**Proceedings of the 17th World Congress**
**The International Federation of Automatic Control**
**Seoul, Korea, July 6-11, 2008**

# On Achieving Periodic Joint Motion for Redundant Robots

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**Abstract:** The consequence of the loss of involutivity of a specific set of vector fields on the periodicity of the joint motion is examined for redundant robots. An output task, defined as a one dimensional periodic closed curve embedded in a two dimensional working surface, is realized through the computation of joint velocities in the configuration space. Depending on the manner in which the joint velocity is computed from the end-effector velocity, the resulting joint motion can become unpredictable and of a chaotical nature, even though the end-effector movement is periodic and predictable. The paper proposes an improvement over classical pseudo-inverse computation of the joint motion by suitably selecting two involutive vector fields. This then leads to a constructive sufficient condition for the periodicity of the joints based on the usage of both the 1-form defining the output manifold and complementary integrable 1-forms. The results are illustrated on a five-link rotary redundant robot (5R robot).

Keywords: Redundancy, Overactuation, Motion Planning, Differential Geometry, Involutivity.

## 1. INTRODUCTION

Redundant robots (i.e. robots having more actuated degrees of freedom than necessary for completely specifying the end-effector position and orientation) show superior flexibility over classical robots insofar as they can accommodate not only for the end effector task but also for a possible necessary change in shape caused by the appearance of an obstacle or obstruction in the surroundings of the robot. The redudancy can be put to good effect to maximize efficiency (torque) resulting in substantial energy savings.

Nevertheless, the augmented number of choices that is at the controller designer disposal makes the control design difficult so as to meet both of these objectives (precision in the output task and versatility in changing the overall configuration when necessary). As a matter of fact, versatility is addressed by many authors using different approaches; Hooper and Tesar [1995], Tatlicioglu et al. [2005], Chettibi et al. [2004] and Nenchev [1989] explore the utilization of performance criterion to solve the redundancy; Gu et al. [1996] looks at posture optimization using potential functions; Hollerbach [1984] studies the optimal kinematic design; Harmeyer and Bowling [2004] studies the artificial augmentation of the jacobian matrix to render it invertible; Gu [2000a] and Gu [2000b] studies extensively the embedding configuration manifold; Boltunov et al. [2005] uses holonomic restrictions to enforce a compact configuration in a complex environment. Should versatility not be explicitly taken into account, then simply fixing some actuators would lead to the relative simplicity of classical robot design in achieving the output task.

The present work considers as an output task the positioning of the end effector so as to track a closed orbit on a working surface in the three dimensional space. Only the position is considered, the orientation is left unspecified.

Versatility is chosen here as the possibility of realizing a periodic motion of *all* joints while not "blocking" specific joints. A repeatable (periodic) motion of the robot in its coordinate frame is highly desirable, while realizing the main task, since the repeatability induces also predictability, Nicolato and Madrid [2005].

In the sequel, a set of vector fields (defining velocities in the joint configuration manifold) is shown to play a key role when using the pseudo-inverse of the jacobian of the forward kinematics in achieving a periodic movement of all joints while realizing the periodic output task.

Using the pseudo-inverse of the jacobian of the forward kinematics is not new. Whitney [1969] suggested first the use of the pseudo-inverse of the jacobian to solve the redundancy, he also first stated the link between the redundancy property and the output task. Liegeois [1977] extended the pseudo-inverse approach to include self-motions using the null-space of the jacobian matrix. Recently, Nicolato and Madrid [2005] focuses on a recursive algorithm to obtain the inverse kinematic of redundant manipulators for which the joint motion is shown (experimentally) to become periodic after convergence.

Section 2 gives classical definitions together with those pertaining to our work. A case study is then presented in Section 3 based on the 5R robot, before exposing the results in Section 4, namely an improvement on classical jacobian inversion (Section 4.2), a sufficient condition for periodicity (Section 4.3) and finally a design procedure ensuring periodicity (Section 4.4). Conclusions and future work are discussed in Section 5.

1 Partially supported by EPFL and FNRS.

## 2. DEFINITIONS AND PROBLEMATIC

Importance is put on the difference between redundancy and overactuation through the notion of an output manifold. Definitions 3, and 5 are new and specific to the work undertaken. They are especially useful for the results given in Sections 4.2 to 4.4.

