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Freeway Density Control Via Model-free Adaptive Ramp Metering Approach

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Abstract: By introducing a new dynamical linearization technology, this paper presents a model-free adaptive control approach for density control of freeway traffic flow via ramp metering, which is consisted with a control input learning law and a parameter updating law. The design and analysis only depends on the I/O data of the freeway traffic system. Furthermore, the control input learning law is extended to a higher-order form by incorporating more control information of previous sampling instants for improving the control performance. Both convergence analysis and simulation results illustrate the validity of the presented methods.

1. INTRODUCTION

Ramp metering has been recognized as one of the most effective ways for combating freeway congestion (Papageorgiou & Kotsialos, 2002). A common objective of ramp control is to regulate the amount of traffic entering a freeway from entry ramps during certain time periods so that the flow on the freeway does not exceed its capacity.

From the viewpoint of system control, ramp metering is a typical regulating problem and numerous control methods have been exploited (Papageorgiou et al, 1991; Zhang et al, 2001; Chang & Li, 2002; Akiyama & Okushima, 2006). However, the freeway traffic flow system is of nonlinearities, coupling, and uncertainties and an accurate model is hardly available in practice. The above mentioned control methods may encounter some difficulties in practice to design and construct for nonlinear processes, such as model uncertainties, unmodelled dynamics, and so on.

In this paper, we explore the possibility of extending the model-free adaptive control (Hou & Huang, 1997; Chi, 2005) to deal with the density control of freeway traffic flow via ramp metering just depending upon the I/O data. The dynamical linearization of the freeway macroscopic traffic flow system is given firstly and then the model-free adaptive control scheme is presented for the density control of freeway traffic flow, which is constituted with a control input updating law and a parameter updating law. Furthermore, the control input updating law is extended to a higher-order form by using a new weighted control input criterion function. Both convergence analysis and simulation results illustrate the validity of the proposed method.

The rest of this paper is organized as follows. Section 2 is the problem formulation. Section 3 provided a new dynamic linearization for the freeway macroscopic traffic flow. Section 4 presents the model-free adaptive control for traffic flow density. Section 5 extends the control input updating law to a higher-order form for improving the control performance furthermore. Case studies with simulations are provided in Section 6. Finally, Section 7 concludes this paper.

2. PROBLEM FORMULATION

2.1 Macroscopic Traffic Model

The spatially discretized traffic flow model, proposed by Papageorgiou et al (1989; 1990), for a single freeway with one on-ramp and one off-ramp on each section is shown in Fig. 1 and (1)-(4) below

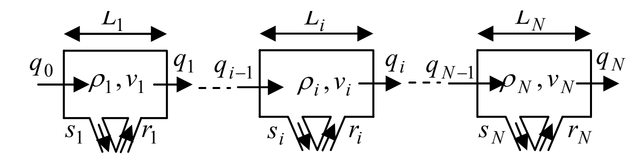

Fig. 1 Sections on a freeway with on/off ramp

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i}[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)], \quad (1)$$

$$q_i(k) = \rho_i(k) v_i(k) \quad (2)$$

$$\begin{aligned} v_i(k+1) = {} & v_i(k) + \frac{T}{\tau}[V(\rho_i(k)) - v_i(k)] \\ & + \frac{T}{L_i} v_i(k)[v_{i-1}(k) - v_i(k)] - \frac{\nu T}{\tau L_i}\frac{[\rho_{i+1}(k) - \rho_i(k)]}{[\rho_i(k) + \kappa]} \end{aligned} \quad (3)$$

$$V(\rho_i(k)) = v_{free}(1 - [\frac{\rho_i(k)}{\rho_{jam}}]^l)^m \quad (4)$$

where T is the sample time interval. $k \in \{0,1,\cdots,K\}$ is the k-th time interval, and $i \in \{1,\cdots,N\}$ is the i-th section of a freeway, and N is the total section number. $\rho_i(k)$: density in section i at time kT (veh/lane/km); $v_i(k)$: space mean speed in section i at time kT (km/h); $q_i(k)$: traffic flow leaving section i and entering section i+1 at time kT (veh/h); $r_i(k)$: on-ramp traffic volume for section i at time kT (veh/h); $s_i(k)$: off-ramp traffic volume for section i at time kT (veh/h), which is regarded as an unknown disturbance; L_i: length of freeway in section i, (km); V_{free} and ρ_{jam} are the free speed and the maximum possible density per lane, respectively. $\tau, \gamma, \kappa, l, m, \omega$ are constant parameters which reflect particular characteristics of a given traffic system and depend on the freeway geometry, vehicle characteristics, drivers' behaviours, etc.

