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THE DESIGN OF A TWO-LEVEL CONTROLLER FOR SUSPENSION SYSTEMS ⋆

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Abstract: In this paper, the design of a two-level controller is proposed for active suspension systems. The required control force is computed by applying a high-level controller, which is designed using a Linear Parameter Varying (LPV) method. The suspension structure contains nonlinear dynamics of the dampers and the springs. The model is augmented with weighting functions specified by the performance demands and the uncertainty assumptions. The actuator generating the necessary control force is a highly nonlinear system, for which a low-level backstepping-based force-tracking controller is designed. The operation of the two-level controller is illustrated through simulation examples.

Keywords: linear parameter varying control, nonlinear control, backstepping, vehicle dynamics, suspension.

1. INTRODUCTION

Active suspensions are used to provide good handling characteristics and to improve ride comfort while harmful vibrations caused by road irregularities and on-board excitation sources act upon the vehicle. The performance of suspension systems is assessed quantitatively in terms of four parameters: passenger comfort, suspension deflection, tire load variation and energy consumption, see Gillespie [1992].

Several methods have been proposed to design active suspension systems. The vast majority of the papers assume that the suspension system can be approximated by a linear model and the control system is designed by linear methods, see e.g. Hrovat [1997], Yamashita et al. [1994], Gáspár et al. [2003b]. Another and smaller part of the papers assume that nonlinearity in suspension systems is dominant and the linearity assumption is not valid in the entire operation domain. The dynamic characteristics of suspension components, i.e. dampers and springs, have nonlinear properties, and they are not time-invariant, but change during the vehicle life cycles, see e.g. Alleyne and Hedrick [1995], Lin and Kanellakopoulos [1995], Karlsson et al. [2000].

A Linear Parameter Varying (LPV) design is proposed for active suspensions which contain nonlinear suspension components. This modeling approach allows us to take into consideration the highly nonlinear effects in the state space description in such a way that the model structure is linear in the states. The design is based on an $\mathcal{H}_\infty$ control synthesis extended to LPV systems that use parameter dependent Lyapunov functions. A weighting strategy is applied to meet performance specifications, i.e. passenger comfort and road holding, guarantee a trade-off between performances that are in conflict with each other and consider the model uncertainties, see e.g. Fialho and Balas [2000], Gáspár et al. [2003a].

Due to the highly nonlinear nature of the suspension actuator its direct inclusion in the state space description of the suspension dynamics is quite complicated. Ignoring the nonlinearities of the actuator leads to unacceptable behavior while applying an LPV approach for the actuator leads to conservative results. In this paper, a design of a two-level controller is proposed for active suspension systems.

For the high-level controller by using an LPV method passenger comfort, road holding and tire deflection are taken into consideration as performance outputs and the control input designed is the control force. In this step the uncertainties of the model are also considered. The designed control force is a required force, which must be created by the hydraulic actuator. One of the difficulties of the control is that even the simplest model of the actuator is a bimodal switching system where the switching depends on one of the state variables. A nonlinear method is proposed for the design of the low-level controller that solves an output tracking problem, i.e. the four required force computed by the high-level controller is tracked by a lower-level controller designed using a backstepping method by setting the valves of the actuators.

The proposed method has several advantageous. The design in the complex case is difficult since the actuator has fast dynamics while the suspension system has slow dynamics, which might lead to numerical difficulties and

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performance degradation. Furthermore, if the suspension design were carried out on the basis of the full-car model, this might lead to numerical problems due to the increased complexity.

The structure of the paper is as follows. In Section 2 the modeling of the active suspension system for control design is presented. In Section 3 the model is augmented with the performance specifications and multiplicative uncertainties. The design of the high-level controller, which generates the control force demand is presented. In Section 4 the design of a lower-level controller, which is based on a control Lyapunov function and backstepping method is presented. In Section 5 the operation of the two–level controller is demonstrated through simulation examples. Finally, Section 6 contains some concluding remarks.

