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LMI based output-feedback controllers: γ-optimal versus linear quadratic ⋆

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Abstract: Solution to the linear quadratic control problem is given in the class of linear dynamic output-feedback full order controllers. Necessary and sufficient conditions for existence of such an optimal controller are stated in terms of linear matrix inequalities provided that initial conditions for controller states to be zero. It is shown that parameters of the optimal controller depend on an initial plant state. As an alternative we introduce γ-optimal controller which minimizes the maximal ratio of the performance index and square of the norm of the initial plant state. Numerical comparison for two kinds of these controllers is presented for inverted and double inverted pendulums.

1. INTRODUCTION

The classical deterministic linear quadratic (LQ) control problem is well known (see Kwakernaak and Sivan (1972)) to be solvable in terms of Riccati equations only when plant state is measurable. The essential step for solving the LQ output-feedback problem was connected with presentation of the LQ performance as H_2-norm of the system transfer matrix (see Doyle et al. (1989), Scherer et al. (1997)). The latter paper provides a result that allows to step from the performance analysis conditions formulated in terms of matrix inequalities to the corresponding linear matrix inequalities (LMIs) for H_∞ and H_2 synthesis problems. This is achieved by a nonlinear bijective transformation of the controller parameters. Another approach to LMI based synthesizing H_∞-controllers utilizes so called elimination lemma to convert performance analysis conditions into LMIs with respect to Lyapunov function matrix and controller parameters separately (see Gahinet and Apkarian (1994)).

Iwasaki et al. (1994) used H_2-presentation to formulate a suboptimal LQ control problem in terms of linear matrix inequalities and synthesized a stabilizing static output-feedback controller which can guarantee a specified level of the LQ performance for all initial conditions of the plant. Such a suboptimal controller was shown to exist if and only if there exists a positive definite matrix satisfying two LMIs, while its inverse matrix satisfies another LMI. The set of these matrices satisfying the LMIs is not convex. That is the main difficulty for solving LQ and other control problems for static or reduced order output-feedback controllers. Many computational algorithms have been developed to overcome this difficulty (see, for example, Iwasaki and Skelton (1995), El Ghaoui et al. (1997), Balandin and Kogan (2004), He and Wang (2006)).

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The present paper deals with synthesizing full order LQ output-feedback controllers. It turns out that this problem can be formulated in terms of LMIs as a convex optimization problem provided that the initial state of the plant is known and that the initial state of the dynamic controller is zero. Necessary and sufficient conditions are established for existence of this optimal controller. It is shown that its parameters depend on the initial state of the plant.

Such a controller can be viewed as an ideal one because the initial state of the plant is not actually available for measurement. As an alternative we introduce γ-optimal output-feedback full order controller which minimizes the maximal ratio of the performance index and square of the norm of the initial plant state. Parameters of this controller do not depend on the initial state of the plant. The problem of γ-optimal control is very close to one stated by Iwasaki et al. (1994). In contrast to last paper, we have a convex optimization problem and can evaluate performance losses of this worst case controller compared with the ideal one. Some numerical experiments with inverted and double inverted pendulums demonstrate the performance degradation of γ-optimal controller with respect to LQ (ideal) controller.

Note that similar ideas emerged in the recent work of Köroğlu and Scherer (2008) devoted to the problem of generalized asymptotic regulation with suboptimal transient response.

2. LQ OUTPUT-FEEDBACK CONTROLLERS

Consider the linear quadratic optimal control problem for the system

$$\begin{aligned} \dot{x} &= Ax + Bu \ , \quad x(0) = x_0 \ , \\ z &= C_1 x + Du \ , \\ y &= C_2 x \end{aligned} \tag{1}$$

in the class of full order dynamic output-feedback controllers

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r y \ , \quad x_r(0) = x_0^r \ , \\ u &= C_r x_r + D_r y \end{aligned} \tag{2}$$

providing asymptotic stability for the closed-loop system and minimizing the quadratic cost

$$J(\Theta) = \int_0^\infty |z|^2 \, dt \ , \tag{3}$$

where $x \in R^{n_x}$ is the plant state, $u \in R^{n_u}$ is the control input, $y \in R^{n_y}$ is the measurable output, $z \in R^{n_z}$ is the controlled output, $x_r \in R^{n_x}$ is the controller state, and

$$\Theta = \begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}$$

are controller parameters to be found.

