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Interpretability-accuracy improvement in a neuro-fuzzy ART based model of a DC motor ⋆

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Abstract:
The aim of this paper is to propose a general methodology applicable to any rule based fuzzy model generated by any precise or linguistic fuzzy algorithm to improve the linguistic-accuracy trade-off. Here, the neuro-fuzzy system FasArt (Fuzzy Adaptive System ART based) is used for its proven model capabilities, as shown in previous papers and works. If does, however, have the usual drawbacks, from the linguistic point of view, of most fuzzy modeling methods found in the scientific literature. A fuzzy model of a DC motor is generated by FasArt, whose performance is a good estimation of the motor's behavior, then this performance is improved by a better interpretability of the knowledge attained and stored by this fuzzy model. The main idea behind this approach is to find a fuzzy model with enough accuracy and an adequate capacity of explanation or interpretability of its data acquired knowledge. The modeling process can thus be seen as knowledge extraction in human or linguistic terms: from a numeric level (data) to a symbolic one (linguistic fuzzy rules).

Keywords: Fuzzy modeling; precise-linguistic modeling; fuzzy rules; interpretability; knowledge extraction.

1. INTRODUCTION

Nowadays, the application of Fuzzy Logic theory in technical and research areas is very common. In scientific literature, it is possible to find methodologies, algorithms and applications based on fuzzy logic theory, or using this theory in combination with other approaches, such as soft computing techniques: neural networks, genetic algorithms, etc.. Applications for control, modeling, pattern recognition, computer vision, signal processing, etc... are known and used. Fuzzy logic systems are applied to solve real world problems (Karray and De Silva (2004); Bonissoene et al. (1999)). The fuzzy approaches are usually implemented as fuzzy rule-based systems, so one of their most relevant components is the fuzzy rule set, in which the knowledge of the problem's solution is stored.

On the other hand, one "contradiction" appears corcerning the fuzzy nature of these rules and, in general, about the way in which the fuzzy logic theory has been applied and used in the scientific bibliography, focusing on fuzzy modeling and other connected techniques.

In accordance with the principles of fuzzy logic theory, the fuzzy rules must describe knowledge by linguistic variables that take values described by linguistic labels/concepts. In general, this rule set must have some properties that can be connected with its understanding and interpretation. When the rule set of most fuzzy logic systems reported in research papers are analyzed, it is very usual to verify that these desired fuzzy properties (understanding and interpretation) are weak performances of these rule sets. This situation does not deal with a concept of "interpretability" compatible with the fuzzy logic theory so, are these rule sets truly fuzzy or not?. Here, two points of view concerning the use of fuzzy logic arise: based on interpretability versus based on accuracy. The first is the "true-original" fuzzy approach, whereas the second uses fuzzy principles to generate a solution with high accuracy but while it loses some relevant fuzzy performances. This is the case of most of the data-driven fuzzy systems given for engineering problems.

This paper is focused on the balancing of both aspects, accuracy and interpretability, for modeling problems. The methodology proposed can be applied to different alternatives to those used here. For this goal, accuracy has not been the only criterion for generating the logic fuzzy system. In addition, interpretability and compactness have been taken into account when improving the model from

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data. A simpler, and very early version, of this work is in Sainz et al. (2007).

The paper is organized as follows: first, a brief description of alternative points of view for fuzzy modeling is given. Next, the methodology used in this paper is described. Then, a very brief description of the FarsArt neuro fuzzy system is carried out, the description of the DC motor plant is introduced and the main results obtained are discussed. Finally, the most interesting conclusions obtained from this work are set out.

2. FUZZY MODELING: ACCURACY VS. INTERPRETABILITY

Fuzzy modeling is an usual approach to develop black and grey box models. Taking into account the previous ideas concerning fuzzy rules, two well known modeling approaches based on fuzzy rules are common:

- Precise fuzzy modeling.
- Fuzzy linguistic modeling.

The differences between both approaches are based on the two contradictory concepts described above and the balance achieved (Setnes et al. (1998b); Casillas et al. (2003b)):

- Interpretability, or the capacity of the fuzzy model to express the behavior of the system in an understandable way. Here, other terms, such as consistency, compactness, transparency, etc., are involved and permit the implementation of this concept.
- Accuracy, or the capacity of the fuzzy model to faithfully represent the modeled system.

