Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Physical-Model-Based Control of Engine Cold Start via Role State Variables

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Abstract: The present paper describes a model representation of multi-cyclic phenomena for a multi-cylinder engine system. The model is simplified for implementation as a practical engine controller. The simplified model with physically meaningful variables can be used in design considering practical objectives and constraints more effectively. The proposed approach consists of two steps. First, an approximate analytical discrete crank angle model (i.e., a periodically time-varying state space model) is derived from the conservation laws. Second, the concept of role state variables is proposed to transform the periodically time-varying state space model into a time-invariant state space model. The stabilizability and optimality of the time-invariant state space model imply those of the periodically time-varying state space model. The time-invariant state space model is used to design cold start feedforward and feedback controllers.

1. INTRODUCTION

As the regulation of automotive performance becomes increasingly strict, the development of a high-efficiency and zero-emissions powertrain has become crucial. However, it is feared that conventional development techniques will exponentially increase the man-hours required for engine control design. In order to solve this problem, the powertrain should be controlled electronically with high performance and concisely, and model-based development should be realized as soon as possible.

The engine control system is redundant because the torque is controlled by multiple inputs, such as throttle angle, fuel injection quantity, and spark timing. These inputs have a time-delay. Specifically, in the port-injection engine, the fuel injection quantity has a delay of one cycle. Moreover, the system has both time-dependent and crank angle-dependent dynamics, that is, a continuous time nonlinear phenomenon in each cylinder is switched by discrete valve opening and closing events. In addition, the system is multi-cyclic, that is, the intake, combustion (compression / expansion), and exhaust strokes are repeated cyclically in each cylinder, where the combustion stroke does not occur simultaneously in multiple cylinders.

In current engine control design, the main control method is based on maps and if-then rules and makes use of the experience of experts. However, in the design and verification processes for a new engine, this requires a great deal of time and patience. Recently, a number of studies [Ohata], [Johansson], [Jurgen] have used empirically combined SISO models, such as the partial physical model and identified ARX models. Nevertheless, the optimality has not been discussed sufficiently because, in these methods, the inputs are calculated without consideration of the interactions among all of the state variables in the engine.

The present study proposes a model representation of multi-cyclic phenomena for multi-cylinder engine systems. Section 2 briefly introduces a benchmark model. Section 3 describes a method of deriving a simple model, which is periodically time-varying, for an engine system with complicated physical phenomena. We also introduce the concept of role state variables, by which the derived model can be transformed into a time-invariant state space model, and discuss the stabilizability of those models. Using the time-invariant state space model, optimal design examples of cold start feedforward and feedback control are demonstrated in Section 4, and Section 5 describes a numerical experiment. Finally, Section 6 presents the conclusions of the present study.

2. BENCHMARK MODEL

The SICE (the Society of Instrument and Control Engineers) Research Committee on Advanced Control of Engines has provided a benchmark model and has set the cold start control as a benchmark problem.

Figure 1 shows the benchmark model, a V6 spark ignition engine, which is composed of six submodels: an air model, a fuel model, a cylinder model, a valve-temperature model, a port-temperature model, and a piston-crank model. These models are expressed according to the following fundamental equations:

Gas equation : $P_a V_a = M_a R T_a, \quad P_{cj} V_{cj} = M_{cj} R T_{cj}$ (1)

Mass conservation :

$$dM_a / dt = m_t - \sum_{j=1}^{6} m_{ivj}, \quad dM_{cj} / dt = m_{ivj} - m_{evj} \tag{2}$$

Energy conservation :

$$\begin{cases} d\left(M_a C_{Va} T_a\right) / dt = m_t C_{Po} T_o - \sum_{j=1}^{6} m_{ivj} C_{Pa} T_a \\ d\left(M_{cj} C_{Vcj} T_{cj}\right) / dt = m_{ivj} C_{Pa} T_a - m_{evj} C_{Pcj} T_{cj} - P_{cj}\left(dV_{cj} / dt\right) + q_{wj} + q_{bj} \\ dT_{pi} / dt = \sum_{j \in J_i} \left(H_1 q_{wj} - H_2\left(T_{pi} - T_o\right)\right) \\ dT_{vj} / dt = H_3 q_{wj} - H_4\left(T_{vj} - T_o\right) \end{cases} \tag{3}$$