The equation of asymptotically stable closed-loop system (1), (2) is of the form

$$\begin{aligned} \dot{x}_c &= A_c x_c \ , \quad x_c(0) = \bar{x}_0 \ , \\ z &= C_c x_c \ , \end{aligned} \tag{4}$$

where $x_c = \mathrm{col}\,(x, x_r)$,

$$A_c = \begin{pmatrix} A + BD_rC_2 & BC_r \\ B_rC_2 & A_r \end{pmatrix} \ , \quad \bar{x}_0 = \begin{pmatrix} x_0 \\ x_0^r \end{pmatrix} \ , \tag{5}$$

$$C_c = (C_1 + DD_rC_2 \quad DC_r) \ .$$

Since $z(t) = C_c e^{A_c t} \bar{x}_0$, then

$$J(\Theta) = \int_0^\infty \bar{x}_0^{\mathrm{T}} e^{A_c^{\mathrm{T}} t} C_c^{\mathrm{T}} C_c e^{A_c t} \bar{x}_0 \, dt \tag{6}$$

or

$$J(\Theta) = \bar{x}_0^{\mathrm{T}} X_0 \bar{x}_0 \ , \tag{7}$$

where

$$X_0 = \int_0^\infty e^{A_c^{\mathrm{T}} t} C_c^{\mathrm{T}} C_c e^{A_c t} \, dt$$

is the solution of the Lyapunov equation

$$A_c^{\mathrm{T}} X + X A_c + C_c^{\mathrm{T}} C_c = 0 \ . \tag{8}$$

Thus, the problem is reduced to minimization of (7) subject to constraint (8) which is a nonlinear matrix equation with respect to unknown variables X and Θ.

Observe that

$$J(\Theta) = \gamma^2 |\bar{x}_0|^2 \ ,$$

and reformulate this problem as follows: given initial state of the closed-loop system $\bar{x}_0 \neq 0$, find

$$\gamma_* = \min\{\gamma \geq 0 : \exists \Theta \quad J(\Theta) \leq \gamma^2 |\bar{x}_0|^2\} \ . \tag{9}$$

Matrix Θ corresponding to γ_* will define parameters of LQ output-feedback controller.

In what follows, instead of this problem we will consider the suboptimal LQ output-feedback problem: given $\bar{x}_0 \neq 0$, find

$$\gamma_* = \inf\{\gamma > 0 : \exists \Theta \quad J(\Theta) < \gamma^2 |\bar{x}_0|^2\} \ . \tag{10}$$

Matrix Θ of the suboptimal LQ controller is defined by

$$J(\Theta) < (\gamma_*^2 + \varepsilon)|\bar{x}_0|^2$$

for arbitrary small $\varepsilon > 0$. In fact, parameters of LQ and suboptimal LQ controllers will differ indefinitely small, so in the sequel we will call both controllers LQ optimal.

To solve the problem (10) we will show that inequality

$$J(\Theta) < \gamma^2 |\bar{x}_0|^2 \tag{11}$$

for given $\gamma > \gamma_*$ can be expressed in terms of LMIs.

Theorem 1. For the system (4) and the cost (3), inequality (11) holds if and only if there exists matrix $Y = Y^{\mathrm{T}} > 0$ such that

$$\begin{pmatrix} A_c Y + Y A_c^{\mathrm{T}} & Y C_c^{\mathrm{T}} \\ C_c Y & -I \end{pmatrix} < 0, \begin{pmatrix} Y & \bar{x}_0 \\ \bar{x}_0^T & \gamma^2 |\bar{x}_0|^2 \end{pmatrix} > 0. \tag{12}$$

Proof. Let the inequality (11) hold, then $\bar{x}_0^{\mathrm{T}} X_0 \bar{x}_0 < \gamma^2 |\bar{x}_0|^2$, where X_0 is the solution of the matrix equation (8). Consider the equation

$$A_c^{\mathrm{T}} X + X A_c + C_c^{\mathrm{T}} C_c + \varepsilon^2 I = 0$$

that has an unique solution $X > X_0$ and choose the parameter ε so that $\bar{x}_0^{\mathrm{T}} X_0 \bar{x}_0 < \bar{x}_0^{\mathrm{T}} X \bar{x}_0 < \gamma^2 |\bar{x}_0|^2$. Thus, we have

$$A_c^{\mathrm{T}} X + X A_c + C_c^{\mathrm{T}} C_c < 0 \ , \quad \bar{x}_0^{\mathrm{T}} X \bar{x}_0 < \gamma^2 |\bar{x}_0|^2 \ . \tag{13}$$