*Definition 1.* (Open Kinematic Chain) An open kinematic chain is a $n$-serial link manipulator arm, whose end-effector, attached to the last link, performs a task in $\mathbb{R}^3$. The end-effector position $Y(t)$ is described with a function of the joints angle (for rotary joint) and length (for prismatic joint). Using the joint-configuration (angle and length) coordinates $(q_1, ..., q_n) \in Q \subset \mathbb{R}^n$, we have $Y = (\, x(q) \;\, y(q) \;\, z(q) \,)^T$ which for notational purposes will be written $Y = \zeta(q)$.

*Definition 2.* (Working Space) The working space $\Gamma^n \subset Q \subset \mathbb{R}^n$ of a $n$-joints manipulator arm, described by its joints configuration $(q_1, ..., q_n)$, corresponds to the end-effector accessibility area in $\mathbb{R}^m$.

$$\Gamma^n = \Big\{ Y \in \mathbb{R}^m : \exists \{q_1, ..., q_n\} \in Q, \\ Y = \zeta(q_1, ..., q_n) \Big\}$$

*Definition 3.* (Output Manifold) The smooth manifold $S$ containing the end-effector trajectories $Y(t)$, is called the output manifold.

*Definition 4.* (Redundant Robot) A manipulator arm, realizing an open kinematic chain of $n$ links, is said to be redundant, when, for a fixed end-effector position $\bar{Y}$, there exist infinite different joints configurations $(q_1, ..., q_n)$. Let $\bar{x}, \bar{y}, \bar{z} \in \mathbb{R}$ be a given end-effector position $\bar{Y} = (\, \bar{x} \;\, \bar{y} \;\, \bar{z} \,)^T \in \mathbb{R}^3$.

$$\Omega_q = \{(q_1, ..., q_n) \in Q \subset \mathbb{R}^n : \zeta(q_1, ..., q_n) = \bar{Y}\}$$
$$\dim(\Omega_q) = (n \times \infty)$$

The redundant manifold $\Omega \subset \mathbb{R}^n$ is the manifold collecting the position-dependent working subspace $\Omega_q \subset Q \subset \mathbb{R}^n$.

The redundancy property of a manipulator is intimately linked to the end-effector task (Whitney [1969] and Liegeois [1977]). This observation motivates the following extension of the redundancy definition, so as to include the output manifold.

*Definition 5.* (Overactuation) A redundant manipulator is said to be overactuated, when its number of internal degrees of freedom $(q_1, ..., q_n) \in Q \subset \Gamma^n$, is larger than its end-effector motion capabilities $(\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)^T \subset \mathbb{R}^3$, on a predefined output manifold $S$.

$$\forall\, Y(t) \in S \subset \Gamma^n, \exists \text{ at least } q(t), \bar{q}(t) : \\ Y(t) = \zeta(q(t)) = \zeta(\bar{q}(t))$$

Whenever the robot is redundant, the jacobian matrix $J$ that realizes the map $\phi$ between the tangent spaces has not full-column rank and therefore can not be exactly inverted. For every trajectory of the end-effector in the working space of the redundant robot, there exists an infinite number of corresponding joint motions. Figure 1 illustrates the main symbols and terminology. The output manifold

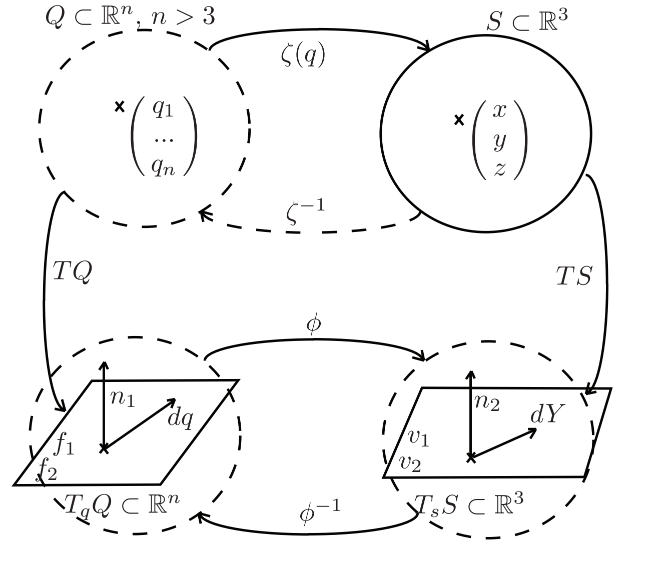



Fig. 1. Direct and inverse kinematic map $\zeta$ between end-effector position and its corresponding joint configuration.

