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Observer-based Fault Estimation for Networked Control Systems with Transfer Delays ⋆

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Abstract: In this paper, diagnosis of actuator/component faults for networked control systems (NCSs) with transfer delays is investigated. First, the linear NCSs with transfer delays are modelled by T-S discrete-time systems with input delays. Next, under certain conditions, a stable adaptive observer is designed for the purpose of fault diagnosis. An extension to a class of nonlinear systems is then made. Finally, a motor example is given to illustrate the efficiency of the proposed method.

1. INTRODUCTION

In networked control systems (NCSs), a controller and spatially distributed sensors/actuators are grouped into network nodes and communicate by exchanging packet-based messages via a network. NCSs have several advantages over the classical control systems, such as reduced installation and maintenance costs, and are thus of large practical interest. However, NCSs require novel control designs to account for networks presence in the closed loop. Modelling, analysis, and design of NCSs have received increasing attention in recent years, see Krtolica et al. (1994); Nešić et al. (2003); Nešić et al. (2004); Silva et al. (2007); Walsh et al. (2001); Walsh et al. (2002); Zhang et al. (2001); Zheng et al. (2006).

Fault can always lead to degradation of the system performance. Fault detection and diagnosis (FDD) and fault tolerant control (FTC) procedures are designed to guarantee that the system goal is still achieved in spite of the faults. Fruitful results can be found in several books Blanke et al. (2003); Chen et al. (1999); Gertler (1998) and many papers, e.g. Berdjag et al. (2006); Chowdhury et al. (2006); Nguang et al. (2007); Nguang et al. (2006) . FDD for NCSs have attracted much attention recently. For some representative works on fault detection (FD) of NCS, we refer the readers to Kambhampati et al. (2006); Sauter et al. (2006); Ye et al. (2004); Zhang et al. (2006); Mao et al. (2007) and the references therein. However, to the best of our knowledge, until now, few results have been reported about fault estimation (FE) for NCSs.

The challenges of model-based FE for NCSs are twofold: 1) there is a lack of appropriate model, especially for nonlinear NCSs. It is difficult for NCSs (in particular for nonlinear NCSs) fault estimation to find a model with sufficient accuracy, under the conditions of network-induced delays and packets loss; and 2) now, in most related work, the NCSs are modelled as discrete-time systems with delays. However, some control theories, such as Lyapunov stability analysis or adaptive observer design for discrete-time systems are not so matured as that for continuous-time systems, and using the techniques developed from continuous systems to deal with discrete systems is not a simple task.

Our objective in this paper is to propose an observer-based FE method for NCSs. The main contributions of this paper is extension the FE result of linear NCSs to a kind of nonlinear NCSs, on which few results are available so far.

The rest of this paper is organized as follows. System description and the modelling method for the NCSs are presented in Section 2. The adaptive observer based fault estimation is derived in Section 3. An extension to a class of nonlinear systems is made in Section 4. An application example is given in Section 5, followed by some concluding remarks in Section 6.

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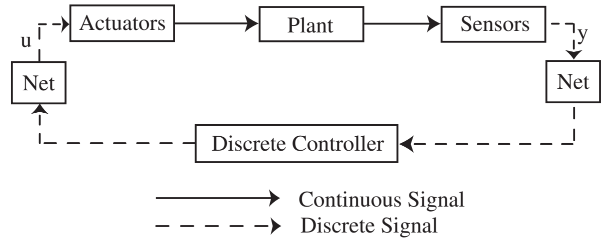

Fig. 1. The block of the networked control system with FTC

2. SYSTEM DESCRIPTION

Consider a NCS as shown in Fig. 1, the continuous-time, state-space model of the linear time-invariant plant dynamics can be described by the following standard form:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \tag{1}$$

$$y(t) = Cx(t) + Df(t) \tag{2}$$

where $x \in R^n$ denotes the state vector, $u \in R^m$ is the control input vector, $y \in R^r$ is the measurable output vector. The pair (A, C) is observable and $\text{rank}(D) = q$, $q \leq r$. It is assumed that the fault vector $f(t) \in R^q$ is norm bounded, i.e. $\| f(t) \| \leq f_0$, where $f_0 \geq 0$.

