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A Mean-Variance Model for Optimal Portfolio Selection with Transaction Costs

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Abstract: On the basis of Markowitz mean-variance framework, a new optimal portfolio selection approach is presented. The portfolio selection model proposed in the approach includes the expected return, the risk, and especially a quadratic type transaction cost of a portfolio. Using this model may yield an optimal portfolio solution that maximizes return, and minimizes risk, as well as also minimizes transaction costs by softening the transaction strength and smoothing the volume of the transacted securities in trading process. The optimization problem appeared in this approach is convex and can be solved by the quadratic programming (QP) routine. A case study demonstrates the effectiveness and the significant performance improvements of the optimal portfolio selection strategy proposed.

1. INTRODUCTION

The main study of the modern portfolio theory is still how to trade off risk and reward in choosing among risky investments under various conditions. The famous Markowitz mean-variance model shows how investors could achieve the lowest possible risk for any given target rate of return (Markowitz, 1952, 1959, 1987). The core of the Markowitz model is to take the expected return of a portfolio as the investment return and the variance of the expected return of a portfolio as the investment risk, and then to derive the maximum investment return by maximizing the expected return of a portfolio for a given risk level, or derive the minimum risk by minimizing the variance of a portfolio for a given specific return level.

On the basis of the Markowitz theory, a number of the effective models, criteria and strategies have been proposed for portfolio selection under certain assumptions (Wang and Xia, 2002). However, many problems still remain to be solved, in which how to effectively cope with the transaction cost, caused by changing the investment from an existing portfolio to a new portfolio according to an optimal portfolio selection strategy, is one of the main concerns. Ignoring the transaction cost in a portfolio selection model often leads to an inefficient portfolio in practice (Patel and Subrahmanyam, 1982). A typical way to consider the transaction cost in a model was to assume that the transaction cost is a V-shaped function of the difference between an existing portfolio and a new portfolio, and is formulated explicitly into the portfolio return (Markowitz, 1987, Wang and Xia, 2002, Perold, 1984). However, this resulted in nonlinear and non-convex function optimization problems (also see in Konno *et al*., 2005, Ehrgott *et al*., 2004, and Wang *et al*., 2006) that usually are easily got stuck in a local optimal solution if using general nonlinear optimization problem solvers.

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In this paper, the transaction cost is still to be assumed as a V-shaped function of the difference between an existing portfolio and a new portfolio. To avoid solving non-convex optimization problems in portfolio selection, and to soften the transaction strength or smooth the transacted securities for avoiding too radical trading, the transaction cost is indirectly controlled in this paper. To this end, a quadratic-type mean-variance model including the expected return, the variance of the return and the trading-cost described by the norm of the transacted securities is built, and a standard quadratic programming (QP) routine can thus be applied to solve the portfolio optimization problem. A case study to a portfolio having two securities whose corresponding indexes are respectively the growth-style index and the value-style index related to the benchmark index in a market is also given to illustrate the effectiveness and the significant performance improvements of the optimal portfolio selection approach proposed in this paper.

2. MARKOWITZ MEAN-VARIANCE MODEL

A portfolio selection problem in the Markowitz mean-variance framework may be written as follows as in Wang and Xia (2002) and Adcock (2002)

$$\begin{aligned}&\min_{w}\left\{\mu w^{\mathrm{T}} V w-(1-\mu)\bar{r}^{\mathrm{T}} w\right\}\\ &\text{subject to } \sum_{i=1}^{n} w_i=1,\ \ w_i\geq 0,\ \ i=1,2,\cdots,n\end{aligned} \tag{1}$$

where $w=\begin{bmatrix}w_1 & w_2 & \cdots & w_n\end{bmatrix}^{\mathrm{T}}$, $\bar{r}=\begin{bmatrix}\bar{r}_1 & \bar{r}_2 & \cdots & \bar{r}_n\end{bmatrix}^{\mathrm{T}}$

$$r=\begin{bmatrix}r_1 & r_2 & \cdots & r_n\end{bmatrix}^{\mathrm{T}},\ \ \bar{r}=E\{r\} \tag{2}$$

$$V=E\left\{(r-\bar{r})(r-\bar{r})^{\mathrm{T}}\right\} \tag{3}$$

and n is the number of risky securities, w_i is the portfolio weight that is the asset proportion invested in security i, $\bar{r}$

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is the expected security return, r is the actual security return, V is the variance of the expected security return, and μ is the risk aversion factor of the investor satisfying $0 \le \mu \le 1$. In optimization problem (1), $\bar{r}^{\mathrm{T}} w$ is the expected portfolio return, and $w^{\mathrm{T}} V w$ is the portfolio risk. The input data of minimization problem (1) are $\bar{r}$ the expected return of securities and V the variance-covariance matrix of the expected returns of securities in a portfolio.

