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Effect of Time-Delay on the Derivative Feedback Control of a 2-Degree-of-Freedom Torsional Bar with Parameter Perturbations

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Abstract: This paper is concerned with the derivative feedback control of a 2-degree-of-freedom (2-DOF) torsional bar with parameter perturbations. The time-delay is introduced into the controller and the maximum allowed time-delay bound is estimated by employing a discretized Lyapunov functional method. It is shown, through a numerical example with simulation results, that suitably introducing time-delay into the controller can indeed improve the performance of the stability of the closed-loop system.

1. INTRODUCTION

Continuous-time control systems have been studied for many years and used successfully in many industrial applications. Time delays are ubiquitous in practical control systems. There are two aspects of time delays in continuous-time control systems. One aspect is that while effectiveness of a control system relies on timely delivery of control signals, small time delays are inherent in industrial sensors and actuators (Moon and Johnson, 1998). Also, in many large scale (distributed) control systems, such as manufacturing plants and power generation plants, communication networks are employed to exchange information and control signals between spatially distributed system components where substantial time delay is inherent (Walsh *et al.*, 2002, Yue *et al.* 2004, 2005).

Traditionally, apart from some scattered research, time delays have been thought of as having a deleterious effect on both the stability and the dynamic performance of the controlled systems. Intensive research has been done in attempting to eliminate them, compensate for them, or nullify their presence. Examples include, e.g. in modelling of internal combustion engines (Cook and Powell, 1998), and steel rolling mill control (Sbarbaro-Hofer, 1993).

In recent years, a new research area has emerged that makes use of time delays for good engineering control purposes, such as stabilizing oscillatory systems (Abdallah *et al.*, 1993), controlling chemical reactors (Schneider *et al,* 1993), actively controlling buildings against earthquakes (Udwadia *et al.*, 2003), and controlling aeroelastic systems in aerospace engineering (Yuan *et al.* 2004). It has been shown that introducing time delay in control is beneficial to achieve superior control performance, e.g. regulating chaos into their unstable periodic orbits (Pyragas 1992, 2002; Chen and Yu, 1999), vibration control of resonators (Olgac and Holm-Hansen, 1994), actively control of buildings (Udwadia *et al.* 2003), and aeroelastic systems control (Yuan *et al.* 2004).

In this paper, we consider the problem of controlling a 2-degree-of-freedom (2-DOF) torsional bar with parameter perturbations. The assumption is that the ideal controller is designed *a priori*. Then time-delay is then introduced in the controller, and we will investigate the effect of time-delay on the performance of the stability of the closed-loop system. The maximum allowed time-delay bound will be estimated by using a discretized Lyapunov functional method. A numerical example will be given to show that suitably introducing time-delay into the controller can improve the performance of the stability of the closed-loop system.

2. MODELING OF A 2-DEGREE-OF-FREEDOM (2-DOF) TORSIONAL BAR

This paper considers a 2-degree-of-freedom (2-DOF) torsional bar (Udwadia *et al.* 2003), which consists of 2 disks that undergo torsional vibrations, see Figure 1. The inertial properties of the disks can be altered by fastening additional weights to them. The control system has four primary components: (1) the real-time controller that generates the input trajectory and computes the control algorithm, (2) the software for defining the controller, (3) the actuator at the lower disc, and (4) the optical sensors. The real-time controller is a digital signal processor-based single-board computer. The servo loop closure involves the computation of the user-supplied control algorithm, and these computations occur at a rate of once every sampling period (0.00442 s). The actuator that actuates the lower disk utilizes a brushless dc motor with electrical commutation. Electrical commutation is accomplished by a sinusoidal switching scheme which has the advantage of reducing the magnitude of torque ripple. A sensor is secured to the motor shaft and reads its position. There are four incremental rotary shaft

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optical encoders on the system. Three are used to sense the position of the rotating disks. They have a resolution of 4000 pulses per revolution. The fourth encoder, with a resolution of 1000 pulses per revolution, is connected to the motor.

