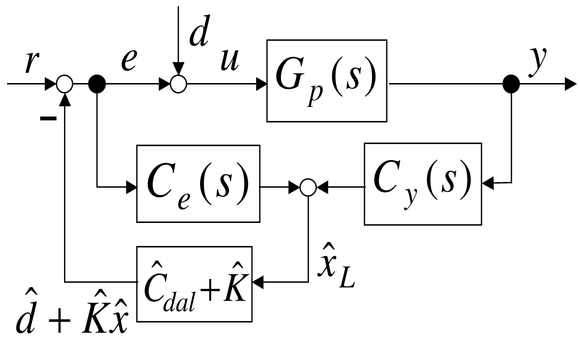

Fig. 3. Block diagram of DOFV

3. DISTURBANCE ATTENUATION SYSTEM

3.1 DOFV

The overall structure of the DOFV is depicted in Fig.3. $G_p(s)$ represents a given plant, $r(s), u(s), y(s)$ and $e(s)$ are reference input, control input, measurement output and error signal, respectively. $\hat{K}$ is the state feedback gain matrix to asign desired poles to the closed loop system.

Plant $G_p(s)$ can be represented by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t), \end{aligned} \tag{3}$$

$$G_p(s) = C\left(sI_n - A\right)^{-1} B. \tag{4}$$

Where $x(t) \in R^n, u(t) \in R, y(t) \in R^m$. The pair (A, B) and (C, A) are controllable and observable respectively.

Consider the disturbance $d(t)$ described by

$$d(t) = k \sin \omega t.$$

Where amplitude k is unknown constant and frequency ω is varied stepwise as stated before but satisfied quasi-static condition, $\dot{\omega} = 0$. Then the equation of disturbance can be rewritten as the following state equations.

$$\begin{aligned} \dot{x}_{dal}(t) &= A_{dal} x_{dal}(t) \\ d(t) &= C_{dal} x_{dal}(t) \end{aligned} \tag{5}$$

Where

$$A_{dal} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}, \; C_{dal} = [1 \quad 0]$$

Combining eqn.(3) with eqn.(5), we obtain the state equations of the augumented system including the dynamics of disturbance as follows.

$$\begin{aligned} \dot{x}_L(t) &= A_L x_L(t) + B_L e(t) \\ y(t) &= C_L x_L(t) \end{aligned} \tag{6}$$

Where

$$x_L(t) = \begin{bmatrix} x(t) \\ x_{dal}(t) \end{bmatrix}, \; A_L = \begin{bmatrix} A & BC_{dal} \\ O_{2\times n} & A_{dal} \end{bmatrix}$$
$$B_L = \begin{bmatrix} B \\ O_{2\times 1} \end{bmatrix}, \; C_L = [\, C \quad O_{m\times 2} \,].$$

And $d(t)$ is represented in terms of augumented state $x_L(t)$ as

$$d(t) = \hat{C}_{dal} x_L(t), \quad \hat{C}_{dal} = [\, O_{1\times n} \quad C_{dal} \,]. \tag{7}$$

Here we assume that the augumented system (6) satisfies following observability conditions of (C_L, A_L).

(1) (C, A) is observable.
(2) $G_p(s)$ has no invariant zeros on the imaginary axis.

To estimate the disturbance we synthesize the Luenberger observer.

$$\begin{aligned} \dot{\hat{x}}_L(t) &= \hat{A}\hat{x}_L(t) + B_L e(t) + G y(t) \\ \hat{d}(t) &= \hat{C}_{dal} \hat{x}_L(t) \end{aligned} \tag{8}$$