Solving the quadratic type optimization problem (1) yields an optimal portfolio selection, which maximizes the portfolio return and minimizes the portfolio risk. However, the trading process often shows too radical transaction of assets, as shown in the case study in this paper. Transaction cost is another important issue in portfolio selection. If applying the widely used V-shaped function of the difference between an existing portfolio and a new portfolio, and formulating it explicitly into the portfolio return, finally one has to solve a non-convex optimization problem. An improved mean-variance model presented in the next section may be utilized to overcome the difficulties mentioned above.

3. A NEW POTFOLIO SELECTION STRATEGY

3.1 Quadratic-Type Mean-Variance Model with Transaction Costs

To use vector and matrix versions to represent the new mean-variance model, first define

$$
\begin{aligned}
x(t) &= \left[x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\right]^{\mathrm{T}} \\
s(t) &= \sum_{i=1}^{n} x_i(t) \qquad (4) \\
u(t) &= \left[u_1(t) \quad u_2(t) \quad \cdots \quad u_n(t)\right]^{\mathrm{T}}
\end{aligned}
$$

$$
\begin{aligned}
\bar{r}(t+1 \mid t) &= \left[\bar{r}_1(t+1 \mid t) \quad \bar{r}_2(t+1 \mid t) \quad \cdots \quad \bar{r}_n(t+1 \mid t)\right]^{\mathrm{T}} \\
r(t+1 \mid t) &= \left[r_1(t+1 \mid t) \quad r_2(t+1 \mid t) \quad \cdots \quad r_n(t+1 \mid t)\right]^{\mathrm{T}} \\
\bar{r}(t+1 \mid t) &= E\left\{r(t+1 \mid t)\right\} \qquad (5)
\end{aligned}
$$

$$
V(t+1 \mid t) = E\left\{\left[r(t+1 \mid t) - \bar{r}(t+1 \mid t)\right]\left[r(t+1 \mid t) - \bar{r}(t+1 \mid t)\right]^{\mathrm{T}}\right\} \qquad (6)
$$

where $x_i(t)$ is the i-th asset component in the portfolio $x(t)$ at time t, $s(t)$ is the gross asset of a portfolio at time t, $u_i(t)$ is the transacted asset with respect to the i-th asset $x_i(t)$, obviously the sum of all transacted assets in a portfolio must be zero, *i.e.* $\sum_{i=1}^{n} u_i(t) = 0$, $\bar{r}_i(t+1 \mid t)$ is the expected return of the i-th asset at time $t+1$, $r(t+1 \mid t)$ is the predictive return at time $t+1$, and $V(t+1 \mid t)$ is the variance-covariance matrix of the expected return.

An improved mean-variance portfolio selection strategy with quadratic type transaction cost term can now be given as follows

$$
\begin{aligned}
&\min_{u(t)} \left\{ \begin{array}{l} \mu\left[x(t)+u(t)\right]^{\mathrm{T}} V(t+1 \mid t)\left[x(t)+u(t)\right] \\ -(1-\mu)\, s(t) \bar{r}(t+1 \mid t)^{\mathrm{T}}\left[x(t)+u(t)\right] + u(t)^{\mathrm{T}} R u(t) \end{array} \right\} \\
&\text{subject to } \sum_{i=1}^{n} u_i(t) = 0, \quad x_i(t) + u_i(t) \ge k_i x_i(t) \qquad (7) \\
&\qquad x_i(t+1) = \left[1 + r_i(t+1)\right]\left[x_i(t) + u_i(t) - k_i \left|u_i(t)\right|\right] \\
&\qquad x_i(0) > 0, \quad i = 1, 2, \cdots, n
\end{aligned}
$$

where $R \in \mathbb{R}^{n \times n}$ is the constant positive-definite weighting matrix, k_i is the constant rate with respect to the i-th asset component, $k_i \left|u_i(t)\right|$ is the transaction cost with respect to the i-th asset, and $r_i(t+1)$ is the real return of the i-th asset component, which will be know at time $t+1$. In portfolio select strategy (7), the expected return is $\bar{r}(t+1 \mid t)^{\mathrm{T}}\left[x(t)+u(t)\right]$, the risk is measured by the variance of the return, *i.e.* $\left[x(t)+u(t)\right]^{\mathrm{T}} V(t+1 \mid t)\left[x(t)+u(t)\right]$, and it is easy to confirm that the two terms above are equivalent to the portfolio return and risk appeared in (1) respectively. The quadratic penalizing term $u(t)^{\mathrm{T}} R u(t)$ in (7) is utilized to soften the transacting strength in order to control the transaction costs and also decrease the risk of too drastic over- or under-trading of assets. Furthermore, the optimization problem (7) is of the quadratic form, and is equivalent to the following model after removing the constant terms