Piston – crank dynamics:

$$I\left(d^2q/dt^2\right)=\tau - B^T(q)\lambda,\quad B(q)(dq/dt)=0 \qquad (4)$$

Fuel dynamics:

$$\begin{cases} dF_{wpj}/dt = X_{pj}u_{fj} - Y_{pj}F_{wpj} \\ dF_{wvj}/dt = X_{vj}u_{fj} - Y_{vj}F_{wvj} \\ F_{cj} = (1 - X_{pj} - X_{vj})u_{fj} + Y_{pj}F_{wpj} + Y_{vj}F_{wvj} \end{cases} \qquad (5)$$

where P: pressure, V: volume, M: mass, T: temperature, m_t, m_{iv}, m_{ev}: flows of throttle, intake valve, and exhaust valve, C_v, C_p: specific heat at constant volume and constant pressure, q_b: combustion energy, q_w: cooling loss, q: state vector of pistions' positions and crank angle, I: matrix of inertia and mass, τ: force, B: constraint space, λ: Lagrange multipliers, F: fuel quantity, u_f: fuel injection quantity, R: gas constant, H_1, H_2, H_3, H_4: constants, X, Y: parameters, and the suffixes mean as follows: $j \in \{1,2,\cdots,6\}$: cylinder's number, $i \in \{1,2\}$: port(bank)'s number, $J_1 = \{1,3,5\}, J_2 = \{2,4,6\}$, a = surge tank, c = cylinder, o = outer air, p = port(bank), v = valve, w = wet.

Note that the cylinder model includes combustion and cooling loss, and the port-temperature model expresses the temperatures of the left and right banks. In addition, the exhaust model is approximated in the atmosphere.

The benchmark model has 13 control inputs: one throttle angle, six fuel injection quantities, six spark timings, and two outputs: engine speed and throttle flow.

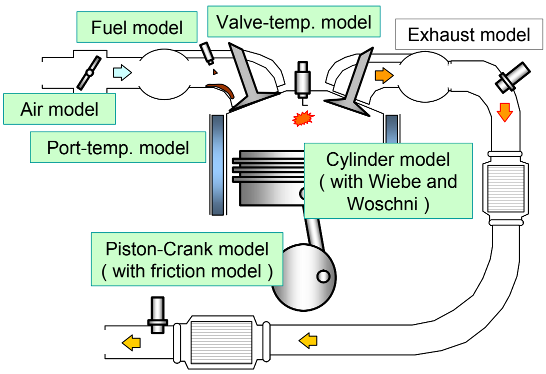

Fig. 1. Benchmark model

3. MODELING

The benchmark model is complex, where both time-dependent and crank angle-dependent dynamics exist over six cylinders. Therefore, a simplified model is needed for a practical controller design from the viewpoint of computational load. Moreover, it is important to maintain the state variables to be physically meaningful in this model reduction process, considering the practical objectives and constraints more effectively. Here, we will choose the sampling points based on the crank angle to derive a simplified discrete crank angle model as follows.

1st step: A set of nonlinear differential equations (1)-(5) are solved by approximate analytical techniques to obtain the state variables at each sampling point. Thus, a nonlinear, periodically time-varying state space model is derived.

2nd step: Using the new concept of role state variables, the periodically time-varying state space model is transformed into a time-invariant state space model.

3.1 Sampling Point and State Variable

The engine system switches the strokes of intake, combustion and exhaust by opening and closing of intake and exhaust valves. We propose that all of the states are calculated at the end of each stroke (switching point) and at the middle of each stroke (middle point). Figure 2 shows the proposed sampling points "k". Therefore, one cycle, i.e., 720 degCA is divided into six sampling points, each of which contains three switching points and three middle points. Note that the discrete crank angle model is discretized approximately every 120 degCA, not precisely every 120 degCA, because these sampling points do not occur at strictly the same time.