Where G is synthesized so that $\hat{A} = A_L - GC_L$ is stable. Since A_L depends on ω, G should be synthesized in terms of ω. Now we assume $\omega_1 \leq \omega \leq \omega_2$. Where ω_1 and ω_2 are known. Letting q_1 and q_2 be

$$q_1 = \frac{\omega_2 - \omega}{\omega_2 - \omega_1}, \; q_2 = \frac{\omega - \omega_1}{\omega_2 - \omega_1}, \; q_1 + q_2 = 1, \; q_1, q_2 \geq 0 \tag{9}$$

then

$$A_L = q_1 A_1 + q_2 A_2, \quad G = q_1 G_1 + q_2 G_2. \tag{10}$$

Finally we obtain polytopic representation of $\hat{A}$.

$$\hat{A} = q_1 (A_1 - G_1 C_L) + q_2 (A_2 - G_2 C_L) \tag{11}$$

Scheduling control theory suggests that $\hat{A}$ is stable if there exist the positive solution X and solutions $M_i (i = 1, 2)$ to the following LMIs.

$$\begin{aligned} X &> O \\ XA_i + A_i^T X - M_i C_L - C_L^T M_i^T &< O \\ M_i &= XG_i, \; i = 1, 2 \end{aligned} \tag{12}$$

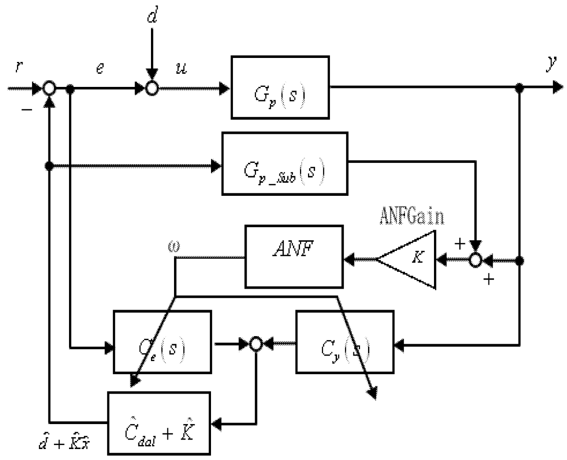

Fig. 4. Block diagram of DOFV with ANF

Furthermore to obtain the observer gain matrix G, including these LMIs we use LMIs represented the conic sector condition which asigns an area of poles(Chilali et al. (1996)). Then we obtain the disturbance observer in terms of ω such that

$$\begin{aligned} C_e(s) &= \left(sI_{n+2} - \hat{A}\right)^{-1} B_L \\ C_y(s) &= \left(sI_{n+2} - \hat{A}\right)^{-1} G(\omega) \end{aligned} \tag{13}$$

3.2 DOFV with ANF

In this subsection we propose the disturbance attenuation system which is combined DOFV with ANF to reduce the adverse effects of the unknown pure sinusoidal disturbance. Generally unknown disturbance cannot be directly used as input signal to frequency estimator. Therefore we use the measurement output influenced by the disturbance as the input signal to the ANF. Estimation performance of ANF is improved as the information of the disturbance is increased. However, since the information of the disturbance in the measurement output is gradually reduced by virtue of DOFV, estimation performance is deteriorated. To overcome this drawback we introduce $G_{p\_sub}(s)$ to feedback estimated disturbance into ANF. $G_{p\_sub}(s)$ can be given arbitrarily if it is proper stable transfer function. But it is much better that $G_{p\_sub}(s)$ has same frequency response as plant transfer function$G_p(s)$. On the other hand, if the plant $G_p(s)$ is unstable, $G_{p\_sub}(s)$ may be designed to be constant. In this case, we have to mediate the conflict between $G_{p\_sub}(s)$ and ANF parameters(ζ, γ). The overall disturbance attenuation system is depicted in Fig.4.

4. SIMULATIONS

4.1 Stable Plant

Stable plant $G_p(s)$ is given by

$$A = \begin{bmatrix} -5.845 & -328.2 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [\, 0 \quad 2100 \,]. \tag{14}$$

Where (A, B) is controllable and (C, A) is observable. $G_p(s)$ has no invariant zeros on the imaginary axis.

