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An Optimal Graph Theoretic Approach to Data Association in SLAM

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Abstract: In this paper, we study the problem of data association in simultaneous localization and mapping (SLAM). Since almost all existing methods for solving the problem are only able to provide suboptimal solutions, we revisit this problem and propose an optimal graph approach to resolve it. We first formulate the problem as integer programming (IP) problem, and then algorithmically prove that the IP is equivalent to a minimum weight bipartite perfect matching problem. Thus, optimally solving the bipartite matching problem is equivalent to optimally resolve the IP problem (i.e., the data association problem). Simulations validate the effectiveness and accuracy of the proposed approach.

Keywords: Bipartite matching, Data association, SLAM, Navigation, Mobile robot

1. INTRODUCTION

Simultaneous localization and mapping (SLAM) is the process of building a map of an environment and concurrently generating an estimate of the robot pose from the sensor readings. SLAM is considered an essential capability for autonomous mobile robots to exploring unknown environments. Since Smith and Cheeseman (1987) first introduced a stochastic solution to the SLAM problem, i.e., *stochastic mapping*, in the late 1980s, rapid and exciting progress in solving the problem has been made over the past decades, resulting in several compelling implementations (e.g. Newman (1999); Williams et al. (2000); Kim and Sukkarieh (2003)). Recent interest in SLAM has focused on the design of estimation algorithms (Montemerlo (2003); Paskin (2002)), sensor data processing (Se et al. (2002)) and theoretic analysis of the performance (Mourikis and Roumeliotis (2006)) etc.

One of most critical and challenging problems in SLAM is the data association, which consists of relating sensor measurements to the landmarks (or features) in the existing map. It is crucial to establish correct correspondences between the sensed landmarks and mapped landmarks for building a consistent map, because any single mismatching may cause the estimator such as extended Kalman filter (EKF) diverge. It is intuitive to consider data association a search problem in the space of measurement-landmark correspondences. However, it is usually intractable to do exhaustive searching, because the complexity of finding correspondences between the measurements and the mapped landmarks is exponential on the number of measurements.

To our best knowledge, almost all the existing methods for solving the data association is *suboptimal*. However, in this paper, we revisit the problem by proposing an *optimal* graph approach. Specifically, the data association in SLAM is first formulated as a 0-1 integer programming (IP) problem. It is well known that optimally solving IP problem generally is NP-hard. Therefore, the relaxation technique is usually adopted (e.g., linear programming relaxation (Zhang et al. (2005))) to obtain suboptimal solutions. We algorithmically prove that the IP problem is equivalent to a minimum weight bipartite perfect matching problem. Hence, we are able optimally solve the bipartite matching problem and thus equivalently optimally resolve the IP problem (i.e., data association).

The remainder of the paper is organized as follows: After an overview of the related work in the next section, the formulation of the data association problem in SLAM is presented in Section 3. In Section 4, we prove the equivalence between the IP problem and a minimum weight bipartite perfect matching. The optimal graph approach to find the matching in bipartite graph is described in Section 5. The simulation results are presented in Section 6. Finally, in Section 7 the conclusions of this work are drawn and future research directions are suggested.

2. RELATED WORK

In stochastic mapping, the problem of data association is resolved by widely employing the gated nearest neighbor (NN) algorithm (Leonard and Durrant-Whyte (2005)). The normalized squared innovation test is used to determine the compatibility, and the Mahalanobis distance is calculated to select the best matchings. The most appealing characteristics of NN is its $O(mn)$ computational complexity besides its conceptual simplicity. Here m is the number of measurements and n is the number of existing landmarks in the map. It performs well in the environments with sparse landmarks. However, in the surroundings with high density of landmarks, the innovations of matching different observations obtained at same locations are correlated, thus, the NN algorithm may accept wrong matchings, which leads to divergence of the estimator.

Dezert and Bar-Shalom (1993) proposed a better solution, i.e., joint probabilistic data association (JPDA). JPDA associates all of the measurements falling inside a suitably chosen validation region of a track to itself by a probabilistic weighting procedure and performs relatively well when spurious measurements are relatively moderate. The limitation however is that it can be computationally prohibitive in terms of calculating weighting probabilities, and the process my corrupt the feature recognition or discrimination information.

