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Towards Model-based Continuous-time Identification of the Human Balance Controller

Peter J Gawthrop * **Liuping Wang** **

* *Centre for Systems and Control and Department of Mechanical Engineering, University of Glasgow, GLASGOW. G12 8QQ Scotland.* ***P.Gawthrop@eng.gla.ac.uk***
** *Discipline of Electrical Energy and Control Systems, School of Electrical and Computer Engineering, RMIT University, Melbourne, Victoria 3000, Australia.* ***liuping.wang@rmit.edu.au***

Abstract: There are a number of competing scientific hypotheses about the structure and parameters of the human control system concerned with balance. System identification techniques have potential to distinguish between such competing hypotheses. As a step towards this goal, the data from an initial series of experiments involving balancing an inverted pendulum by a human via a joystick was analysed using a recently-developed two-stage continuous-time identification method.

Keywords: Identification and validation; Model formulation, experiment design; Kinetic modelling and control of biological systems

1. INTRODUCTION

The investigation of physiological control systems in general, and the control of standing in particular have been the subject of research over an extended period. One topic of debate is whether physiological control mechanisms can be modelled as technological control systems and, if so, what control algorithm is used (Fitzpatrick *et al.*, 1996). The so called proportional-integral-derivative (PID) control algorithm, alongstanding process control algorithm, was suggested some time ago (Johansson *et al.*, 1988) and has received a lot of attention recently (Peterka, 2002; Maurer and Peterka, 2005; Pavol, 2005).

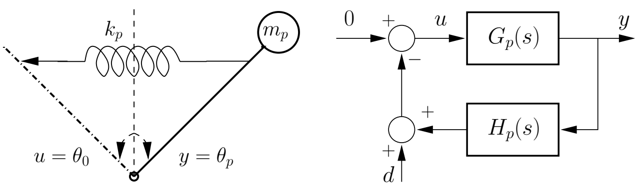
Systems with a pure input delay of t_d sec are common in technological systems and also seem to be a plausible model for some physiological control systems. A key insight, attributed to Smith (Smith, 1959), is that a *predictor*, based on an *internal system model)* can eliminate the time delay from the feedback loop (though not the overall response) thus reducing controller design and performance analysis to the delay-free case. It is plausible that physiological control systems have built in model-based prediction (McRuer, 1980; Miall *et al.*, 1993; Wolpert *et al.*, 1998; Bhushan and Shadmehr, 1999; Neilson and Neilson, 2005; Loram *et al.*, 2006). As the human balance system is open-loop unstable, some predictors such as that of Smith (1959) are not applicable; a state-space based approach akin to that of Kleinman (1969) is used here.

These competing hypotheses must be tested using experimental data. Given a set of experimental data, two broad classes of approach can be distinguished: a detailed examination of (suitably averaged) small sections of data (Loram and Lakie, 2002*b*), and the more engineering based approach of system identification (Johansson *et al.*, 1988; Fransson *et al.*, 2003; Peterka, 2002). This paper suggests the application of a quite recent two-stage approach to system identification (Gawthrop and Wang, 2005; Wang and Gawthrop, 2008) which in a sense combines the two broad classes: the first stage "compresses" a set of data to yield a non-parametric system model in the form of either an impulse response or a frequency response; the second step estimates system parameters from the non-parametric mode.

Identification of physiological control systems from unperturbed measured data has two problems: the controller is embedded in a closed-loop system and the need to estimated disturbance models can both lead to ambiguity in interpretation of the results. These two pitfalls are avoided here by using an external measured perturbation to the system and by identifying the entire closed-loop dynamics.

There are a number of different representations of controllers, including the transfer function representation based on Laplace transforms and the state-space approach based on differential equations. In the context of this paper, it is important that the controller corresponding to each hypothesis is represented and implemented within the *same* control engineering framework thus avoiding apparent differences solely due to implementation artifacts. This paper uses a state-space framework within which PID control and predictive control control are embedded in a uniform way.

The outline of the paper follows. Section 2 looks at closed-loop control of an inverted pendulum and derives the corresponding closed-loop impulse responses for non-predictive and predictive controllers. Section 3 uses the two-stage identification process to fit the closed-loop impulse response of the parametrised predictive controller to the identified impulse response. Section 4 draws some preliminary conclusions.