The first approach, precise fuzzy modeling, usual generates very good accurate models, but the interpretability of its fuzzy rules is very poor, which means that the knowledge contained into these rules is not accessible or understandable for an "expert" on the problem/system domain.

The main reason for this lack of interpretability is the way in which the modeling process is carried out. One only parameter is taken into account, so this process is only ruled by this parameter: the error between real and estimated system behaviors is usually used. If other points of view concerning interpretability aspects (such as compactness, consistency, etc.) are taken into account in this modeling process, then these other performances can be improved.

On the other hand, the linguistic approach permits models with good interpretability, but their accuracy is very low. This means that the fuzzy rules of the model are not accurate enough but can be "understood" by domain experts.

Both modeling approaches have drawbacks of either accuracy or interpretability. Therefore, one interesting goal is to achieve a good balance or compromise solution between accuracy and interpretability, obtaining a sufficiently accurate model with a good level of explanation (Figure 1). Thus, the research in this field is triying to find a solution to the poor performance of each case. Precise fuzzy modeling aims to obtain a better level of interpretability, improving this aspect in its fuzzy rule set (Casillas et al. (2003a); Setnes et al. (1998b)) in order to "explain" the information embedded in the system data, while the linguistic approach tries to achieve better model accuracy from its fuzzy rule set (Casillas et al. (2003b)).

Thus, if an adequate trade-off accuracy-interpretability was achieved, in addition to the advantages of generating a "classic" good model, the data-driven fuzzy modeling can be considered as a knowledge extraction method. This is able to describe the information structure embedded in the system data in understandable terms, which is very relevant in real-world problems.

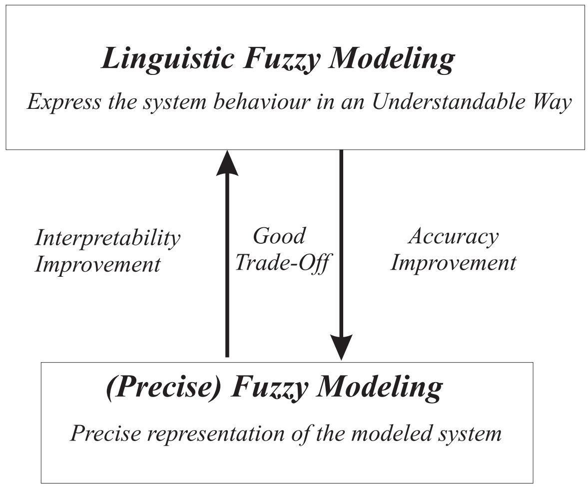

Fig. 1. Precise Fuzzy Modeling vs. Linguistic Fuzzy Modeling

In the scientific bibliography, this compromise is tackled from several points of view:

- Structure of the fuzzy rules,
- Simplification and reduction of fuzzy sets and rules,
- Interpretability constraints for tuning fuzzy rules,
- etc...

Two main approaches are used when rule reduction is involved (Casillas et al. (2003b); Setnes and Babuska (2001); Setnes et al. (1998a); Yen and Wang (1999)):

- Similarity-driven rule base simplification.
- Rule reduction with orthogonal transforms.

In this paper, the first approach, based on similarity, is used. Our method is based on two steps: in the first, a (precise) fuzzy model is generated taking advantage of a well-known method for fuzzy modeling found in the technical literature. There is no restriction on which algorithm can be used. The advantages of that are obvious: the algorithms already exist, they have been checked, generate precise models and there are a lot of alternatives... so new fuzzy model algorithms are not mandatory. In this paper an ART based neuro-fuzzy system has been used in this stage: FasArt (Fuzzy Adaptive ART based) (Cano Izquierdo et al. (2001)). This neuro fuzzy system has been applied to several real problems in previous works (Sainz et al. (2005, 2004)), and is an excellent precise fuzzy modeling instance to check the performance of this proposal, because it shows they usual accuracy advantages and weak interpretability of the precise fuzzy algorithms.