is written $S$ and embedded in $\mathbb{R}^3$. At a given point of $S$, the tangent space is labelled $T_sS$. Here $s$ is a point of $S$. The corresponding point in $\mathbb{R}^3$ is also labelled $s$ and corresponds to $(x, y, z)^T$. The context will make it clear which of the two is considered. The configuration manifold is noted $Q$ with the tangent space at the configuration $q$ written $T_qQ$. $\zeta(.)$ is a submersion for which $\phi$ is the corresponding surjective map from the tangent space $T_qQ$ to the tangent space $T_{\zeta(q)}S$, where $\zeta(q) = s$. $TS$ and $TQ$ are the respective tangent bundles.

Throughout the paper, focus is put on realizing a periodic movement of the end effector in the output manifold $S$ while trying as best as possible to also achieve periodic movements of all joints in $Q$. To meet both objectives, without loosing the redundant property (i.e. keeping $n > 3$), is not a trivial matter.

## 3. CASE STUDY

From now on (and without loss of generality), the particular case of a five rotary-link robot is considered. Additionally, an example of an output trajectory is chosen for comparison purposes.

### *3.1 5R Robot*

Consider an open kinematic chain constituted of five serially linked rotary joints (5R) realizing a periodic positioning task $Y(t)$ in $\mathbb{R}^3$, see Figure 2. Only end-effector trajectories that can be embedded in a smooth manifold $S$ are considered.

### *3.2 Example of a Main-Task Design*

As the main task, the end-effector trajectory in space is parameterized with time. The manipulator output manifold $S$, containing the end-effector trajectory, is selected to be a sphere. The tangent bundle $(TS)$ associated with the output manifold contains the velocity-vector of displacement $\mathrm{d}Y/\mathrm{d}t = \dot{Y}(t)$.

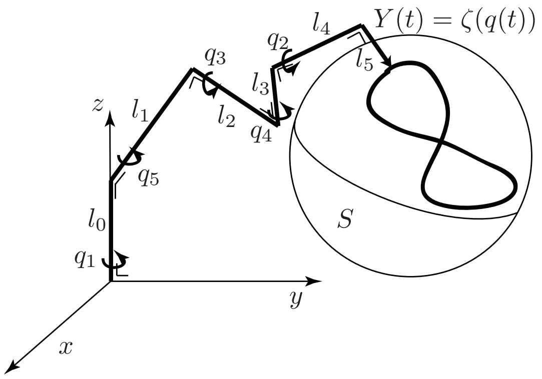



Fig. 2. Five rotary (5R) joints ($q_1$ to $q_5$) serially linked ($l_0$ to $l_5$) manipulator arm.

As an example, a Lissajou curve parameterized by the following equations is selected as the main positioning task of the manipulator arm.

$$x_p = A\cos(\omega t + \varphi)$$
$$y_p = A\cos(2\omega t + \varphi)$$

Using the rotation matrices $R_x(a), R_y(b), R_z(c)$ defined by Goldstein [1980] (pp. 146-147 and 608), the planar curve is mapped onto the output manifold (sphere) through the local diffeomorphism defined by:

$$\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix} = R_x(a)R_y(b)R_z(c)\begin{pmatrix} R\cos(x_p)\cos(y_p) \\ R\cos(x_p)\sin(y_p) \\ R\sin(x_p) \end{pmatrix} + \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} \quad (1)$$

where $c_x$, $c_y$, $c_z$ represent the sphere center coordinates and $R$ its radius.