Disturbance is given by (1) and frequency is varied stepwise. The initial conditions and ANF parameters ζ, γ are given as follows

$$\left.[z(t) \;\; \dot{z}(t) \;\; \omega(t)]^T\right|_{t=0} = [0 \;\; 0 \;\; 2\pi]^T ,$$
$$\gamma = 0.1, \quad \zeta = 0.45. \qquad (15)$$

Observer gain matrix G_1 and G_2 are given by LMIs in eqn.(12) and LMIs represented conic sector (sector angle 88°) condition. These parameters are

$$G_1^T = [\,12.2655 \quad 0.2880 \quad 276.644 \quad 32.0694\,],$$
$$G_2^T = [\,11.3457 \quad 0.2660 \quad 251.640 \quad 12.7449\,].$$

$G_{p\_sub}(s)$ is set to be equal to $G_p(s)$. Since A is stable, $\hat{K} = O_{1\times 2}$. Fig.5 shows estimation results by ANF. Solid line shows estimated frequencies and broken line shows diaturbance frequencies. This result illustrates that estimation is caried out successfully. Fig.6 shows measurement output controlled by the proposed disturbance attenuation system. Fig.7 shows measurement output without control, namely, disturbance response. These results demonstrate the effectiveness of the proposed disturbance attenuation system.

4.2 Unstable Plant

State space representation of unstable plant $G_p(s)$ is

$$A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix},\; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\; C = [\,1 \quad 0\,]. \qquad (16)$$

This plant has unstable poles$\{1, 2\}$. Since (A, B) is controllable, there exists $\hat{K}$ such that poles of the closed loop system is asigned to $\{-5, -6\}$. Then, we obtain $\hat{K} = [\,-28 \quad 14\,]$. $G_{p\_sub}(s)$ is set to constant1000. Observer gain matrix G_1 and G_2 are given by LMIs in eqn.(12) and LMIs represented conic sector (sector angle 89°) condition. These are

$$G_1^T = [\,0.0119 \quad -0.4355 \quad -7.9343 \quad -0.6331\,] \times 10^5,$$
$$G_2^T = [\,0.0118 \quad -0.4316 \quad -7.8289 \quad -0.3900\,] \times 10^5.$$