The sampling period of the NCS is T, sensors are time-driven, controller and actuators are event-driven. In NCSs, the sensors data packets reach the controller, and the controller data packets arrive at the actuators via network channel, whose load and limited communication bandwidth can cause transfer delays. The network-induced delays include the sensors-controller delay τ_{sc} and controller-actuators delay τ_{ca}. Then, the overall network delay, which is also the transfer delay, can be computed by $\tau = \tau_{sc}+\tau_{ca}$.

Assumption 1 (Zheng et al. (2006)). The transfer delay of the data packet, which is received by the actuator at the instant kT, is $\tau_k (\in \mathcal{N})$ periods and $\max(\tau_k) = n$.

Similarly as that in Zheng et al. (2006), considering the effect of delay τ and sampling period T, the above plant model is transformed into a T-S discrete-time model under the Assumption 1:

$$x(k+1) = \sum_{i=1}^{n} \mu_i(k) \Big\{ \bar{A}x(k) + \bar{B}u(k-i) + \bar{E}f(k) \Big\} \tag{3}$$

$$y(k) = Cx(k) + Df(k) \tag{4}$$

where $x(k) = x(kT)$, $y(k) = y(kT)$, $f(k) = f(kT)$, $\bar{A} = e^{AT}$, $\bar{B} = \int_T^{T+1} e^{At} Bdt$, $\bar{E} = \int_0^T e^{A(T-t)} Edt$. Furthermore, we can obtain that the pair $(\bar{A}, C)$ is observable and $\| f(k) \| \leq f_0$, with $f_0 \geq 0$. $\mu_i(k)$ is the membership function, representing the probability of $\tau_k = i$, i.e., $\mu_i(k) = Prob(\tau_k = i)$. It satisfies $\sum_{i=1}^{n} \mu_i(k) = 1$, $0 \leq \mu_i(k) \leq 1$, $\forall i = 1, 2, \ldots, n$. More details about this modelling method can be found in Zheng et al. (2006).

Remark 1 . We use the form in T-S model to describe the input signal as $\sum\limits_{i=1}^{n} \mu_i(k)\bar{B}u(k-i)$, which means in current time, the input could be one from the set $\{u(k-i), i = 1, \ldots, n\}$ and $\mu_i(k)$ represented the probability of $u(k-i)$.

3. AN ADAPTIVE DIAGNOSTIC OBSERVER DESIGN

The fault detection observer can be designed as follows:

$$\hat{x}(k+1) = \sum_{i=1}^{n} \mu_i(k) \Big\{ \bar{A}\hat{x}(k) + \bar{B}u(k-i) + K[\hat{y}(k) - y(k)] \Big\} \tag{5}$$

$$\hat{y}(k) = Cx(k) \tag{6}$$

where $\hat{x}(k) \in R^n$ is the observer state vector. K is selected such that $(\bar{A} - KC)$ is a stable matrix. If there is no fault, the estimation error will converge to zero. The proof of convergence of the observer error dynamics is omitted due to the page limit.

In order to assure the sensitivity of the residual, $r(k) = y(k) - \hat{y}(k)$, to the fault, the related transfer function $D + C[sI - (A - KC)]^{-1}E$ should be non-zero. Moreover, the residual evaluation function is selected as (Zhong et al. (2005)):

$$J(r) = \sum_{k=k_0}^{k=k_0+L} r^T(k)r(k)$$

where k_0 denotes the initial evaluation time instant. L denotes the evaluation time steps. Based on this, the occurrence of faults can be detected by

$$\left. \begin{array}{l} J(r) = 0, \ \textit{no fault occurs} \\ J(r) \neq 0, \ \textit{a fault has occurred} \end{array} \right\}$$

Prior to the design of an adaptive diagnostic observer, the following assumption is also made.

Assumption 2 . There exist positive definite matrices $P \in R^{n\times n}$, $Q \in R^{n\times n}$ and a matrix $K \in R^{n\times r}$ such that

$$2(\bar{A} - KC)^T P(\bar{A} - KC) - P + 3(D^\dagger C)^T D^\dagger C = -Q$$

where $D^\dagger$ is the left-inverse of the matrix D.