2. CONTROL-ORIENTED MODELING OF THE SUSPENSION SYSTEM

The full-car vehicle model, which is shown in Figure 1, comprises five parts: the sprung mass and four unsprung masses. Let the sprung and unsprung masses be denoted by m_s, m_{uf}, and m_{ur}, respectively. All suspensions consist of a spring, a damper and an actuator to generate a pushing force between the body and the axle. The suspension stiffness and the tire stiffness are denoted by k_s and k_t, respectively. The front and rear suspension dampers are denoted by b_s. Let the front and rear displacement of the sprung mass on the left and right side be denoted by x_{1fl}, x_{1rl} and x_{1fr}, x_{1rr}. Let the front and rear displacement of the unsprung mass on the left and right side be denoted by x_{2fl}, x_{2rl}, x_{2fr}, and x_{2rr}. In the full-car model, the disturbances, w_{fl}, w_{rl}, w_{fr}, w_{rr} are caused by road irregularities. The input signals, f_{fl}, f_{rl}, f_{fr}, f_{rr} are generated by the actuators.

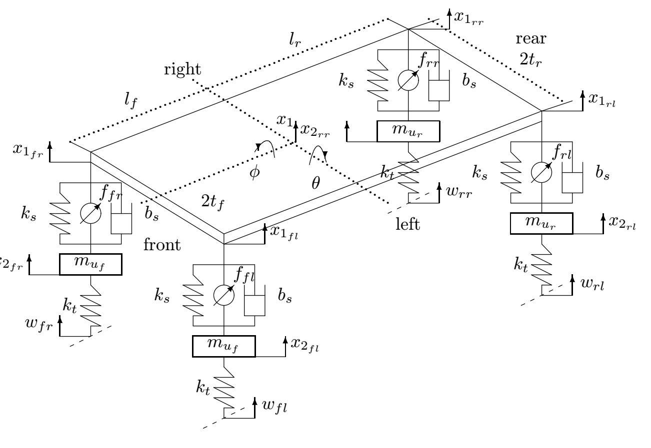

Fig. 1. The full car model

The full-car model is seven degrees-of-freedom: The sprung mass is assumed to be a rigid body and has freedoms of motion in the vertical, pitch and roll directions. The x_1 is the vertical displacement at the center of gravity, θ is the pitch angle and ϕ is the roll angle of the sprung mass, respectively. Each unsprung mass has freedom of motion in the vertical direction, x_{2fl}, x_{2rl}, x_{2fr}, x_{2rr}. The vehicle dynamical model, i.e. the heave motion, the pitch motion, the roll motion, the front and rear tires, is as follows:

$$\begin{aligned} m_s\ddot{x}_1 = & F_{kfl} + F_{kfr} + F_{krl} + F_{krr} + \\ & F_{bfl} + F_{bfr} + F_{brl} + F_{brr} \\ & - f_{fl} - f_{fr} - f_{rl} - f_{rr} \end{aligned} \tag{1}$$

$$\begin{aligned} I_\theta\ddot{\theta} = & l_f F_{kfl} + l_f F_{kfr} - l_r F_{krl} - l_r F_{krr} + \\ & l_f F_{bfl} + l_f F_{bfr} - l_r F_{brl} + l_r F_{brr} \\ & - l_f f_{fl} - l_f f_{fr} + l_r f_{rl} + l_r f_{rr} \end{aligned} \tag{2}$$

$$\begin{aligned} I_\phi\ddot{\phi} = & t_f F_{kfl} - t_f F_{kfr} + t_r F_{krl} - t_r F_{krr} + \\ & t_f F_{bfl} - t_f F_{bfr} + t_r F_{brl} - t_r F_{brr} \\ & - t_f f_{fl} + t_f f_{fr} - t_r f_{rl} + t_r f_{rr} \end{aligned} \tag{3}$$

$$m_{uf}\ddot{x}_{2_{fl}} = -F_{kfl} - F_{tfl} - F_{bfl} + f_{fl} \tag{4}$$

$$m_{uf}\ddot{x}_{2_{fr}} = -F_{kfr} - F_{tfr} - F_{bfr} + f_{fr} \tag{5}$$

$$m_{ur}\ddot{x}_{2_{rl}} = -F_{krl} - F_{trl} - F_{brl} + f_{rl} \tag{6}$$

$$m_{ur}\ddot{x}_{2_{rr}} = -F_{krr} - F_{trr} - F_{brr} + f_{rr} \tag{7}$$

where the following linear approximations are applied: $x_{1_{fl}} = x_1 + l_f\theta + t_f\phi$, $x_{1_{fr}} = x_1 + l_f\theta - t_f\phi$, $x_{1_{rl}} = x_1 - l_r\theta + t_r\phi$, and $x_{1_{rr}} = x_1 - l_r\theta - t_r\phi$.