The initial conditions of ANF and parameters of ANF are

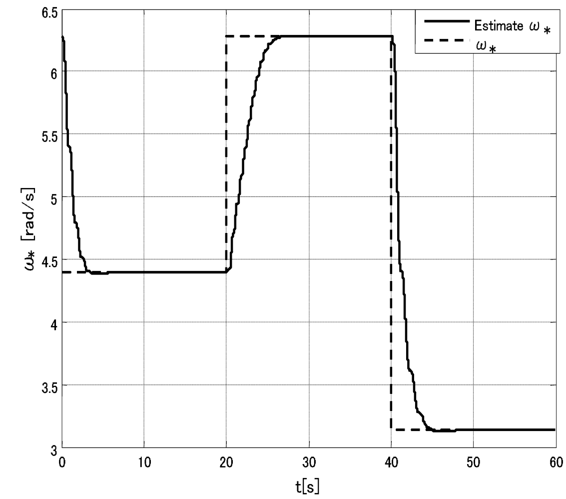

Fig. 5. Estimation of angular velocity ω_* for stable plant

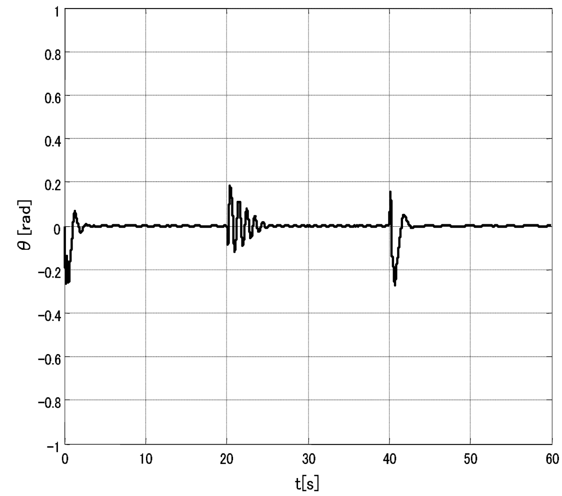

Fig. 6. Disturbance response with proposed control for stable plant

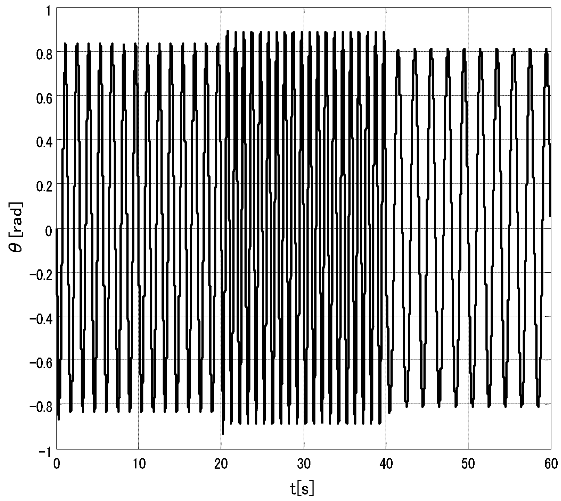

Fig. 7. Disturbance response without control for stable plant

$$\left.[z(t) \;\; \dot{z}(t) \;\; \omega_*]^T\right|_{t=0} = [0 \;\; 0 \;\; 2\pi \times 7]^T ,$$
$$\gamma = 0.1, \quad \zeta = 0.45. \qquad (17)$$

Fig.8 shows estimation results by ANF. Solid line shows estimated frequencies and broken line shows diaturbance frequencies. This result illustrates that estimation is caried out successfully similar to the stable plant. Fig.9 shows measurement output controlled by the proposed disturbance attenuation system. Fig.10 shows measurement output without control, namely, disturbance response. These results demonstrate the effectiveness of the proposed disturbance attenuation system also in the case of the unstable plant.

5. EXPERIMENTS

In this section we apply the proposed disturbance attenuation system to experimental equipment.

5.1 Experimental System

A sketch of the experimental system is depicted in Fig.11. Fig.12 shows also a detail of the system. Two disks are driven by DC servo motors separately. One of these