In the second step, the model obtained is improved, a posteriori, in order to achieve a new one with a better performance in interpretability aspects. In Taha and Ghosh (1999), some of these evaluation criteria corcerning rule interpretability are described:

- Granularity, details of the system's "decision-taking".
- Comprehensiveness: the amount of embedded knowledge captured by the rules.
- Compressibility: the number of rules and number of premises in each extracted rule.
- Transparency of the extracted rules: how well the decisions or conclusions are explained.
- Stability and robustness.
- Complexity and scalability: computational issues.

In Casillas et al. (2003b), another set of properties are described to achieve that goal:

- Coverage property: every element of the universe of discourse has to have at least no null membership value.
- Normality property: each membership function must fully match, at least, one value of the universe of discourse.
- Distinguishability property: each fuzzy set should have a clear definition of the universe of discourse and the associated linguistic term should have a clear meaning.

If a good level of interpretability is wanted, then the fuzzy rules, and their membership functions, fuzzy partitions, etc... generated defining the rule-based fuzzy model must reasonably show the above properties. If not, these elements, and therefore the model, should be improved to achieve a better performance.

In this paper, the interpretability is based on the completeness and distinguishability concepts, that permit a clear meaning to be assigned to each fuzzy rule and set (Mikut et al. (2005); Huang and Xing (2002); Chen and Linkens (2004); Jin and Sendhoff (2003); Jin (2000)). These two concepts can be expressed as follows:

$$\gamma_1 < Similarity(A_i, A_{i+1}) < \gamma_2$$

$$Similarity(A_i, A_{i+1}) = \frac{|A_i \cap A_{i+1}|}{|A_i \cup A_{i+1}|} = \\ = \frac{\sum_i^j [\mu_{a_i}(x_j) \wedge \mu_{a_{i+1}}(x_j)]}{\sum_i^j [\mu_{a_i}(x_j) \vee \mu_{a_{i+1}}(x_j)]} \tag{1}$$

if γ_1 is small but not zero, then the completeness is guaranteed. However, if γ_2 is sufficiently far from 1, then a good distinguishability is reached.

In this way, the incoherent and redundant rules also present in the model must be eliminated or, at least, reduced. Then, a better level of compactness and coherency, and therefore a better level of interpretability, will be achieved.

2.1 Neuro-Fuzzy system FasArt

The FasArt model (Cano Izquierdo et al. (2001); Sainz Palmero et al. (2000)) is a neuro fuzzy system based on the Adaptive Resonance Theory (ART): Fuzzy ARTMAP (Carpenter et al. (1992)).

FasArt introduces an equivalence between the activation function of each FasArt neuron and a membership function. In this way, FasArt is equivalent to a Mamdani fuzzy ruled-based system with: Fuzzification by single point, Inference by product, Defuzzification by average of fuzzy set centers. A full description of this model can be found in Cano Izquierdo et al. (2001); Sainz Palmero et al. (2000).

The FasArt system has been used in several previous works (Sainz et al. (2005, 2004)) for modeling, fault detection, pattern recognition, etc... with reasonable results when its accuracy as a fuzzy model is involved, but when other aspects, such as rule interpretability, are focused then the problems described in previous sections appears: proliferation of rules, of fuzzy sets, etc... so this system is an adequate instance for checking this proposal, taking advantage of the knowledge learnt and stored by FasArt for each problem involved.

3. METHODOLOGY

The methodology used in this paper is made up of two general steps, each of which can be adapted by the user:

1. Generation of a fuzzy model: using any of the algorithms described in the scientific bibliography for either precise fuzzy modeling or linguistic fuzzy modeling. This paper is focused on precise fuzzy modeling, so the neuro fuzzy system FasArt is used, but this stage is open to any fuzzy rule-based algorithm.
2. Improvement of the fuzzy model through its fuzzy rule set and membership functions. So a more compact and interpretable model is obtained, avoiding non-relevant, redundant, and incoherent rules, no complete fuzzy partitions, etc.... This task is based on the optimization of a cost function that includes any performance or aspect wanted for the model, each one balances by the user according to the desired relevance. This paper is based on accuracy and interpretability:

$$F(Accuracy,\ Interpretability) = \\ \lambda_1 * Accuracy + \lambda_2 * Interpretability \tag{2}$$

These two concepts have been implemented by the following criteria or indexes :

- Accuracy, the model error is used to check the model is accurate enough:

$$ECM = \frac{1}{N}\sum_{i=1}^{N}(y_i - y_i')^2 \tag{3}$$

- Interpretability, in this paper this concept has been made by:

- Number of rules, a low rule number is desired for a better interpretability level of the rule set.
- Similarity amongst rules, based on Eq. (1) this index must be minimized to improve rule distinguishability. So, when the similarity of the rule set is low, its distinguishability will be easier and better.