By multiplying the first inequality of (13) with $Y = X^{-1}$ on the left and on the right, we get

$$Y A_c^{\mathrm{T}} + A_c Y + Y C_c^{\mathrm{T}} C_c Y < 0 \ , \quad \bar{x}_0^{\mathrm{T}} Y^{-1} \bar{x}_0 < \gamma^2 |\bar{x}_0|^2 \ .$$

Finally, taking into account Schur lemma we arrive at (12).

Now, let inequalities (12) hold. Then inequalities (13) with $X = Y^{-1}$ hold as well and, hence, $J(\Theta) = x_0^{\mathrm{T}} X_0 x_0 < x_0^{\mathrm{T}} X x_0 < \gamma^2 |x_0|^2$. This completes the proof.

Further, let us present the matrices of the closed-loop system in the form

$$A_c = A_0 + \mathcal{B} \Theta \mathcal{C} \ , \quad C_c = C_0 + \mathcal{D} \Theta \mathcal{C} \ ,$$

where

$$A_0 = \begin{pmatrix} A & 0_{n_x \times n_x} \\ 0_{n_x \times n_x} & 0_{n_x \times n_x} \end{pmatrix}, \mathcal{B} = \begin{pmatrix} 0_{n_x \times n_x} & B \\ I_{n_x} & 0_{n_x \times n_u} \end{pmatrix},$$

$$\mathcal{C} = \begin{pmatrix} 0_{n_x \times n_x} & I_{n_x} \\ C_2 & 0_{n_y \times n_x} \end{pmatrix}, C_0 = (C_1 \quad 0_{n_z \times n_x}), \tag{14}$$

$$\mathcal{D} = (0_{n_z \times n_x} \quad D).$$

By inserting these expressions into the first inequality of (12) we present it in the form

$$\Psi + P^{\mathrm{T}} \Theta^{\mathrm{T}} Q + Q^{\mathrm{T}} \Theta P < 0 \ , \tag{15}$$

where

$$\Psi = \begin{pmatrix} A_0 Y + Y A_0^{\mathrm{T}} & Y C_0^{\mathrm{T}} \\ C_0 Y & -I \end{pmatrix} \ , \tag{16}$$

$$P = (\mathcal{C} Y \quad 0) \ , \quad Q = (\mathcal{B}^{\mathrm{T}} \quad \mathcal{D}^{\mathrm{T}}) \ .$$

Then by elimination lemma (see, for instance, Gahinet and Apkarian (1994)), inequality (15) holds for some Θ if and only if

$$W_P^{\mathrm{T}} \begin{pmatrix} A_0Y + YA_0^{\mathrm{T}} & YC_0^{\mathrm{T}} \\ C_0Y & -I \end{pmatrix} W_P < 0 \ , \qquad W_Q^{\mathrm{T}} \begin{pmatrix} A_0Y + YA_0^{\mathrm{T}} & YC_0^{\mathrm{T}} \\ C_0Y & -I \end{pmatrix} W_Q < 0 \ , \tag{17}$$

where W_P and W_Q denote any bases of the null spaces of the matrices P and Q, respectively. Observe that

$$P = (\mathcal{C}Y \quad 0) = G \begin{pmatrix} Y & 0 \\ 0 & I \end{pmatrix} \ , \quad G = (\mathcal{C} \quad 0) \ .$$

Hence

$$W_P = \begin{pmatrix} Y^{-1} & 0 \\ 0 & I \end{pmatrix} W_G \ ,$$

where W_G denotes any basis of the null space of the matrix G. Consequently, the first inequality of (12) is equivalent to following two LMIs

$$W_G^{\mathrm{T}} \begin{pmatrix} A_0^TX + XA_0 & C_0^{\mathrm{T}} \\ C_0 & -I \end{pmatrix} W_G < 0 \ , \qquad W_Q^{\mathrm{T}} \begin{pmatrix} A_0Y + YA_0^{\mathrm{T}} & YC_0^{\mathrm{T}} \\ C_0Y & -I \end{pmatrix} W_Q < 0 \ , \tag{18}$$

where $X = Y^{-1}$.