Neira and Tardos (2001) proposed using a joint compatibility test based on the branch and bound (JCBB) search with a acceptable computational cost in indoor environments. JCBB takes groups of feature observation associations into consideration in the context of searching for the hypothesis with the maximum number of compatible pairs. However, the resultant exponential search space, despite the branch and bound pruning, renders the method computationally intensive for real-time implementation.

Nieto et al. (2003) proposed a real-time data association method for FastSLAM (Montemerlo (2003)) by applying the multiple hypotheses tracking (MHT) method in a variety of outdoor environments. MHT is the most structured approach employing the idea of delay decision for multitarget tracking and data association (Reid (1979)). It forms a number of hard association hypotheses from several scans of data, and delays the association decision to a later time when more information becomes available. The method in (Montemerlo (2003)) splits each particle representing a map in FastSLAM into further particles for keeping association hypotheses. The particles with wrong data associations are expected to die out in the resampling state. Bailey et al. (2000) considered relative distances and angles between points and lines in two laser scans and used graph theory to find the largest number of compatible pairings between the measurements and existing features.

More recently, Zhang et al. (2005) formulated the problem of data association in SLAM as a linear programming (LP) relaxation and thus obtained the suboptimal correspondences by solving the LP problem. In the same fashion of formulating data association into an optimization problem, Wijesoma et al. (2006) introduced a multidimensional assignment based method to resolve the problem. However, all above mentioned methods only provide suboptimal solutions, while, in this work, we seek a theoretically optimal approach to solve the problem.

3. PROBLEM FORMULATION

Data association of SLAM is a decision process of associating measurements with existing landmarks in the stochastic map. We start by formulating the problem as a 0-1 integer programming (IP) or 2D assignment problem. A mathematical framework of SLAM based on EKF (Dissanayake et al. (2001)) will be applied.

3.1 Formulation of IP Problem

At time step k, denote a set of measurements collected in the latest scan by $Z(k)$ and a set of landmarks existing in the map so far by $F(k)$, i.e.,

$$Z(k) \triangleq \{z_i(k) : i = 0, 1, 2, ..., n_k\} \tag{1}$$

$$F(k) \triangleq \{l_j(k) : j = 0, 1, 2, ..., m_k\} \tag{2}$$

where n_k is the number of actual measurements at time k and m_k the number of existing landmarks in the map up to time k. Note that $z_0(k)$ and $l_0(k)$ are the dummy elements in the case of a false alarm or new landmark is detected. Next, we introduce a 0-1 decision variable.

$$x_{ij}(k) \triangleq \begin{cases} 1, & \text{if } z_i(k) \text{ associated with } l_j(k) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $i = 0, 1, 2, ..., n_k$ and $j = 0, 1, 2, ..., m_k$. Two special cases shall be emphasized here: $x_{i0}^k = 1$ stands that i^{th} measurement can not be assigned to any of the existing landmarks in the map and therefore assigned with a dummy one which may be false alarm or new landmark, while $x_{0j}^k = 1$ implies that j^{th} landmark in the map doesn't have any possible measurement associated with it in the current scan. It should be noted that in tracking typically we do not define the $j = 0$ case. We do so in order to get all equality constraints which will be become much clearer after the discussion below.

There are two important physical constraints imposed on the data association problem as Li et al. (1999) discussed. (i) *Single source constraint*: Each actual measurement $z_i(k)$ $(i = 1, 2, ..., n_k)$ can be assigned to at most one landmark. However, the dummy measurement $z_0(k)$ can be assigned to multiple landmarks in the case of false detection or new detected landmark. We therefore set it free to have the following equality constraint.

$$\sum_{j=0}^{m_k} x_{ij}(k) = 1, \forall i = 1, 2, ..., n_k \tag{4}$$

and (ii) *Single return constraint*: Each landmark $l_j(k)$ $(j = 1, 2, ..., m_k)$ can produce at most one measurement in current scan. Clearly not all landmarks can return measurements. In other words, some existing landmarks are undetected in current scan and hence we assign these undetected landmarks dummy reports.