  There are two alternatives to measure this index:

  (a) Averaged similarity value of the rule set.

$$Average(Similarity(R_i, R_j)) \\ \forall R_i, R_j \in FuzzyRuleSet \tag{4}$$

(b) Threshold values, number of times that values are achieved in the rule set.

$$\frac{Card(Similarity(R_i, R_j) \geq \beta_{Threshold})}{(RuleNumber - 1)!} \quad \forall R_i, R_j \in FuzzyRuleSet \tag{5}$$

- Redundant rules, these must be avoided, so their number have to be minimized. Then, interpretability will be more feasible and accuracy may even be improved. This factor is calculated taking into account the $Similarity(Rule_i, Rule_j)$.
- Incoherent rules, idem to previous.
- Coverage of the fuzzy partitions, to achieve complete fuzzy partition is desirable.

$$NoCompletePartition = = \frac{Card(NoCompleteFuzzyPartition)}{NumberFuzzyPartitions} \tag{6}$$

Finally, the resulting function is:

$$\begin{aligned} &F(Accuracy, Interpretability) = \lambda_1 * Error \\ &+\lambda_2 * RuleNumber \\ &+\lambda_3 * RuleSetSimilarity \\ &+\lambda_4 * IncoherentRuleNumber \\ &+\lambda_5 * RedundatRuleNumber \\ &+\lambda_6 * NoCompletePartition \end{aligned} \tag{7}$$

Similar to step 1 of this methodology, step 2 is open to eliminating some factors or adding new indexes in order to set up the cost function according to the desired performance for the fuzzy model.

Here, the genetic algorithm based on GAOT (GAOT) is used to carry out this step, but other optimization techniques could be employed.

This approach is more open, simple and computer economic than other "global" approaches (i.e. fuzzy genetic approach) that define new algorithms to generate the desired fuzzy model, most of them are only focused on TSK systems. Here, advantages of existing and checked fuzzy modeling algorithms are taken to improve their original performances, without restrictions about the fuzzy algorithm employed, or the performance indexes their balance.

4. LABORATORY PLANT

The work described in this paper uses a well-known laboratory plant: Amira DR-300 equipment as seen in Figure 2. The technical specifications of this system can be seen in $http://www.amira.de/dr300\_engl.html$. Here this plant has been used, but other plants or problems could have been chosen for testing this proposal. There are not restrictions.

5. EXPERIMENTS AND RESULTS

This system has two coupled DC motors to work as motor and load respectively. To be described by the fuzzy rule-based models, this DC motor was excited by several types of input behaviors. Three input/output variable sets were acquired containing 25 secs each of the motor operation.

- Input variables: Voltage (U), ΔU and Current (I_{arm}).
- Output variable: Angular speed (ω).

These variables were chosen for their simplicity and uniformity with some other theoretical models used in previous work. Here, the U input signal is: $U(s) = \frac{(s+2)}{(s^2+s+1)} * Step * Ramp$, ΔU and I_{arm} (Fig. 3), that is used for one of the usual motor tasks.

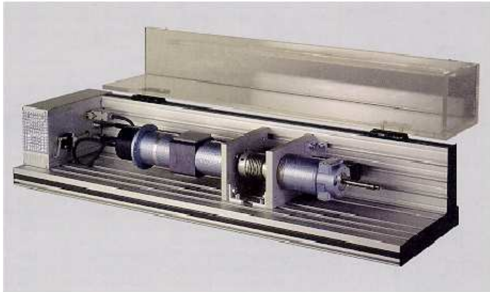

Fig. 2. Laboratory Plant: Motor Amira DR-300

	Model 1	Model 2
Rule Number	16	505
Error	0.0022	0.0045
Similarity (Eq. 4)	0	0.0139
Similarity (Eq. 5)	0.3142	0.3726
Redundancy (Eq. 4)	0	0.0078
Redundancy (Eq. 5)	0	0.0070
Inconsistency	0	0
Coverage	Yes	Yes

Table 1. Performance of FasArt models.