2.2 Boundary

We assume that the traffic flow rate entering section 1 during the time period kT and $(k+1)T$ is $q_0(k)$ and the mean speed of the traffic entering section 1 is equal to the mean speed of section 1, i.e. $v_0(k) = v_1(k)$. We also assume that the mean speed and traffic density of the traffic exiting section N+1 are equal to those of section N, i.e. $v_{N+1}(k) = v_N(k)$, $\rho_{N+1}(k) = \rho_N(k)$. Boundary conditions can be summarized as follows

$$\rho_0(k) = q_0(k)/v_1(k), \tag{5}$$

$$v_0(k) = v_1(k), \tag{6}$$

$$\rho_{N+1}(k) = \rho_N(k), \tag{7}$$

$$v_{N+1}(k) = v_N(k), \qquad \forall k. \tag{8}$$

2.3 Control Objective

The control objective is to seek an appropriate on-ramp traffic flow, $r_i(k)$ for the i-th on-ramp locally driving the traffic density $\rho_i(k)$ of section i to track the desired traffic density $\rho_{i,d}(k)$ of section i, i.e. the tracking error $e_i(k) = \rho_{i,d}(k) - \rho_i(k)$ converges to zero asymptotically as k approaches to infinity. Obviously, due to the highly nonlinear and uncertain nature of traffic flow model, such a control profile cannot be calculated directly from the model.

3. DYNAMICAL LINEARIZATION OF MACROSCOPIC TRAFFIC FLOW MODEL

The macroscopic traffic flow model described by equations (1) and (2) can be written in the following form

$$\begin{aligned}\rho_i(k+1) = (1 - \frac{T}{L_i} v_i(k))\rho_i(k) + \frac{T}{L_i} v_{i-1}(k)\rho_{i-1}(k) \\ + \frac{T}{L_i} r_i(k) - \frac{T}{L_i} s_i(k),\end{aligned} \tag{9}$$

thus, we can also rewrite (9) as

$$\begin{aligned}\rho_i(k+1) = a_i(k)\rho_i(k) + b_i(k)\rho_{i-1}(k) \\ + c_i(k)r_i(k) - c_i(k)s_i(k),\end{aligned} \tag{10}$$

with $a_i(k) = 1 - \frac{T}{L_i} v_i(k), b_i(k) = \frac{T}{L_i} v_{i-1}(k), c_i(k) = \frac{T}{L_i}$.

Assumption 1: The partial derivative of dynamical system (10) with respect to control input $r_i(k)$ is exist and continuous.

Remark 1: It shall be noted that the macroscopic traffic flow model (1)-(4) is continuous differentiable in all arguments, thus Assumption 1 holds in nature.

Assumption 2: The system (10) is generalized Lipschtz, that is, for any k and $\Delta r_i(k) \neq 0$,

$$|\Delta\rho_i(k+1)| \leq M|\Delta r_i(k)|, \tag{11}$$

where M is a constant, $\Delta\rho_i(k+1) = \rho_i(k+1) - \rho_i(k)$, and $\Delta r_i(k) = r_i(k) - r_i(k-1)$.

Remark 2: Assumption 2 is some limitation on the rate of the system output change when the control input increases. Clearly, it is satisfied with freeway traffic system in practice. Furthermore, we just need the existence of such a constant M without requiring the exact value.

Theorem 1: For dynamical system (10) under assumptions 1 and 2, there must exist a parameter variable $\theta_i(k)$ such that when $\Delta r_i(k) \neq 0$,

$$\Delta\rho_i(k+1) = \theta_i(k)\Delta r_i(k), \tag{12}$$

and $|\theta_i(k)| \leq M$.