$$\sum_{i=0}^{n_k} x_{ij}(k) = 1, \forall j = 1, 2, ..., m_k \tag{5}$$

Our objective is to exactly match the sensor observations with the existing landmarks in the map. Similar to the multitarget tracking problem (Li et al. (1999)), the cost of a feasible association of measurement with existing landmark or new landmarks (or false alarm) as the negative logarithm of the normalized joint probability of such an association. Define the set of all possible association pairs at current time step $\Omega(k) \triangleq \{(i, j) : z_i(k) \in Z(k), l_j(k) \in F(k), i, j \neq 0\}$, and a partition of the set $\omega = \{\omega_T, \omega_F\}$, where ω_T denotes the set of measurements associated with existing landmarks in the map while ω_F is the set of measurements associated with new landmarks or false alarms. Note that we do not distinguish the cases of new landmark and false alarm. We instead assume all measurements not associated in current scan are new landmarks, because if spurious landmarks are detected, they will be removed from the map if they do not appear in the following scans. Thus, ω_F only stands for the case of association with new landmarks. Therefore, the likelihood of detecting new landmarks is simply approximated with 1, i.e., $\Lambda(\omega_F) = 1$. For the true associations, we have

$$\Lambda(\omega_T) = \prod_{(i,j)\in\omega_T} q(zl) \tag{6}$$

where zl denotes the true association of $z_i(k)$ and $l_j(k)$, and $q(zl)$ is a Gaussian probability density function (pdf) of the measurement variable $z_i(k)$, i.e.,

$$q(zl) \sim \mathcal{N}(z_i(k) : \hat{z}_j(k|k-1), S_i(k)) \tag{7}$$

where $\hat{z}_j(k|k-1)$ is the estimate of landmark $l_j(k)$, and $S_i(k)$ is the covariance matrix of the residual $(z_i(k) - \hat{z}_j(k|k-1))$. Both quantities are obtained in the update step in EKF SLAM. Therefore, the likelihood of the partition ω can be calculated as follows (the time index is temporarily dropped to preserve the clarity of the presentation):

$$\Lambda(\omega) = \Lambda(\omega_T)\Lambda(\omega_F) = \prod_{(i,j)\in\omega_T} q(zl) \tag{8}$$

$$= \prod_{(i,j)\in\omega_T} \frac{1}{\sqrt{\det(2\pi S_i)}} \exp\{-\frac{1}{2}[z_i - \hat{z}_j]^T S_i^{-1}[z_i - \hat{z}_j]\}$$

In order to ensure the likelihood is consistent, normalization of $\Lambda(\omega)$ is necessary. It is easy to realize that the normalized likelihood will be the same as $\Lambda(\omega)$ because itself is already consistent. Our objective is rephrased to find the one with maximum likelihood $\Lambda(\omega)$ among all possible partitions ω, which is equivalent to minimizing the negative log-likelihood $-\Lambda(\omega)$, i.e., $J(\omega) \triangleq -\ln\Lambda(\omega) = -\sum_{(i,j)\in\omega} \ln q(zl)$. By defining the cost coefficient

$$c_{ij}(k) \triangleq \begin{cases} 0, & (i,j)\in\omega_F \\ -\ln q(zl), & (i,j)\in\omega_T \end{cases} \tag{9}$$

the data association of SLAM can be formulated as the following 0-1 IP problem:

$$\mathbf{\Pi_1} : \min \sum_{(i,j)\in\omega} c_{ij}(k)x_{ij}(k)$$
$$\text{s.t. Eqs. } (3), (4) \text{ and } (5) \tag{10}$$

Notice that any solution to this IP corresponds to a matching and therefore this is a valid formulation of the minimum weight perfect matching problem in bipartite graphs (West (2000)), which will be elaborated later (cf. Section 4).

3.2 Formulation of LP Relaxation

Consider now the linear programming (LP) obtained by simply dropping the integrality constraints:

$$\mathbf{\Pi_2} : \min \sum_{(i,j)\in\omega} c_{ij}(k)x_{ij}(k)$$
$$\text{s.t. } x_{ij} \geq 0, \text{ and Eqs. (4) and (5)} \tag{11}$$

