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A Decoupling Derivative-based Approach for Hammerstein System Identification

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Abstract: This paper proposes a non iterative algorithm for the identification of Hammerstein model, using the sampled output data obtained from the step response, giving a continuous-time model for the linear part and a point-wise estimation of the nonlinear one. Key in the derivation of the results is the algebraic derivative method in the frequency domain yielding exact formula in terms of multiple integrals of the output signal, when placed in the time domain. By investigating the connection between such integrals and parameters to be estimated, a set of three linear regression equations is proposed. The first equation is used to estimate the structure of poles in the linear part, the second to estimate a point of the nonlinearity, the third to estimate the structure of zeros in the linear part. No a priori knowledge of the structure of the nonlinearity is required. The proposed algorithm is numerically robust, since it is based only on least squares estimation. Simulation results validate the proposed algorithm.

1. INTRODUCTION

Mathematical modeling of real-life systems is a very common methodology in science and engineering. It is used both as a means for achieving deeper knowledge about a system and as an engineering tool, e.g., as a basis for simulations or for design of controllers. Sometimes, it is possible to construct a model of a system from physical laws and principles. However, in other cases this is not possible, either because of a lack of knowledge of the studied system or because physical modeling is considered too time consuming. In these cases, system identification can be a way of solving the modeling problem. System identification is concerned with characterizing an unknown system using observations of the system's input and output signals. Most dynamical systems can be better represented by nonlinear models, which are able to describe the global behavior of the system over the whole operating range, rather than by linear ones that are only able to approximate the system around a given operating point. Therefore, in order to have more accurate characterization of the system, even if it is more complicate, nonlinear models are preferable to the linear one. The main reason that leads to the identification of nonlinear systems is their widespread application in many fields especially in control engineering. One of the most frequently studied classes of nonlinear models are the so called block-oriented nonlinear models, which consist of the interconnection of Linear Time Invariant (LTI) systems and static (memoryless) nonlinearities. The more common model structures in this class are the Hammerstein models and Wiener models composed of a nonlinear static characteristic followed by a linear dynamic transfer function, and vice versa. Identification of this model is attractive because its structure is very simple and it can describe a nonlinearity of dynamical system efficiently, Dolanc et al. [2005]. For example, they are used to describe the acoustic channel of a nonlinear system concatenated with a linear faded echo path (Ngia et al. [1998]), to code speech signals (Turunen et al. [2003]), to control the position of electrohydraulic servo systems (Knohl et al. [2000]), to model the static and dynamic characteristics of the solid oxide fuel cells (Jurado et al. [2006], Jurado [2006], Jurado et al. [2005]), to identify chaotic dynamical systems (Xu et al. [2001]), to model and control heart rate modulation during treadmill exercise (Su et al. [2006]), to identify marine thruster dynamics (Leonessa et al. [2001]), to identify the human thermoregulatory system (Rollins et al. [2006]), to identify nonlinear distortion models (Picard et al. [2003]), to identify thermal systems (Chaari et al. [2006]), to identify vestibulo-ocular reflex (Kukreja et al. [2005]). Clearly the above list of applications is not complete; for a more exhaustive one, the reader is referred to Giannakis et al. [2001] and the references therein. As noted, this system structure is common in many real-life applications and it is thus natural that identification of Hammerstein systems has been an active research field for quite some time. Many techniques have been proposed in the literature for the black-box estimation of Hammerstein systems from given input/output data. These techniques mainly distinguish themselves in the way the static nonlinearity is represented and in the type of optimization problem that is finally obtained. The already existent methods can be roughly divided into seven categories: iterative methods, non iterative methods, stochastic methods, nonlinear least squares methods, separable least squares methods, blind methods, over-parameterization methods, (see Bai [2003] and the references therein). In this paper, an attempt to approximate a nonlinear system, in the form of Hammerstein model, by devising a novel systematic algorithm that works directly with the step response data, to produce a continuous-time transfer function model of the process and a point-wise estimation of the nonlinearity, is proposed. In contrast to classical parametric approaches, no specific model structure is imposed on the nonlinearity. Hence, the presented technique combines a nonparametric approach for the identification of the nonlinear static