Proof: From system (10), we have

$$\begin{aligned}\Delta\rho_i(k+1) &= a_i(k)\rho_i(k) + b_i(k)\rho_{i-1}(k) \\ &\quad + c_i(k)r_i(k) - c_i(k)s_i(k) \\ &\quad - a_i(k)\rho_i(k) - b_i(k)\rho_{i-1}(k) \\ &\quad - c_i(k)r_i(k-1) + c_i(k)s_i(k) \\ &\quad + a_i(k)\rho_i(k) + b_i(k)\rho_{i-1}(k) \\ &\quad + c_i(k)r_i(k-1) - c_i(k)s_i(k) \\ &\quad - a_i(k-1)\rho_i(k-1) - b_i(k-1)\rho_{i-1}(k-1) \\ &\quad - c_i(k-1)r_i(k-1) + c_i(k-1)s_i(k-1) \\ &= c_i(k)\Delta r_i(k) + \xi_i(k),\end{aligned} \tag{13}$$

where

$$\begin{aligned}\xi_i(k) = a_i(k)\rho_i(k) + b_i(k)\rho_{i-1}(k) + c_i(k)r_i(k-1) \\ - c_i(k)s_i(k) - a_i(k-1)\rho_i(k-1) \\ - b_i(k-1)\rho_{i-1}(k-1) - c_i(k-1)r_i(k-1) + c_i(k-1)s_i(k-1).\end{aligned}$$

Consider an equation with $\eta_i(k)$

$$\xi_i(k) = \eta_i(k)\Delta r_i(k), \tag{14}$$

when $\Delta r_i(k) \neq 0$, we must have the solution $\eta_i(k)$ of equation (14).

Let $\theta_i(k) = c_i(k) + \eta_i(k)$, equation (12) can be obtained directly from (13) and (14). And in terms of Assumption 2, $|\theta_i(k)| \le M$.

Remark 3: In fact $\theta_i(k)$ is a most complicated unknown function about the past inputs and system states. In this paper, we will give its prediction or estimation values by using the information of previous sampling times.

Remark 4: In fact, the traffic dynamics may be expressed as a general nonlinear form (Papageorgiou & Kotsialos, 2002)

$$\boldsymbol{x}(k+1) = f[\boldsymbol{x}(k), \boldsymbol{r}(k), \boldsymbol{d}(k)],$$

where the state vector $\boldsymbol{x}$ comprises all traffic densities and mean speeds, as well as all ramp queues; the control vector $\boldsymbol{r}$ comprises all controllable ramp volumes; the disturbance vector $\boldsymbol{d}$ comprises all on-ramp demands and turning rates. According to Hou (1997), we can still give its responding linearization form similar to (12). The macroscopic traffic flow model (1)-(4) above is a special case and only serves as a simulation evaluation in this paper. The design and analysis of the MFAC does not require any information of the traffic flow model as shown in the following section.

4. MODEL-FREE ADAPTIVE CONTROL OF TRAFFIC FLOW DENSITY

For briefness in writing, we omit the subscript i in the flowing equations in the case of not causing confusion.

4.1 Model-free Adaptive Controller Design

Rewrite (12) as

$$\rho(k+1) = \rho(k) + \theta(k)\Delta r(k). \tag{15}$$

Define an index function of control input as

$$J(r(k)) = |e(k+1)|^2 + \lambda |r(k) - r(k-1)|^2, \tag{16}$$

where λ is a positive weighting factor.

According to (15), a new expression of $J(r(k))$ is given as follows

$$\begin{aligned} J(r(k)) = &\left|\rho_d(k+1) - \rho(k) - \theta(k)(r(k) - r(k-1))\right|^2 \\ &+ \lambda |r(k) - r(k-1)|^2. \end{aligned} \tag{17}$$

Using the optimal condition $\frac{1}{2}\frac{\partial J}{\partial r(k)} = 0,$ we have

$$r(k) = r(k-1) + \frac{\alpha_k \theta(k)}{\lambda + |\theta(k)|^2}(\rho_d(k+1) - \rho(k)), \tag{18}$$

where α_k is a step-size constant series, which is added to make the generality of the algorithm (18) and will be used in analytical stability proof later.