This is the LP relaxation of the above IP problem ($\mathbf{\Pi_1}$), which has been explored by Zhang et al. (2005). In an LP, the variables can take fractional values and therefore there are many feasible solutions to the set of constraints above which do not correspond to matchings. But we only care about the optimum solutions. The set of feasible solutions to the constraints in $\mathbf{\Pi_2}$ forms a polytope, and when we optimize a linear constraint over a polytope, the optimum will be attained at one of the corners or extreme points of the polytope. In general, even if all the coefficients of the constraint matrix in an LP are either 0 or 1, the extreme points of an LP are not guaranteed to have all coordinates integral (This is of no surprise since the general IP problem is NP-hard, while LP is polynomially solvable). As a result, there is no guarantee that the optimum solution of IP ($\mathbf{\Pi_1}$) is equal to optimum solution of its LP relaxation ($\mathbf{\Pi_2}$). However, $\mathbf{\Pi_2}$ provides a lower bound of $\mathbf{\Pi_1}$. Moreover, the following lemma is easy to prove.

Lemma 1. If an optimum solution to $\mathbf{\Pi_2}$ is integral, then it must also be an optimum solution to $\mathbf{\Pi_1}$.

Proof. The integral optimum solution to $\mathbf{\Pi_2}$ satisfies all the constraints of $\mathbf{\Pi_1}$. □

4. MINIMUM WEIGHT BIPARTITE PERFECT MATCHING

By assigning infinite costs to the edges not present, we assume that the bipartite graph is complete. The minimum cost (or weight) perfect matching problem is often described by the following account: There are n jobs to be processed on n machines and one would like to process exactly one job per machine such that the total cost of processing the jobs is minimized. Analogue to this story, with help of the dummy variables, the IP formulation of data association in SLAM, $\mathbf{\Pi_1}$ (cf. (10)), can be considered as a minimum weight bipartite perfecting matching problem. Instead of using optimization method directly to solve the data association problem, we shall employ graph approaches to solve the equivalent bipartite matching problem. In the case of the perfect matching problem, the constraint matrix has a very special form and one can show that the optimality of the solutions can be preserved. To do so, we start by stating a very crucial result (cf. Lemma 2) as well as a purely algorithmic proof in the following.

Lemma 2. Any extreme point of the polytope of $\mathbf{\Pi_2}$ is a 0-1 vector and, hence, is the incidence vector of a perfect matching.

Proof. To prove algorithmically, we construct a *primal-dual* algorithm for solving the minimum weight perfect matching problem. Suppose in a specific case of the bipartite matching problem, we have measurement u_i and landmark v_j such that $u_i + v_j \leq c_{ij}$. The dual of the LP relaxation, $\mathbf{\Pi_2}$, can be obtained as follows:

$$\mathbf{\Pi_3} : \max \sum_{(i,j)\in\omega} (u_i + v_j)$$
$$\text{s.t. } u_i + v_j \leq c_{ij} \tag{12}$$

The dual constraints can be interpreted as $w_{ij} \geq 0$, where $w_{ij} = c_{ij} - u_i - v_j$. If, for any instance, we could always find a feasible solution u, v to the dual $\mathbf{\Pi_3}$ and hence a perfect matching such that equalities in Eq. (3) hold (i.e. the cost of the perfect matching is equal to the value of the dual solution). Thus, we would know that the matching found is optimum. Given a solution u, v to the dual, a perfect matching would satisfy equality if it contains only edges (i,j) such that $w_{ij} = c_{ij} - u_i - v_j = 0$. This is what is referred to as complementary slackness. However, for a given u, v, we may not be able to find a perfect matching among the edges with $w_{ij} = 0$. The algorithm performs a series of iterations. It always maintains a dual feasible solution and tries to find an almost primal feasible solution x satisfying complementary slackness. The fact that complementary slackness is imposed is crucial in any primal-dual algorithm.

More precisely, the algorithm works as follows. It first starts with any dual feasible solution, say $u_i = 0$ for all i and $v_j = \min_{i\in\omega} c_{ij}$ for all j. In a given iteration, the algorithm has a dual feasible solution (u, v) or say (u, v, w). Imposing complementary slackness means that we are interested in matchings which are subgraphs of $B = \{(i, j) : w_{ij} = 0\}$. If B has a perfect matching then the incidence vector of that matching is a feasible solution in $\mathbf{\Pi_2}$ and satisfies complementary slackness with the current dual solution and, hence, must be optimal. To check whether B has a perfect matching, one can use the cardinality matching method. If the maximum matching output is not perfect, then the algorithm will use information from the optimum vertex cover C^* to update the dual solution in such a way that the value of the dual solution increases. Recall that we are maximizing the dual.