The equations of motion of the two masses in Figure 2 are as follows:

$$\begin{cases} J_1\ddot{\theta}_1(t)+c_1\dot{\theta}_1(t)+k_1[\theta_1(t)-\theta_2(t)]=T_c(t) \\ J_2\ddot{\theta}_2(t)+c_2\dot{\theta}_2(t)+k_1[\theta_2(t)-\theta_1(t)]+k_2\theta_2(t)=0 \end{cases} \quad (1)$$

where J_i, $i=1,2$, are the mass moments of inertia of the disks; c_i, $i=1,2$, are the respective viscous damping coefficients; k_i, $i=1,2$, are the stiffness coefficients; $\theta_i(t)$, $i=1,2$, are the angular displacements of the disks; and $T_c(t)$ is the actuator torque, respectively.

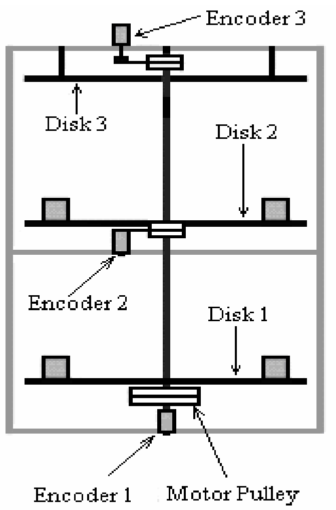


Fig. 1. Experimental apparatus

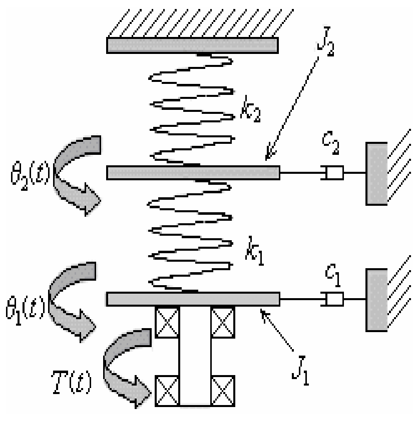


Fig. 2. Model of experimental apparatus

Let $x_1(t)=\theta_1(t)$, $x_2(t)=\dot{x}_1(t)=\dot{\theta}_1(t)$, $x_3(t)=\theta_2(t)$, $x_4(t)=\dot{x}_3(t)=\dot{\theta}_2(t)$, and $u(t)=T_c(t)$. Then system (1) can be written as

$$\dot{x}(t)=Ax(t)+Bu(t) \quad (2)$$

where $x(t)=\begin{pmatrix}x_1(t) & x_2(t) & x_3(t) & x_4(t)\end{pmatrix}^T$ and

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{J_1} & -\frac{c_1}{J_1} & \frac{k_1}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{J_2} & 0 & -\frac{(k_1+k_2)}{J_2} & -\frac{c_2}{J_2} \end{pmatrix},\ B=\begin{pmatrix} 0 \\ \frac{1}{J_1} \\ 0 \\ 0 \end{pmatrix}.$$

In practice, there always exists an uncertainty in the values of the system's parameters. The uncertainty usually arises from situations where the system parameters cannot be measured accurately or variations of the system's parameters are caused by fatigue, structural degradation and so on. Therefore, taking account into the perturbation of system's parameters yields the following system

$$\dot{x}(t)=[A+\Delta A(t)]x(t)+[B+\Delta B(t)]u(t) \quad (3)$$

where $\Delta A(t)$ and $\Delta B(t)$ are real matrix functions representing time-varying parameter uncertainties. The uncertainties are assumed in the form

$$[\Delta A(t)\ \ \Delta B(t)]=LF(t)[E_a\ \ E_b] \quad (4)$$

where $F(t)\in\mathbb{R}^{p\times q}$ is an unknown continuous time-varying matrix function satisfying

$$\sigma_{\max}(F(t))\le 1 \quad (5)$$

and L, E_a and E_b are known real constant matrices which characterize how the uncertainty enters the nominal matrices *A* and *B*.