Remark 2 . The existence condition of the matrix $D^\dagger$ is that the dimension of the output should not be smaller than that of the fault, i. e. $q \leq r$.

To diagnose the actuator/component fault after its detection, the following observer is constructed

$$\hat{x}(k+1) = \sum_{i=1}^{n} \mu_i \Big\{ \bar{A}\hat{x}(k) + \bar{B}u(k-i) + \bar{E}\hat{f}(k) + K[y(k) - \hat{y}(k)] \Big\} \tag{7}$$

$$\hat{y}(k) = C\hat{x}(k) + D\hat{f}(k) \tag{8}$$

where $\hat{x}(k) \in R^n$ is the observer state vector and $\hat{f}(k)$ is an estimate of $f(k)$.

Denote

$$e_x(k) = x(k) - \hat{x}(k), \quad e_y(k) = y(k) - \hat{y}(k), \tag{9}$$

$$e_f(k) = f(k) - \hat{f}(k) \tag{10}$$

then it can be obtained that

$$e_x(k+1) = (\bar{A} - KC)e_x(k) + (\bar{E} - KD)e_f(k) \quad (11)$$

$$e_y(k) = Ce_x(k) + De_f(k) \quad (12)$$

The following theorem produces a convergent adaptive diagnostic algorithm for estimating the fault f.

Theorem 1. Under Assumption 2, the observer described by (7) and (8) and the following fault estimation algorithm

$$\hat{f}(k+1) = \Gamma D^\dagger e_y(k) - \Gamma\theta\hat{f}(k) \quad (13)$$

where $\Gamma = \Gamma^T > 0$ and θ is chosen such that

$$\lambda_{min}(Q) - \lambda_{max}[(D^\dagger C)^T \Gamma D^\dagger C] > 0$$

and

$$\lambda_{min}(\Gamma^{-1}) - \lambda_{max}[2(\bar{E} - KD)^T P(\bar{E} - KD) + (\theta+1)^2\Gamma + (\theta^2+1)\Gamma^2] > 0$$

can guarantee that the system (11) - (12) is stable.

Proof. Consider the following Lyapunov function

$$V(k) = e_x^T(k)Pe_x(k) + e_f^T(k)\Gamma^{-1}e_f(k) \quad (14)$$

From (13), we can obtain

$$\begin{aligned} e_f(k+1) &= f(k+1) - \hat{f}(k+1) \\ &= f(k+1) - \Gamma D^\dagger e_y(k) + \Gamma\theta\hat{f}(k) \\ &= f(k+1) + \Gamma\theta f(k) - \Gamma D^\dagger e_y(k) - \Gamma\theta e_f(k) \end{aligned}$$

Further, according to (11) - (12), its difference with respect to time is

$$\begin{aligned} &\Delta V(k+1) \\ &= V(k+1) - V(k) \\ &= e_x^T(k+1)Pe_x(k+1) - e_x^T(k)Pe_x(k) \\ &\quad + e_f^T(k+1)e_f(k+1) - e_f^T(k)e_f(k) \\ &= e_x^T(k)(\bar{A} - KC)^T P(\bar{A} - KC)e_x(k) \\ &\quad + 2e_x^T(k)(\bar{A} - KC)^T P(\bar{E} - KD)e_f(k) \\ &\quad + e_f^T(k)(\bar{E} - KD)^T P(\bar{E} - KD)e_f(k) - e_x^T(k)Pe_x(k) \\ &\quad + [f(k+1) + \Gamma\theta f(k)]^T\Gamma^{-1}[f(k+1) + \Gamma\theta f(k)] \\ &\quad + (Ce_x(k) + De_f(k))^T(D^\dagger)^T\Gamma D^\dagger(Ce_x(k) + De_f(k)) \\ &\quad + \theta^2 e_f^T(k)\Gamma e_f(k) - 2[f(k+1) + \Gamma\theta f(k)]^T\theta e_f(k) \\ &\quad - 2[f(k+1) + \Gamma\theta f(k)]^T D^\dagger(Ce_x(k) + De_f(k)) \\ &\quad + 2\theta e_f^T(k)\Gamma D^\dagger(Ce_x(k) + De_f(k)) \\ &\quad - e_f^T(k)\Gamma^{-1}e_f(k) \end{aligned} \quad (15)$$

It is easy to show that

$$2uMv \leq \frac{1}{\mu}u^T Mu + \mu v^T Mv, \quad u \in R^n, \quad v \in R^n \quad (16)$$

holds for any constant $\mu > 0$ and a positive definite matrix M.