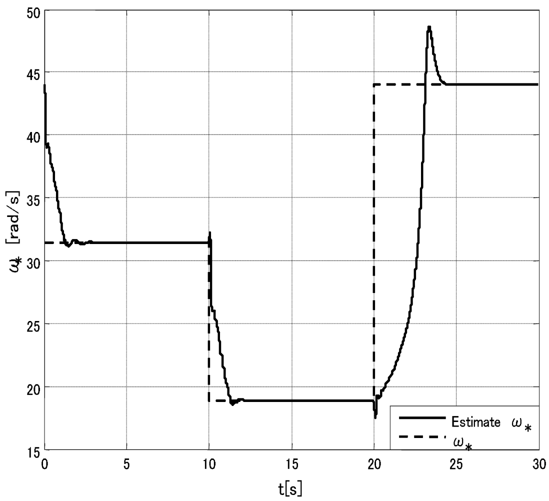

Fig. 8. Estimation of angular velocity ω_* for unstable plant

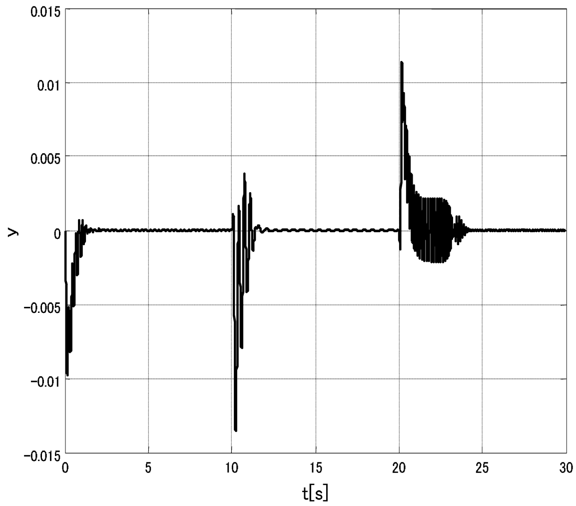

Fig. 9. Disturbance response with proposed control for unstable plant

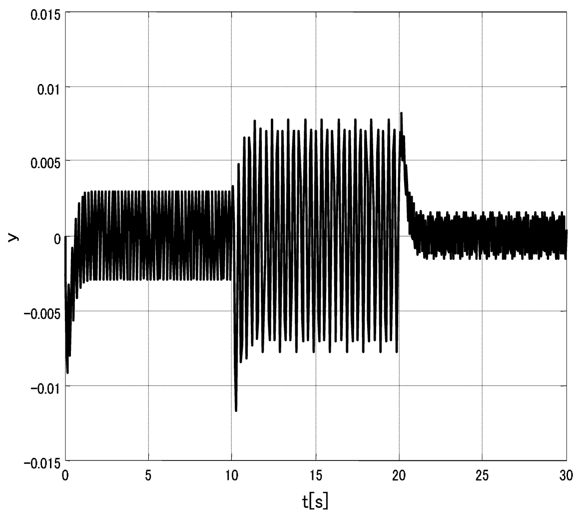

Fig. 10. Disturbance response without control for unstable plant

motors is served as control actuator. The other is served as disturbance source. Since force transmissions between each motors and each disks are caried out by timing belts(GATES Power Grip 260XL037) without elasticity, this experimental sysytem can be considered to be single inertia system with single input and single disturbance. And also matching condition is satified.

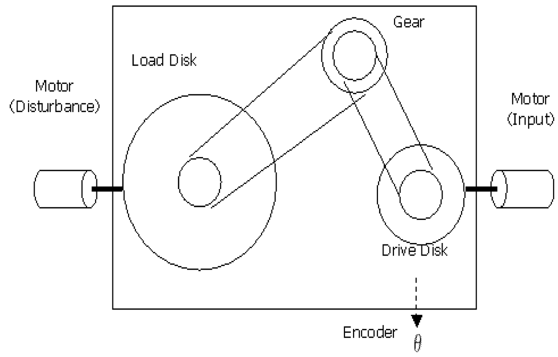

Fig. 11. Sketch of the experimental plant

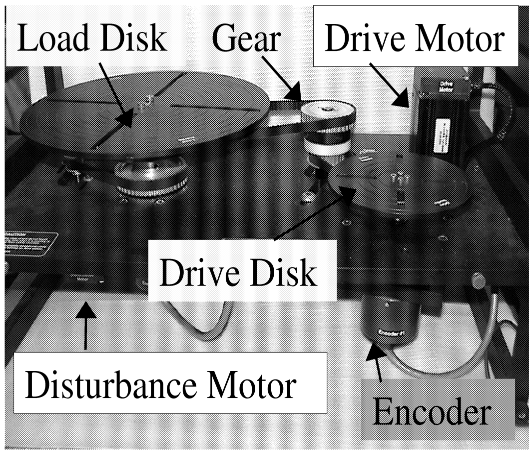

Fig. 12. Experimental system

5.2 Modeling of Plant

To derive dynamical model of the experimental system we examine relationship between sinusoidal input signal to the control motor and measurement output signal obtained from the encoder mounted on the axis of the drive disk. Frequency of the sinusoidal input is varied from 0.1[Hz] to 2.5[Hz]. By the identification method based on frequency response the dynamical model of the experimental system is obtained as

$$G_p(s) = \frac{2100}{s^2 + 5.845s + 3.282 \times 10^2}. \quad (18)$$

State space representation of this plant is shown in eqn.(14). Therefore, all design parameters of the control system are equal to those used in simulations of the stable plant in section 4.1.