The velocity reference $\mathrm{d}Y/\mathrm{d}t$ is obtained through the time derivative of equation (1). The following initial conditions are set for all simulations: $q_1(0) = 0.298$, $q_2(0) = 0.165$, $q_3(0) = 0.260$, $q_4(0) = 1.24$, $q_5(0) = 4.37$. Sphere: $R = 1.5$, $c_x = 0$, $c_y = -0.5$, $c_z = 0.2$; Manipulator: $l_0 = l_1 = l_2 = l_3 = l_4 = 1$, $l_5 = 0.5$; Task: $\omega = 0.4$, $A = 0.6$, $\phi = \frac{\pi}{2}$, $a = \frac{\pi}{4}, b = \frac{\pi}{4}, c = 0$.

## 4. ON ACHIEVING PERIODICITY

This section gives the main results of the paper. Section 4.1 reviews the classical jacobian inversion and its limitation in achieving a periodic joint movement. We then propose in Section 4.2 an improvement through the computation of an involutive basis of the tangent bundle of the output manifold $TS$ (to be more precise, in a large enough submanifold of $TS$ of the same dimension). Section 4.3 gives a sufficient condition using the Poincaré-Bendixson theorem and the involutive property. Finally, Section 4.4 states a clear procedure to enforce periodic trajectories of the joints while realizing the main task. The case study illustrates the concepts as the theory proceeds.

### *4.1 Direct Jacobian Inversion*

The pseudo-inverse $J^+ = (JJ^T)^{-1}J^T$ of the jacobian of the forward kinematics can be used to express one choice of joint velocities associated with a given velocity of the end effector.

$$\mathrm{d}q/\mathrm{d}t = J^+ \mathrm{d}Y/\mathrm{d}t \quad (2)$$

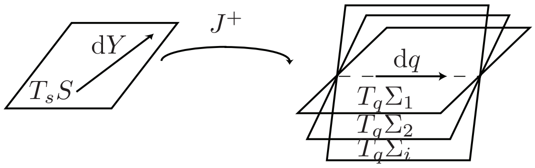
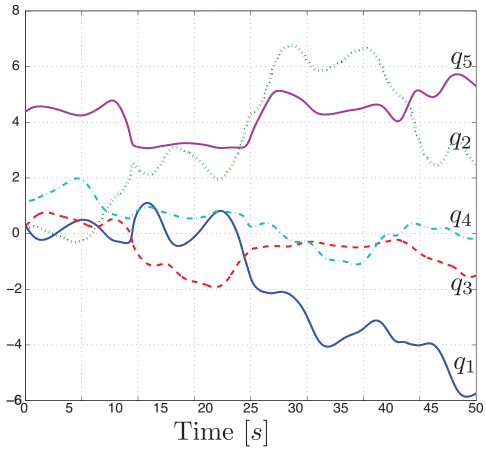
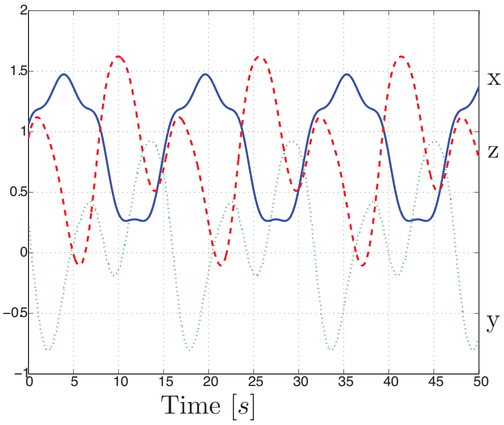



Fig. 3. The inverse map based on $J^+$ selects locally *one of* the existing tangent planes $T_q\Sigma_i$.

The results of integrating Equation (2) from the initial conditions given in Section 3 for the case study are shown in Figure 4.




(a) Joints Evolution through time.




(b) Output evolution through time: the periodic Lissajou is perfectly realized.

Fig. 4. Direct pseudo-inversion of the velocity vector leads to an erratic unpredictable behavior of the jonts, even if the Lissajou periodic task is perfectly realized.

Even though the robot performs perfectly the main task (see Figure 4(b)), the displacement along the Lissajou curve induces a completely chaotic and unpredictable motion in the joint space (see Figure 4(a)).

The choice of joint velocity is fortunately not unique for a prescribed motion of the end effector (thanks to redundancy). The next section explores another possibility based on taking advantage of the distinction between overactuation and redundancy (that is, by explicitly using the defining equations of the output manifold).