The discrete crank angle model has 35 state variables: seven pressures and seven masses in the surge tank and each cylinder, eight temperatures of the valve of each cylinder and of the left and right banks, engine speed, 12 fuel amounts adhering to the valve of each cylinder, and the port of each cylinder.

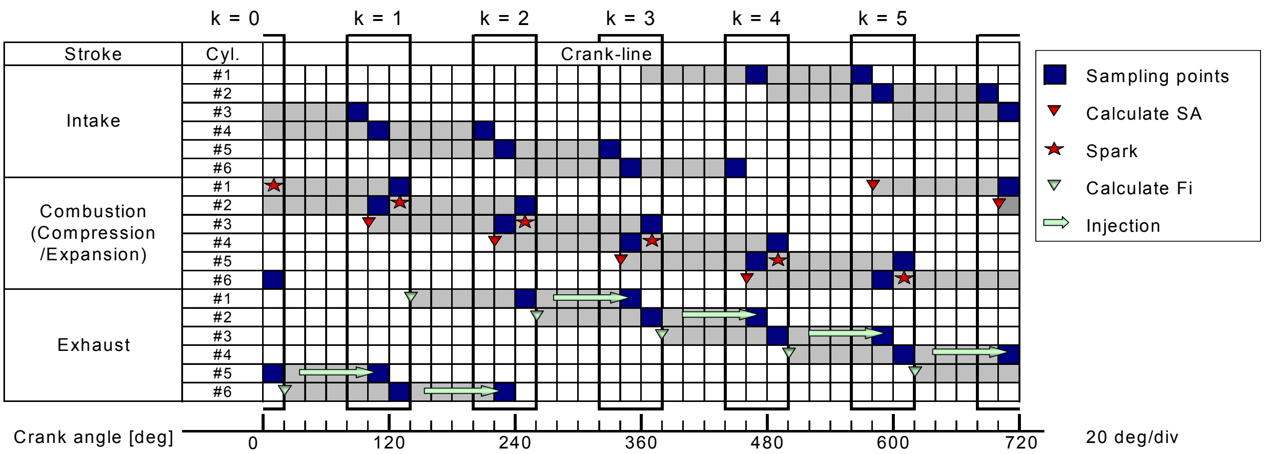

Fig. 2. Sampling points

3.2 Periodically Time-Varying State Space Model

It is difficult to obtain the exact behaviors at each sampling point because the behaviors are subject to the nonlinear differential equations (1)-(5). Therefore, we derive a periodically time-varying state space model using approximate analytical techniques as below. Note that the detailed description is omitted due to space limitations.

- The mass and pressure in the surge tank and each cylinder during the intake stroke and the exhaust stroke are obtained from the gas equation (1) and the conservation law (2) and (3) using stationary approximation at every sampling point. The derived model can strictly distinguish the case of two cylinders at the intake (exhaust) stroke at the same time from that of one cylinder at the intake (exhaust) stroke.
- The pressure in each cylinder and the piston work during the combustion stroke are obtained from the gas equation (1) and the conservation law (3) using the approximated cooling loss model. Note that mass and pressure at the opening of the exhaust valve and the piston work are expressed using only the states at the closing of the intake valve without using the states under combustion.
- The square of engine speed is convenient for approximating analytical techniques for Lagrange equations (4) with constraints, which express the reciprocating dynamics of the six pistons, the rotational dynamics of the crank, and their interlock.
- The temperature of the valve of each cylinder and the temperature of the left and right banks are derived from the conservation law (3) using approximate analytical techniques for the cooling loss model and integral terms. Note that the valve temperature at the opening of the exhaust valve is expressed using only the states at the closing of the intake valve without using the valve temperature under combustion.
- The fuel model is easy to discretize from the original model (5) in the benchmark model.