Since $\theta(k)$ is unknown and not available, we present the learning control law as

$$r(k) = r(k-1) + \frac{\alpha_k \hat{\theta}(k)}{\lambda + |\hat{\theta}(k)|^2}(\rho_d(k+1) - \rho(k)), \tag{19}$$

where $\hat{\theta}(k)$ is to learning parameter $\theta(k)$ and updated in terms of the optimal solution of the following criterion index function

$$J(\hat{\theta}(k)) = \left|\Delta\rho(k) - \hat{\theta}(k)\Delta r(k-1)\right|^2 + \mu\left|\hat{\theta}(k) - \hat{\theta}(k-1)\right|^2, \tag{20}$$

where $\mu > 0$ is a positive weighting factor.

Using the optimal condition $\frac{1}{2}\frac{\partial J}{\partial \hat{\theta}(k)} = 0,$ we can obtain the parameter updating law as follows

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) \\ &+ \frac{\beta_k \Delta r(k-1)}{\mu + |\Delta r(k-1)|^2} \times (\Delta\rho(k) - \hat{\theta}(k-1)\Delta r(k-1)), \end{aligned} \tag{21}$$

where $\mu > 0$ is the positive weighting factor in (20), $\beta_k \in (0 \quad 2)$ is a step-size constant series added to make the generality of the algorithm (21), and $\hat{\theta}(0)$ can be chosen arbitrarily.

In order to make the condition $|\Delta r(k)| \ne 0$ be hold for any k and parameter estimation algorithm (21) have stronger ability in tracking variable parameter, we present a reset algorithm as follows

$$\hat{\theta}(k) = \hat{\theta}(0), \text{ if } \hat{\theta}(k) \le \varepsilon \text{ or } |\Delta r(k-1)| \le \varepsilon, \tag{22}$$

where ε is a small positive constant.

Remark 5: For the proposed MFAC scheme, what we need is to tune the parameters α_k and β_k in a small range with properly fixed values of λ and μ , without requiring any other priori knowledge of the dynamic system.

4.2 Convergence Analysis

Assumption 3: The parameter $\theta(k)$ satisfies that $\theta(k) \ge 0$ (or $\theta(k) \le 0$) $\forall t$, and $\theta(k) = 0$ holds only at finite time instant k.

Remark 6: This assumption is similar to the limitation of control input direction. In fact, many practical systems can satisfy this assumption. For example, the traffic flow density will increase (or not decrease at least) when the on-ramp metering traffic volume increases in practice.

Theorem 2: For freeway traffic control system (10) satisfying assumptions 1-3, when the desired density value $\rho_d(k) = \rho_d$ is a constant, the presented model-free adaptive scheme, (19), (21), and (22), guarantees that

(a) The parameter estimation value $\hat{\theta}(k)$ is bounded.

(b) The traffic flow density converges to the desired value as k approaches to infinity.

(c) The control signals are bounded.

Proof: The proof consists of three parts. Part 1 derives the boundedness of the parameter estimation value $\hat{\theta}(k)$. Part 2 proves the exponential convergence. The boundedness of control signals is shown in part 3.

Part 1. The Boundedness of $\hat{\theta}(k)$

Case 1: When $|\Delta r(k-1)| \le \varepsilon$, by (22), clearly $\hat{\theta}(k)$ is bounded.

Case 2: When $|\Delta r(k-1)| > \varepsilon$, subtracting $\theta(k)$ from both sides of (21), we have

$$\begin{aligned}\tilde{\theta}(k) &= \tilde{\theta}(k-1) - (\theta(k) - \theta(k-1)) \\ &\quad + \frac{\beta_k \Delta r(k-1)}{\mu + |\Delta r(k-1)|^2} \times (\Delta\rho(k) - \hat{\theta}(k-1)\Delta r(k-1)),\end{aligned} \tag{23}$$

where $\tilde{\theta}(k) = \hat{\theta}(k) - \theta(k)$.

Let $\Delta\theta(k) = \theta_k(k) - \theta(k-1)$. Substituting (12) into (19), and taking absolute value, we have

$$|\tilde{\theta}(k)| \le \left|1 - \frac{\beta_k (\Delta r(k-1))^2}{\mu + |\Delta r(k-1)|^2}\right| |\hat{\theta}(k-1)| + 2M. \tag{24}$$

Noting that for $|\Delta r(k-1)| > \varepsilon$, $\mu > 0$, and $\beta_k \in (0,\ 2)$,

$$0 < 1 - \frac{\beta_k (\Delta r(k-1))^2}{\mu + |\Delta r(k-1)|^2} \le d_1 < 1 \tag{25}$$

holds. So the boundedness of $|\hat{\theta}(k)|$ is a direct result of (24) and Theorem 1.