$$
\begin{aligned}
&\min_{u(t)} \left\{ \begin{array}{l} \frac{1}{2} u(t)^{\mathrm{T}}\left[2R + 2\mu V(t+1 \mid t)\right] u(t) \\ + \left[2\mu x(t)^{\mathrm{T}} V(t+1 \mid t) - (1-\mu)\, s(t) \bar{r}(t+1 \mid t)^{\mathrm{T}}\right] u(t) \end{array} \right\} \\
&\text{subject to } \sum_{i=1}^{n} u_i(t) = 0, \quad x_i(t) + u_i(t) \ge k_i x_i(t) \qquad (8) \\
&\qquad x_i(t+1) = \left[1 + r_i(t+1)\right]\left[x_i(t) + u_i(t) - k_i \left|u_i(t)\right|\right] \\
&\qquad x_i(0) > 0, \quad i = 1, 2, \cdots, n
\end{aligned}
$$

Notice that the absolute value function in (8) does not destroy the quadratic type structure of model (8), so the minimization problem (8) can be easily solved by a standard QP algorithm.

3.2 Estimation of Future Return and its Variance

To obtain the estimation of the expected return and its variance, first, a set of the first order AR models are used to estimate the future values of individual indexes corresponding to individual securities in a portfolio, and then the estimates of the future return and its variance are calculated based on the predictions of the indexes and the past sample data in this paper. Assume that for the i-th asset $(i = 1, 2, \cdots, n)$, its corresponding trading-index at current time t is $I_i(t)$ that is known, and the predictive index difference at time $t+1$ is $\Delta I_i(t+1 \mid t)$, which is estimated by the following AR model

$$\begin{aligned}
&\Delta I_i(t+1|t) = a_{i0}(t) + a_{i1}(t)\Delta I_i(t) + e_i(t+1)\\
&\Delta I_i(t+1|t) = I_i(t+1|t) - I_i(t) \\
&\Delta I_i(t) = I_i(t) - I_i(t-1)
\end{aligned} \tag{9}$$

where $a_{i0}(t)$ and $a_{i1}(t)$ are the parameters of the model, and $e_i(t+1)$ is a white noise. The parameters of model (9) are estimated by the following gradually forgetting least squares method

$$\begin{bmatrix} \hat{a}_{i0}(t) \\ \hat{a}_{i1}(t) \end{bmatrix} = \Theta(t)^{+} Y(t) \tag{10}$$

$$\begin{cases} \Theta(t)^{+} = \left[\Theta(t)^{\mathrm{T}}\Theta(t)\right]^{-1}\Theta(t)^{\mathrm{T}} \\ \Theta(t) = \begin{bmatrix} 1 & \Delta I_i(t-1) \\ 1 & \Delta I_i(t-2) \\ \vdots & \vdots \\ 1 & \Delta I_i(t-m) \end{bmatrix}, \quad Y(t) = \begin{bmatrix} \Delta I_i(t) \\ \Delta I_i(t-1) \\ \vdots \\ \Delta I_i(t-m+1) \end{bmatrix} \end{cases} \tag{11}$$

where $m > 2$ is the efficient data horizon, and $\Theta(t)^{+}$ is the pseudo-inverse of $\Theta(t)$, calculated using the singular value decomposition (SVD) for overcoming ill-conditioned problems, which will improve the robustness of the numerical computation. According to model (9-11), the prediction of the index difference $\Delta I_i(t+1|t)$ is then given by

$$\Delta\hat{I}_i(t+1|t) = \hat{a}_{i0}(t) + \hat{a}_{i1}(t)\Delta I_i(t) \tag{12}$$

From the estimated future individual index-differences corresponding to individual assets in a portfolio, the estimation of the expected return of each asset in a portfolio is thus calculated by the following formula

$$\bar{r}_i(t+1|t) = \frac{\Delta\hat{I}_i(t+1|t)}{I_i(t)} \tag{13}$$

The latest estimate of variance-covariance matrix of the portfolio return is approximated by the sample variance-covariance matrix obtained from the past and the predicted data in this paper, which is given by

$$\tilde{V}(t+1|t) = \frac{1}{m+1}\left\{ \begin{aligned} &\left[\bar{r}_i(t+1|t) - \bar{r}_{t,m}\right]\left[\bar{r}_i(t+1|t) - \bar{r}_{t,m}\right]^{\mathrm{T}} \\ &+\sum_{i=1}^{m}\left[r(t-i+1) - \bar{r}_{t,m}\right]\left[r(t-i+1) - \bar{r}_{t,m}\right]^{\mathrm{T}} \end{aligned} \right\} \tag{14}$$

$$\bar{r}_{t,m} = \frac{1}{m+1}\left\{\bar{r}_i(t+1|t) + \sum_{i=1}^{m} r(t-i+1)\right\} \tag{15}$$