From combining each discrete crank angle model as outlined above, we obtain the following periodically time-varying state space model,

$$\begin{aligned}\hat{x}(k+1) &= f_{\bar{k}}(\hat{x}(k),\hat{u}(k)) \\ y(k) &= h(\hat{x}(k),\hat{u}(k))\end{aligned} \tag{6}$$

where the notations are used as follows:

$\hat{x} = \begin{bmatrix}\hat{M}^T & \hat{P}^T & \Omega & \hat{T}_v^{\,T} & \hat{T}_p^{\,T}\end{bmatrix}^T \in R^{23}$: state variable,

$y = \begin{bmatrix}\Omega & m_t\end{bmatrix}^T \in R^2$: output,

$\hat{u} = \begin{bmatrix}\hat{u}_t & \hat{u}_s^{\,T} & \hat{u}_f^{\,T}\end{bmatrix}^T \in R^{13}$: real input, $\bar{k} = k \bmod 6$,

$\hat{M}, \hat{P} \in R^7$: mass, pressure in the surge tank and all cylinders,

$\hat{T}_v \in R^6$: temperature of all cylinders' valve,

$\hat{T}_p \in R^2$: temperature of left bank and right bank,

$\Omega, m_t \in R$: the square of engine speed, throttle flow,

$\hat{u}_t \in R$: throttle angle, $\hat{u}_s \in R^6$: spark timing of all cylinders,

$\hat{u}_f \in R^6$: fuel injection quantities of all cylinders.

Note that the torque, the thermal efficiency, and the specific fuel consumption can also be expressed by the state variable $\hat{x}$ and the input $\hat{u}$.

Figure 3 shows the entire model with Eq. (6), a fuel model, and some unit delays of inputs. The fuel model is a periodically time-varying state space model in which each cylinder has fuel injection at a different sampling point. Note that the total delay is five samples in the fuel model because the torque is generated one cycle after the fuel injection quantity is specified as the opening exhaust valve. The delays of the throttle angle and the spark timing indicate that throttle opening and spark are executed one sample after specified.

Figure 4 shows the validation of the obtained model (Fig. 3). These errors are mainly caused by approximating the cooling loss. Notice that the computational load of the obtained model is less than 1/100 that of the benchmark models (1)-(5).