3. A TIME-DELAYED DERIVATIVE CONTROLLER AND STABILITY CRITERIA

In this paper we are interested in studying the effect of time-delay on the control of a 2-degree-of-freedom (2-DOF) torsional bar. For this purpose, we introduce the time-delay derivative controller as

$$u(t)=-\rho x_4(t-r)=-\begin{pmatrix}0 & 0 & 0 & \rho\end{pmatrix}x(t-r)=-Cx(t-r) \quad (6)$$

where ρ is the control gain and $r>0$ is the time-delay, respectively. From (3) and (6) we have

$$\dot{x}(t)=[A+\Delta A(t)]x(t)-[B+\Delta B(t)]Cx(t-r) \quad (7)$$

with initial condition

$$x(\theta)=\phi(\theta),\ \forall\theta\in[-r,0]. \quad (8)$$

In the following, the control gain ρ is assumed to have been designed *a priori*. We are to find the maximum allowed time-delay bound such that the system described by (7) and (8) is robustly stable.

Since there exist time-varying uncertainties in (7), one can not employ the frequency-domain approach, such as the eigenvalue analysis method, to determine the time-delay bound. We use the Lyapunov-Krasovskii functional approach in the time-domain. A discretized Lyapunov functional method will be employed to study the robust stability of system (7)-(8).

Choose a Lyapunov-Krasovskii functional $V(\phi)$ of a quadratic form

$$V(\phi)=\frac{1}{2}\phi^T(0)P\phi(0)+\phi^T(0)\int_{-r}^{0}Q(\xi)\phi(\xi)d\xi$$
$$+\frac{1}{2}\int_{-r}^{0}[\int_{-r}^{0}\phi^T(\xi)R(\xi,\eta)\phi(\eta)d\eta]d\xi$$
$$+\frac{1}{2}\int_{-r}^{0}\phi^T(\xi)S(\xi)\phi(\xi)d\xi \quad (23)$$

where

$P\in\mathbb{R}^{4\times4}$, $P=P^T$; $Q:[-r,0]\to\mathbb{R}^{4\times4}$;

$S:[-r,0]\to\mathbb{R}^{4\times4}$, $S^T(\xi)=S(\xi)$;

$R:[-r,0]\times[-r,0]\to\mathbb{R}^{4\times4}$, $R(\eta,\xi)=R^T(\xi,\eta)$.

Choose Q, R and S to be continuous piecewise linear, i.e.

$$Q^i(\alpha)\overset{\Delta}{=}Q(\delta_{i-1}+\alpha h)=(1-\alpha)Q_{i-1}+\alpha Q_i \quad (24a)$$
$$S^i(\alpha)\overset{\Delta}{=}S(\delta_{i-1}+\alpha h)=(1-\alpha)S_{i-1}+\alpha S_i \quad (24b)$$
$$R(\delta_{i-1}+\alpha h,\delta_{j-1}+\eta h)=R^{ij}(\alpha,\eta)$$
$$\overset{\Delta}{=}\begin{cases}(1-\alpha)R_{i-1,j-1}+\eta R_{ij}+(\alpha-\eta)R_{i,j-1}, & \alpha\ge\eta\\(1-\eta)R_{i-1,j-1}+\alpha R_{ij}+(\eta-\alpha)R_{i-1,j}, & \alpha<\eta\end{cases} \quad (24c)$$

for $0\le\alpha\le1$, $0\le\eta\le1$, where

$\delta_i=-r+ih$, $i=0,1,2,\cdots,N$; $h=r/N$

i.e., N is the number of divisions of the interval $[-r,\ 0]$, and h is the length of each division.

Applying Lemma 1 in Han *et al.* (2003), we have the following result.