(a) Inverted Pendulum (b) Closed-loop system

Fig. 1. Experimental system

2. SYSTEM AND CONTROL MODELLING

A simple model of human standing is equivalent to the control of an inverted pendulum (the body) via a spring (tendons and muscle) (Loram and Lakie, 2002*b*, Figure 1). It is convenient to represent such a model by Figure 1(a) where the input u is the effective input angle θ_0 and the output y is the pendulum angle θ_p and the length of the pendulum is l. The system can be modelled with three parameters:

- the inertia about the pivot J_p
- the effective gravitational spring k_g and
- the ratio α of the effective spring constant to the gravitational spring.

The feedback structure is given in Figure 1(b) where H_p is the human controller and d a disturbance signal. The closed-loop system of Figure 1(b) has one input (d) and two outputs (y and u).

2.1 Inverted Pendulum

Parameter	Value
c	0.850
k_p	146
J_p	15.0

Table 1. System parameters

As discussed previously (Loram and Lakie, 2002*b*), the inverted pendulum of Figure 1(a) can be modelled by the second order differential equation:

$$J_p\ddot{\theta} - (1-c)k_p\theta = ck_p(u_0 + d) \quad (1)$$

were J_p is the pendulum moment of inertia, $k_p = mgh$ the effective gravitational spring constant, c the stiffness of the muscle/tendon effective spring, $u_0 = \theta_0$ the "bias" input and d an added disturbance that will be used later. It is known (Loram and Lakie, 2002*a*) that $c < 1$ so that (1) represents an unstable system. Parameter values appear in Table 1.

A standard control engineering approach to introduce controller *integral action* (the I of PID) is to augment the system with an integrator; in this case define a new control signal u

$$u = \dot{u}_0 = \dot{\theta}_0 \quad (2)$$

Note that (2) is equivalent to:

$$u_0(t) = \int_0^t u(t')dt' + u_0(0) \quad (3)$$

The second order differential equation (1), together with the new control signal, can be written as the state-space system:

$$\dot{x} = Ax + B(u + \dot{d}) \quad (4)$$
$$y = Cx \quad (5)$$

where

$$x = \begin{bmatrix}\dot{\theta} & \theta & \theta_0 + d\end{bmatrix}^T \quad (6)$$

$$A = \begin{bmatrix}0 & (1-c)k_p & ck_p \\ 1 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}; \; B = \begin{bmatrix}0\\0\\1\end{bmatrix}; \; C = \begin{bmatrix}0\\1\\0\end{bmatrix}^T \quad (7)$$

2.2 PID control

Give a system in state-space form (4), the standard state-space controller generates the control signal u by multiplying the column vector x with the *state-feedback* row vector k

$$u = -kx \quad (8)$$
$$k = [k_1 \; k_2 \; k_3] \quad (9)$$

Using the standard approach, (9) is substituted into (4) to give:

$$\dot{x} = A_c x + B_d d \quad (10)$$
$$y = Cx \quad (11)$$
$$u = -kx \quad (12)$$

where the *closed-loop* matrix A_c is given by

$$A_c = A - Bk \quad (13)$$

Subject to certain conditions (satisfied in this particular case), k can be designed to place the closed loop system poles (eigenvalues of A_c) anywhere in the complex plane subject to the poles being either real or in complex-conjugate pairs. In particular, a k can always be chosen to stabilise the unstable system (1).

There are many possible ways to choose k. In particular, the *Linear-quadratic* approach (Kwakernaak and Sivan, 1972) chooses k to minimise the cost function:

$$J = \int_0^\infty x^T(t)Qx(t) + Ru(t)^2 dt \quad (14)$$

For the purposes of this paper the cost parameters Q and R are chosen as:

$$Q = \begin{bmatrix}q_v & 0 & 0 \\ 0 & q_p & 0 \\ 0 & 0 & 0\end{bmatrix}; \; R = 1 \quad (15)$$

The two positive numbers q_v and q_p weight pendulum angular velocity and position respectively.