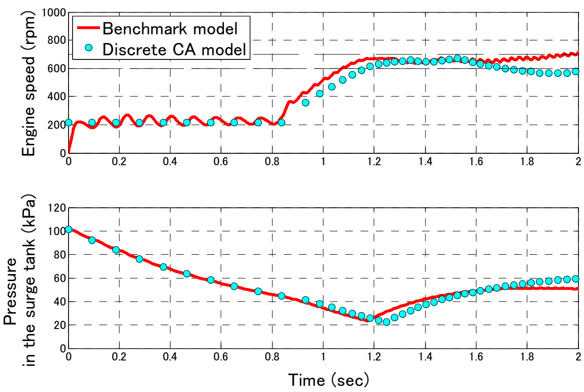

Fig. 4. Validation

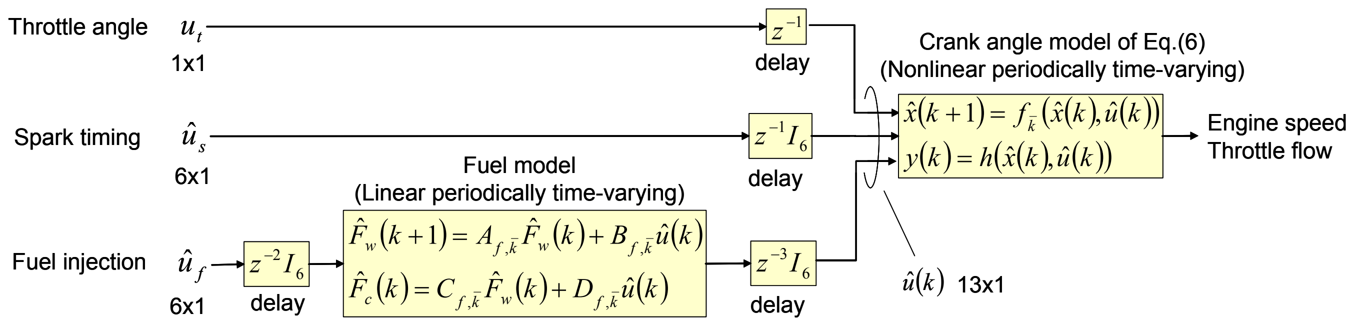

Fig. 3. Periodically time-varying model with fuel model and unit delays

3.3 Time-Invariant State Space Model

3.3.1 Role State Variable

Table 1a summarizes Fig. 2, showing the relation among the cylinder number, the sampling point, and the number of strokes. In Table 1a, $\hat{\xi}_j$ is the state variables of mass, pressure, and valve temperature in the # j cylinder. It is easy to see that the state transition function at sampling points k to k+1 is different from that at sampling points k+1 to k+2, because the stroke of $\hat{\xi}_j$ at sampling points k to k+1 is different from that at sampling points k+1 to k+2. This is why Eq. (6) is time-varying and periodical.

Next, we introduce a new state variable. The relation shown in Table 1b is the same as that shown in Table 1a, except with respect to how to choose the state variables. In Table 1b, the state variable ξ_1 is defined in terms of the mass, pressure, and valve temperature of the cylinder at the middle of the combustion stroke (mC). Similarly, state variables ξ_2 to ξ_6 are defined as mass, pressure, and valve temperature of the cylinder under the corresponding strokes. We refer to these state variables ξ as role state variables.

The state transition function of ξ_i at sampling points k to k+1 is the same as that at sampling points k+1 to k+2 because the state variable ξ_i is always in the same stroke. Therefore, the periodically time-varying state space model of Eq. (6) can be transformed into a time-invariant state space model if the role state variables are used.

When the role state variables are used, there exist only three inputs: the spark timing, the fuel injection quantity, and the throttle angle. We refer to these inputs as role inputs. The role input of spark timing is a reduced input from six spark timings of all of the cylinders, as well as the role input of fuel injection. Therefore, it is important to grasp the correspondence between the role inputs and the real inputs. Note that the role input of the throttle angle is equal to the real input.

Using the role state variables and the role inputs, the periodically time-varying state space model of Eq. (6) is transformed into the following time-invariant state space model,

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)) \end{aligned} \qquad (7)$$

where

$$x = \begin{bmatrix} M^T & P^T & \Omega & T_v^T & T_p^T \end{bmatrix}^T \in R^{23} : \text{role state variable}$$
$$u = \begin{bmatrix} u_t & u_s & u_f \end{bmatrix} \in R^3 : \text{role input}.$$

Figure 5 shows an entire model with Eq. (7), the fuel model, and the unit delays of inputs.

The concept of the role state variables has been discussed here in the case of six-cylinders. Note, however, that the discussion holds for any number of cylinders.

Table 1. Relation between state variables and role state variables:
mI: Middle of Intake stroke, mC: Middle of Combustion stroke, mE: Middle of Exhaust stroke
eI: End of Intake stroke, eC: End of Combustion stroke, eE: End of Exhaust stroke, #: cylinder number

a. state variables

State Variables \ Sampling Point	0	1	2	3	4	5	6	7	...
$\hat{\xi}_1$ at #1	mC	eC	mE	eE	mI	eI	mC	eC	...
$\hat{\xi}_2$ at #2	eI	mC	eC	mE	eE	mI	eI	mC	...
$\hat{\xi}_3$ at #3	mI	eI	mC	eC	mE	eE	mI	eI	...
$\hat{\xi}_4$ at #4	eE	mI	eI	mC	eC	mE	eE	mI	...
$\hat{\xi}_5$ at #5	mE	eE	mI	eI	mC	eC	mE	eE	...
$\hat{\xi}_6$ at #6	eC	mE	eE	mI	eI	mC	eC	mE	...

b. role state variables

Role State Variables \ Sampling Point	0	1	2	3	4	5	6	7	...
ξ_1 at mC	#1	#2	#3	#4	#5	#6	#1	#2	...
ξ_2 at eI	#2	#3	#4	#5	#6	#1	#2	#3	...
ξ_3 at mI	#3	#4	#5	#6	#1	#2	#3	#4	...
ξ_4 at eE	#4	#5	#6	#1	#2	#3	#4	#5	...
ξ_5 at mE	#5	#6	#1	#2	#3	#4	#5	#6	...
ξ_6 at eC	#6	#1	#2	#3	#4	#5	#6	#1	...