Theorem 2: For continuous piecewise linear Q, S and R as described by (24) and for $\Delta A(t)$ and $\Delta B(t)$ satisfying (4), the system described by (7)-(8) is robustly stable if the following LMIs hold

$$\begin{pmatrix}P & \tilde{Q}\\ \tilde{Q}^T & \frac{1}{h}\tilde{S}+\tilde{R}\end{pmatrix}>0 \quad (25)$$

$$\Xi(t)=\begin{pmatrix}G_{11}(t) & -G_{12}(t) & D_1^s(t) & D_1^a(t)\\ -G_{12}^T(t) & G_{22}(t) & D_2^s(t) & D_2^a(t)\\ D_1^{sT}(t) & D_2^{sT}(t) & \frac{1}{h}S_d+R_d & 0\\ D_1^{aT}(t) & D_2^{aT}(t) & 0 & \frac{3}{h}S_d\end{pmatrix}>0 \quad (26)$$

for $\Delta A(t)$ and $\Delta B(t)$ satisfying (4), where

$$\tilde{S}=diag(S_0\ S_1\ S_2\ \cdots\ S_{N-1}\ S_N)$$

$$\tilde{R}=\begin{pmatrix}R_{00} & R_{01} & \cdots & R_{0N}\\ R_{10} & R_{11} & \cdots & R_{1N}\\ \vdots & \vdots & \ddots & \vdots\\ R_{N0} & R_{N1} & \cdots & R_{NN}\end{pmatrix}$$

$$\tilde{Q}=[Q_0\ Q_1\ \cdots\ Q_N]$$

$$G_{11}(t)=-\{P[A+\Delta A(t)]+[A+\Delta A(t)]^TP+S_N+Q_N+Q_N^T\}$$

$$G_{12}(t)=P\{-[B+\Delta B(t)]C\}-Q_0,\ G_{22}(t)=S_0$$

$$S_d=diag(S_{d1}\ S_{d2}\ \cdots\ S_{dN})$$

$$S_{di}=\frac{1}{h}(S_i-S_{i-1})$$

$$R_d=\begin{pmatrix}R_{d11} & R_{d12} & \cdots & R_{d1N}\\ R_{d21} & R_{d22} & \cdots & R_{d2N}\\ \vdots & \vdots & \ddots & \vdots\\ R_{dN1} & R_{dN2} & \cdots & R_{dNN}\end{pmatrix}$$

$$R_{dij}=\frac{1}{h}(R_{ij}-R_{i-1,j-1});\ i,j=1,\ 2,\ \cdots,\ N$$

$$D_j^s(t)=[D_{j1}^s(t)\ D_{j2}^s(t)\ \cdots\ D_{jN}^s(t)];\ \ j=1,\ 2;$$

$$D_{1i}^s(t)=-\frac{1}{2}[A+\Delta A(t)]^T(Q_i+Q_{i-1})+\frac{1}{h}(Q_i-Q_{i-1})-\frac{1}{2}(R_{i,N}^T+R_{i-1,N}^T)$$

$$D_{2i}^s(t)=\frac{1}{2}C^T[B+\Delta B(t)]^T(Q_i+Q_{i-1})+\frac{1}{2}(R_{i,0}^T+R_{i-1,0}^T)$$

$$D_j^a(t)=[D_{j1}^a(t)\ D_{j2}^a(t)\ \cdots\ D_{jN}^a(t)];\ \ j=1,\ 2;$$

$$D_{1i}^a(t)=\frac{1}{2}[A+\Delta A(t)]^T(Q_i-Q_{i-1})+\frac{1}{2}(R_{i,N}^T-R_{i-1,N}^T)$$

$$D_{2i}^a(t)=-\frac{1}{2}C^T[B+\Delta B(t)]^T(Q_i-Q_{i-1})-\frac{1}{2}(R_{i,0}^T-R_{i-1,0}^T).$$

For norm-bounded, and possibly time-varying, uncertainty, using the technique in Han *et al.* (2003), the following result can be derived.

Theorem 3: The uncertain system (7)-(8) is robustly stable if there exist real 4×4 matrices $X=X^T$, Y_i, $W_i=W_i^T$ ($i=0,1,2,\cdots,N$) and $Z_{ij}=Z_{ji}^T$ ($i,j=0,1,2,\cdots,N$) such that

$$\begin{pmatrix}X & \tilde{Y}\\ \tilde{Y}^T & \frac{1}{h}\tilde{W}+\tilde{Z}\end{pmatrix}>0 \quad (27)$$

$$\begin{pmatrix}H_{00} & -H_{01} & 0 & \Gamma_0^s & \Gamma_0^a\\ -H_{01}^T & H_{11} & -H_{12} & \Gamma_1^s & \Gamma_1^a\\ 0 & -H_{12}^T & H_{22} & \Gamma_2^s & \Gamma_2^a\\ \Gamma_0^{sT} & \Gamma_1^{sT} & \Gamma_2^{sT} & \frac{1}{h}W_d+Z_d & 0\\ \Gamma_0^{aT} & \Gamma_1^{aT} & \Gamma_2^{aT} & 0 & \frac{3}{h}W_d\end{pmatrix}>0 \quad (28)$$