Part 2: The exponential convergence

Using (12) and (19), the tracking error gives

$$\begin{aligned}|e(k+1)| &= |\rho_d - \rho(k+1)| = |\rho_d - \rho(k) - \theta(k)\Delta r(k)| \\ &= \left|1 - \frac{\alpha_k \theta(k)\hat{\theta}(k)}{\lambda + |\hat{\theta}(k)|^2}\right| |e(k)|.\end{aligned} \tag{26}$$

From Assumption 3 and reset algorithm (22), we know that $\theta(k)\hat{\theta}(k) > 0$. Thus we can choose α_k and λ appropriately such that

$$0 < 1 - \frac{\alpha_k \theta(k)\hat{\theta}(k)}{\lambda + |\hat{\theta}(k)|^2} \le d_2 < 1 \tag{27}$$

holds except for some finite sample instants k. Then the exponential convergence of tracking error is got immediately.

Part 3. Boundedness of Control Signals

From the learning law (19), we get that

$$|\Delta r(k)| \le \Gamma |e(k)|, \tag{28}$$

where $\Gamma = \dfrac{\alpha_k \Gamma_1}{\lambda + \delta^2}$ with Γ_1 being the upper boundedness of $\hat{\theta}(k)$.

Noted that

$$\begin{aligned}|r(k)| &\le |r(k) - r(0)| + |r(0)| \le |\Delta r(k)| \\ &\quad + |\Delta r(k-1)| + \cdots + |\Delta r(1)| + |r(0)|,\end{aligned} \tag{29}$$

thus from (26)-(29), we have

$$\begin{aligned}|r(k)| &\le \Gamma(|e(k)| + |e(k-1)| + \cdots + |e(1)| + |e(0)|) + |r(0)| \\ &\le \Gamma \frac{d_2}{1-d_2} |e(0)| + |r(0)|.\end{aligned} \tag{30}$$

Clearly the values of initial input $r(0)$ and tracking error $e(0)$ can be chosen bounded, so the control input $r(k)$ is bounded for all k.

5. EXTENSION TO HIGHER-ORDER LEARNING LAW

An adaptive controller differs from an ordinary controller in that the controller parameters are variable, and there is a mechanism for adjusting these parameters on-line based on signals in the system. It is true that the more past information is exploited, the better control performance can be achieved. However, simply using more past information does not necessarily mean a better performance. The performance is dependent not only on how much information is used but also on whether the information is crucial and how the information is fused together effectively. In this section, the optimality technology is applied to design the higher-order learning adaptive control law.

Consider again a new index function of control input as follows

$$J(r(k), \boldsymbol{w}_k) = |e(k+1)|^2 + \lambda \left| r(k) - \sum_{i=1}^{l} w_{k,i} r(k-i) \right|^2, \tag{31}$$

where $\boldsymbol{w}_k = (w_{k,1}, w_{k,2}, \cdots, w_{k,l})^T$ is the vector of weighting coefficient for control input, and $\sum_{i=1}^{l} w_{k,i} = 1$; λ is a positive weighting factor punishing the variation of control input; $r(k-1), r(k-2), \cdots, r(k-l)$ are the inputs of the previous l sampling instants, which are known at the k-th sampling instant.

Following the same steps that lead to (19) in Section 4, we give the control learning law as follows

$$r(k)=\frac{\hat{\theta}^2(k)}{\lambda+\left|\hat{\theta}(k)\right|^2}r(k-1)+\frac{\lambda}{\lambda+\left|\hat{\theta}(k)\right|^2}\sum_{i=1}^{l}w_{k,i}r(k-i)$$
$$+\frac{\hat{\theta}(k)}{\lambda+\left|\hat{\theta}(k)\right|^2}(\rho_d(k+1)-\rho(k)). \tag{32}$$

Then the high-order model-free adaptive control scheme is constructed by (21), (22) and (32), and its validity is verified by the following theorem.