To express (18) in terms of the plant parameters, partition X and Y as

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{pmatrix} \ , \quad Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{pmatrix} \ .$$

Meanwhile, since

$$G = \begin{pmatrix} 0_{n_x \times n_x} & I_{n_x} & 0_{n_x \times n_z} \\ C_2 & 0_{n_y \times n_x} & 0_{n_y \times n_z} \end{pmatrix} \ ,$$

$$Q = \begin{pmatrix} 0_{n_x \times n_x} & I_{n_x} & 0_{n_x \times n_z} \\ B^{\mathrm{T}} & 0_{n_u \times n_x} & D^{\mathrm{T}} \end{pmatrix} \ ,$$

bases of null spaces of these matrices will be of the form

$$W_G = \begin{pmatrix} W_{C_2} & 0 \\ 0 & 0 \\ 0 & I \end{pmatrix} \ , \quad W_Q = \begin{pmatrix} W_Q^{(1)} \\ 0 \\ W_Q^{(2)} \end{pmatrix} \ ,$$

where $((W_Q^{(1)})^T \ (W_Q^{(2)})^T)^T$ is any basis of the null space of $(B^T \ D^T)$. This reduces (18) to

$$W_G^{\mathrm{T}} \begin{pmatrix} A^{\mathrm{T}}X_{11} + X_{11}A & A^{\mathrm{T}}X_{12} & C_1^{\mathrm{T}} \\ \star & 0 & 0 \\ \star & \star & -I \end{pmatrix} W_G < 0 \ ,$$

$$W_Q^{\mathrm{T}} \begin{pmatrix} Y_{11}A^{\mathrm{T}} + AY_{11} & AY_{12} & Y_{11}C_1^{\mathrm{T}} \\ \star & 0 & Y_{12}^{\mathrm{T}}C_1^{\mathrm{T}} \\ \star & \star & -I \end{pmatrix} W_Q < 0 \ ,$$

where $\star$ denotes the corresponding block of the symmetrical matrix. Observe that the second rows of W_G and W_Q are identically zero, these conditions are reduced to

$$M^{\mathrm{T}} \begin{pmatrix} A^{\mathrm{T}}X_{11} + X_{11}A & C_1^{\mathrm{T}} \\ C_1 & -I \end{pmatrix} M < 0 \ , \qquad N^{\mathrm{T}} \begin{pmatrix} Y_{11}A^{\mathrm{T}} + AY_{11} & Y_{11}C_1^{\mathrm{T}} \\ C_1Y_{11} & -I \end{pmatrix} N < 0 \ , \tag{19}$$

where M and N are matrices whose columns form bases of the null spaces of $(C_2 \ 0)$ and $(B^{\mathrm{T}} \ D^{\mathrm{T}})$, respectively.

Furthermore, according to Frobenius formula, $Y = X^{-1}$ implies

$$Y_{11} = (X_{11} - X_{12}X_{22}^{-1}X_{12}^T)^{-1} \tag{20}$$

which shows there exist reciprocal matrices $X > 0$, $Y > 0$ with given blocks $X_{11} = X_{11}^{\mathrm{T}} > 0$, $Y_{11} = Y_{11}^{\mathrm{T}} > 0$ if and only if $X_{11} - Y_{11}^{-1} \geq 0$, i.e.

$$\begin{pmatrix} X_{11} & I \\ I & Y_{11} \end{pmatrix} \geq 0 \ . \tag{21}$$

In the case of strict inequality (21), blocks Y_{12} and Y_{22} of the corresponding matrix Y can be chosen, for example, as it follows from formula

$$X_{11} = (Y_{11} - Y_{12}Y_{22}^{-1}Y_{12}^T)^{-1} \ ,$$

in the form

$$Y_{12} = Y_{22} = Y_{11} - X_{11}^{-1} \ . \tag{22}$$

Second inequality in (12) in view of Schur lemma is equivalent to

$$\gamma^2(|x_0|^2 + |x_0^r|^2) - \begin{pmatrix} x_0 \\ x_0^r \end{pmatrix}^T \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0^r \end{pmatrix} > 0 \ . \tag{23}$$