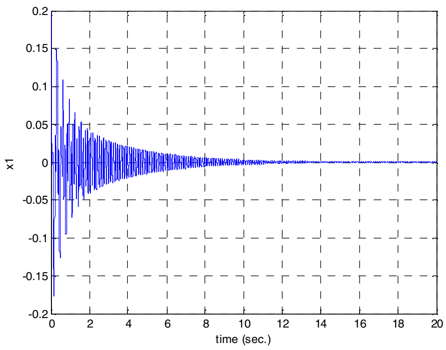
4. NUMERICAL RESULTS AND SIMULATIONS

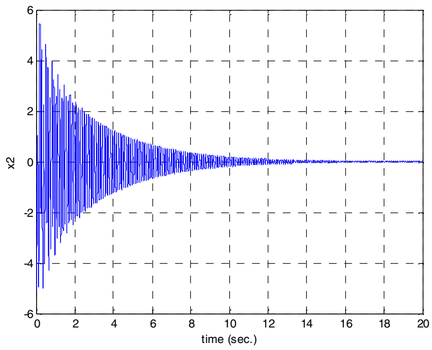
In the following, the following system's parameter setting from Udwadia *et al.* (2003) is used for simulation studies.

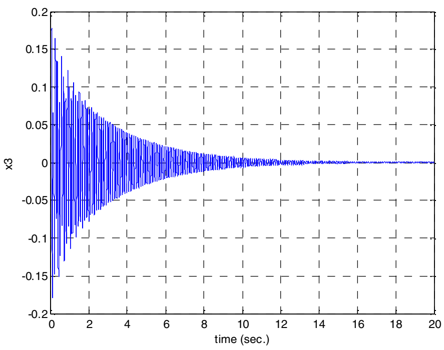
TABLE II SYSTEM'S PARAMETERS

System parameter	Experimental Value
J_1	0.00252 kg m^2
J_2	0.00194 kg m^2
k_1	2.830 N m/rad
k_2	2.697 N m/rad
c_1	0.00659 N m s/rad
c_2	0.00229 N m s/rad

Suppose that the control gain is designed as $\rho = 0.008$. We calculate the maximum allowed time-delay for asymptotic stability of the closed-loop system (7) as $r_{\max} = 0.10$. Figure 3 shows the simulation results for $r_{\max} = 0.10$ while Figure 4 gives the simulation results for $r = 0$, which means that there is no time-delay in the controller. One can clearly see that suitably introducing time-delay in the controller can significantly improve the performance of the stability of the closed-loop system.