According to Assumption 2 and from (15) - (16), one can further obtain that

$$\begin{aligned} &\Delta V(k+1) \\ &\leq e_x^T(k)[2(\bar{A} - KC)^T P(\bar{A} - KC) - P \\ &\quad + 3(D^\dagger C)^T D^\dagger C + (D^\dagger C)^T\Gamma D^\dagger C]e_x(k) \\ &\quad + e_f^T(k)\Big\{2(\bar{E} - KD)^T P(\bar{E} - KD) + (\theta+1)^2\Gamma \\ &\quad + (\theta^2+1)\Gamma^2 + (\frac{\theta}{\sigma_1} + \frac{\theta}{\sigma_2})I_{q\times q} - \Gamma^{-1}\Big\}e_f(k) \\ &\quad + [f(k+1) + \Gamma\theta f(k)]^T(\Gamma^{-1} + (\sigma_1 + \sigma_2 + 1)I_{q\times q}) \\ &\quad \times [f(k+1) + \Gamma\theta f(k)] \\ &\leq -c_1 \parallel e_x(k) \parallel^2 - c_2 \parallel e_f(k) \parallel^2 + c_3 f_0^2 \end{aligned} \quad (17)$$

where

$$\begin{aligned} c_1 &= \lambda_{min}(Q) - \lambda_{max}[(D^\dagger C)^T\Gamma D^\dagger C] > 0 \\ c_2 &= \lambda_{min}(\Gamma^{-1}) - (\frac{\theta}{\sigma_1} + \frac{\theta}{\sigma_2}) \\ &\quad - \lambda_{max}[2(\bar{E} - KD)^T P(\bar{E} - KD) + (\theta+1)^2\Gamma \\ &\quad + (\theta^2+1)\Gamma^2] > 0 \\ c_3 &= \lambda_{max}[(\Gamma^{-1} + (\sigma_1 + \sigma_2 + 1)I_{q\times q})(\theta\Gamma + I_{q\times q})^2] > 0 \end{aligned}$$

$\Gamma = \Gamma^T > 0$ is a weighting matrix. σ_1 and σ_2 are chosen such that $c_1 > 0$ and $c_2 > 0$.
On the other hand, from (14), we have

$$V(k) \leq \lambda_{max}(P) \parallel e_x(k) \parallel^2 + \lambda_{max}(\Gamma^{-1}) \parallel e_f(k) \parallel^2 \quad (18)$$

Substituting (18) into (17) yields

$$\Delta V(k) \leq -\alpha_1 V + c_3 f_0^2 \quad (19)$$

where $\alpha_1 = \frac{min(c_1,c_2)}{max[\lambda_{max}(P), \lambda_{max}(\Gamma^{-1})]}$. It can be seen that the following inequality holds for $(e_y(k), \hat{f}(k)) \in S_1$, with $S_1 = \left\{(e_y(k), \hat{f}(k)) \middle| \frac{\lambda_{min}(P)}{2\|C\|} \parallel e_y(k) \parallel^2 + \frac{\rho_1}{2} \parallel \hat{f}(k) \parallel^2 > \rho_1 f_0^2 + \frac{c_3 f_0^2}{\alpha_1}\right\}$, $\rho_1 = 1 - \frac{\|D\|^2\lambda_{min}(P)}{\|C\|}$

$$\begin{aligned} V(k) &\geq \lambda_{min}(P) \parallel e_x(k) \parallel^2 + \parallel e_f(k) \parallel^2 \\ &\geq \frac{\lambda_{min}(P)}{2 \parallel C \parallel} \parallel e_y(k) \parallel^2 + \rho_1 \parallel e_f(k) \parallel^2 \\ &\geq \frac{\lambda_{min}(P)}{2 \parallel C \parallel} \parallel e_y(k) \parallel^2 + \rho_1[\frac{1}{2} \parallel e_f(k) \parallel^2 - \parallel f_0 \parallel^2] \\ &\geq \frac{c_3 f_0^2}{\alpha_1} \end{aligned}$$

From the above inequality and (19), it can be seen that

$$\Delta V(k) < 0 \quad \text{for} \quad (e_y(k), \hat{f}(k)) \in S_1$$

As a result, the dynamic system described by (11) and (12) is stable. This completes the proof.