The suspension damping force, the suspension spring force, the tire force, respectively, are as follows:

$$\begin{aligned} F_{bij} = & b_s^l(\dot{x}_{2ij} - \dot{x}_{1ij}) - b_s^{sym}|\dot{x}_{2ij} - \dot{x}_{1ij}| \\ & + b_s^{nl}\sqrt{|\dot{x}_{2ij} - \dot{x}_{1ij}|}\,\mathrm{sgn}(\dot{x}_{2ij} - \dot{x}_{1ij}), \end{aligned} \tag{8}$$

$$F_{kij} = k_s^l(x_{2ij} - x_{1ij}) + k_s^{nl}(x_{2ij} - x_{1ij})^3, \tag{9}$$

$$F_{tij} = k_t(x_{2ij} - d_{ij}), \tag{10}$$

and f_{ij} are the forces of the actuator, where $ij \in \{fl, fr, rl, rr\}$. Here, parts of the nonlinear suspension damper b_s are b_s^l, b_s^{nl} and b_s^{sym}. The b_s^l coefficient affects the damping force linearly while b_s^{nl} has a nonlinear impact on the damping characteristics. b_s^{sym} describes the asymmetric behavior of the characteristics. Parts of the nonlinear suspension stiffness k_s are a linear coefficient k_s^l and a nonlinear one, k_s^{nl}.

The state vector x is selected as follows:

$$x = [q\ x_u\ \dot{q}\ \dot{x}_u]^T, \tag{11}$$

with $q = [x_1\ \theta\ \phi]^T$ and $x_u = [x_{2fl}\ x_{2fr}\ x_{2rl}\ x_{2rr}]^T$.

In the LPV modeling parameters, which are directly measured or can be calculated from the measured signals, must be selected. Two expressions concerning the front and rear displacement of the unsprung mass on the left and right side and their velocities are selected as scheduling variables:

$$\rho_{bij} = sgn(\dot{x}_{2ij} - \dot{x}_{1ij}), \tag{12}$$

$$\rho_{kij} = (x_{2ij} - x_{1ij})^2. \tag{13}$$

with $ij \in \{fl, fr, rl, rr\}$. Parameter ρ_{bij} depends on the relative velocity, parameter ρ_{kij} is equal to the relative displacement. In practice, the relative displacement is a measured signal. The relative velocity is then determined by numerical differentiation from the measured relative displacement. Thus, in the LPV model of the active suspension system eight parameters are selected as scheduling variables.

The state space representation of the LPV model is as follows:

$$\dot{x} = A(\rho)x + gu, \tag{14}$$

where $u = [f_{fl}\ f_{fr}\ f_{rl}\ f_{rr}]^T$.

2.1 Modeling of the actuator dynamics

The actuator which generates the necessary force for the suspension system is a four-way valve-piston system. For a given quarter-car model denote by f the the force of the actuator and by $z(= \dot{x}_2 - \dot{x}_1)$ the relative velocity. Then

$$f = A_P P_L, \tag{15}$$

where A_P is the area of the piston and P_L is the pressure drop across the piston with respect to the front and rear suspensions. The derivative of P_L is given by

$$\dot{P}_L = -\beta P_L + \alpha A_P z + \gamma Q, \tag{16}$$

in which Q is the hydraulic load flow, α, β, γ are constants and

$$Q = sgn\left[P_S - sgn(x_v)P_L\right]\sqrt{|P_S - sgn(x_v)P_L|}x_v, \tag{17}$$

with the supply pressure P_S and the displacement of the spool valve x_v. The cylinder velocity acts as a coupling from the position output of the cylinder to the pressure differential across the piston. It is considered a feedback term, which has been analyzed by Merritt [1967], Alleyne and Liu [2000]. The displacement of the spool valve is controlled by the input to the servo-valve u:

$$\dot{x}_v = \frac{1}{\tau}\left(-x_v + u\right). \tag{18}$$

where τ is a time constant.