Theorem 3: For freeway traffic control system (10) satisfying assumptions 1-3, when the desired density value $\rho_d(k+1)=\rho_d$ is a constant, the presented higher-order model free adaptive control scheme, i.e., (21), (22) and (32), guarantees that the traffic flow density converges to the desired value as *k* approaches to infinity.

Proof: The boundedness of the parameter estimation $\hat{\theta}(k)$ can be obtained by the same analysis as previous section. Now, we show the convergence of the tracking error.

Since $r(k)$ is the optimal control input and $\boldsymbol{w}_k$ is the optimal weight coefficient vector, so the following inequality is true

$$J(r(k),\boldsymbol{w}_k)\le J(r(k-1),\boldsymbol{\Lambda}_0), \tag{33}$$

where $\boldsymbol{\Lambda}_0=(1,0,\cdots,0)^T$.

By (31), we have

$$J(r(k-1),\boldsymbol{\Lambda}_0)=J(r(k-1),(1,0,\cdots,0)^T)=\left|e(k)\right|^2. \tag{34}$$

According to (31), (33), and (34), we have

$$\left|e(k+1)\right|^2\le J(r(k),\boldsymbol{w}_k)\le\left|e(k)\right|^2. \tag{35}$$

This implies that $\lim_{k\to\infty}\left|e(k)\right|^2$ exists. Consequently, $\lim_{k\to\infty}J(r(k),\boldsymbol{w}_k)$ also exists and equals to $\lim_{k\to\infty}\left|e(k)\right|^2$, that is

$$\lim_{k\to\infty}\left|e(k)\right|^2=\lim_{k\to\infty}J(r(k),\boldsymbol{w}_k)<\infty. \tag{36}$$

By the relationship of (31) and (36), we have

$$\lim_{k\to\infty}\lambda\left|r(k)-\sum_{i=1}^{l}w_{k,i}r(k-i)\right|^2$$
$$=\lim_{k\to\infty}J(r(k),\boldsymbol{w}_k)-\lim_{k\to\infty}\left|e(k+1)\right|^2=0. \tag{37}$$

Because λ is a positive constant, (37) results in

$$\lim_{k\to\infty}(r(k)-\sum_{i=1}^{l}w_{k,i}r(k-i))=0. \tag{38}$$

According to (32), we have

$$\lim_{k\to\infty}\left(r(k)-\sum_{i=1}^{l}w_{k,i}r(k-i)\right)=\lim_{k\to\infty}\frac{\hat{\theta}(k)}{\lambda+\left|\hat{\theta}(k)\right|^2}e(k)$$
$$+\lim_{k\to\infty}\frac{\hat{\theta}^2(k)}{\lambda+\left|\hat{\theta}(k)\right|^2}\left(r(k-1)-\sum_{i=1}^{l}w_{k,i}r(k-i)\right). \tag{39}$$

In terms of (38) and (39), we have

$$\lim_{k\to\infty}\frac{\hat{\theta}(k)}{\lambda+\left|\hat{\theta}(k)\right|^2}e(k)=0. \tag{40}$$

The boundedness of $\hat{\theta}(k)$ has been derived in Section 4. In terms of the reset algorithm (22), we have $\hat{\theta}(k)>\varepsilon$, so $\frac{\hat{\theta}_k(t)}{\lambda_2+\hat{\theta}_k(t)^2}$ is bounded and away from zero. By virtue of (40), we can obtain that $\lim_{k\to\infty}e_k(t+1)=0$.

6. ILLUSTRATIVE EXAMPLE

In order to verify the effectiveness of the MFAC approach, we simulate a freeway traffic flow process with the desired density $\rho_d=30veh/km$ per lane, in the presence of a large exogenous disturbance (modelled by an exiting flow in an off-ramp).

Consider a long segment of freeway that is subdivided into 12 sections. The length of each section is 0.5km. The initial traffic volume, entering section 1, is time-varying with $1500+5\sin(\pi k/10)$ vehicles per hour. The initial density and mean speed of each section, and the parameters used in this model are listed in Table 1.