Corollary 1 . The pair $(e_y(k), \hat{f}(k))$ is uniformly bounded and converges to $\bar{S}_1$ exponentially at a rate greater than $e^{-\alpha_1 k}$, with $\bar{S}_1 = \left\{(e_y(k), \hat{f}(k)) \middle| \frac{\lambda_{min}(P)}{2\|C\|} \parallel e_y(k) \parallel^2 + \frac{\rho_1}{2} \parallel \hat{f}(k) \parallel^2 \leq \rho_1 f_0^2 + \frac{c_3 f_0^2}{\alpha_1}\right\}$.

Remark 3 . The estimation errors of the fault and the state are uniformly bounded and can be made small by choosing proper matrices Γ, Q and θ, see Jiang et al. (2002). Furthermore, the accurate estimation of the fault and the state can be obtained if the fault is constant (i.e. $\parallel f(k+1) - f(k) \parallel = 0$) after some transient period.

4. EXTENSION TO NONLINEAR SYSTEMS

In the above sections, we assume that the system is linear. However, many industrial systems are nonlinear in nature. Therefore, the development of nonlinear fault detection and diagnosis schemes plays a significant role in practical applications. In this section, we extend the fault diagnosis algorithms in above sections to a class of nonlinear NCSs.

4.1 System description

Consider a NCS as shown in Fig. 1, and the continuous-time, state-space model of the nonlinear time-invariant plant dynamics can be described as follows:

$$\dot{x}(t) = Ax(t) + g(t, x(t), u(t)) + Ef(t) \tag{20}$$
$$y(t) = Cx(t) + Df(t) \tag{21}$$

where $g(\cdot, \cdot, \cdot)$ is a nonlinear continuous function which is L_g Lipschitz with respect to its second arguments. It is assumed that $g(t, 0, 0) = 0$, $\forall t \in R$, other notations are the same as those in section 2.

Under Assumption 1 and Euler approximate method, similar to that in Section 2, system (20) - (21) can be discretized as follows:

$$\begin{aligned} x(k+1) &= x(k) + T\Big\{ Ax(t) \\ &\quad + \sum_{i=1}^{n} \mu_i(k) g(k, x(k), u(k-i)) + Ef(k) \Big\} \\ &= \sum_{i=1}^{n} \mu_i(k) \Big\{ \bar{A}x(k) + Tg(k, x(k), u(k-i)) + \bar{E}f(k) \Big\} \\ y(k) &= Cx(k) + Df(k) \end{aligned}$$

with $\bar{A} = I_{n\times n} + TA$, $\bar{E} = TE$.

Remark 4 . The order terms greater than 2 of the above Euler approximate method can be omitted in practice, since the sampling period T guarantees the accuracy of the modelling, see Nešić et al. (1999). From the theoretical point of view, this term could be considered as the modelling uncertainty with the bound determined from the corresponding physical vector. Many estimation schemes, e.g. observer, neural network are applicable on such uncertainty. Since the robust fault estimation is not the focus in this paper, the terms are omitted here.

4.2 Fault diagnosis

Similar to those in Section 3, the following observer is constructed

$$\hat{x}(k+1) = \sum_{i=1}^{n} \mu_i(k) \Big\{ \bar{A}\hat{x}(k) + Tg(k, \hat{x}(k), u(k-i)) + \bar{E}\hat{f}(k) + K[y(k) - \hat{y}(k)] \Big\} \tag{22}$$

$$\hat{y}(k) = C\hat{x}(k) + D\hat{f}(k) \tag{23}$$

where $\hat{x}(k) \in R^n$ is the observer state vector and $\hat{f}(k)$ is an estimate of $f(k)$.