5.3 Experimantal Results

We implement the disturbance attenuation system proposed in section 3 to the experimental system with sampling time 2.6[ms]. Disturbance is same as that used in subsection 4.1. Fig.13 shows estimation results by ANF.

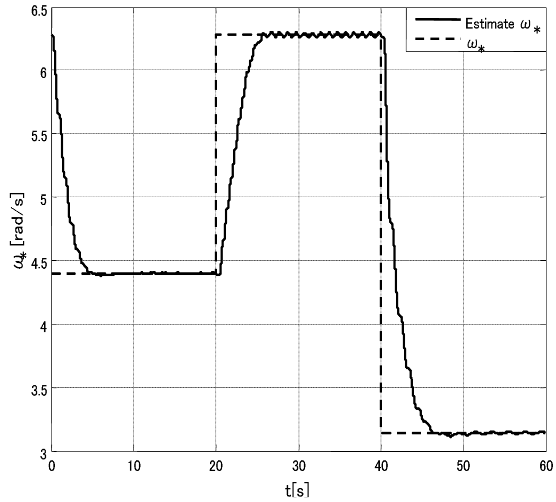

Fig. 13. Estimation of angular velocity

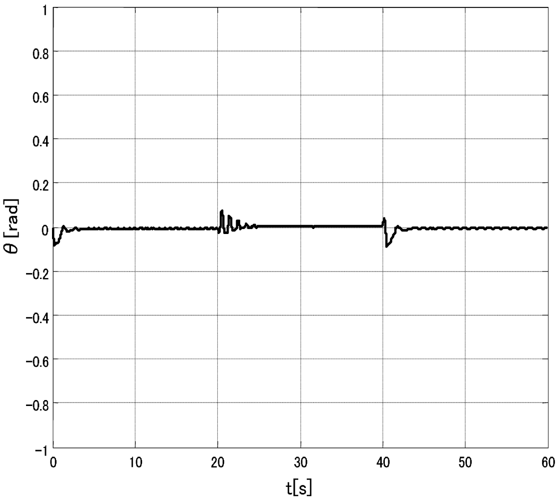

Fig. 14. Disturbance response with proposed control

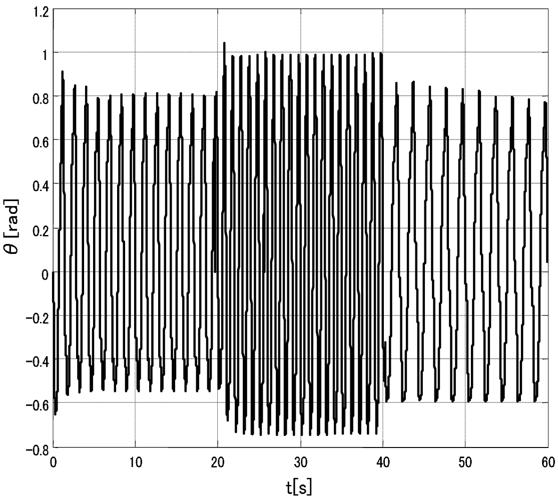

Fig. 15. Disturbance response without control

Solid line shows estimated frequencies and broken line shows diaturbance frequencies. This result illustrates that estimation is caried out successfully similar to the simulations. Fig.14 shows measurement output controlled by the proposed disturbance attenuation system. Fig.15 shows measurement output without control, namely, disturbance response. These results demonstrate the effectiveness of the proposed disturbance attenuation system similar to the simulation.

6. CONCLUDING REMARK

In this study to reduce the adverse effects caused by the pure sinusoidal disturbance we proposed the disturbance attenuation system which is combined DOFV with ANF. Even though the disturbance cannot be measured directly, the proposed system can estimate the disturbance frequency exactly from information of the mesurement output. This estimated frequency is applied to the DOFV and it cancels the disturbance on a input channel. Simulations and experiments demonstrate the effectiveness and usefulness of the proposed disturbance attenuation system.

Authors would like to thank Mr.Ikeba supporting maintenance of the experimental system and implementation of the proposed system to the experimental system.

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