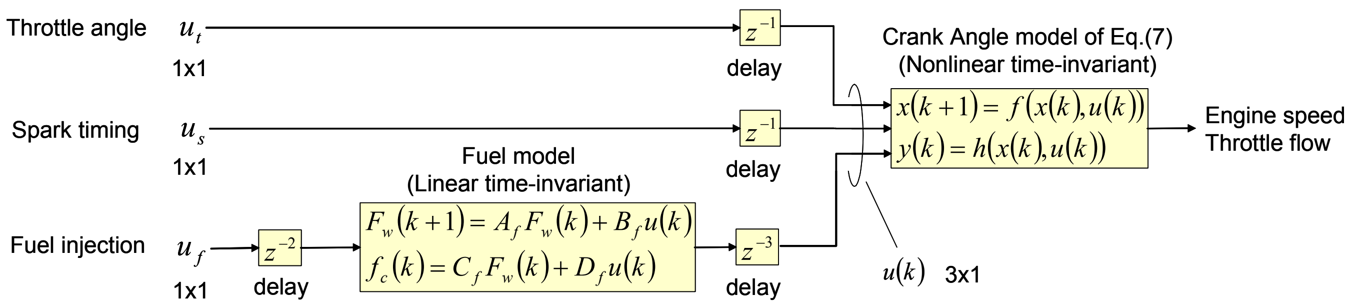

Fig. 5. Time-invariant state space model with fuel model and unit delays

3.3.2 Permutation Matrix

This section clarifies the relation between the role state variables and the state variables, as well as the relation between the role inputs and the real inputs.

From the relation between the role state variables ξ_i and the state variables $\hat{\xi}_j$ in Table 1, it is easy to see that using the following permutation matrix:

$$Q = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 1 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 1 \\ 1 & 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \in R^{6\times 6}, \tag{8}$$

the transformation between $\xi(k)$ and $\hat{\xi}(k)$ is given by

$$\xi(k) = Q^{\bar{k}}\hat{\xi}(k) \tag{9}$$

where $\bar{k} = k \bmod 6$. Similarly, the relation between the role inputs and the real inputs is given by

$$\begin{cases} u_s(k) = r_s Q^{\bar{k}} \hat{u}_s(k) \\ u_f(k) = r_f Q^{\bar{k}} \hat{u}_f(k) \end{cases} \tag{10}$$

where
$$r_s = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$r_f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

3.3.3 Stabilizability

We consider some linearized models around a steady state to clarify the relation between the stabilizability of the time-invariant state space model (7) with the role state variables and that of the periodically time-varying state space model (6).

Equation (11) indicates the linearized state space models derived from the time-invariant state space model (7) and the periodically time-varying state space model (6), in which Δu, Δx, and Δy denote the perturbations of the input, state, and output, respectively, from a steady state. Here, we assume that in a steady state, the fuel quantity in a cylinder is equal to the fuel injection quantity, and thus the fuel model, the valve-temperature model, and the port-temperature model can be disregarded.