Let the set L (for labeling) of vertices which can be reached by a directed path from an exposed vertex in measurement set Z. In particular, there is then no edge of B between $Z \cap L$ and $F \cap L$, where we remind that F is the mapped feature set. In other words, for every $i \in (Z \cap L)$ and every $j \in (F - L)$, we have $w_{ij} > 0$. Let $\delta = \min_{i\in(Z\cap L), j\in(F-L)} w_{ij}$. By the above argument, $\delta > 0$. The dual solution is updated as follows:

$$u_i = \begin{cases} u_i, & i \in Z - L \\ u_i + \delta, & i \in Z \cap L \end{cases} \tag{13}$$

$$v_j = \begin{cases} v_j, & j \in F - L \\ v_j - \delta, & j \in F \cap L \end{cases} \tag{14}$$

One easily check that this dual solution is feasible, in the sense that the corresponding vector w satisfies $w_{ij} \geq 0$ for all i and j. The difference between the values of the new dual solution and the old dual solution is equal to:

$$\begin{aligned} &\delta(|Z \cap L| - |F \cap L|) \\ =&\delta(|Z \cap L| + |Z - L| - |F - L| - |F \cap L|) \\ =&\delta(n - |C^*|) \end{aligned} \tag{15}$$

where Z has size of n and C^* is the optimum vertex cover for the bipartite graph with edge set B. But by assumption $|C^*| < n$, implying that the value of the dual solution strictly increases.

This procedure is repeated until the algorithm terminates. At that point, we have an incidence vector of a perfect matching and also a dual feasible solution which satisfy complementary slackness. They must therefore be optimal and this proves the existence of an integral optimum solution to $\mathbf{\Pi_2}$. Since, by carefully choosing the cost function, one can make any extreme point be the unique optimum solution to the linear program. Now we need to prove that the algorithm indeed terminates. Notice that at least one more vertex of F must be reachable from an exposed vertex of Z (and no vertex of F becomes unreachable), since an edge $e = (i, j)$ with $i \in (Z \cap L)$ and $j \in (F - L)$ now has $w_{ij} = 0$ by our choice of δ. This also gives an estimate of the number of iterations. In at most n iterations, all vertices of F are reachable or the matching found has increased by at least one unit. Therefore, after $\mathcal{O}(n^2)$ iterations, the matching found is perfect. This completes the proof. □

Now we reach the core of our findings, which is the equivalence between IP problem and minimum weight bipartite perfect matching problem.

Lemma 3. Solving $\mathbf{\Pi_1}$ is equivalent to solve a corresponding minimum weight bipartite perfect matching.

Proof. With Lemma 1 and 2, the optimum solution of the minimum weight bipartite perfect matching is also optimum solution to $\mathbf{\Pi_1}$. □

Therefore, instead of solving the original IP problem (i.e., $\mathbf{\Pi_1}$) directly, we resolve the minimum weight bipartite perfect matching problem to obtain the optimum solution to the data association.

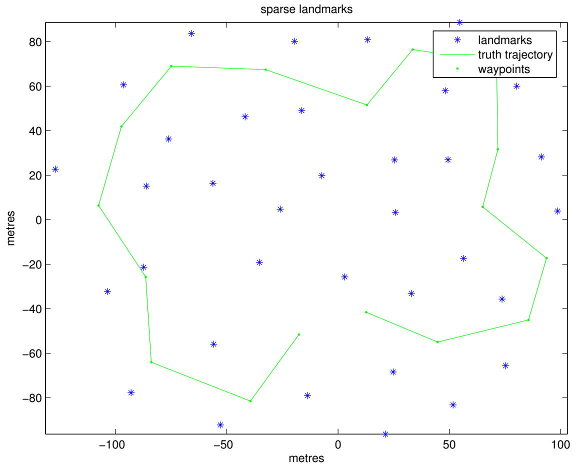
5. ALGORITHM BASED ON WEIGHTED BIPARTITE MATCHING

In this section, we focus on finding the minimum weight matchings in the bipartite matching. The general idea is straightforward: start with any empty matching, and repeatedly discover augmenting paths.