In this particular case, k (16) has three elements and so the control signal u is given by

$$u = -[k_1\dot{\theta} + k_2\theta + k_3(\theta_0 + d)] \quad (16)$$

Using (1) to replace θ_0 on the right-hand side of (16) and and (2) to replace u on the left-hand side of (16):

$$\theta_0 = -[k_1\theta + (k_2 + \frac{1-c}{c}k_3)\int \theta dt + \frac{J_p}{ck_p}k_3\dot{\theta}] \quad (17)$$

The standard PID controller is of the form:

$$\theta_0 = -K_p[\theta + \frac{1}{T_i}\int \theta dt + T_d\dot{\theta}] \quad (18)$$

Comparing (18) and (17) gives:

$$K_p = k_1; \; T_i = \frac{k_1}{k_2 + \frac{1-c}{c}k_3}; \; T_d = \frac{J_p}{ck_pk_1}k_3 \quad (19)$$

Impulse and frequency response Two useful non-parametric representations of the closed loop system relating d to y and u are the *impulse responses* $g_y(t)$ and $g_u(t)$. Because the model (4) includes the derivative of the disturbance d care is needed when replacing $d(t)$ by the impulse function $\delta(t)$ (Lundberg *et al.*, 2007).

To be compatible with the predictive control considered in this paper, it is assumed that the system is open-loop for an infinitesimal time interval after $t = 0$, the end of this interval is denoted $0+$. Following the analysis of Kailath (1980)(sec. 2.3), the state of the open-loop system at $t = 0+$ (4) when $d(t) = \delta(t)$ is:

$$x(0+) = B_d = AB \tag{20}$$

If the loop is closed at this point, the initial condition of the closed loop system (10) becomes B_d and the state evolves as:

$$x(t) = e^{A_c t} B_d \tag{21}$$

Using (11) and (12), it follows that the closed-loop impulse response of the system output and input are

$$g_y(t) = Ce^{A_c t} B_d \tag{22}$$

$$g_u(t) = -ke^{A_c t} B_d \tag{23}$$

2.3 Observer-based PID

PID control in the form of (18) has a serious drawback; it requires the *derivative* of the system output $y = \theta$. For this reason, practical PID controllers use a low-pass filtered version of this derivative. Exactly the same criticism can be made of the state-feedback controller (9), and moreover, the third state contains d which is not available to the controller. For these reasons, the state feedback is normally used in conjunction with a state *observer*. The idea is quite simple, use the system model (4) to generate an approximation $\hat{x}$ to the state x and then feedback the corresponding output error

$$\dot{\hat{x}} = A\hat{x} + Bu - L(\hat{y} - y) \tag{24}$$

$$\hat{y} = C\hat{x} \tag{25}$$

where L is the observer gain vector. Equations (24) and (25) can be rewritten as

$$\dot{\hat{x}} = A_o\hat{x} + Bu + Ly \tag{26}$$

where $A_o = A - LC$. Again, given certain conditions satisfied here, L can be chosen anywhere in the complex plane subject to the poles being either real or in complex-conjugate pairs. Again an LQ approach to choosing L can be used (Kwakernaak and Sivan, 1972). Here unit measurement noise is assumed whereas the states are perturbed by noise of variance q_o.

The state feedback (9) is replaced by:

$$u = -k\hat{x} \tag{27}$$

Combining (26) and (27), the controller can be rewritten in standard state-space form as:

$$\dot{\hat{x}} = A_{oc}\hat{x} + Ly \tag{28}$$

$$u = -k\hat{x} \tag{29}$$

where $A_{oc} = A - Bk - LC$.

Impulse and Frequency Response Defining the state observation error $\tilde{x} = \hat{x} - x$, subtracting (4) from (24) gives

$$\dot{\tilde{x}} = A_o\tilde{x} - B\dot{d} \tag{30}$$

$$\tilde{u} = -k\tilde{x} \tag{31}$$

The closed loop system (10) is replaced by:

$$\dot{x} = A_c x + B\dot{d} + B\tilde{u} \tag{32}$$

$$y = Cx \tag{33}$$

Defining the combined state of (4) and (30) as

$$X = \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} \tag{34}$$

equations (10) and (30) can be combined as:

$$\dot{X} = A_{co}X + B_{co}\dot{d} \tag{35}$$

$$y = C_{co}X \tag{36}$$

$$u = -k_{co}X \tag{37}$$

where:

$$A_{co} = \begin{bmatrix} A_c & -Bk \\ 0 & A_o \end{bmatrix};\ B_{co} = \begin{bmatrix} B \\ -B \end{bmatrix} \tag{38}$$