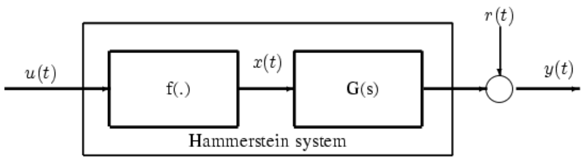

Fig. 1. Hammerstein system.

characteristic and a parametric approach for the linear dynamical part. In our approach we propose to identify in a unique experiment the structure of poles in the linear system $G(s)$, one point of the nonlinearity $f(\cdot)$ and finally the structure of zeros in $G(s)$. By applying a set of step inputs of different amplitude A, it is possible to point-wise estimate the nonlinear function $f(\cdot)$. Our approach, based on algebraic derivatives in frequency domain, allows to obtain a relation which involves only the elementary symmetric functions on the poles of the linear system $G(s)$, decoupling from the nonlinearity and zeros of $G(s)$. The identification scheme is designed to determine exactly the model of the plant dynamics and a set of N different couples $(u, f(u))$, where N is arbitrarily chosen by the user. Given the previous couples, one can build up an $(N-1)$th degree polynomial or polygonal approximation of the static gain characteristic. The aim of the proposed method is to identify, without iterative procedures, the unknown parameters of $G(s)$, having in mind an its end-use for controller design, and to point-wise reconstruct the nonlinearity by using the response of step inputs with different amplitudes. Although good parameter identification requires the application of a frequency-rich input and the standard solution in practice is provided by the use of pseudo-random binary sequences (Landau [1990]), we however design a method based on the measurement of the process step response because of the simple physical interpretation, and its easy implementation in industrial environments. The effectiveness of the proposed method is demonstrated by numerical examples.

2. MAIN RESULTS

Let consider the Hammerstein model depicted in Fig. 1, where the block $f(\cdot)$ represents the nonlinear static element and $G(s)$ is the linear part of the process. The linear dynamic block $G(s)$ is assumed a n_p-order plant

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{m} \gamma_i s^i}{\sum_{i=0}^{n_p} \sigma(n_p, i) s^{n_p - i}}, \quad m \leq n_p - 1 \tag{1}$$

where

$$\begin{cases} \sigma(n_p, k) = (-1)^k \displaystyle\sum_{1 \leq \pi_1 < \ldots < \pi_k \leq n_p} p_{\pi_1} p_{\pi_2} \ldots p_{\pi_k}, \; k = 1, \ldots, n_p, \\ \sigma(n_p, 0) = 1, \end{cases} \tag{2}$$

is the kth order elementary symmetric function associated with the system poles $\{p_1, p_2, \ldots, p_{n_p}\}$.

Let

$$U(s) = \frac{A}{s}, \tag{3}$$

be the Laplace transform of the input $u(t)$, then

$$X(s) = \frac{f(A)}{s} \tag{4}$$

and

$$Y(s) = G(s)\frac{f(A)}{s} = \frac{\xi_0}{s} + \sum_{k=1}^{n_p} \frac{\xi_k}{s - p_k}. \tag{5}$$

Since any pair $(\alpha f(u), G(s)/\alpha)$, $\alpha \neq 0$, would produce identical input and output measurements, then the gain of $G(s)$ can be fixed to be unit, i.e.

$$\gamma_0 = \sigma(n_p, n_p). \tag{6}$$

It is straightforward to note that $f(A) = \xi_0$.

We propose a three-step identification algorithm. In the first step, the denominator $D(s)$ of the linear part is identified. With the help of the algebraic derivative method in the frequency domain (Fliess et al. [2003]), we show that the identification of $D(s)$ is decoupled from the numerator $N(s)$. Moreover, this decoupling is independent of the nonlinearity which could be discontinuous and unknown. In the second step, the value of ξ_0 is estimated. Finally the numerator of $G(s)$ is identified by using a linear least-squares method. An analogous approach has already been utilized for nonlinear parametric identification in (Fliess et al. [2005]), and for nonparametric identification in (Fliess et al. [2006]).