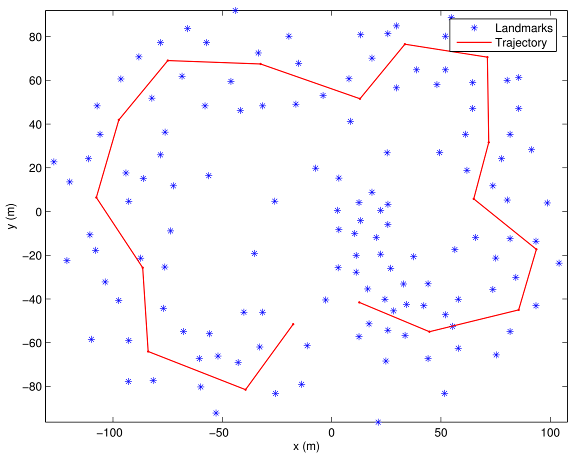
Several essential definitions are first delivered (West (2000)). Given a matching M in a bipartite graph $G = (V, E)$, a simple path in G is called an *augmenting path* with respect to M if its two vertices are both unmatched and its edges are alternative in $E - M$ and M. Let p be an augmenting path with respect to M, and P denote the set of edges in path p, then $M \oplus P \triangleq (M - P) \cup (P - M)$ is called *symmetric difference* of M and P. One can verify the following properties of $M \cup P$: (i) it is a matching, and (ii) $|M \cup P| = |M| + 1$. The total weight of matching M is $w(M) = \sum_{e\in M} w(e)$. Suppose M' be a set of edges. An incremental weight $\Delta M'$ is defined as $\Delta M' = w(M' \cap M) - w(M' - M)$. From this definition, for an augmenting path p with respect to M, ΔP gives the net change in the weight of the matching after augmenting p, i.e.,

$$w(M \cup P) = w(M) + \Delta P \tag{16}$$

The minimum weight matchings are found iteratively. Specifically, the matching M is initialized to be empty. At each iteration, M is increased by finding an augmenting path of minimum weight. The procedure stops till no augmenting path with respect to M can be found. Johnson and Mcgeoch (1993) already proved that the process yields a minimum weight matching if repeatedly performing augmentations by using augmenting paths of minimum incremental weight. In order to search augmenting paths with respect to matching M systematically and efficiently, a search starts by constructing alternating paths from the unmatched points. As an augmenting path must have one unmatched endpoint in Z and the other in F, in general, the search starts by growing alternating paths only from unmatched vertices of Z, and may search for all possible alternating paths from unmatched vertices of Z simultaneously in a breadth-first manner. In this work,



(a) with sparse landmarks



(b) with dense landmarks

Fig. 1. Simulation setup. A robot equipped with range-bearing senor moves on the planned trajectory at constant velocity of $v = 2$ m/sec.