### 4.2 Improving the Periodicity

In order to improve the periodicity of joint movements, the structure of the output manifold is used for constructing the displacement vector $\mathrm{d}Y/\mathrm{d}t$.

Consider the exact 1-form $\omega_1$ associated with the output manifold $S$ through the gradient of its equation $h(x, y, z) = (x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 - R^2$, that is

$$\begin{aligned}\omega_1 &= \mathrm{d}h \\ &= 2(x - c_x)\mathrm{d}x + 2(y - c_y)\mathrm{d}y + 2(z - c_z)\mathrm{d}z.\end{aligned}$$

*Proposition 1.* Except for the points on the equatorial line $(y - c_y)^2 + (z - c_z)^2 = R^2$ and $x = c_x$, the vector fields

$$v_1 = \begin{pmatrix} -(z - c_z) \\ 0 \\ (x - c_x) \end{pmatrix}, \quad v_2 = \begin{pmatrix} -(y - c_y) \\ (x - c_x) \\ 0 \end{pmatrix}$$

(i) span a two dimensional subspace and (ii) are involutive. Therefore except on this line, they locally define an integrable distribution.

*Proof*: The only way for $v_1(s)$ and $v_2(s)$ (with $s = (x \quad y \quad z)^T$) not to span a two dimensional subspace is that $x = c_x$. Using the defining equation for $S$ gives $(y-c_y)^2+(z-z_c)^2 = R^2$ which gives an equatorial line. By construction $< \omega_1; v_1 > = 0$ and $< \omega_1; v_2 > = 0$. Because $d\omega_1 = 0$ by exactness, Formula (1.25) of Olver [1995] yields

$$\begin{aligned}0 &= < d\omega_1; v_1, v_2 > \\ &= v_2 < \omega_1; v_1 > -v_1 < \omega_1; v_2 > - < \omega_1; [v_1, v_2] > \\ &= - < \omega_1; [v_1, v_2] > .\end{aligned}$$

so that the bracket $[v_1, v_2]$, being annhilated by the same 1-form $\omega_1$ as for $v_1$ and $v_2$, belongs to the span of $v_1$ and $v_2$, which means that $v_1$ and $v_2$ are involutive vector fields whose distribution is locally integrable. □

*Remark 1.* The Euler characteristic of $S$ is an obstruction in finding global expressions for $v_1$ and $v_2$ (see Fulton [1995]). The best one can obtain are smooth vectorfields that only vanish at a single point (instead of the whole equatorial line).

As long as the above singular points can be avoided, which is the case for the chosen output task, the end-effector motion $\mathrm{d}Y/\mathrm{d}t$, defined by the time derivative of equation (1), is then expressed in this involutive basis using the two scalars $\alpha$ and $\beta$.

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \alpha v_1 + \beta v_2 \tag{3}$$

Finally, the joint motions are obtained through the local inverse map from $T_sS$ to $T_qQ$ based on the pseudo-inverse of the jacobian matrix $J^+$ (Figure 1). With $f_1 = J^+v_1$ and $f_2 = J^+v_2$ the joint velocity becomes

$$\frac{\mathrm{d}q}{\mathrm{d}t} = \alpha f_1 + \beta f_2 \tag{4}$$

A simulation with the 5DOF serial manipulator realizing a Lissajou curve on a sphere is undertaken (see Figure 5).

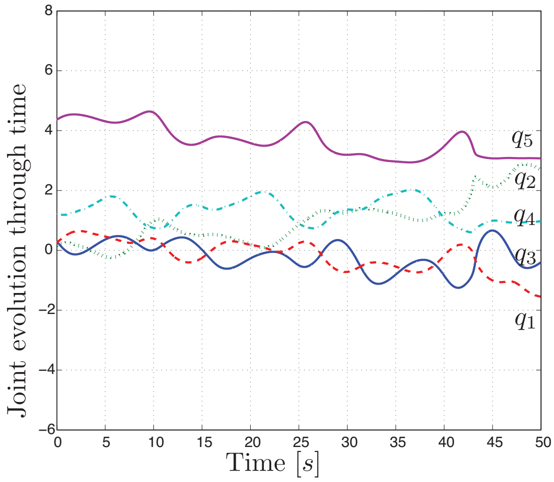



Fig. 5. An involutive basis of the output manifold helps in reducing the unpredictable behavior. (Since the Lissajou task is perfectly realized, as in Figure 4(b), it is not shown again here.)