4. CASE STUDY

4.1 Two Special Style Stocks Indexes Related to Benchmark Index and the Portfolio to be Studied

This case study is based on monthly data from December 1979 to April 2001 for the portfolio with two special styles stocks from a stock market. The indexes of two stocks are the value-style index and the growth-style index respectively related to a benchmark index. There is a particular relationship between the benchmark index and the two stocks indexes, that is, to the benchmark index, if one of the two style-indexes is good, another one is almost bad. This implies that if one switches his/her assets well between the two style-indexes, he/she can obtain a return better than the return gotten from the benchmark index alone. The benchmark index, the value-style index and the growth-style index are shown in Fig.1 respectively. In this case study, each month, the estimate of expected return and variance-covariance of the portfolio are computed by (10)-(15). The optimal portfolios are built and held for one month in the cases without transaction costs and with 0.2% transaction costs, by using the Markowitz mean-variance model (1) and the improved mean-variance model (8) proposed in this paper respectively. The optimization problem (1) or problem (8) is solved by the '*quadprog*' function in Matlab Optimization Toolbox.

Assume that the initial value of gross asset of the portfolio to be studied is 200, in which the initial value of both value-style stock and growth-style stock are all 100 identical with their corresponding indexes' initial values as showed in Fig. 1. For comparison, assume also that one buys a stock whose index is just the benchmark index showed in Fig. 1 and the initial value are also 200, and thus the benchmark stock's assets vary only with the benchmark index. The portfolio's or the benchmark stock's gross asset and its corresponding mean year rate-of-return with respect to the initial asset (*i.e.* 200) in trading process are used for evaluating the investment performance, which is illustrated in Figs. 2-13.

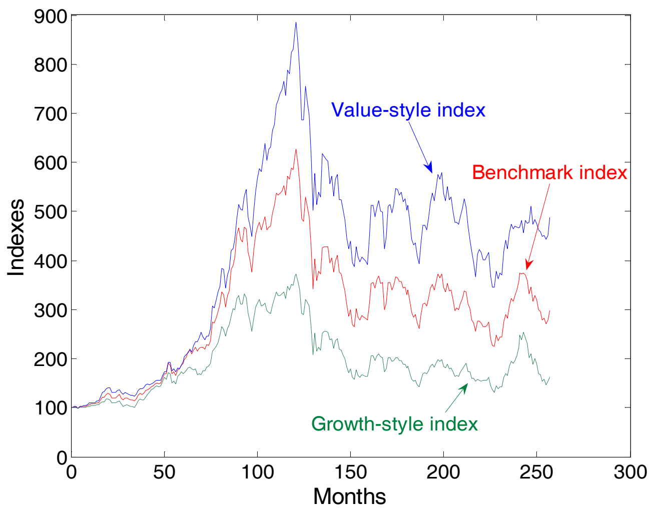

Fig. 1. Benchmark index and two special style stocks indexes; monthly data from December 1979 to April 2001.

4.2 Portfolio Selection Results Using the Markowitz Mean-Variance Method

Figs. 2-4 show the portfolio selection results yielded by the mean-variance model (1) without transaction costs where the used risk aversion factor $\mu = 0.7$ and the efficient data horizon $m = 30$. Figs. 2-3 give the comparisons of gross asset and mean year rate-of-return between the optimal portfolio built by model (1) without transaction costs and the benchmark stock mentioned in Section 4.1. In Figs. 2-3, also in Figs. 5-6, Figs. 8-9, and Figs. 11-12, the red line marked by 'Optimal portfolio' is the gross asset or the mean year rate-of-return of an optimal portfolio, the blue line marked by 'Benchmark' is the asset or the mean year rate-of- return of the benchmark stock, and the green line marked by 'Benchmark + 1.8% year rate' is the asset or the mean year rate-of-return of a target fund whose mean year rate-of-return is the benchmark stock's mean year rate-of-return plus 1.8%. Fig. 3 shows that in the last half part of trading, the mean year rate-of-return of the portfolio built by model (1) without transaction costs is about 0.9% higher than that of the benchmark stock, which is not so good from the practical point of view. Fig. 4 gives the transacted assets of two stocks of the portfolio built by model (1) without transaction costs, from which one can see that the too radical over- or under-transacting of assets are appeared because there are no any restrictions directly adding on the transacted-assets.