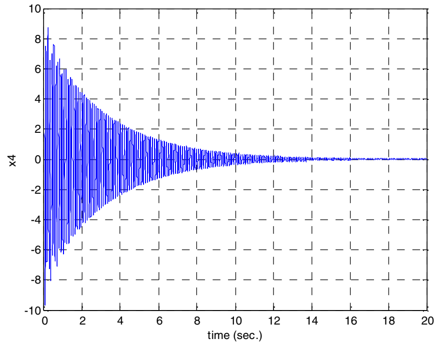
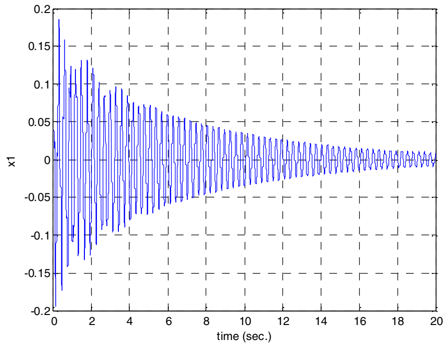
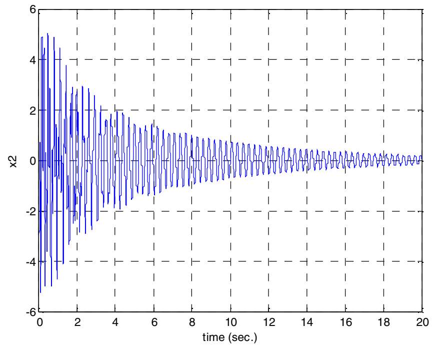
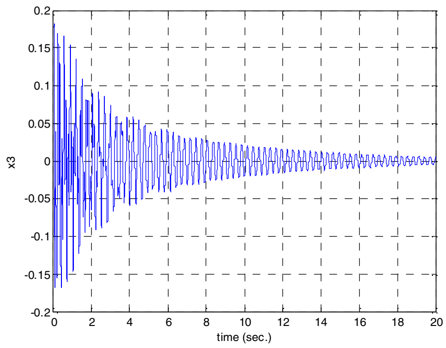


Fig. 3. The trajectories of states x_1, x_2, x_3, x_4 *for* $r_{\max} = 0.10$.







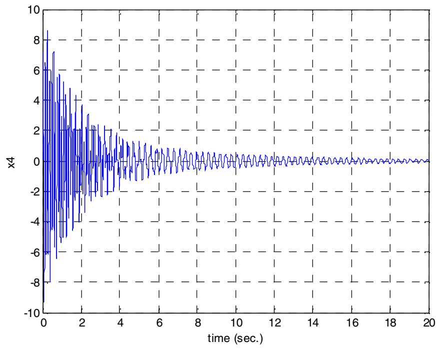
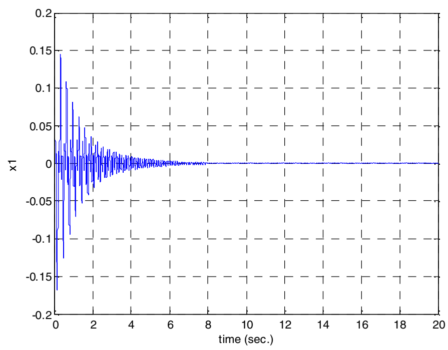


Fig. 4. The trajectories of states x_1, x_2, x_3, x_4 for $r = 0$.

We now analyze the effect of time-delay on the robustness of the stability of the closed-loop system with parameter perturbations. For the sake of simplicity, we first consider the case where there is *only* a single uncertainty $\Delta J_1(t) = \alpha_1 F_1(t)$, $|F_1(t)| \le 1$, in J_1. The allowed time delay bounds can be calculated. Table III lists the maximum allowed time delay bounds for different α_1. For example, Figures 5 and 6 show the results for $r_{\max} = 0.09$ and $r = 0$, respectively with $\alpha_1 = 0.0001$. It again shows that that introducing a suitable time-delay in the controller can indeed improve the performance of the stability of the closed-loop system with parameter perturbations. Similarly, we can also consider cases where there is *only* a single uncertainty in J_2, k_1, k_2, c_1 and c_2, respectively. It should be pointed out that we can consider the cases that there are more than one uncertainties in J_1, J_2, k_1, k_2, c_1 and c_2, simultaneously. Due to page limitation, it is omitted here.

TABLE III
The Maximum Time Delay Bound for $\Delta J_1(t) = \alpha_1 F_1(t)$, $|F_1(t)| \le 1$