Using the same notations of $e_x(k)$, $e_y(k)$ and $e_f(k)$ as in Section 3, the observation error and output error equations are given by

$$\begin{aligned} e_x(k+1) &= (\bar{A} - KC)e_x(k) + (\bar{E} - KD)e_f(k) \\ &\quad + \sum_{i=1}^{n} \mu_i(k) TG(e_x(k), u(k-i)) \end{aligned} \tag{24}$$
$$e_y(k) = Ce_x(k) + De_f(k) \tag{25}$$

where

$G(e_x(k), u(k-i)) \triangleq g(x(k), u(k-i)) - g(\hat{x}(k), u(k-i))$

Further $G(e_x(k), u(k-i)) \leq L_g e_x(k)$.

Assumption 3 . There exist positive definite matrices $P \in R^{n\times n}$, $Q \in R^{n\times n}$ and a matrix $K \in R^{n\times r}$ such that

$$\begin{aligned} &3(\bar{A} - KC)^T P(\bar{A} - KC) + 3T^2 L_g^2 P - P \\ &+3(D^{\dagger}C)^T D^{\dagger}C = -Q \end{aligned}$$

where $D^{\dagger}$ is the left-inverse of the matrix D.

Theorem 2. Under Assumption 3, the observer described by (22) and (23) and the following diagnostic algorithm

$$\hat{f}(k+1) = \Gamma D^{\dagger} e_y(k) - \Gamma\theta\hat{f}(k) \tag{26}$$

where $\Gamma = \Gamma^T > 0$ and θ is chosen that

$$\lambda_{min}(Q) - \lambda_{max}[(D^{\dagger}C)^T \Gamma D^{\dagger}C] > 0$$

and

$$\begin{aligned} &\lambda_{min}(\Gamma^{-1}) - \lambda_{max}[2(\bar{E} - KD)^T P(\bar{E} - KD) \\ &\qquad +(\theta+1)^2\Gamma + (\theta^2+1)\Gamma^2] > 0 \end{aligned}$$

can guarantee system (24) - (25) is stable.

Proof. Consider the following Lyapunov function

$$V(k) = e_x^T(k)Pe_x(k) + e_f^T(k)\Gamma^{-1}e_f(k) \tag{27}$$

According to (24), (25) and (26), its difference with respect to time is

$$\begin{aligned} &\Delta V(k+1) \\ &= V(k+1) - V(k) \\ &= e_x^T(k+1)Pe_x(k+1) - e_x^T(k)Pe_x(k) \\ &\quad + e_f^T(k+1)e_f(k+1) - e_f^T(k)e_f(k) \\ &= e_x^T(k)(\bar{A} - KC)^T P(\bar{A} - KC)e_x(k) - e_x^T(k)Pe_x(k) \\ &\quad + e_f^T(k)(\bar{E} - KD)^T P(\bar{E} - KD)e_f(k) \\ &\quad + T^2[\sum_{i=1}^{n} \mu_i G(e_x(k), u(k-i))]^T P \\ &\qquad \times[\sum_{i=1}^{n} \mu_i G(e_x(k), u(k-i))] \end{aligned}$$

$$
\begin{aligned}
&+2Te_x^T(k)(\bar{A}-KC)^T P(\sum_{i=1}^{n}\mu_i G(k,e_x(k),u(k-i)))\\
&+2e_x^T(k)(\bar{A}-KC)^T P(\bar{E}-KD)e_f(k)\\
&+2T(\sum_{i=1}^{n}\mu_i G(k,e_x(k),u(k-i)))^T P(\bar{E}-KD)e_f(k)\\
&+[f(k+1)+\Gamma\theta f(k)]^T\Gamma^{-1}[f(k+1)+\Gamma\theta f(k)]\\
&+(Ce_x(k)+De_f(k))^T(D^\dagger)^T\Gamma D^\dagger(Ce_x(k)+De_f(k))\\
&+\theta^2 e_f^T(k)\Gamma e_f(k)-2[f(k+1)+\Gamma\theta f(k)]^T\theta e_f(k)\\
&-2[f(k+1)+\Gamma\theta f(k)]^T D^\dagger(Ce_x(k)+De_f(k))\\
&+2\theta e_f^T(k)\Gamma D^\dagger(Ce_x(k)+De_f(k))-e_f^T(k)\Gamma^{-1}e_f(k)
\end{aligned}
$$