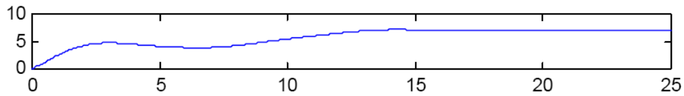

Fig. 3. Input variable: Voltage (U)

The experiments try to obtain fuzzy models with an adequate trade-off between accuracy and interpretability, in accordance with criteria described above. Cross validation has been used for these experiments.

Step 1: Based on FasArt system, two models were generated, each one with different fuzzy complexities and fuzzy performances:

- Model 1, FasArt parameters: $\rho_A = \rho_B = 0.9$ $\gamma_A = \gamma_B = 11$.
- Model 2, FasArt parameters: $\rho_A = \rho_B = 1$ $\gamma_A = \gamma_B = 11$.

Table 1 shows the performances of both original FasArt models. In model 1: the number of rules is low, the error is low, the similarity amongst rules could be better, there is no redundancy and the fuzzy partitions are complete. But in model 2, the number of rules is very high, the error is low, the similarity and redundancy could be improved, and the fuzzy partitions are complete.

Step 2: The improvement is carried out by a genetic optimization involving the fuzzy rule set of the previous models. Five initial populations were used for each alternative of similarity, incoherence and redundancy formulation (Eqs. 4 and 5) and the usual genetic operators were used. For each initial population and similarity-redundancy alternative, the improvement experiment is done three times. The population individuals are encoded by Gray code containing one fuzzy logic system with

	F_1	F_2	F_3	F_4	F_5
Error	0.0024	0.0025	0.0023	0.0024	0.0022
Rule Number	9	8	9	9	11
Similarity		0.3044	0.3093	0.3006	0.2973
Redundancy			0	0	0
Incoherency				0	0
Coverage					yes

Table 2. Improved Fuzzy Model 1 (Similarity based on Eq. 4)

	F_1	F_2	F_3	F_4	F_5
Error	0.0038	0.0042	0.0043	0.0042	0.0042
Rule Number	72	111	126	130	123
Similarity		0.0038	0.0024	0.0024	0.0029
Redundancy			0.0010	0.0008	0.0007
Incoherency				0	0
Coverage					yes

Table 3. Improved of Fuzzy Model 2 (Similarity based on Eq. 5 and $\beta_{Threshold} = 0.8$)

	F_1	F_2	F_3	F_4	F_5
Error	0.0038	0.0038	0.0041	0.0042	0.0043
Rule Number	72	81	107	89	100
Similarity		0.3233	0.3333	0.3255	0.3308
Redundancy			0.0006	0.0005	0.0006
Incoherency				0	0
Coverage					yes

Table 4. Improved Fuzzy Model 2 (Similarity based on Eq. 4)

variable rule number. Each initial population contains $\sqrt(RuleNumber)$ individuals, including at least one individual similar to the original fuzzy system.

On the other hand, in order to avoid convergence problems when the fuzzy system to be encoded is too large, each individual is made up of "n" fuzzy sub-systems each containing $\frac{size}{n}$ of the original system. The evolution of each one of them is independent.

An incremental process was used during the experiments to define the cost function to be used in each case. Each time, one of the interpretability indexes from section 2 was added to test its influence and improve the fuzzy performance of the fuzzy model:

$$\begin{aligned} F_i(Accuracy, Interpretability) = \quad & \lambda_1 * Error + \\ + \sum_{j=2}^{i+1} \lambda_j * InterpretabilityIndex & \end{aligned} \tag{8}$$

In Eq. 8, every factor has been considered with the same relevancy: $\lambda_j = 1$. Finally, 60 experiments were done for each F_j.