$\alpha_1 = 0.0001$	$r_{\max} = 0.09$
$\alpha_1 = 0.0002$	$r_{\max} = 0.08$



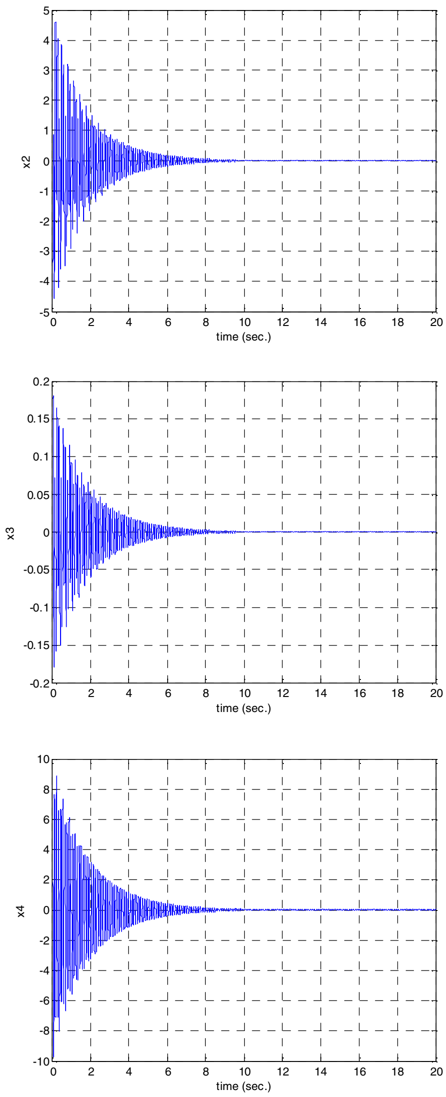
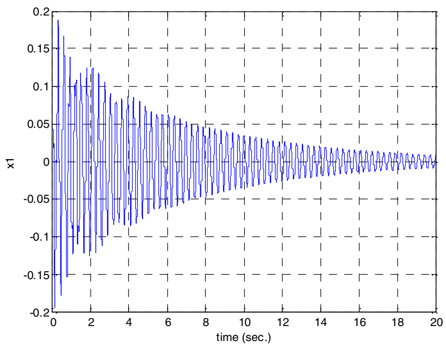


Fig. 5. The trajectories of states x_1, x_2, x_3, x_4 *for* $\alpha_1 = 0.0001$, $r_{\max} = 0.09$.



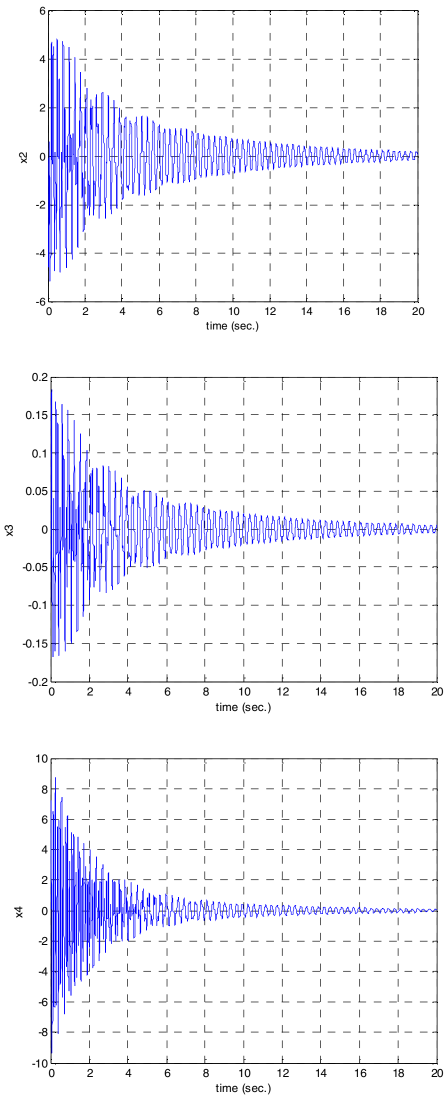


Fig. 6. The trajectories of states x_1, x_2, x_3, x_4 *for* $\alpha_1 = 0.0001$, $r = 0$.

4. CONCLUSION

The effect of time-delay on the derivative feedback control for a 2-degree-of-freedom torsional bar with parameter perturbations has been investigated. The maximum allowed time delay bound has been estimated by employing the delay bi--decomposition method and the discretized Lyapunov functional method. The simulation results have illustrated that suitably introducing time-delay into the controller can improve the performance of the stability of the closed-loop system.

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