$$C_{co} = [C\ 0];\ k_{co} = [k\ k] \tag{39}$$

Following the approach of Section 2.2.1, the closed-loop impulse response is:

$$g_y = C_{co}e^{A_{co}t}B_{dco} \tag{40}$$

$$g_u = -k_{co}e^{A_{co}t}B_{dco} \tag{41}$$

where:

$$B_{dco} = \begin{bmatrix} B_d \\ -B_d \end{bmatrix} \tag{42}$$

Similarly, the frequency responses are:

$$G_y(j\omega) = C_{co}[j\omega I - A_{co}]^{-1}B_{dco} \tag{43}$$

$$G_u(j\omega) = -k_{co}[j\omega I - A_{co}]^{-1}B_{dco} \tag{44}$$

2.4 Predictive control

As discussed in the introduction, it is plausible that physiological control systems have built in model-based prediction. A state-space formulation of predictive control (similar to that of Sage and Melsa (1971) and Kleinman (1969)) is now given.

The state-space system (4) is replaced by the following time-delayed version:

$$\dot{x}(t + t_d) = Ax(t + t_d) + B(u(t) + \dot{d}(t + t_d)) \tag{45}$$

$$y(t + t_d) = Cx(t + t_d) \tag{46}$$

The observer/state-feedback controller (27) is replaced by:

$$u(t) = -k\hat{x}(t + t_d|t) \tag{47}$$

where $\hat{x}(t + t_d|t)$ is the *predicted* state given by:

$$\hat{x}(t + t_d|t) = e^{At_d}\hat{x}(t) + \int_0^{t_d} e^{At'}Bu(t - t')dt' \tag{48}$$

Noting that the solution of (45) starting from time t is:

$$x(t + t_d) = e^{At_d}x(t) + \int_0^{t_d} e^{At'}B(u(t - t') + \dot{d}(t + t_d - t'))dt' \tag{49}$$

It follows that the *prediction error* $\tilde{x}(t + t_d|t)$ is given by:

$$\tilde{x}(t + t_d|t) = e^{At_d}\tilde{x}(t) + \tilde{d}(t + t_d) \tag{50}$$

where:

$$\tilde{d}(t + t_d) = k\int_0^{t_d} e^{At'}B\dot{d}(t + t_d - t')dt' \tag{51}$$

Combining (45), (47) and (50) gives the closed-loop system:

$$\dot{x}(t + t_d) = A_c x(t + t_d) - Bke^{At_d}\tilde{x}(t) - B\tilde{d}(t + t_d) \tag{52}$$

$$y(t + t_d) = Cx(t + t_d) \tag{53}$$

Impulse and Frequency Response In a similar fashion to Section 2.3.1, define the combined state of (52) and (30) as

$$X(t) = \begin{bmatrix} x(t + t_d) \\ \tilde{x}(t) \end{bmatrix} \tag{54}$$

equations (45) and (30) can be combined as:

$$\dot{X} = A_{co}X + B_{cod}(\dot{d} + \tilde{d}) \tag{55}$$

$$y = C_{co}X \tag{56}$$

$$u = -k_{co}X \tag{57}$$

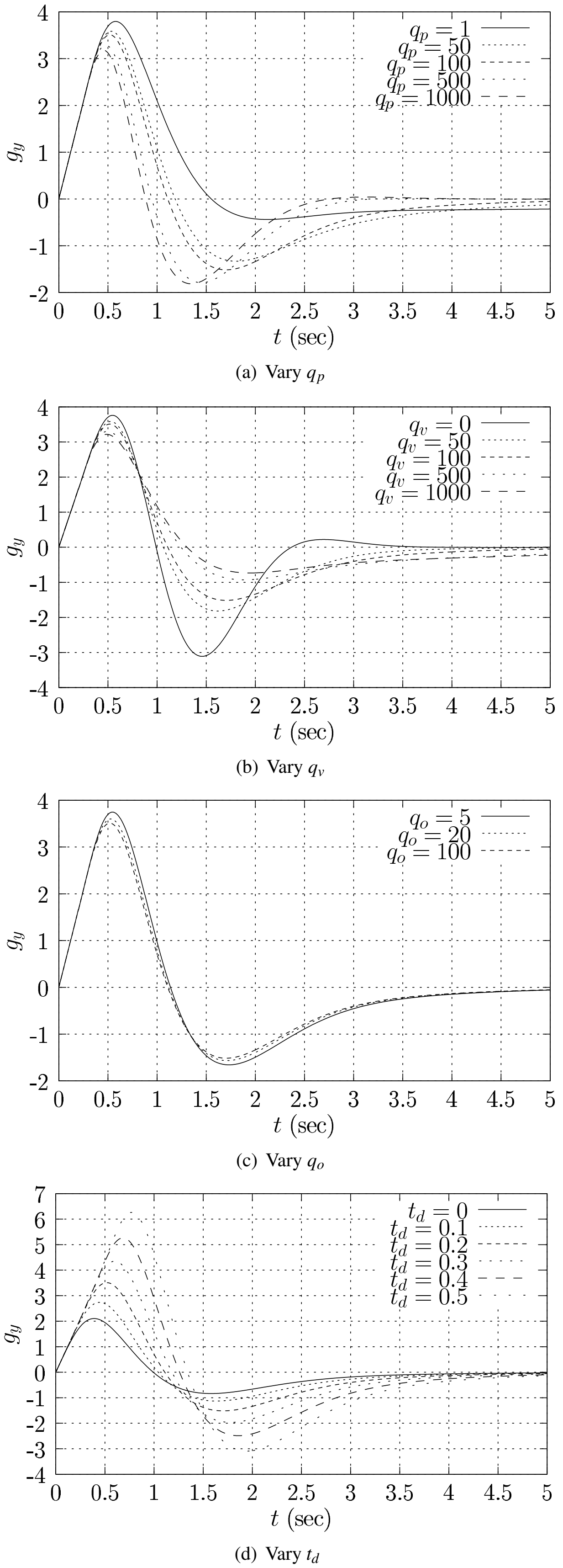


Fig. 2. Theoretical closed-loop disturbance impulse response g_y for predictive control (62).

where:

$$A_{co} = \begin{bmatrix} A_c & -Bke^{At_d} \\ 0 & A_o \end{bmatrix};\ B_{cod} = \begin{bmatrix} e^{At_d}B_d \\ -B_d \end{bmatrix} \quad (58)$$

$$C_{co} = [C\ 0];\ k_{co} = \begin{bmatrix} k & ke^{At_d} \end{bmatrix} \quad (59)$$

The impulse response of the closed-loop system is computed in two parts.

(1) When $t \leq t_d$, the control signal is zero and the open loop system is given by (4) with $u(t) = 0$. Following the discussion in Section 2.2.1, the means that:

$$x(t) = e^{At}B_d \quad (60)$$

where $B_d = AB$ as given by (20).

(2) When $t > t_d$, equation (52). From (60), it follows that $x(t_d+) = e^{At_d}B_d$; using observer error equation (30) and the arguments of Section 2.2.1 it follows that $\tilde{x}(0+) = -B_d$. Hence, from the arguments of section 2.2.1 that when $d = \delta$ and using (54) that the initial condition for (55) is:

$$X(0+) = B \begin{bmatrix} x(t_d)+ \\ \tilde{x(0+)} \end{bmatrix} = \begin{bmatrix} e^{At_d}B_d \\ -B_d \end{bmatrix} \quad (61)$$

When $d(t) = \delta(t)$, it follows that $\dot{d}(t) = 0$ when $t > t_d$ and so that, from (51), $\tilde{d} = 0$. In this case, the impulse responses are given by (40) and (40) but reinterpreted in terms of this section.

To summarise.

$$g_y(t) = \begin{cases} Ce^{At}B_d & \text{for } t \leq t_d \\ C_{co}e^{A_{co}(t-t_d)}B_{dco} & \text{for } t > t_d \end{cases} \quad (62)$$

$$g_u(t) = \begin{cases} -ke^{At}B_d & \text{for } t \leq t_d \\ -k_{co}e^{A_{co}(t-t_d)}B_{dco} & \text{for } t > t_d \end{cases} \quad (63)$$

The corresponding frequency responses are:

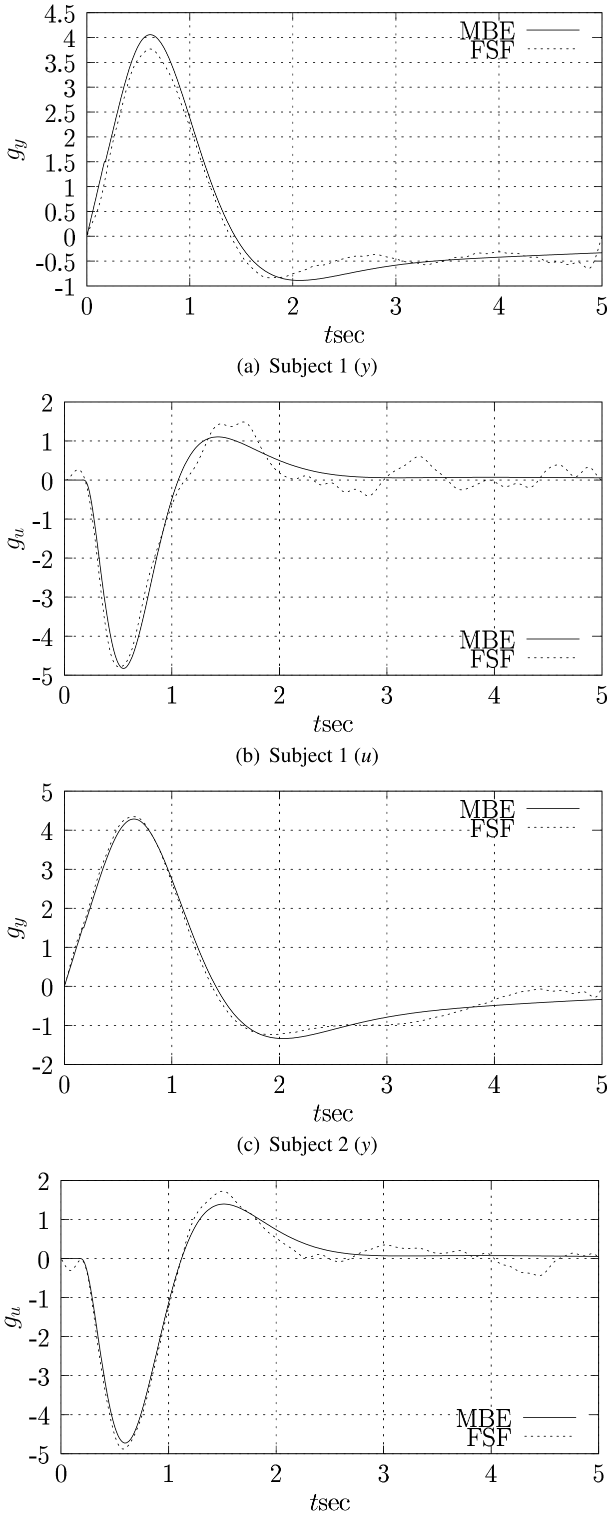
$$G_y(j\omega) = C[j\omega I - A]^{-1}[I - e^{-(j\omega I - A)t_d}]B_d + e^{-jwt_d}C_{co}[j\omega I - A_{co}]^{-1}B_{dco} \quad (64)$$

$$G_u(j\omega) = -k[j\omega I - A]^{-1}[I - e^{-(j\omega I - A)t_d}]B_d - e^{-jwt_d}k_{co}[j\omega I - A_{co}]^{-1}B_{dco} \quad (65)$$

Figure 2 shows the theoretical closed-loop disturbance impulse response g_y for predictive control (62). The three controller design parameters (15) q_p(position weight), q_v(velocity weight), and q_o (observer weight) are varied in (a)–(c); the delay t_d is varied in (d). The nominal values are $q_p = q_v = q_o = 100$, $t_d = 0.18$sec. The response is relatively insensitive to the two controller parameters if above 50; this is because the ultimate performance is limited by the the time-delay t_d. The response is relatively insensitive to observer gain as long as it is large enough; this is because the states can be accurately observed within a short time when using a high observer gain. The response is mainly determined by the time-delay. Ideally, the response to a disturbance would be zero; therefore the larger response associated with larger t_d implies worse performance. If $t_d > 0$, it is impossible to reduce the response to zero. The impulse responses g_u (63) show a similar pattern.