It is assumed that during the operation $P_S > P_L$. With this assumption, eq. (17) reads

$$Q = \begin{cases} x_v\sqrt{P_S - P_L}, \; x_v \geq 0 \\ x_v\sqrt{P_S + P_L}, \; x_v < 0 \end{cases} \tag{19}$$

which defines a state-dependent bimodal switching system for the actuator dynamics, see e.g. Alleyne and Liu [2000].

Let x_5 and x_6 denote P_L and x_v, respectively. Then, the actuator model can be written separately as

$$\dot{x}_5 = -\beta x_5 + \alpha A_P z + \gamma Q, \tag{20}$$

$$\dot{x}_6 = -\frac{1}{\tau}x_6 + \frac{1}{\tau}u_a. \tag{21}$$

3. DESIGN OF A HIGH-LEVEL CONTROLLER BASED ON AN LPV METHOD

In order to improve passenger comfort it is important to keep the effects of the road disturbance on the heave acceleration small. Structural features of the vehicle place a hard limit on the amount of suspension deflection available for reducing the acceleration of the vehicle body. Hence it is also important to keep the effect of the disturbance on the suspension deflection sufficiently small. In order to reduce the dynamic tire load deflection, the effect of the disturbance on tire deflection should also be kept small. The control force limitation is incorporated into the design procedure in order to avoid large control forces.

Consider the closed-loop system in Figure 2, which includes the feedback structure of the model G and controller K, and elements associated with the uncertainty models and performance objectives. In the diagram, u is the control input, which is generated by actuators, y is the measured output, which contains the relative displacement between the sprung mass and the unsprung mass, n is the measurement noise. In the figure, w is the disturbance signal, which is caused by road irregularities. $\tilde{z}$ represents the performance outputs: passenger comfort (heave accelerations, pitch and roll angle accelerations), the suspension deflections, the wheel relative displacements and the control forces.

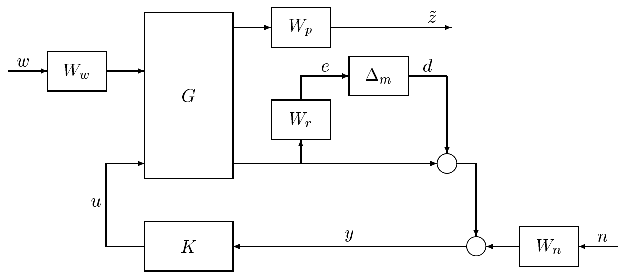

Fig. 2. The closed-loop interconnection structure

The purpose of weighting functions $W_{p,az}$, $W_{p,sd}$, $W_{p,td}$ and $W_{p,F}$ is to keep the heave accelerations, suspension deflections, wheel travels, and control inputs small over the desired operation range. These weighting functions chosen for performance outputs can be considered as penalty functions, i.e. weights should be large in a frequency range where small signals are desired and small where larger performance outputs can be tolerated. Thus, $W_{p,az}$ and $W_{p,sd}$ are selected as

$$W_{p,az}(\rho_{kij}) = \phi_{az}(\rho_{kij}) \cdot 0.5\frac{\frac{s}{350}+1}{\frac{s}{10}+1}, \tag{22}$$

$$W_{p,sd}(\rho_{kij}) = \phi_{sd}(\rho_{kij}) \cdot \frac{\frac{s}{350}+1}{\frac{s}{10}+1}. \tag{23}$$

Here, it is assumed that in the low frequency domain disturbances at the heave accelerations of the body should be rejected by a factor of ϕ_{az} and at the suspension deflection by a factor of ϕ_{sd}.

The trade-off between passengers comfort and suspension deflection is due to the fact that is not possible to keep them together simultaneously. A large gain ϕ_{az} and a small gain ϕ_{sd} correspond to a design that emphasizes passenger comfort. On the other hand, choosing ϕ_{az} small and ϕ_{sd} large corresponds to a design that focuses on suspension deflection. In the *LPV* controller ρ_{kij} is the relative displacement between the sprung and the unsprung masses. ρ_{kij} is used to focus on minimizing either the vertical acceleration or the suspension deflection response, depending on the magnitude of the vertical suspension deflection.