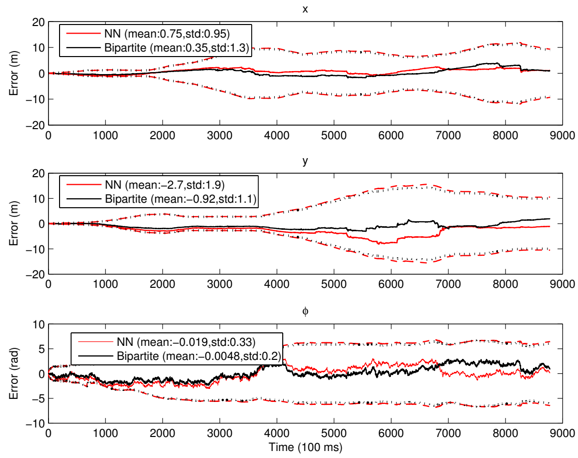


Fig. 2. Estimation errors of robot pose in the environment with sparse landmarks

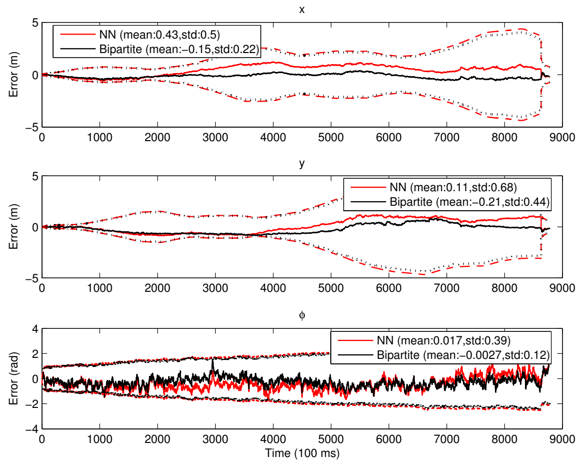


Fig. 3. Estimation errors of robot pose in the environment with dense landmarks

the approach proposed by Johnson and Mcgeoch (1993) is employed to compute the minimum weight matching in the bipartite graph, which consists of two basic steps: (i) finding a shortest path augmentation from a subset of vertices in Z to a subset of vertices in F, and (ii) performing the shortest augmentation.

6. SIMULATION RESULTS

The simulations were designed to evaluate the performance of the proposed approach applied in EKF-SLAM. We implemented the simulation experiments based on the simulator written by Bailey [1]. To demonstrate the capability of the proposed graph approach to improve the accuracy of the data association and thus the estimation, we particularly compared the performance with NN data association, which is one of most widely used methods in EKF-SLAM. Two different scenarios were considered: one is with sparse landmarks, while the other has relatively denser landmarks (cf. Fig. 1). The velocity of the robot is kept constant at $v = 2$ m/sec, while its rotational velocity is obtained by calculating the changing rate of the orientation from the current location to the next one in order to best fit the generated trajectory. The robot pose has zero initial uncertainty. The standard deviation of the velocity measurement noise is equal to $\sigma_v = 0.1$ m/sec and the standard deviation of the errors in the orientation estimates is equal to $\sigma_\omega = 0.0524$ rad/sec. Similarly, the standard deviations of the exteroceptive measurement noise (i.e., range and bearing) are $\sigma_r = 0.1$ m and $\sigma_b = 0.0524$ rad. The maximum sensing range of the sensor is set to 5 m. The resulting estimation errors of robot pose are shown in Figs. 2 and 3, respectively. Figs. 4 and 5 depict the estimation errors of the landmarks. As seen from these figures, the bipartite matching data association performs consistently, since the estimation errors are all well bounded in the 3σ regions, thus validating the effectiveness of the proposed algorithm. Moreover, in terms of the accuracy, the proposed graph approach attains better results, in that it has smaller covariance than NN, especially in the dense environment.

7. CONCLUSIONS AND FUTURE WORK

Almost all existing solutions to data association problem in SLAM are suboptimal. In this paper, however, we formulated the problem as an equivalent minimum weight

[1] The simulator is available online `http://www-personal.acfr.usyd.edu.au/tbailey/software/slam_simulations.htm`