$$\begin{aligned} &\begin{cases} \Delta x(k+1) = A\Delta x(k) + B\Delta u(k) \\ \Delta y(k) = C\Delta x(k) \end{cases} \\ \Leftrightarrow &\begin{cases} \Delta \hat{x}(k+1) = \hat{A}(k)\Delta \hat{x}(k) + \hat{B}(k)\Delta \hat{u}(k) \\ \Delta y(k) = \hat{C}(k)\Delta \hat{x}(k) \end{cases} \end{aligned} \tag{11}$$

where

$$\Delta x(k) = Q_x^{\bar{k}}\Delta\hat{x}(k), \qquad \hat{A}(k) = Q_x^{-\overline{k+1}} A Q_x^{\bar{k}},$$
$$\Delta u(k) = R_u Q_u^{\bar{k}}\Delta\hat{u}(k), \qquad \hat{B}(k) = Q_x^{-\overline{k+1}} B R_u Q_u^{\bar{k}}$$
$$\Delta\hat{u}(k) = Q_u^{-\bar{k}} R_u^T \Delta u(k), \qquad \hat{C}(k) = C Q_x^{\bar{k}},$$

$$Q_x = \begin{bmatrix} Q_a & 0 & 0 & 0 \\ 0 & Q_a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & I_7 \end{bmatrix} \in R^{22\times 22}, Q_a = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & Q \end{array}\right] \in R^{7\times 7},$$

$$R_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_f \end{bmatrix} \in R^{3\times 13}, Q_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} \in R^{13\times 13}.$$

Theorem: In Eq. (11), the periodically time-varying state space model is stabilizable if the time-invariant state space model is stabilizable.

Proof: If the time-invariant state space model is stabilizable, there exists a gain matrix F such that $A + BF$ is stable.

Substituting $\Delta u(k) = F\Delta x(k)$ into $\Delta\hat{u}$ yields $\Delta\hat{u}(k) = \hat{F}(k)\Delta\hat{x}(k)$ where $\hat{F}(k)$ is given by

$$\hat{F}(k) \equiv Q_u^{-\bar{k}} R_u^T F Q_x^{\bar{k}}. \tag{12}$$

The periodically time-varying state space model is then given by

$$\Delta\hat{x}(k+1) = \left\{\hat{A}(k) + \hat{B}(k)\hat{F}(k)\right\}\Delta\hat{x}(k) \tag{13}$$

where

$$\hat{A}(k) + \hat{B}(k)\hat{F}(k) = Q_x^{-\overline{k+1}}(A + BF)Q_x^{\bar{k}}. \tag{14}$$

The stability of the periodically time-varying state space model is decided according to the transition matrix from arbitrary sampling point k to sampling point k+6, i.e., by one cycle

$$\Delta\hat{x}(k+6) = \Phi(k+6,k)\Delta\hat{x}(k) \tag{15}$$

where from Eqs. (13) and (14), it is easy to see that

$$\Phi(k+6,k) = Q_x^{-\bar{k}}(A+BF)^6 Q_x^{\bar{k}}. \tag{16}$$

Therefore, it is clear that $\Phi(k+6,k)$ is stable when $A+BF$ is stable, which means that the theorem has been proven. □

This theorem implies that the stabilization problem of the periodically time-varying state space model can be reduced to that of the time-invariant state space model. In addition, it is easy to see that a similar theorem holds with respect to detectability. Therefore, we can use the control theory of time-invariant systems in the design of an engine feedback controller.

4. DESIGN EXAMPLE

The benchmark problem mainly sets up the following control specifications.

・The engine speed, 650±50 rpm, should be reached in 1.5 seconds after cold start.

・The overshoot of the engine speed should be as small as possible.

・The engine speed should converge to 650 rpm, and so the steady state engine speed, 650 rpm, should be asymptotically stable.

Figure 6 shows the design flow diagram.

1st step: The steady states and inputs are obtained numerically using the nonlinear time-invariant state space model of Eq. (7), in which a set of 15 nonlinear algebraic equations should be solved numerically.

2nd step: An optimal feedforward control inputs for the 1.5 seconds after cold start is searched using Eq. (7), a performance index (square sum of the engine speed error), and some constraints (engine speed, inputs, misfire, and stall).

3rd step: An LQI controller is obtained by using the linear time-invariant state space model of Eq. (11) and considering the unit delays of the inputs.

Figure 7 shows the controller designed in the present study.