The parameter dependence of the gains is characterized by the constants ρ_1 and ρ_2 in the following way:

$$\phi_{az}(\rho_{kij}) = \begin{cases} 1 & \text{if} \quad |\rho_{kij}| < \rho_1 \\ \dfrac{|\rho_{kij}| - \rho_2}{\rho_1 - \rho_2} & \text{if} \quad \rho_1 \leq |\rho_{kij}| \leq \rho_2 \\ 0 & \text{otherwise} \end{cases}, \tag{24}$$

$$\phi_{sd}(\rho_{kij}) = \begin{cases} 0 & \text{if} \quad |\rho_k| < \rho_1 \\ \dfrac{|\rho_{kij}| - \rho_1}{\rho_2 - \rho_1} & \text{if} \quad \rho_1 \leq |\rho_{kij}| \leq \rho_2 \\ 1 & \text{otherwise} \end{cases}. \tag{25}$$

The selection of the parameter dependence of the gains is the reason why an LPV model is formulated even in a linear model is used in the basis of the control design. The uncertainties of the model are represented by W_r and Δ_m.

W_r is assumed to be known, and Δ_m is assumed to be unknown with $\|\Delta_m\|_\infty < 1$. Design models used for active suspension control typically exhibit high fidelity at lower frequencies, but they degrade at higher frequencies. Thus, W_r is selected as $W_r = 2.25\frac{S+20}{S+450}$.

The solution of an LPV problem is based on the set of infinite dimensional LMIs being satisfied for all $\rho \in \mathcal{F}_\mathcal{P}$, thus it is a convex problem, Rough and Shamma [2000], Wu [2001]. In practice, this problem is set up by gridding the parameter space and solving the set of LMIs that hold on the subset of $\mathcal{F}_\mathcal{P}$, see Balas et al. [1997]. The number of grid points depends on the nonlinearity and the operation range of the system. For the interconnection structure, $\mathcal{H}_\infty$ controllers are synthesized for 5 values of ρ_1 in a range $[-2, 2]$ and 5 values of ρ_2 in a range $[0, 1]$.

4. NONLINEAR DESIGN OF A LOW-LEVEL CONTROLLER

We assume that the reference for F (which is a linear function of x_5) is given by the linear controller. The goal is to asymptotically track this reference with the actuator dynamics. Since the actuator subsystem and the suspension subsystem form a cascade of a nonlinear and a linear system, the backstepping methodology is an appropriate choice for our control goal. Backstepping is a control Lyapunov function-based nonlinear controller design method Sepulchre et al. [1997]. We will use the notations of van der Schaft [2000] where backstepping is presented from the viewpoint of the theory of interconnected passive systems.

The model of the suspension and actuator system with zero disturbance is written in the following form

$$\dot{z} = Az + B\xi_1, \tag{26}$$

$$\dot{\xi}_1 = a_1(z, \xi_1) + b_1(\xi_1)\xi_2, \tag{27}$$

$$\dot{\xi}_2 = a_2(\xi_2) + b_2 u_a, \tag{28}$$

where $z = [x_1\ x_2\ x_3\ x_4]^T$ with relative displacements x_1, x_2 and relative velocities $x_3(= \dot{x}_1)$, $x_4(= \dot{x}_2)$, moreover, $\xi_1 = x_5$, $\xi_2 = x_6$, and

$$a_1(z, \xi_1) = -\beta x_5 + \alpha A_P(x_4 - x_3), \tag{29}$$

$$b_1(\xi_1) = \begin{cases} \gamma\sqrt{P_S - x_5}, & \xi_2 = x_6 \geq 0 \\ \gamma\sqrt{P_S + x_5}, & \xi_2 = x_6 < 0 \end{cases}, \tag{30}$$

$$a_2(\xi_2) = -\frac{1}{\tau}x_6, \tag{31}$$

$$b_2 = \frac{1}{\tau}. \tag{32}$$

Let us assume that there exists a smooth feedback function $K(z)$ (possibly in LPV form) such that the closed loop system

$$\dot{z} = Az + BK(z) \tag{33}$$

is asymptotically stable with control Lyapunov function $V(z)$.