Thus, to solve the problem (10), in accordance with Theorem 1 one should find a minimal value of γ for which LMIs (19), (21) and (23) are feasible provided that equality (20) holds. Due to this very equality the problem under study cannot be solved, generally speaking, in terms of LMIs. However, in the particular case when the initial state of controller is zero, i.e. $x_r(0) = 0$, inequality (23) is reduced to $x_0^T X_{11} x_0 < \gamma^2 |x_0|^2$ which is LMI in variables X_{11} and γ^2 only and does not involve unknown matrices X_{12} and X_{22}.

Now, we are in position to formulate the main result.

Theorem 2. LQ output-feedback controller of the form (2) with $x_r(0) = 0$ exists if and only if LMIs

$$M^{\mathrm{T}} \begin{pmatrix} A^{\mathrm{T}}X_{11} + X_{11}A & C_1^{\mathrm{T}} \\ C_1 & -I \end{pmatrix} M < 0 \ , \tag{24}$$

$$N^{\mathrm{T}} \begin{pmatrix} Y_{11}A^{\mathrm{T}} + AY_{11} & Y_{11}C_1^{\mathrm{T}} \\ C_1Y_{11} & -I \end{pmatrix} N < 0 \ , \tag{25}$$

$$\begin{pmatrix} X_{11} & I \\ I & Y_{11} \end{pmatrix} \geq 0 \ , \tag{26}$$

$$x_0^T X_{11} x_0 < \gamma^2 |x_0|^2 \tag{27}$$

are feasible in variables $X_{11} = X_{11}^{\mathrm{T}} > 0$, $Y_{11} = Y_{11}^{\mathrm{T}} > 0$ and $\gamma^2 > 0$.

Given x_0, LQ output-feedback controller can be obtained numerically as follows: one should find the minimal γ_*^2 and corresponding matrices X_{11}, Y_{11} satisfying LMIs (24)-(27) in Theorem 2, then reconstruct matrix Y by using, for example, (22) and, finally, find matrix Θ of the controller as a solution to LMI (15) under Y constructed.

Note that by Finsler lemma from (24) it follows that

$$\begin{pmatrix} A^{\mathrm{T}} X_{11} + X_{11} A - \mu C_2^T C_2 & C_1^{\mathrm{T}} \\ C_1 & -I \end{pmatrix} < 0$$

for some $\mu > 0$. It means that the left upper block of the latter matrix is definitely negative, i.e. the pair (A, C_2) should be detectable. Analogously, (25) implies stabilizability of the pair (A, B).

3. γ-OPTIMAL OUTPUT-FEEDBACK CONTROLLER

Along with the LQ problem (10) let us consider the problem of minimizing the maximal ratio of the performance index and square of the norm of the initial plant state: find

$$\gamma_* = \inf\{\gamma > 0 : \exists \Theta \quad J(\Theta) < \gamma^2 |x_0|^2 \quad \forall\, x_0 \neq 0\} \ . \quad (28)$$

Define γ-optimal control law of the form (2) with $x_r(0) = 0$ providing

$$J(\Theta) < (\gamma_*^2 + \varepsilon)|x_0|^2 \quad \forall\, x_0 \neq 0$$

for arbitrary indefinitely small $\varepsilon > 0$. This control law can be interpreted as minimax one since it minimizes the worst relative performance value, i.e. when the initial plant state results in the maximal relative performance value.

In contrast to the above case of LQ controller, inequality (27) should hold now for all nonzero initial plant states, which leads to inequality

$$X_{11} < \gamma^2 I \ . \quad (29)$$

Theorem 3. γ-optimal output-feedback controller of the form (2) with $x_r(0) = 0$ exists if and only if LMIs (24)-(26), (29) are feasible in variables $X_{11} = X_{11}^{\mathrm{T}} > 0$, $Y_{11} = Y_{11}^{\mathrm{T}} > 0$ and $\gamma^2 > 0$.

Matrix of parameters Θ of γ-optimal controller is computed as provided by the procedure described above for LQ controller.