Figs. 5-7 show the portfolio selection results yielded by the mean-variance model (1) with 0.2% transaction costs, where the risk aversion factor $\mu = 0.7$, and the efficient data horizon $m = 30$. Figs. 5-6 shows the comparisons of gross asset and mean year rate-of-return between the optimal portfolio built by model (1) with 0.2% transaction costs and the benchmark stock. From Figs. 5-7, it is clear that the built optimal portfolio is even worse than the benchmark stock in most of the cases of trading, because too drastic over- or under-transacting of assets appears in the trading process. This can be seen in Fig. 7 and yields a larger negative effect on the portfolio under the condition of deducting transaction costs in trading.

4.3 Portfolio Selection Results Using the Improved Mean-Variance Modeling Approach Proposed

Figs. 8-10 show the portfolio selection results yielded by the proposed mean-variance model (8) without transaction costs, where the risk aversion factor $\mu = 0.7$, the weighting matrix $R = 0.02I_2$, $I_2 \in \mathbb{R}^{2\times 2}$ represents the 2×2 identity matrix, the cost rate $k_i = 0$, and the efficient data horizon $m = 7$. Figs. 8-9 give the comparisons of gross asset and mean year rate-of-return between the optimal portfolio built by model (8) without transaction costs and the benchmark stock mentioned in Section 4.1. From Figs. 8-9 one can see that in the last half part of transacting, the gross asset and the mean year rate-of-return of the optimal portfolio built by model (8) without transaction costs are very close to that of the target fund introduced in Section 4.2.

Compared with the portfolio built by model (1) also without transaction costs showed in Figs. 2-3, the portfolio built by the proposed model (8) without transaction costs given in Figs. 8-9 reveals much better performance. This is because an additional penalty for the transacted assets is added to the mean-variance model (7) for avoiding too radical over- or under- transacting of assets, which is particularly useful in general case that the estimate of expected return and variance-covariance of a portfolio may be not so accurate, and thus such radical over- or under-transacting of assets will lead to a larger loss of portfolio performance, especially in case with transaction costs, as showed in Figs. 2-7. Comparing Fig. 10 and Fig. 4, it reveals that the transacting-strength of assets in Fig. 10 is largely softened and smoothed by using strategy (8). Therefore, the performance of the portfolio built by model (8) is far better than that of the portfolio built by model (1) as showed in Figs. 8-9 and Figs. 2-3 under the condition of using the same estimation approach to the expected return and variance-covariance.

Figs. 11-13 give the portfolio selection results yielded by the proposed mean-variance model (8) with 0.2% transaction costs, where the risk aversion factor $\mu = 0.7$, the weighting matrix $R = 0.02I_2$, the constant cost-rate $k_i = 0.2\%$, and the efficient data horizon $m = 7$. In Fig. 12, the mean year rate-of-return of the optimal portfolio built by model (8) with 0.2% transaction costs is 1.06-2.35% higher than the mean year rate-of-return of the benchmark stock in the last half part of transacting; this is quite good from the practical point of view. Furthermore, comparing Fig. 13 and Fig. 7, one can see that the variation of transacted assets of the portfolio built by the proposed model (8) are mach smoother than that of the portfolio built by model (1), this is the reason for that the portfolio performance yielded by model (8) is far better than the portfolio performance yielded by model (1) as seen from the comparison between Figs. 11-12, and Figs. 5-6.

5. CONCLUSIONS

This paper proposed a novel mean-variance approach for optimal portfolio selection with transaction costs, which effectively avoided too drastic transaction of assets during portfolio selection process, and could be easily solved by a standard quadratic programming algorithm. This implies that the local minimum problem does not exist in the strategy proposed. The first order AR model applied for predicting the expected return and variance of a portfolio is simple but efficient. However, using other model better than AR model may yields better portfolio if using the mean-variance approach proposed in this paper.

REFERENCES

Adcock, c.j. (2002). Portfolio optimization. In: *Neural Networks and the Financial Markets* (J. Shadbolt and J.G. Taylor (Eds)), 221-246. Springer, London.

Ehrgott, M., K. Klamroth and C. Schwehm (2004). Decision aiding an MCDM approach to portfolio optimization. *European Journal of Operational Research*, **155**, 752-770.

Konno, H., K. Akishino and R. Yamomoto (2005). Optimization of long-short portfolio under non-convex transaction costs. In: *Special issue, Optimization and Risk Modelling* (G. Mitra (Ed)), *Computational Optimization and Application*, **32**, 112-132.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, **7**, 77-91.

Markowitz, H. (1959). *Portfolio Selection, Efficient Diversification of Investment*. Basil Blackwell, New York.

Markowitz, H. (1987). *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. Blackwell Publishers, Cambridge.