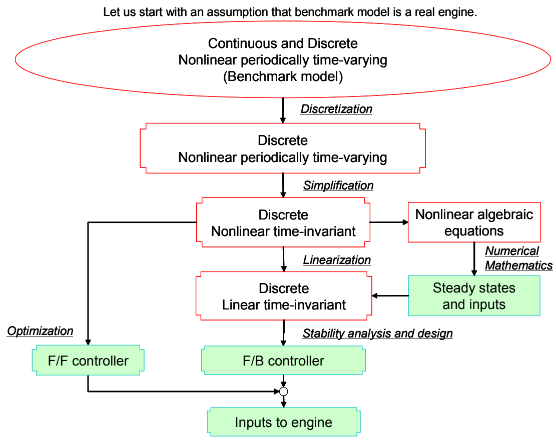

Fig. 6. Design flow diagram

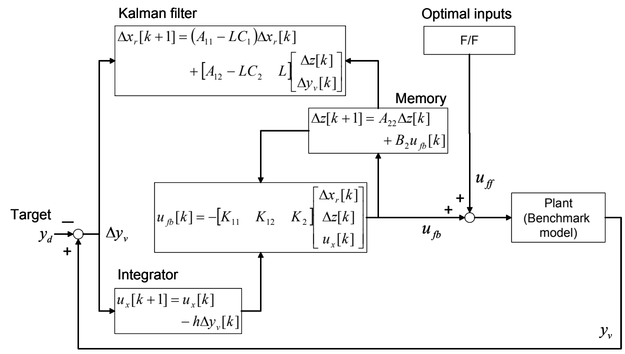

Fig. 7. Cold start controller

5. NUMERICAL EXPERIMENT

Using the benchmark model and the designed controller given in Fig. 7, numerical experiments for cold start control were conducted. Figure 8 shows results that satisfy the specifications of the benchmark problem. In Fig. 8, the feedback control was executed from the sixth cycle (approximately 1.8 seconds), at which time the temperatures of the valves of all of the cylinders were near steady state. Moreover, the feedforward control inputs were kept at the steady state values after this time. Note that in Fig. 8, the spark timing and the fuel injection quantity are plotted for every sample, i.e., for different cylinders.

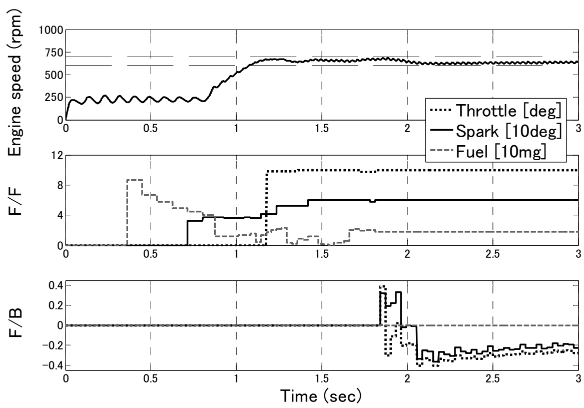

Fig. 8. Results

6. CONCLUSIONS

The present paper proposes a model representation of multi-cyclic phenomena for a multi-cylinder engine system using the new concept of role state variables in order to design an optimal multiple inputs. The features of this model representation are as follows.

・The periodically time-varying state space model is equivalently transformed into the time-invariant state space model using the role state variables. This holds for any number of cylinders.

・The stabilizability of the time-invariant state space model implies the stabilizability of the periodically time-varying state space model, as well as detectability.

・The time-invariant state space model enables an optimal design for periodical engine system.

・The time-invariant state space model and the permutation matrix introduce some simplification in the program structure when the controller is implemented.

In the future, model predictive control will be applied explicitly to consider the constraints of inputs.

REFERENCES

Ohata, A. (1990). An Adaptive Fuel Injection Control with Internal Model in Automotive Engines. *IECON'90*, pp.78-83.

Johansson, R. and A. Rantzer (2003). *Nonlinear and Hybrid Systems in Automotive Control*, Springer-Verlag London.

Jurgen, R.K. (2004). Electronic Engine Control Technologies 2nd Edition, SAE.