Table 1: Initial values and parameters associated with the traffic model

	Section											
	1	2	3	4	5	6	7	8	9	10	11	12
$\rho_i(0)$ veh/ lane/km	30	30	30	30	30	30	30	30	30	30	30	30
$v_i(0)$ km/h	50	50	50	50	50	50	50	50	50	50	50	50
Parameters	v_{free}	ρ_{jam}	l	m	κ	τ	T	γ	$q_0(k)$	$r_i(0)$	α	
	80 km/h	80 veh/lane/km	1.8	1.7	13 veh/km	0.01h	0.00417h	35 km^2/h	1500 veh/h	0 veh/h	0.95	

There are one on-ramp and one off-ramp in the segment, located in section 7 and section 4, respectively. The traffic demand pattern (on-ramp) and the outflow pattern (off-ramp) are shown in Fig. 2. They were chosen to simulate a traffic scenario during rush hour.

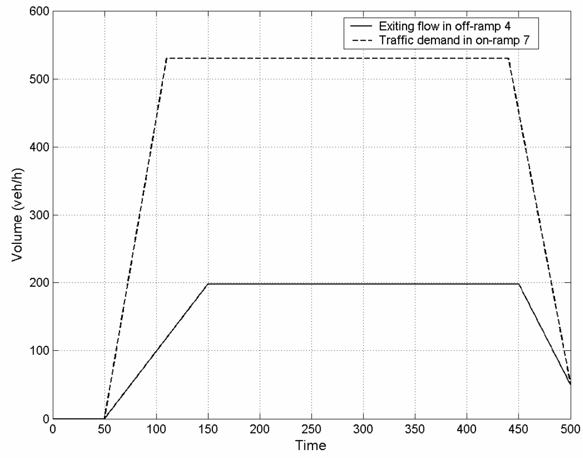

Fig. 2. Traffic demand in on-ramp 7 and the unknown existing flow in off-ramp 4.

For the purpose of comparison, three cases are considered.

Case I (one-order MFAC). The parameters of one-order MFAC are chosen to be $\alpha_k = 20$, $\beta_k = 0.0001$ $\mu = 0.01$, $\lambda = 0.001$, $\varepsilon = 0.00005$.

Case II (two-order MFAC). The parameters of two-order MFAC are set to be $\beta_k = 0.0001$ $\mu = 0.01$, $\lambda = 0.0001$, $\varepsilon = 0.00005$, $w_1 = 0.6$, $w_2 = 0.4$.

Case III (ALINEA).

The compared result is shown in Fig. 3. Apparently, the density performance by means of the two-order MFAC is better than the one-order MFAC, and the one-order MFAC is better than ALINEA.

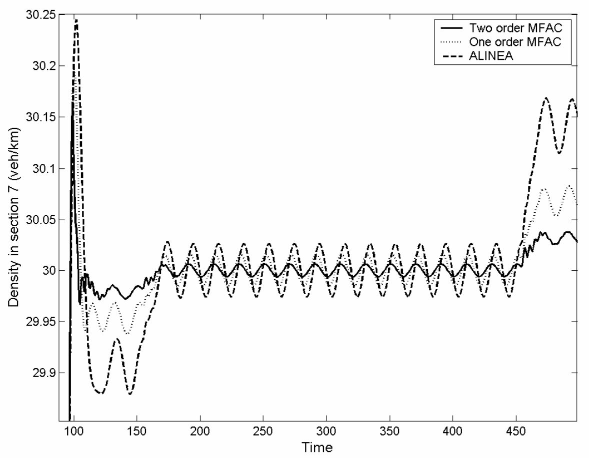

Fig. 3. The comparison among ALINEA, one-order MFAC, and two-order MFAC.

7. CONCLUSIONS

Through this paper, we show that the MFAC provides a new ramp metering control method that is suited for the traffic density control problems. The main advantage of MFAC-based ramp metering is that the design and analysis of the control system only depends on the I/O data of the freeway traffic system. Hence, the modelling uncertainties can be rejected completely and the traffic performance can be improved as a result. Also we provide an extension of MFAC, and a higher-order learning control law is presented and proved for improving the control performance further by using more control information in previous sampling instants. Convergence analysis and case studies with intensive simulations on a macroscopic level freeway model confirm the validity of the proposed approaches.

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