Patel, N.R. and M. G. Subrahmanyam (1982). A simple algorithm for optimal portfolio selection with fixed transaction costs. *Management Science*, **28**, 303-314.

Perold, A.F. (1984). Large-scale portfolio optimization. *Management Science*, **30**, 1143-1160.

Wang, S. and Y. Xia (2002). *Portfolio Selection and Asset Pricing*. Springer, Berlin.

Wang, S.M., J.C. Chen, H.M. Wee and K.J. Wang (2006). Non-linear Stochastic Optimization Using Genetic Algorithm for Portfolio Selection. *International Journal of Operations Research*, **3**, 16-22.

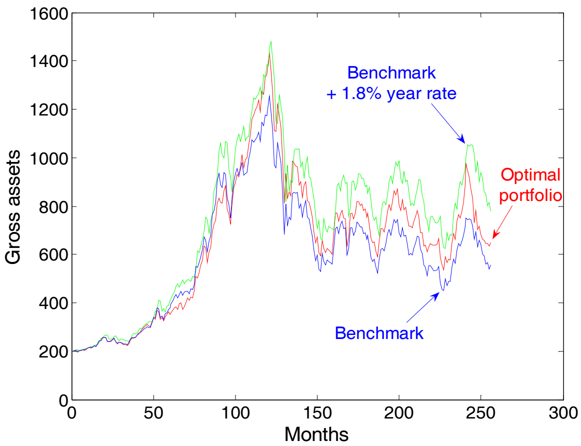

Fig. 2. Gross asset for the optimal portfolio based on two stocks built by mean-variance model (1) without transaction costs.

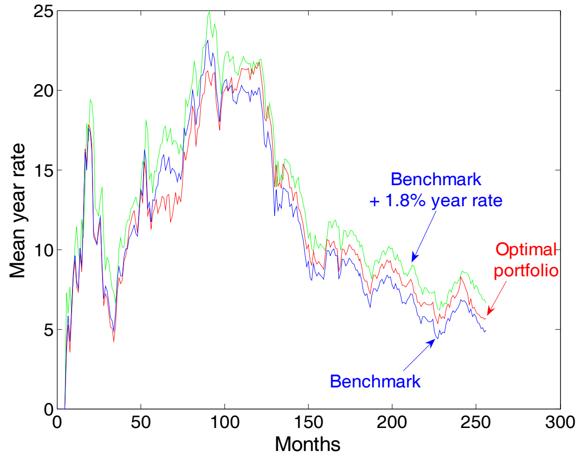

Fig. 3. Mean year rate of return for the optimal portfolio built by mean-variance model (1) without transaction costs.

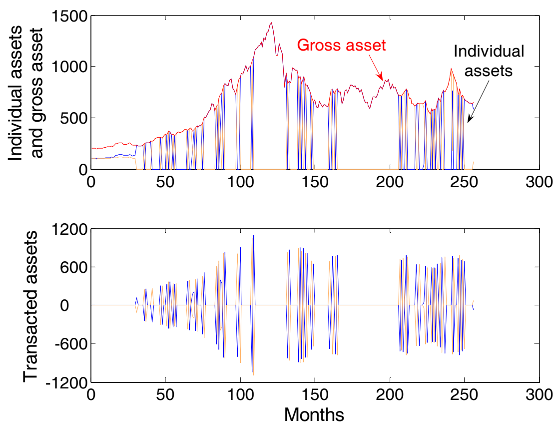

Fig. 4. Two stocks assets and the corresponding transacted-assets for the optimal portfolio built by mean-variance model (1)

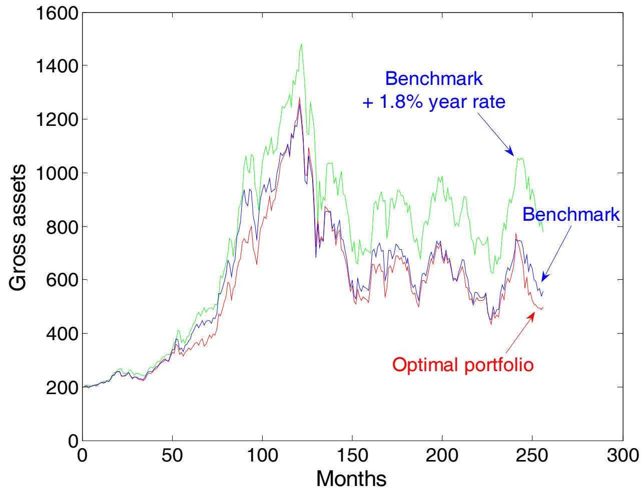

Fig. 5. Gross asset for the optimal portfolio based on two stocks built by mean-variance model (1) with 0.2% transaction costs.