According to Assumption 3 and (16), one can further obtain that

$$\Delta V(k+1)\le -c_{11}\parallel e_x(k)\parallel^2 -c_{12}\parallel e_f(k)\parallel^2 +c_{13}f_0^2 \tag{28}$$

where

$$
\begin{aligned}
c_{11} &= \lambda_{min}(Q)-\lambda_{max}[(D^\dagger C)^T\Gamma D^\dagger C]>0\\
c_{12} &= \lambda_{min}(\Gamma^{-1})-(\frac{\theta}{\sigma_3}+\frac{\theta}{\sigma_4})\\
&\quad -\lambda_{max}[3(\bar{E}-KD)^T P(\bar{E}-KD)+(\theta+1)^2\Gamma\\
&\qquad +(\theta^2+1)\Gamma^2]>0\\
c_{13} &= \lambda_{max}[(\Gamma^{-1}+(\sigma_3+\sigma_4+1)I_{q\times q})(\theta\Gamma+I_{q\times q})^2]>0
\end{aligned}
$$

σ_3 and σ_4 are chosen such that $c_{12}>0$ and $c_{13}>0$.
On the other hand, from (27), we have

$$V(k)\le\lambda_{max}(P)\parallel e_x(k)\parallel^2 +\lambda_{max}(\Gamma^{-1})\parallel e_f(k)\parallel^2 \tag{29}$$

Substituting (29) into (28) yields

$$\Delta V(k)\le -\alpha_{12}V+c_{13}f_0^2 \tag{30}$$

where $\alpha_{11}=\frac{min(c_{11},c_{12})}{max[\lambda_{max}(P),\lambda_{max}(\Gamma^{-1})]}$. It can be seen that the following inequality holds for $(e_y(k),\hat{f}(k))\in S_1$, with $S_1=\left\{(e_y(k),\hat{f}(k))\middle|\frac{\lambda_{min}(P)}{2\|C\|}\parallel e_y(k)\parallel^2+\frac{\rho_1}{2}\parallel\hat{f}(k)\parallel^2>\rho_1 f_0^2+\frac{c_{13}f_0^2}{\alpha_1}\right\}$, $\rho_1=1-\frac{\|D\|^2\lambda_{min}(P)}{\|C\|}$

$$
\begin{aligned}
V(k) &\ge \lambda_{min}(P)\parallel e_x(k)\parallel^2+\parallel e_f(k)\parallel^2\\
&\ge \frac{\lambda_{min}(P)}{2\parallel C\parallel}\parallel e_y(k)\parallel^2+\rho_1\parallel e_f(k)\parallel^2\\
&\ge \frac{\lambda_{min}(P)}{2\parallel C\parallel}\parallel e_y(k)\parallel^2+\rho_1[\frac{1}{2}\parallel e_f(k)\parallel^2-\parallel f_0\parallel^2]\\
&\ge \frac{c_{13}f_0^2}{\alpha_{11}}
\end{aligned}
$$

From the above inequality and (30), it can be seen that

$$\Delta V(k)<0 \quad \text{for} \quad (e_y(k),\hat{f}(k))\in S_1$$

As a result, the dynamic system described by (24) and (25) is stable. This completes the proof.

Corollary 2 . The pair $(e_y(k),\hat{f}(k))$ is uniformly bounded and converges to $\bar{S}_1$ exponentially at a rate greater than $e^{-\alpha_{11}k}$, with $\bar{S}_1=\left\{(e_y(k),\hat{f}(k))\middle|\frac{\lambda_{min}(P)}{2\|C\|}\parallel e_y(k)\parallel^2+\frac{\rho_1}{2}\parallel\hat{f}(k)\parallel^2\le\rho_1 f_0^2+\frac{c_{13}f_0^2}{\alpha_1}\right\}$.