Along with LQ and γ-optimal output-feedback controllers, it is interesting to introduce an average LQ output-feedback controller of the form (2) with zero initial controller state. This controller provides minimum of the expected value of the performance index when initial plant state x_0 is assumed to be a zero mean random vector satisfying $E\{x_0 x_0^T\} = I$. In this case, we have

$$E\{J(\Theta)\} < \gamma^2 E|x_0|^2 = \gamma^2 n_x$$

and, consequently, the average LQ output-feedback controller exists if and only if LMIs (24)-(26) and

$$\mathrm{tr}\, X_{11} < \gamma^2 n_x$$

are feasible in variables $X_{11} = X_{11}^{\mathrm{T}} > 0$, $Y_{11} = Y_{11}^{\mathrm{T}} > 0$ and $\gamma^2 > 0$.

4. NUMERICAL COMPARISON OF LQ, AVERAGE LQ, AND γ-OPTIMAL OUTPUT-FEEDBACK CONTROLLERS

Let us compare LQ and γ-optimal controllers for a linear model of an inverted controlled pendulum

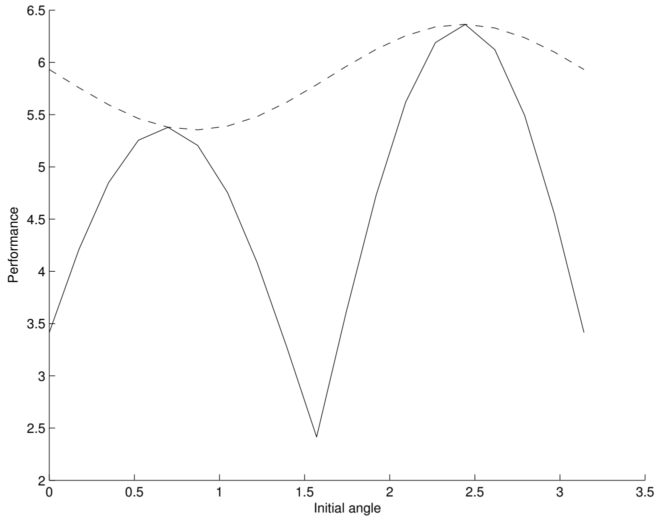

Fig. 1. Performance values corresponding to LQ (solid line) and γ-optimal (dashed line) controllers for the inverted pendulum without damping

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_1 + u, \\ &y = x_1, \\ &z_1 = x_1, \quad z_2 = x_2, \quad z_3 = u \ . \end{aligned}$$

The initial conditions of the plant are chosen in the form

$$x_1(0) = \cos\varphi \ , \quad x_2(0) = \sin\varphi \ , \quad \varphi \in [0, \pi] \ .$$

The plot of optimal values of LQ performance as a function of angle φ under associated LQ controllers is shown in Fig.1 by solid line, while the plot of performance values under γ-optimal controller is shown by dashed line. For example, for $\varphi_1 = 2\pi/3$ and $\varphi_2 = \pi$ we calculated the following parameters of LQ controllers

$$\Theta_1 = \begin{pmatrix} -5.5208 & -1.1936 & 2.7717 \\ -0.2436 & -4.3578 & -3.1919 \\ 2.6829 & -4.3578 & -6.5957 \end{pmatrix} ,$$

$$\Theta_2 = 10^8 \cdot \begin{pmatrix} 0.3670 & 0.2595 & 0.4025 \cdot 10^{-8} \\ -1.4829 & -1.0485 & 1.0000 \cdot 10^{-8} \\ -1.4829 & -1.0485 & -5.4847 \cdot 10^{-16} \end{pmatrix} .$$

These data allow to conclude that LQ output-feedback controller considerably depends on the initial conditions of the plant. On the other hand, parameters of γ-optimal controller are as follows

$$\Theta = \begin{pmatrix} -4.0451 & -0.6275 & 1.8914 \\ 0.1658 & -3.1425 & -2.3116 \\ 1.6776 & -3.0625 & -4.5003 \end{pmatrix} .$$

As it follows from Fig.1, performance values corresponding to LQ and γ-optimal controllers may significantly differ. However, next example of an inverted pendulum with damping

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = x_1 - x_2 + u, \\ &y = x_1, \\ &z_1 = x_1, \quad z_2 = x_2, \quad z_3 = u \ , \end{aligned}$$

shows that this difference may be not too much (Fig.2).