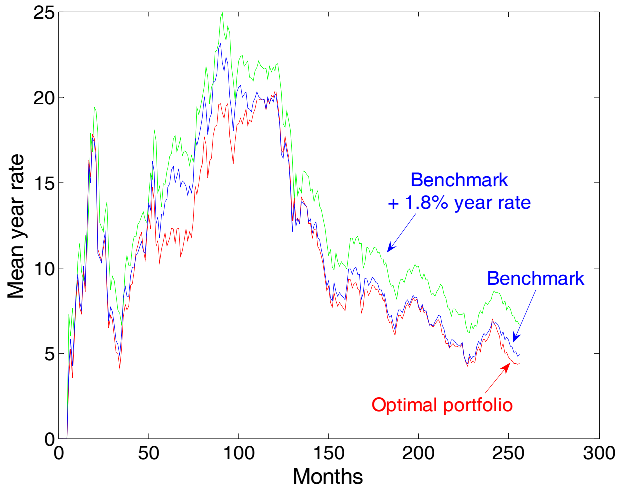

Fig. 6. Mean year rate of return for the optimal portfolio built by mean-variance model (1) with 0.2% transaction costs.

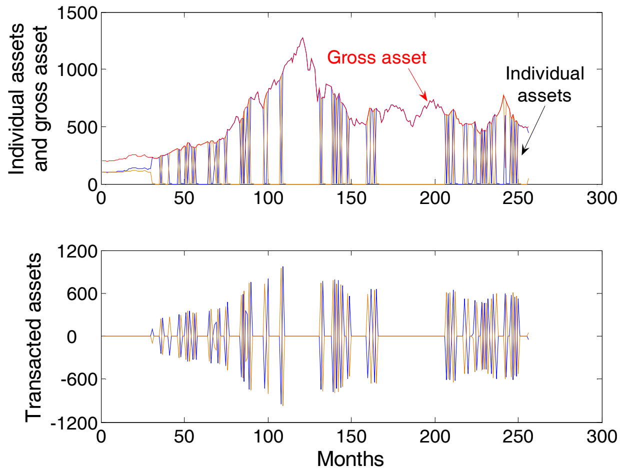

Fig. 7. Two stocks assets and the corresponding transacted-assets for the optimal portfolio built by mean-variance model (1) with 0.2% transaction costs.

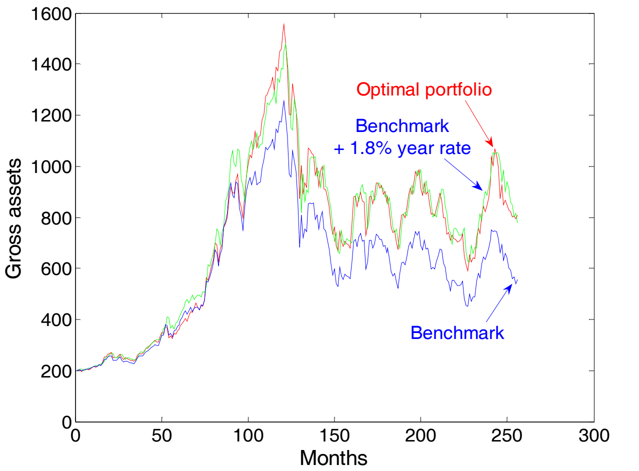

Fig. 8. Gross asset for the optimal portfolio based on two stocks built by the proposed mean-variance model (8) without transaction costs.

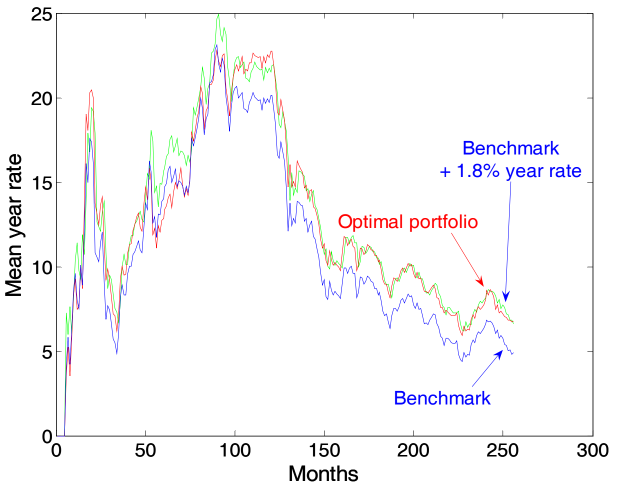

Fig. 9. Mean year rate of return for the optimal portfolio built by the proposed mean-variance model (8) without transaction costs.