The backstepping design for the actuator subsystem can be performed in two steps. In the first step, let us consider ξ_2 as a virtual input and $y_1 = \xi_1 - K(z)$ as a virtual output. Since ξ_1 is not a manipulable input, we would like to construct a feedback that guarantees the tracking of $K(z)$ with ξ_1. It is reasonable therefore to define the tracking error to be linear and stable, i.e., $\dot{y}_1 = -k_1 y_1$, $k_1 > 0$. From this (using eqs. (26)–(27)), the desired time-function for ξ_2 can be computed as a nonlinear feedback of the form

$$\begin{aligned} \xi_{2,des} &= \alpha_1(z, \xi_1) \\ &= \frac{1}{b_1(\xi_1)}[-a_1(z, \xi_1) \\ &+ \frac{\partial K}{\partial z} \cdot (Az + B\xi_1) - k_1(\xi_1 - K(z))]. \end{aligned} \tag{34}$$

In the second step, the following virtual output is defined: $y_2 = \xi_2 - \alpha_1(z, \xi_1)$. For the tracking error, a stable linear dynamics is also prescribed in this case: $\dot{y}_2 = -k_2 y_2$, $k_2 > 0$. Using equations (26)–(28), we can now express the physically manipulable actuator input u_a as a function of z and ξ in the following form

$$\begin{aligned} u_a &= \alpha_2(z, \xi_1, \xi_2) \\ &= \frac{1}{b_2} \cdot [-a_2(\xi_2) + \frac{\partial \alpha_1}{\partial z} \cdot (Az + B\xi_1) \\ &+ \frac{\partial \alpha_1}{\partial \xi_1}(a_1(z, \xi_1) + b_1(\xi_1)\xi_2) - k_2(\xi_2 - \alpha_1(z, \xi_1))]. \end{aligned} \tag{35}$$

By applying the above design, the closed loop system will be asymptotically stable with control Lyapunov function $S(z) = V(z) + \frac{1}{2}y_1^2 + \frac{1}{2}y_2^2$ (see Sepulchre et al. [1997]). It is important to note that the obtained feedback law (35) is a state-dependent switching function because of the switching term $b_1(\xi_1)$ (see eq. (30)).

Since the actual feedback law generated by the LPV controller is a rather complicated function of the state variables, and we do not know the road excitation disturbances in advance, the above controller design procedure cannot be implemented in its original theoretical form. Therefore in the next section we will consider the more realistic assumption, when the reference for x_5 is computed by the high level LPV controller, and for the trajectory tracking, the time derivatives of the reference signals are computed numerically.

The reference for x_5 computed for the LPV controller is denoted by $x_{5,ref}$. To simplify the forthcoming calculations, let us use the following notations

$$g_{a1}(x) = -\beta x_5 + \alpha A_p(x_4 - x_3), \tag{36}$$

$$f_{a1}(x_5) = \sqrt{P_S - x_5}, \tag{37}$$

$$f_{a2}(x_5) = \sqrt{P_S + x_5}. \tag{38}$$

This way, equation (20) can be written as

$$\dot{x}_5 = g_{a1}(x) + \gamma Q. \tag{39}$$

The required tracking error dynamics is defined as

$$\dot{x}_5 - \dot{x}_{5,ref} = -k_1(x_5 - x_{5,ref}) \quad \text{with} \quad k_1 > 0. \tag{40}$$

From (40) yield the following form:

$$\gamma x_6 f_{a1,2}(x_5) = -g_{a1}(x) + \dot{x}_{5,ref} - k_1(x_5 - x_{5,ref}). \tag{41}$$

The reference for x_6 is given by

$$x_{6,ref} = \begin{cases} \dfrac{-g_{a1}(x) + \dot{x}_{5,ref} - k_1(x_5 - x_{5,ref})}{\gamma f_{a1}(x_5)} & \text{if} \quad x_6 \geq 0 \\ \dfrac{-g_{a1}(x) + \dot{x}_{5,ref} - k_1(x_5 - x_{5,ref})}{\gamma f_{a2}(x_5)} & \text{if} \quad x_6 < 0 \end{cases}. \tag{42}$$

The tracking error dynamics for $x_{6,ref}$ is written as

$$\dot{x}_6 - \dot{x}_{6,ref} = -k_2(x_6 - x_{6,ref}) \quad \text{if} \quad k_2 > 0. \tag{43}$$