One more example is a double inverted pendulum described by equations

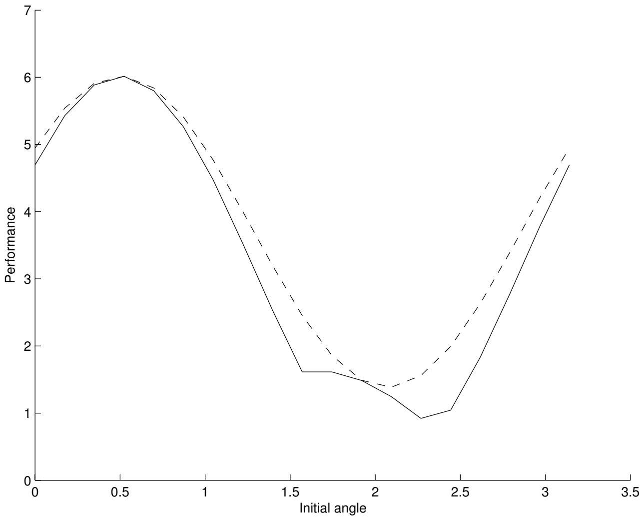

Fig. 2. Performance values corresponding to LQ (solid line) and γ-optimal (dashed line) controllers for the inverted pendulum with damping

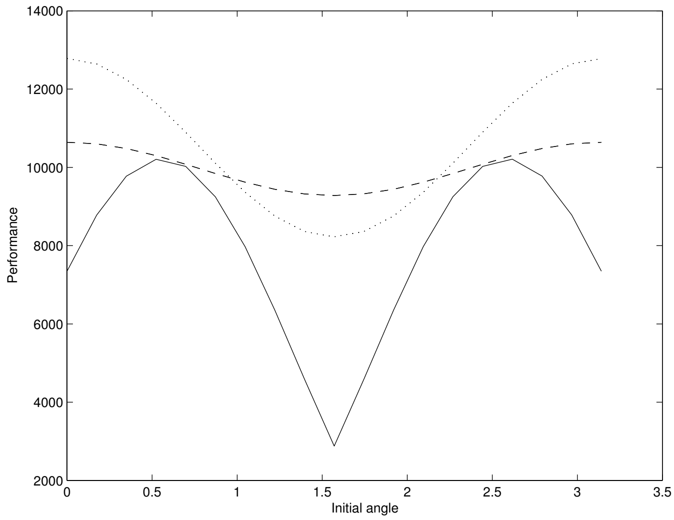

Fig. 3. Performance values corresponding to LQ (solid line), γ-optimal (dashed line), and averaged LQ (dotted line) controllers for the double inverted pendulum

$$\begin{aligned}
&\dot{x}_1 = x_3,\\
&\dot{x}_2 = x_4,\\
&\dot{x}_3 = 2x_1 - x_2 + u,\\
&\dot{x}_4 = -2x_1 + 2x_2,\\
&y = x_1,\\
&z_i = x_i,\ i = \overline{1,4},\ z_5 = u\ .
\end{aligned}$$

The initial conditions of the plant are chosen in the form

$$x_1(0) = \cos\varphi,\ x_3(0) = \sin\varphi,\ x_2(0) = x_4(0) = 0,\ \varphi \in [0, \pi]\ .$$

The plot of optimal values of LQ performance as a function of angle φ under associated LQ controllers is shown in Fig.3 by solid line, while the plots of performance values under γ-optimal controller and average LQ output-feedback controller are shown by dashed and dotted lines, respectively.

5. CONCLUSION

In this paper, the LQ output-feedback control problem is solved in the class of linear dynamic full order controllers with zero initial state. Based on LMI technique, necessary and sufficient conditions for existence of LQ output-feedback controller are derived and a computational procedure to find their parameters is given. It is shown that parameters of this controller essentially depend on the initial plant state.

As an alternative, γ-optimal control law is introduced which minimizes the maximal relative performance. In contrast to LQ output-feedback controller, parameters of γ-optimal controller do not depend on the initial state of the plant. Numerical results show that, in some cases, performance losses under γ-optimal controllers may be insignificant.

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