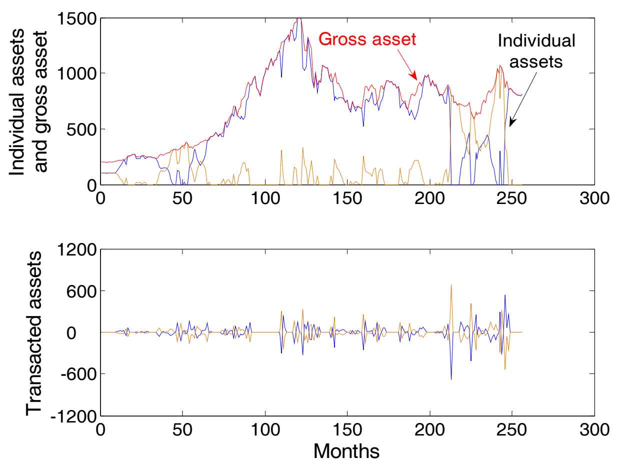

Fig. 10. Two stocks assets and the corresponding transacted-assets for the optimal portfolio built by the proposed mean-variance model (8) without transaction costs.

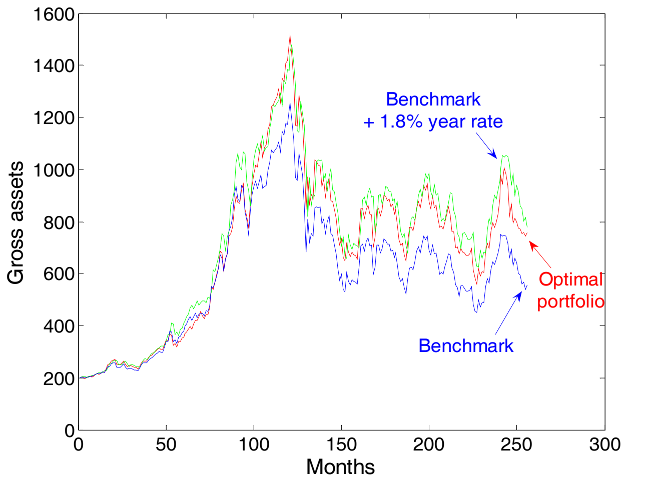

Fig. 11. Gross asset for the optimal portfolio based on two stocks built by the proposed mean-variance model (8) with 0.2% transaction costs.

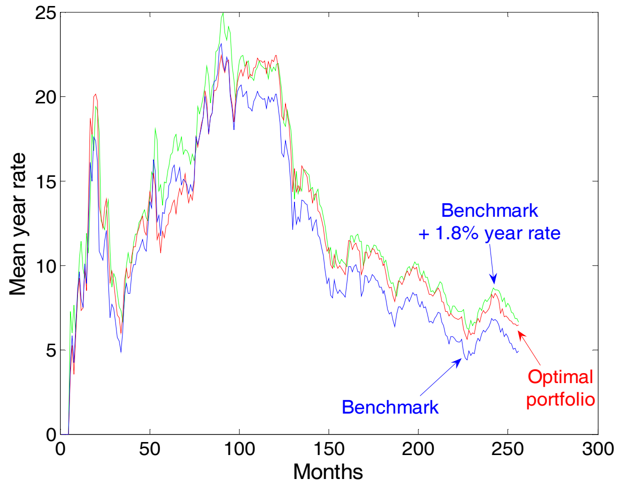

Fig. 12. Mean year rate of return for the optimal portfolio built by the proposed mean-variance model (8) with 0.2% transaction costs.

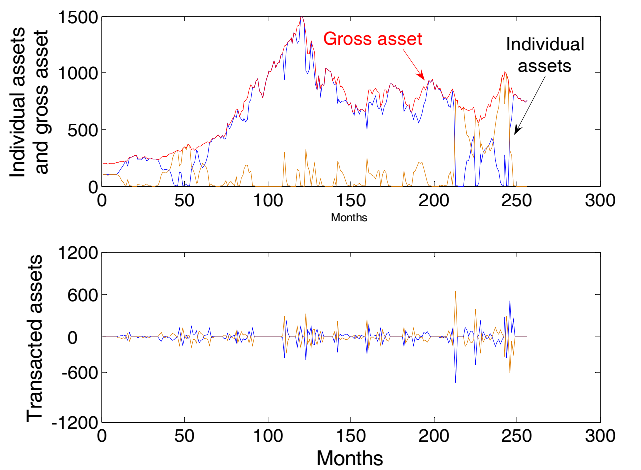

Fig. 13. Two stocks assets and the corresponding transacted-assets for the optimal portfolio built by the proposed mean-variance model (8) with 0.2% transaction costs.