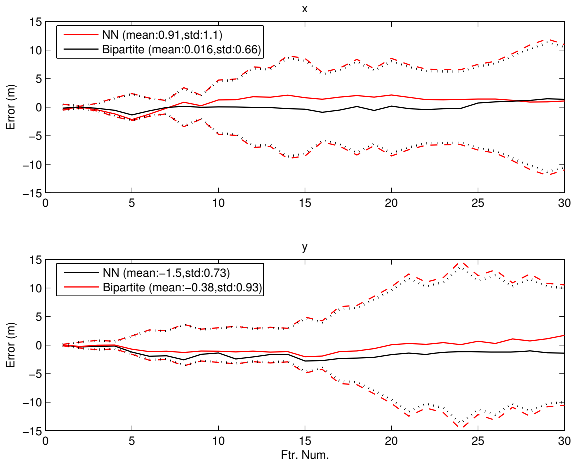


Fig. 4. Estimation errors of landmarks in the environment with sparse landmarks

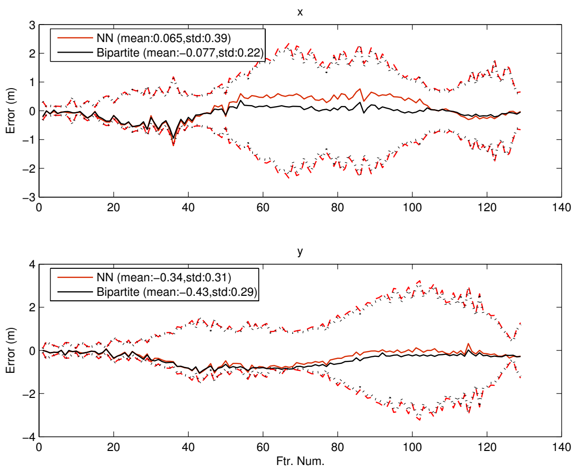


Fig. 5. Estimation errors of landmarks in the environment with dense landmarks

bipartite perfect matching problem which can be optimally solved, thus obtaining the optimal solution to the data association problem. The simulation results also validated the proposed graph approach. More thorough comparison studies with the existing methods in the literature are undergoing. As for the future work, we plan to explore new ways to improve the efficiency of the bipartite matching based algorithm.

REFERENCES

T. Bailey, E. M. Nebot, J. K. Rosenblatt, and H. F. Durrant-Whyte. Data association for mobile robot navigation: a graph theoretic approach. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 2512–2517, 2000.

J. Dezert and Y. Bar-Shalom. Joint probabilistic data association for autonomous navigation. *IEEE Transactions on Aerospace Electronic Systems*, 29(4):1275–1286, 1993.

M. Dissanayake, P. Newman, H. Durrant-Whyte, S. Clark, and M. Csorba. A solution to the simultaneous localization and map building (slam) problem. *IEEE Transactions on Robotics and Automation*, 17(3):229–241, 2001.

D. S. Johnson and C. C. Mcgeoch. Network flows and matching: first dimacs implementation challenge. *Series in DIMACS*, 12, 1993.

J. H. Kim and S. Sukkarieh. Airborne simultaneous localisation and map building. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 406–411, Taipei, Taiwan, September 2003.

J. J. Leonard and H. F. Durrant-Whyte. An efficient data association approach to simultaneous localization and map building. *International Journal of Robotics Research*, 24:49–60, 2005.

X. Li, Z.-Q. Luo, K. M. Wong, and E. Bosse. An interior point linear programming approach to two-scan data association. *IEEE Transaction on Aerospace and Electronic Systems*, 35:474–490, 1999.

M. Montemerlo. *FastSLAM: A factored solution to the simultaneous localization and mapping problem with unknown data association.* PhD thesis, Carnegie Mellon University, 2003.

A.I. Mourikis and S.I. Roumeliotis. Predicting the performance of cooperative simultaneous localization and mapping (c-slam). *International Journal of Robotics Research*, 25(12):1273–1286, 2006.

J. Neira and J. Tardos. Data association in stochastic mapping using the joint compatibility test. *IEEE Transactions on Robotics and Automation*, 17(6):890–897, 2001.

P. Newman. *On the structure and solution of the simultaneous localization and mapping problem.* PhD thesis, University of Sydney, 1999.

J. Nieto, J. Guivant, E. Nebot, and S. Thrun. Realtime data association for fastslam. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 2512–2517, Taipei, Taiwan, September 2003.

M. Paskin. *Thin Junction Tree Filters for Simultaneous Localization and Mapping.* PhD thesis, UC Berkeley, 2002.

D. B. Reid. An algorithm for tracking multiple targets. *IEEE Transactions on Automatic Control*, 24:843–854, 1979.

S. Se, D. Lowe, and J. Little. Mobile robot localization and mapping with uncertainty using scale-invariant visual landmarks. *International Journal of Robotics Research*, 21(8):735–758, 2002.

R. Smith and P. Cheeseman. On the representation and estimation of spatial uncertainty. *International Journal of Robotics Research*, 5(4):56–68, 1987. ISSN 0278-3649.

D. B. West. *Introduction to Graph Theory, 2 edition.* Prentice Hall, 2000.

W. S. Wijesoma, L. D. L. Perera, and M. D. Adams. Toward multidimensional assignment data association in robot localization and mapping. *IEEE Transactions on Robotics*, 22(2):350–365, 2006.

S. B. Williams, P. Newman, M. Dissanayake, and H. Durrant-Whyte. Autonomous underwater simultaneous localisation and map building. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 1793–1798, San Francisco, CA, April 2000.

S. Zhang, L. Xie, and M. Adams. An efficient data association approach to simultaneous localization and map building. *International Journal of Robotics Research*, 24 (1):49–60, 2005.