This gives

$$-\frac{1}{\tau}x_6 + \frac{1}{\tau}u_a - \dot{x}_{v,ref} = -k_2(x_6 - x_{6,ref}), \qquad (44)$$

from which the following expression for the physical input u_a is deduced:

$$u_a = \frac{\frac{1}{\tau}x_6 + \dot{x}_{v,ref} - k_2(x_6 - x_{6,ref})}{1/\tau}. \qquad (45)$$

In order to practically implement the control law, we need to compute the time derivatives of $x_{5,ref}$ and $x_{6,ref}$, which can be done in a number of ways depending on the measurement noise conditions and the required precision. The controller parameters k_1 and k_2 determine the convergence speed of the virtual outputs y_1 and y_2, respectively.

The asymptotic stability of the closed loop system in the original theoretical case follows from the structure of the controller and the control Lyapunov function can be easily determined van der Schaft [2000]. The first solution is to write an LPV state-space realization of the whole closed loop system and find a parameter-dependent or a parameter-independent Lyapunov function by trying to solve the corresponding set of LMIs (see, e.g. Scherer and Weiland [2000]). The second (more conservative) method uses the fact that the closed loop system is a standard feedback interconnection of two systems: the mechanical suspension subsystem together with the LPV controller and the linearized actuator subsystem together with the reference tracking controller. In this case, we can apply the well-known small-gain theorem (see, e.g. Zhou et al. [1996] or van der Schaft [2000]) to prove the overall stability of the closed loop system.

5. SIMULATION EXAMPLES

In the first example the tracking properties of the low-level controller is presented. The force required by the high-level controller at the front and the left-hand side of the vehicle is generated by the low-level actuator. The upper plot of Figure 4 shows that the generated force approximates the required force with high accuracy. The middle plot shows that the tracking error is below 1 %. The tracking is tested in an uncertain case, i.e., when the parameters b_s^l, k_s^l and k_t are assumed to be uncertain and the percentage of the variation around their nominal value is 10 %. The lower plot shows that the tracking error is below 5 %.

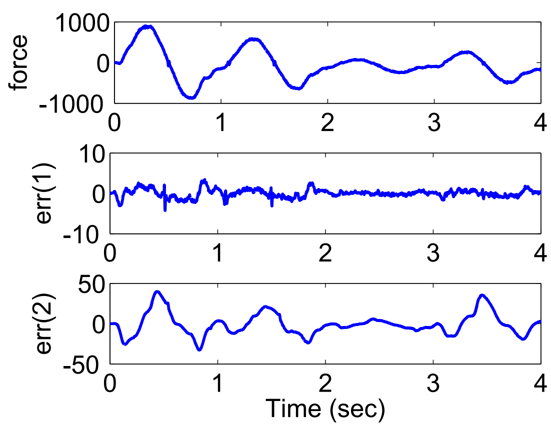

Fig. 3. Analysis of the tracking

In the second example the controlled system is tested on a bad-quality road, on which four bumps disturb the vehicle motion: the bumps are 8 *cm*, 6 *cm*, 2 *cm* and 4 *cm*. The time responses of the heave acceleration, the relative displacement, the wheel travel, and the control force at the front and the left-hand side of the vehicle are illustrated in Figure 4. The solid line corresponds to the force required by the LPV controller in which the performance specifications are taken into consideration. The dashed lines illustrate the result of the controllers designed by the backstepping method. The force tracking is less accurate than in the previous open-loop case, since the small error between the required and the generated forces during the closed-loop operation results in larger deviation from the required force.