By gaining an advantage from the algebraic derivative method in the frequency domain the following result can be easily obtained (Coluccio et al. [2007]):

$$\sum_{i=0}^{n_p} \sum_{j=i}^{n_p+1} \binom{n_p + 1 - i}{j - i} \binom{n_p + 1}{j} (n_p + 1 - j)! s^{j-i} \times \frac{d^j Y(s)}{ds^j} \sigma(n_p, i) = 0. \tag{7}$$

It is worth to note, that the application of any filter, $H(s)$, in eq. (7) does not change the result. Although the division by s^{n_p+1} is sufficient to eliminate all the derivations implicit in the multiplication by power of s, we use the division by s^{n_p+2} because it allows to introduce at least an integral effect on each term which contains the signal $y(t)$, then by taking the inverse Laplace transform one has:

$$\sum_{i=0}^{n_p} \beta(n_p, i, t) \sigma(n_p, i) = 0 \tag{8}$$

with

$$\beta(n_p, i, t) = \sum_{j=i}^{n_p+1} \binom{n_p + 1 - i}{j - i} \binom{n_p + 1}{j} (n_p + 1 - j)! \times \left[\int^{(n_p+2+i-j)} (-1)^j t^j y(t) \right], \tag{9}$$

where we denote by

$$\int^{(j)} \phi(t)$$

the multiple integrals expression

$$\int_0^t \int_0^{x_1} \ldots \int_0^{x_{j-1}} \phi(x_j) dx_j \ldots dx_1,$$

with the definition

$$\int^{(1)} \phi(t) = \int_0^t \phi(x_1)dx_1.$$

It is interesting to observe that high frequency zero mean disturbances on the process output are filtered by the integration operations, so that their contribution to $\beta(n_p, i, t),\ i = 0, ..., n_p$ is negligible. The excellent robustness with respect to noises is explained in (Fliess [2006]). Nevertheless, low-frequency noise and offset errors could cause estimation errors in the proposed method. This is a common problem to any identification method which uses step tests. In fact, the test signal should enable us to inject as much energy as possible, or the experiment needs a high signal-to-noise ratio, but sometimes, in industrial processes, it is not possible to inject frequency-rich input. This implies that offsets should be small. If there exist inherent offsets, they may cause significant estimation error in the unknown parameters.

Once $\sigma(n_p, i),\ i = 1, ..., n_p$ are successfully computed by eq. (8), we can tackle the problem of computing the value of ξ_0. By eq. (5) it is effortless to derive the following expressions for ξ_0:

$$\frac{(-1)^{n_p} n_p! \sigma(n_p, n_p)}{s^{n_p+1}}\xi_0 = \sum_{i=0}^{n_p}\sum_{j=i}^{n_p} \binom{n_p}{j}\binom{n_p-i}{j-i}(n_p-j)!s^{j-i}\frac{d^jY(s)}{ds^j}\sigma(n_p, i). \tag{10}$$

When the inverse Laplace transform, after the division by s^{n_p+1}, is applied to eq. (10), one has:

$$\xi_0 = \frac{(-1)^{n_p}\Gamma(2n_p+2)}{\Gamma(n_p+1)\sigma(n_p,n_p)t^{2n_p+1}}\sum_{i=0}^{n_p}\sum_{j=i}^{n_p}\binom{n_p}{j}\binom{n_p-i}{j-i}(n_p-j)! \times \int^{(n_p+1+i-j)}(-1)^j t^j y(t)\sigma(n_p, i). \tag{11}$$