3. DATA ANALYSIS

The experimental setup involved human subjects controlling a simulated inverted pendulum via a joystick; full detail are given elsewhere (Loram *et al.*, 2006). All the controllers discussed



(a) Subject 1 (y)

(b) Subject 1 (u)

(c) Subject 2 (y)

(d) Subject 2 (u)

Fig. 3. Estimated output Angle(y) and input (u). Model-based estimation (MBE) was used to fit the two controller parameters (q_p and q_v) to the output (y) and input (u) impulse responses derived from the raw data using Frequency Sampling Filters (FSF). The results for two human subjects are shown.

Subject	q_p	q_v	t_d
Subject 1	1.78	28.17	0.18
Subject 2	1.89	11.81	0.18

Table 2. Estimated Controller Parameters.

here are parametrised by the controller gain k (8), (27)& (47) and the observer gain L (24). There are at least three possible ways of parameterising the controllers for the model-based identification of Section 3.

(1) Use the (six) parameters of k and L directly. There are many other possibilities
(2) Use pole-placement control and use the pole positions as parameters.
(3) Use a linear-quadratic formulation and use the cost-function weights as parameters. In the sequel, three parameters were used:
  k the position and velocity were weighted by q_p and q_v respectively and the control signal u (not u_0) by unity.
  L the states were weighted by the unit matrix and the output by q_o.

The simulated and experimental data was analysed using the two stage approach of Gawthrop and Wang (2005) and Wang and Gawthrop (2008). In particular:

(1) The closed-loop impulse responses g_y and g_u were estimated using the Frequency-sampling (FSF) Filter approach (Gawthrop and Wang, 2005). A constrained version (Wang *et al.*, 2005) was used to constrain the impulse responses to be zero a time zero. The use of constraints in both time and frequency domains to set known values needs more investigation. In the sequel, the FSF cutoff frequency was $f_c = 5$Hz and the impulse length was $T_s =$ 5sec – again, more investigation needed.
(2) Controller parameters were fitted using the explicit formulae for g_y (62) and g_u (63) to compute the impulse responses for a set of *estimated* parameters $\hat{\Theta}$. As the estimation problem is not linear in the parameters, these estimated parameters were adjusted using a non-linear optimisation approach due to Kelley (1999) – this method has the important advantage that upper and lower parameter bounds can be set. The optimisation criterion used was:

$$J(\hat{\Theta}) = \sum_{i=0}^{N} (\hat{g}_y(ih, \hat{\Theta}) - g_y(ih))^2 \tag{66}$$

$$+ (\hat{g}_u(ih, \hat{\Theta}) - g_u(ih))^2 \tag{67}$$

where $\hat{g}_y(t, \hat{\Theta})$ and $\hat{g}_u(t, \hat{\Theta})$ are the impulse responses generated from (62) and (63) using the parameters $\hat{\Theta}$ at time t and $h = 0.01$sec is the experimental sample interval. $N = 10000$ was used here. In the sequel

$$\hat{\Theta} = [\hat{q}_p \ \hat{q}_v \ \hat{t}_d]^T \tag{68}$$

As the responses are quite insensitive to q_o, $q_o = 20$ was fixed. As pointed out by a referee, this fact is probably related to loop-transfer recovery(LTR) (Maciejowski, 1989).

The results appear in Figure 3 which show that meaningful impulse responses can be derived from the raw data using the FSF approach and that these impulse responses can indeed be fitted with a parametrised predictive controller using the MBE approach. Similar results were found by using frequency-domain fitting.

4. CONCLUSION

The two step identification method has been successfully applied to data pertaining to human closed-loop control of an inverted pendulum. These initial results are encouraging, but more work is needed to draw scientific conclusions. In particular, the method will be applied to measurements taken on humans during standing and the impulse repose of controllers corresponding to alternative strategies derived and compared.

For example, an alternative control strategy, intermittent control, arises from physiological considerations (Craik, 1947; Neilson and Neilson, 2005) and again has received recent attention (Loram and Lakie, 2002*b*; Loram *et al.*, 2006). There have also been recent publications on intermittent control in the engineering literature (Ronco *et al.*, 1999; Gawthrop and Wang, 2006; Gawthrop and Wang, 2007). Intermittent control involves prediction and is therefore a natural extension of the predictive control discussed here; future work will embed intermittent control within the analysis and identification methods of this paper.

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