5. AN ILLUSTRATIVE EXAMPLE

One of the modes from a switched reluctance motor (SRM) system investigated in Spong et al. (1987) is employed to illustrate our approach. $x=[\theta_m,\omega_m]^T$ is the state, where θ_m and ω_m denote the angular position and velocity of the motor.

The simplified system model is expressed as follows:

$$
\begin{aligned}
\dot{\theta}_m &= \omega_m\\
\dot{\omega}_m &= -\frac{\kappa_e}{J_m}sin(\theta_m)-\frac{b}{J_m}\omega_m+\frac{c}{J_m}u
\end{aligned}
$$

where J_m denotes the inertia of the motor. $\kappa_e>0$ is the elasticity constant. u is the voltage applied to the motor, with b and c being the related constants.

The parameters are $J_m=0.935$, $\kappa_e=0.311$, $b=2.23$, $c=35.31$. We further have

$$
A=\begin{bmatrix}0 & 1\\ 0 & -2.385\end{bmatrix},\quad B=\begin{bmatrix}0\\ 37.765\end{bmatrix},
$$

$$
g(x)=\begin{bmatrix}0\\ -0.333\sin(x_2)\end{bmatrix},\quad C=[\,1\ \ 2\,]
$$

The actuator fault is considered with $E=[-1\quad -2]^T$, $D=1$, as follows:

$$
f(t)=\begin{cases}0 & 0s\le t<2s\\ \sin(4\pi t) & 2s\le t<10s\end{cases}
$$

Assume that the sampling time $T=0.01s$, $n=\max(\tau_k)=3$, $\mu_1=0.2571$, $\mu_2=0.4776$, $\mu_3=0.2653$. After discretizing, we obtain a T-S model: Rule $i(i=1,2,3)$: if τ_k is i, then the NCS model is

$$
\begin{aligned}
\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix} &= \begin{bmatrix}0 & 0.01\\ 0 & -0.02385\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}\\
&+\begin{bmatrix}0\\ -0.00333\sin(x_2)\end{bmatrix}+\begin{bmatrix}0\\ 37.765\end{bmatrix}u(k-i)\\
&+\begin{bmatrix}-0.01\\ -0.02\end{bmatrix}f(k)u(k)\\
y(k) &= [\,1\ \ 2\,]\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}+f(k)u(k)
\end{aligned}
$$

The matrix K is chosen as

$$
K=\begin{bmatrix}0.03\\ -0.018\end{bmatrix}
$$

Let's take $\Gamma=0.16$, $\theta=0.0281$. Fig. 2 shows the fault estimation performance, from which we can see that $\hat{f}(k)$ follows $f(k)$ rapidly with a very small overshoot.

6. CONCLUSION

In this paper, we discuss the fault estimation problem for NCSs with both linear and a kind of nonlinear plants.

The considered NCSs are modelled as T-S systems, which is suitable for adaptive observer-based fault estimation. Then, we use the Lyapunov function to prove that the proposed fault diagnosis method can make the estimation error converge to the given region. Simulation results of a motor system are given to verify the effectiveness of the proposed method.

Further, the purpose of the designed fault estimation method is to achieve the active fault-tolerant control. Ref. Nešić et al. (1999) has provided a sufficient condition for stabilization of NCSs via discrete-time approximations, which is used to model the NCSs. So the FE method proposed in this paper can be employed to reconfigure the controller to recover the system performance, that is also our future work.

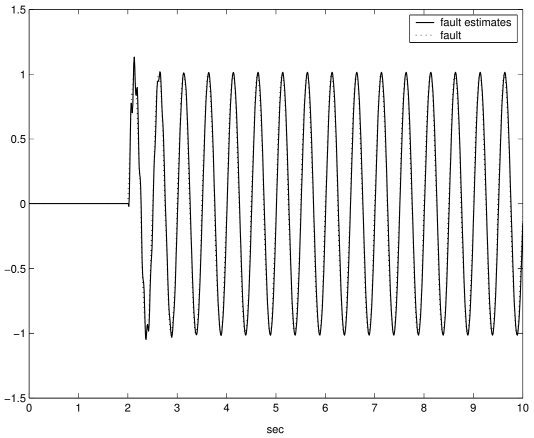

Fig. 2. Fault estimation

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