Equations (11) can be viewed as a time varying dynamic filter and they must be valid for every t. Although the right estimation of the unknown parameter takes place in a fraction of time, in a noisy environment is preferable to estimate the value of ξ_0 as the filter value in the last time instant of the observation window, namely T_{obs} (Fliess et al. [2005]). Finally, in order to estimate parameters $\gamma_i,\ i = 0, ..., m$, we consider the equality

$$\sum_{k=0}^{n_p}\sigma(n_p,k)\frac{Y(s)}{s^{k+1}} = \xi_0\sum_{k=0}^{m}\gamma_k\frac{1}{s^{n_p+2-k}}, \tag{12}$$

easily obtained by eqs. (1) and (5). Taking the inverse Laplace transform of eq. (12), the following expression is obtained

$$\sum_{k=0}^{n_p}\sigma(n_p,k)\int^{(k+1)} y(t) = \xi_0\sum_{k=0}^{m}\bar{\gamma}_k t^{n_p+1-k} \tag{13}$$

with $\bar{\gamma}_k = \frac{\gamma_k}{(n_p+1-k)!}$.

Note that the estimation based on eqs. (8) and (11) could be biased for long observation windows. Indeed in the integrals the noise will be multiplied by powers of t before integration, and the noise induced by that process is likely to increase with the length of the time window. To overcome such drawback is convenient to express eqs. (9) and (11) in terms of multiple integrals of the measured signal. The following result is useful to this aim:

Proposition 1. Let $y(t)$ a function which can be expanded in Mc-Laurin series, then:

$$\int^{(q)} t^k y(t) = \sum_{i=0}^{k}(-1)^i\binom{i+q-1}{q-1}\binom{k}{i}i!\ t^{k-i} \times \int^{(i+q)} y(t), \quad q \geq 1, \quad k \geq 0 \tag{14}$$

Proof. By considering the s-th term of the Mc-Laurin serie of $y(t)$, it follows that

$$\int^{(q)} t^{k+s} = \sum_{i=0}^{k}(-1)^i\binom{i+q-1}{q-1}\binom{k}{i}i!\ t^{k-i}\int^{(i+q)} t^s. \tag{15}$$

Since

$$\int^{(q)} t^{k+s} = \frac{(k+s)!}{(k+s+q)!}t^{k+s+q},$$

then eq. (15) becomes

$$\frac{(k+s)!}{(k+s+q)!}t^{k+s+q} = \sum_{i=0}^{k}(-1)^i\binom{i+q-1}{q-1}\binom{k}{i}i! \times t^{k-i}\frac{s!}{(s+i+q)!}t^{s+i+q}. \tag{16}$$

By standard arguments (Knuth [1973]), eq. (14) follows. ■

Therefore eqs. (9) and (11) becomes respectively:

$$\beta(n_p,i,t) = \sum_{z=0}^{n_p+1}(-1)^z\frac{\Gamma(2n_p+3-z)}{\Gamma(z+1)\Gamma(n_p+2-z)}t^z\int^{(n_p+2+i-z)} y(t), \tag{17}$$

$$\xi_0 = \frac{(-1)^{n_p}\Gamma(2n_p+2)}{\Gamma(n_p+1)\sigma(n_p,n_p)}\sum_{i=0}^{n_p}\sum_{z=0}^{n_p}(-1)^z\frac{\Gamma(2n_p+1-z)}{\Gamma(z+1)\Gamma(n_p+1-z)} \times \sigma(n_p,i)t^{z-2n_p-1}\int^{(n_p+1+i-z)} y(t). \tag{18}$$

Remark 1. Note that since an algebraic derivative method in the frequency domain is used, an advantage of this approach is that it can be used for irregular sampling.