In Tables 2, 3 and 4, the best results for each case are shown, considering the several alternatives for the Similarity parameter (Eqs. 4 and 5).

Table 2 shows the results for the FasArt Model 1. This step has generated a fuzzy model with a better accuracy-interpretability balance: the number of rules is lower (16→9), the level of similarity has been improved (0.31→0.30) and the fuzzy partitions obtained from the new fuzzy rule set are still complete. The cost of this interpretability improvement has been a slightly higher error (0.0022→0.0024). In this case, where the original model is compact enough (16 rules), its improvement has been possible with a reasonable accuracy cost, while is kept very low. If Eq. 5 were used only the rule number could be improved, reducing this number to 9.

Tables 3 and 4 show the results for the more complex FasArt Model 2. In this case, the improvement obtained is more relevant than for Model 1: the number of rules has been greatly reduced (505 →72-130), the similarity has been improved relevantly (0.0139→0.0024-0.0038) (0.37→0.33 - 0.32), and the rule redundancy has been reduced (0.0078→0.007-0.0010) (0.0070→0.0006), keeping full coverage on the fuzzy partitions. In this case it should be noticed that the model error has also been reduced ($0.0045 \leftarrow 0.0038 - 0.0043$), so the improvement achieved has not only concerned linguistic aspects, but the accuracy of the fuzzy model obtained can also be improved.

6. CONCLUSIONS

This paper introduces a 2-step approach to obtain rule-based fuzzy models showing well balanced performances from the points of view of accuracy and interpretability. The goal is to achieve a good trade-off, or compromise solution, between both aspects (precise vs. linguist fuzzy performances). In this way, not only a sufficiently accurate model generated, as is usual in engineering applications, but also fuzzy models with more interpretable fuzzy rules and sets. So these models are more easily understood. The modeling with these performances could be considered as a knowledge extraction process, from a numeric level (data) to a symbolic one (fuzzy "linguist" rules).

The method proposed is made up of two steps:

- **Step 1:**, a rule-based fuzzy model is generated, using any fuzzy modeling algorithm.
- **Step 2:**, an improvement of the previous fuzzy system is done by optimization, taking as cost function: $F(Accuracy, Interpretability) = \lambda_1 * Accuracy + \lambda_2 * Interpretability$.
  In this function, every performance desired for the logic fuzzy model is included, each with its own relevancy implemented by user tuned weight.

The advantages of this proposal are: simplicity, generality and economy. This is an open approach that uses, and takes advantage of, any existing rule-based fuzzy modeling algorithm from scientific literature.

On the other hand, step 2 is open to setting up the improvement of the fuzzy model from any point of view, the relevance of each one being tuned by the user, to obtain the desired performance for the fuzzy model. This method can be applied to Mamdani or TSK systems without restrictions, while in scientific literature, it is usual in applications for TSK systems only.

In this paper, this approach has been applied to the modeling of a DC motor based on the neuro-fuzzy system FasArt, that introduces the most usual problems described in section 2 from the linguistic-interpretability point of view. The improvement criteria used are described in section 3 and function 7.

The results obtained have shown an improvement of the two initial models: in the first model, even though it is a compact model, it has been possible to reduce the number of rules by up to 43%, and the similarity by 3.23%. On the other hand the accuracy was reduced by 9%, but this cost is not very important, taking into account its magnitude in this case. In the second model, very much complex, the number of rules has been reduced by up to 80.42%, the similarity by up to 73.19%, the redundancy by up to 8.62%, providing improved accuracy. The error has been reduce by 8.7%. In both cases the fuzzy partitions obtained are complete. In this case, both aspects or points of view, accuracy and interpretability, have been improved simultaneously.

These results show that the approach introduced in this paper permits fuzzy models to be obtained with a better balance between accuracy and interpretability, simultaneously improving the simplicity of the model which is another interpretability factor. This better balance, confirmed by the results, does not necessarily mean worse accuracy for the fuzzy model. It is possible to improve the accuracy and the interpretability of the fuzzy model jointly.

Future work will focus on the application of this approach to industrial processes, such as in Sainz et al. (2005, 2004) to obtain fuzzy models with an adequate explanation capacity of the process involved.

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