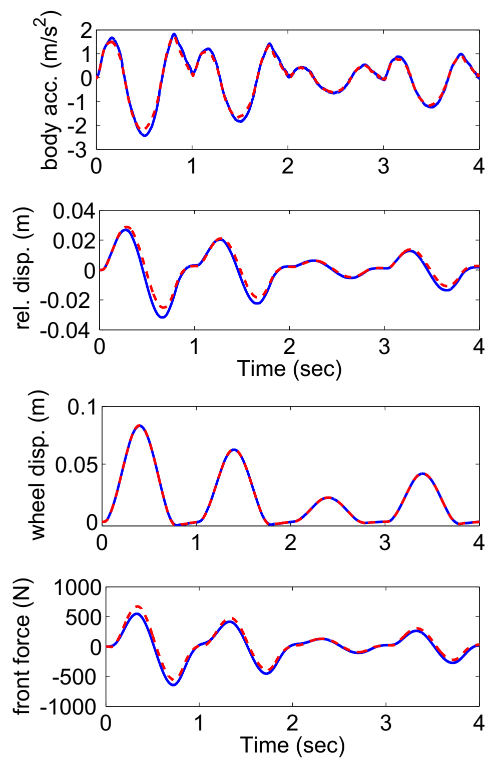

Fig. 4. Time responses of the designed system

The tracking properties of the controllers are acceptable. The realization of the control force by using backsteping method is illustrated in Figure 5. The figure shows the pressure drop across the piston, the displacement of the spool valve and the control signal, see equation (18).

In the final example the tracking properties are tested in an uncertain case, i.e., when the parameters b_s^l, k_s^l and k_t are assumed to be uncertain and the percentage of the variation around their nominal value is 10 %. The solid line corresponds to the force required by the LPV controller in Figure 6. The dashed line illustrates the force of the controller designed by the backstepping method. The controller is able to generate the required force even in uncertain cases. As the handling of uncertainty is an important consideration we recommend the backstepping control method for carrying out the low-level task.

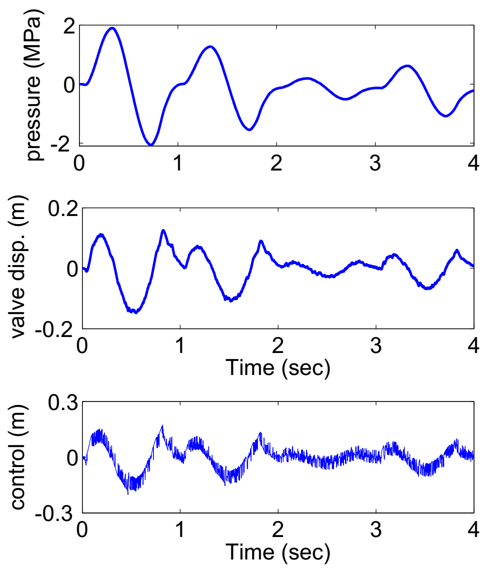

Fig. 5. Time responses of the actuator based on backstepping method

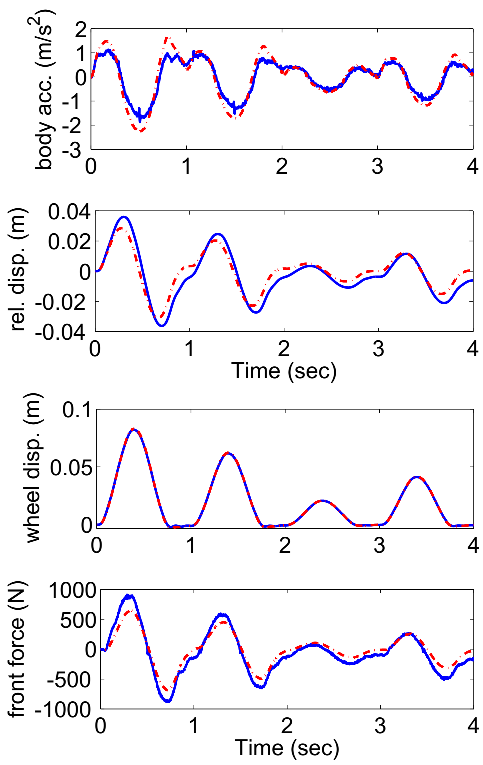

Fig. 6. Time responses of the controlled system with parametric uncertainties

6. CONCLUSIONS

In this paper a combined controller has been presented for the control of active suspension systems. An LPV-based controller is used to compute the required input force, which is tracked using a nonlinear controller for the actuator subsystem. For actuator control a low-level backstepping-based force-tracking controller has been designed. It has been found that the low-level controller shows an adequate tracking performance and it is also able to handle the parametric uncertainties. The successful combination of the linear and nonlinear control design methods is possible mainly because of the different time-domain performance requirements in the two subsystems. The main advantage of the proposed solution is its ability to meet complex control performance criteria together with the handling of switching nonlinear actuator dynamics.

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