3. COMPUTATIONAL ASPECTS

In this section, some computational aspects will be analyzed. In order to get an estimation in the least-squares sense for $\{\sigma(n_p,k)\}_{k=1}^{n_p}$, $\{\gamma_k\}_{k=0}^{m}$, and ξ_0, note that eqs. (8), (13), (18) holds for all t. Let $\hat{t} = \{t_1, t_2, ..., t_n\}$ be a set of time instants where eqs. (8), (13), (18) will be evaluated. Eq. (8) can be expressed in matrix form as:

$$V_1\sigma = u_1 \tag{19}$$

where

$$V_1(i,j) = \beta(n_p, j, t_i), \quad i = 1, ..., n,\ j = 1, ..., n_p, \tag{20}$$

$$\sigma = \{\sigma(n_p,k),\ k = 1, ..., n_p\}, \tag{21}$$

and

$$u_1(i) = \beta(n_p, 0, t_i), \quad i = 1, ..., n. \tag{22}$$

An estimation $\hat{\sigma}$ in the least-squares sense is given by

$$\hat{\sigma} = \left(V_1^T V_1\right)^{-1} V_1^T u_1. \tag{23}$$

As far as the eq. (18) is concerned, it can be expressed as:

$$\xi_0 = \alpha \begin{bmatrix} 1 & \sigma^T \end{bmatrix} V_2 u_2, \tag{24}$$

where

$$\alpha = (-1)^{n_p} \frac{\Gamma(2n_p+2)}{\Gamma(n_p+1)\sigma(n_p,n_p)}, \tag{25}$$

$$V_2(i,j) = \int^{(n_p+1+i-j)} y(t), \quad i,j = 1,...,n_p+1, \tag{26}$$

and

$$u_2(i) = (-1)^{i+1} \frac{\Gamma(2n_p+2-i)}{\Gamma(i)\Gamma(n_p+2-i)} t^{i-2n_p-2}, \quad i = 1,...,n_p+1. \tag{27}$$

Since the static gain of $G(s)$ is fixed to be unit, then γ_0 must be equal to $\sigma(n_p,n_p)$ and eq. (13) becomes

$$\sum_{k=0}^{n_p} \sigma(n_p,k) \int^{(k+1)} y(t) - \xi_0 \frac{\sigma(n_p,n_p)}{\Gamma(n_p+2)} t^{n_p+1} = = \xi_0 \sum_{k=1}^{m} \bar{\gamma}_k t^{n_p+1-k} \tag{28}$$

which can be expressed in matrix form as:

$$U_3 \bar{\gamma} = \frac{1}{\xi_0} V_3 [1 \quad \sigma^T]^T - \frac{\sigma(n_p,n_p)}{\Gamma(n_p+2)} z_3 \tag{29}$$

where

$$U_3(i,j) = t_i^{n_p+1-j}, \quad i = 1,...,n, \ j = 1,...,m, \tag{30}$$

$$\bar{\gamma} = \{\bar{\gamma}_k, \ k = 1,...,m\}, \tag{31}$$

$$V_3(i,j) = \lim_{t \to t_i} \int^{(j+1)} y(t), \quad i = 1,...,n, \ j = 0,...,n_p, \tag{32}$$

and

$$z_3(i) = t_i^{n_p+1}, \quad i = 1,...,n. \tag{33}$$

An estimation $\hat{\bar{\gamma}}$ in the least-squares sense is given by

$$\hat{\bar{\gamma}} = (U_3^T U_3)^{-1} U_3^T \left(\frac{1}{\xi_0} V_3 [1 \quad \sigma^T]^T - \frac{\sigma(n_p,n_p)}{\Gamma(n_p+2)} z_3 \right). \tag{34}$$

We would like to suggest that the multiple integrals of the signal could be accomplished by means of time-varying linear filter:

$$\begin{aligned} \dot{x}(t) &= Fx(t) + gy(t), \\ z(t) &= Hx(t), \end{aligned} \tag{35}$$

where $x(t) \in \mathbb{R}^{2n_p+2}$ is the state vector

$$x(t) = \left[\int^{(2n_p+2)} y(t), \int^{(2n_p+1)} y(t), ..., \int^{(1)} y(t) \right]^T \tag{36}$$

which contains the multiple integrals of the input signal $y(t)$, $z(t) \in \mathbb{R}^{2n_p+2}$ is the output vector which coincides exactly with the state vector and the matrices $F \in \mathbb{R}^{(2n_p+2)\times(2n_p+2)}$, $g \in \mathbb{R}^{(2n_p+2)}$ have the following expressions:

$$F = \begin{bmatrix} \mathbf{0_{2n_p+1}} & \mathbf{I_{2n_p+1}} \\ 0 & \mathbf{0^T_{2n_p+1}} \end{bmatrix}, \tag{37}$$

$$g = \begin{bmatrix} \mathbf{0_{2n_p+1}} \\ 1 \end{bmatrix} \tag{38}$$

with $\mathbf{0_n}$ a $(n \times 1)$ vector of zeros, and the output matrix H is a $(2n_p+2) \times (2n_p+2)$ identity matrix.

Moreover, such numerical integrations could be avoided by considering a polynomial $p(t)$ of degree $l-1$ which fits $y(t)$ in the least-squares sense:

$$p(t) = \sum_{k=1}^{l} a_k t^{k-1}. \tag{39}$$

In this case, by taking into account eq. (39) and standard properties of gamma functions (Gatteschi [1973]), eq. (8) can be rewritten as

$$\sum_{i=0}^{n_p} \tilde{\beta}(n_p,i,t)\sigma(n_p,i) = 0, \tag{40}$$

where

$$\tilde{\beta}(n_p,i,t) = \sum_{k=1}^{l+i-n_p-1} (-1)^{n_p+1} \frac{\Gamma(n_p+1+k-i)\Gamma(n_p+1+k)}{\Gamma(2n_p+3+k)\Gamma(k)} \times a_{k+n_p+1-i} t^{2n_p+2+k}. \tag{41}$$

The estimation, $\tilde{\xi}_0$, of ξ_0, in terms of the coefficients of the polynomial $p(t)$ is

$$\tilde{\xi}_0 = \frac{\Gamma(2n_p+3)}{2\Gamma(n_p+2)\sigma(n_p,n_p)} \times \sum_{i=0}^{n_p} \sum_{k=1}^{l+i-n_p} \frac{\Gamma(n_p+k-i)\Gamma(n_p+k) a_{n_p+k-i} t^{k-1}}{\Gamma(2n_p+1+k)\Gamma(k)} \sigma(n_p,i). \tag{42}$$

When numerical integrals are solved by using the integrations on the polynomial $p(t)$, eq. (13) can be rewritten as:

$$\sum_{i=0}^{n_p} \sum_{k=1}^{l} a_k \frac{\Gamma(k) t^{k+i}}{\Gamma(k+i+1)} \sigma(n_p,i) = \xi_0 \sum_{k=0}^{m} \bar{\gamma}_k t^{n_p+1-k}. \tag{43}$$

By imposing $\gamma_0 = \sigma(n_p,n_p)$, eq. (43) becomes:

$$\begin{aligned} &\sum_{i=1}^{n_p+1} \sum_{k=1}^{l} a_k \frac{\Gamma(k) t^{k+i-1}}{\Gamma(k+i)} \sigma(n_p,i-1) \\ &- \frac{\xi_0 \sigma(n_p,n_p) t^{n_p+1}}{\Gamma(n_p+2)} = \xi_0 \sum_{k=1}^{m} \bar{\gamma}_k t^{n_p+1-k}. \end{aligned} \tag{44}$$

4. SIMULATION EXPERIMENTS

In this section, in order to investigate the effectiveness of the proposed method, some simulation results are presented. Our algorithm was tested on two different nonlinear systems. For each simulation a zero mean white noise $r(t)$ with a signal-to-noise ratio

$$SNR = \sqrt{\frac{\sum_{i=1}^{n} y(t_i)^2}{\sum_{i=1}^{n} r(t_i)^2}} = 20 \tag{45}$$

was added to the signal measurements.

4.1 Example 1

In the first example we propose a comparison between the proposed approach and that one proposed in Bai [2003]. The choice of comparing our method and the Bai's one is due to the fact that it is among the fews which perform on continuous time. Bai discusses Hammerstein model identification in frequency domain by exploring the fundamental frequency and harmonics generated by the unknown nonlinearity. The unknown true linear part considered is given by:

$$G(s) = \frac{6(s+1)}{s^2 + 5s + 6}, \tag{46}$$

and the nonlinear one has the following polynomial expression:

$$f(u) = u^2 + u. \tag{47}$$

Although the two algorithms perform both on continuous time, they are deeply different, then in order to make the comparison possible a different parameters setting is necessary. As far as the parameters setting for our algorithm is concerned, we chosen the total number of samples $n = 1000$, the observation window $T_{obs} = 2$, the sampling time $T_s = 0.2e-2$, $m = 1$ and $n_p = 2$. To identify the Hammerstein model, $N = 20$ experiments were conducted with different step amplitude $A_i \in [-3, 3]$, $i = 1, ..., N$. In order to have an accurate estimation of $\{\sigma(n_p, k)\}_{k=1}^{n_p}$ we generalized eq. (19) to the case of N experiments, i.e. the matrix V_1 and the vector u_1 are constructed so that to contain the information of all experiments conducted. In each experiment the parameter $\xi_0 = f(A_i)$ is estimated. Finally, the estimation of the structure of zeros in $G(s)$ is obtained by using eq. (34) with $\{\bar{\sigma}(n_p, i)\}_{i=1}^{n_p}$, also generalized to the case of N experiments. As far as the parameters setting for Bai's algorithm is concerned, we referred to Bai [2003]; in particular $N = 20$ experiments were conducted with different sinusoidal input $u(t) = cos(\omega_k t)$, $\omega_k \in [0.1, 5]$, $k = 1, ..., N$; in each experiment the total number of samples considered is $n = 1000$, the observation time is $T_{obs} = 2\pi L/\omega_k$, $k = 1, ..., N$, $L = 100$, the sampling time is $T_s = T_{obs}/n$. Note that our method, to the contrary of the Bai's one, in each experiment does not need to have a long observation time and variable in each experiment as well as a variable sampling time.

Remark 2. Note that the observation time T_{obs}, in Bai's approach, varies in the interval $[125.7, 6283.2]$ while in the proposed approach it is fixed to 2.

In order to validate each method, 1000 iterations have been performed, and the global input/output behavior of the estimated models has been investigated by giving in input, to each estimated plant, a white noise with zero mean and variance $\sigma^2 = 1$, so that to consider approximately all amplitude used and a wide range of frequencies. Finally, the goodness of the two approaches was measured in terms of minimum, mean, maximum and variance of the following index:

$$J = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(y(kT_s) - \hat{y}(kT_s)\right)^2} \tag{48}$$

where $\{y(kT_s)\}_{k=1}^{n}$ is the true output data sequence of the system and $\{\hat{y}(kT_s)\}_{k=1}^{n}$ is the estimated one. From Table 1 it is possible to observe the obtained results from both methods.

	$min(J)$	$\mu(J)$	$max(J)$	$\sigma^2(J)$
CF	$4.98e-4$	$2.81e-3$	$8.53e-3$	$1.91e-6$
Bai	$5.44e-2$	$7.28e-2$	$9.51e-2$	$4.30e-5$

Table 1. Min, mean, max and variance of J over 1000 tests.

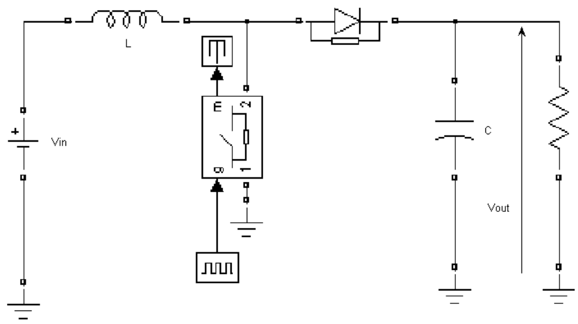

Fig. 2. Example 2: Basic scheme of the Boost converter.

4.2 Example 2

In this example, a Hammerstein model is proposed for describing the behavior of switching converters with particular reference to a DC/DC Boost converter. A basic scheme of the DC/DC Boost converter is given in Fig. 2. The output voltage of the converter V_{out}, is greater than the input voltage V_{in}; the static gain, i.e. the output to input voltage ratio, depends on the duty cycle of the signal supplying the switch. The values of the electrical components of the Boost converter are: $R = 10\Omega, L = 0.67mH, C = 200\mu F$. Two different simulations have been performed. In the first simulation, in order to determine the static characteristic and the linear part of the Hammerstein model, $N = 20$ experiments where conducted with $V_{in} \in [3, 5]$ Volts and duty cycle fixed to 70%. For both simulations, the switching frequency is fixed to $15kHz$, the sampling time $T_s = 6.67e-6$, $m = 0$ and $n_p = 2$. As far as the observation window is concerned, in the first simulation is $T_{obs} = 0.2e-1$ and in the second one is $T_{obs} = 0.7e-2$. In the first simulation, the estimated nonlinear and linear part are respectively:

$$f(u) = 0.0016u^2 + 3.3157u - 0.7883, \tag{49}$$

and

$$G(s) = \frac{6.66\,\mathrm{e}\,5}{s^2 + 551.17s + 6.66\,\mathrm{e}\,5}. \tag{50}$$

In order to validate the identified Hammerstein model, a signal with amplitude uniformly distributed in $[5, 10]$ was applied to the Boost converter, obtaining the results shown in Fig. 3. The proposed approach gives good results as it can be verified from the comparison between the behavior of the Boost converter and that of its Hammerstein model. In the second simulation, $N = 50$ experiments where conducted with $V_{in} = 5$ Volt and duty cycle variable in $[0.45, 0.55]$. In this case, the estimated nonlinear and linear part are given, respectively, by:

$$f(u) = 0.0075u^2 - 0.5409u + 17.5155, \tag{51}$$

and

$$G(s) = \frac{1.88\,\mathrm{e}\,6}{s^2 + 450.84s + 1.88\,\mathrm{e}\,6}. \tag{52}$$

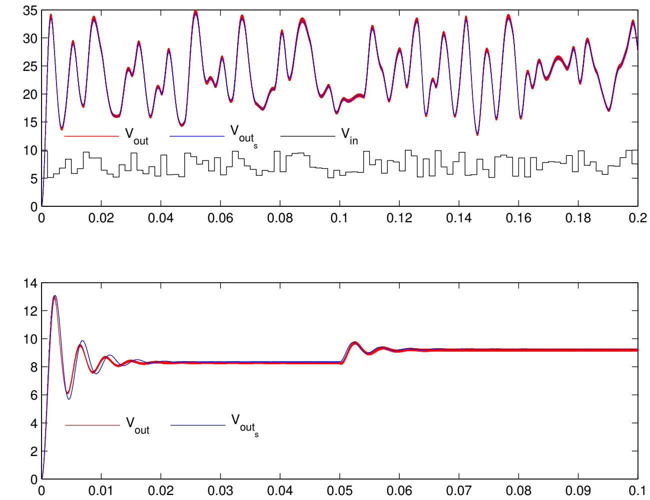

Fig. 3. Validation of the identified Hammerstein models.

The validation of the identified Hammerstein model has been done by varying the duty cycle from $\delta = 0.45$ to $\delta = 50$, obtaining the results shown in Fig. 3.

5. CONCLUDING REMARKS

In this paper, we have proposed a non-iterative method for the identification of Hammerstein models by using step tests. The approach is based on three regression equations which involve multiple integrals of the output signal. By using the derivative method in frequency domain, it has been possible to decouple the structure of poles in the linear part from the structure of zeros and the nonlinear part. No information on the form of the nonlinearity is assumed. By performing several experiments, with different amplitudes of step input, it is possible to point-wise reconstruct the static characteristic of the nonlinear block. The method does not require complex calculations and to wait that the output response reaches its steady state.

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