The displayed trajectories still fail to be periodic. The results are nevertheless better than those obtained using the first method, since the resulting trajectories are closer to periodicity than with the direct Jacobian inversion of the tangent vector of the Lissajou curve (that is, introducing an involutive basis of the tangent bundle $TS$ helps in reducing the erratic and unpredictable characteristics of the motion in the joint space).

### 4.3 A Sufficient Condition for Joint Periodicity

The following theorem uses the Poincaré-Bendixson theorem to achieve a sufficient condition for periodicity.

*Theorem 1.* Let $f_1$ and $f_2$ be two involutive vector fields such that $f_1(q)$ and $f_2(q)$ belong to $T_qQ$ for all $q$. Moreover, assume that for all $q$, $v_1 = J(q)f_1(q)$ and $v_2 = J(q)f_2(q)$ are independent vectors in $T_{\phi(q)}S$. Now, if the following two conditions hold, namely:

- The integral manifold of $f_1$ and $f_2$ is simply connected and compact.
- The end-effector movement $Y(t)$ is periodic.

then the joint motion obtained after decomposing (3) and integrating (4) is also periodic or converges to a limit cycle.

*Proof*: Because by hypothesis ($f_1$ and $f_2$ are involutive), the integral manifold of $f_1$ and $f_2$ exists. Therefore, $q(t)$ is confined to a 2-dimensional submanifold of $Q$. Because this submanifold is also assumed to be simply connected, the hypotheses of Poincaré-Bendixson theorem are satisfied. The result then follows after noticing that, by hypothesis (the end-effector trajectory is periodic and $f_1$ and $f_2$ are in correspondence with $v_1$ and $v_2$), convergence to an equilibrium point is excluded. Moreover divergence of $q(t)$ to infinity as $t \to \infty$ is prohibited since the manifold is assumed compact (i.e. closed and bounded). □

In Section 4.2 although the basis of the tangent bundle $TS$ is ensured by involutive vector fields $v_1$ and $v_2$, the vector fields $f_1$ and $f_2$ could very well not be involutive. It is well known that, whenever $z = \phi(x)$ is a diffeomorphism, then the bracket commutes with the push forward:

$$[\frac{\partial \phi}{\partial x} v_1 \circ \phi^{-1}(z), \frac{\partial \phi}{\partial x} v_2 \circ \phi^{-1}(z)] = \frac{\partial \phi}{\partial x}[v_1, v_2] \circ \phi^{-1}(z)$$

However, when $\phi$ is a submersion, as it is the case here (the jacobian of $\phi$ is a surjective map; it has a non trivial kernel), then the Lie-bracket does not necessarily commute with the pseudo-inverse.

*Lemma 1.* Whenever $\phi$ is a submersion, then $\phi^+[v_1, v_2]$ is not necessarily equal to $[\phi^+ v_1, \phi^+ v_2]$.

*Proof*: If both expressions in the lemma were equal then the involutivity of $v_1$ and $v_2$ would imply the one of $\phi^+ v_1$ and $\phi^+ v_2$ (by linearity of the pseudo-inverse). However, this is not necessarily the case as the following example testifies. Consider the submersion $\phi : q \to x$ from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by $x_1 = q_1$ and $x_2 = q_2 + q_1 q_3$. Let $v_1 = (1 \quad 0)^T$ and $v_2 = (0 \quad 1)^T$ define two trivially involutive vector fields. Now, $f_1 = \phi^+ v_1 = (1 + q_3^2 \quad q_3 \quad q_1 q_3)^T$ and $f_2 = \phi^+ v_2 = (q_3(2 + q_1^2 + q_3^2) \quad 1 + q_1^2 + q_3^2 \quad q_1(1 + q_1^2 + q_3^2))^T$ are not involutive since

$$\det[f_1, f_2, [f_1, f_2]] = -(1 + q_1^2)^2 \neq 0.$$

□

Because the trajectories displayed in Figure 5 are not periodic in the joint space $q$, the vector fields $f_1$ and $f_2$ are either: (i) not involutive or (ii) involutive and the underlying submanifold $\Sigma$ is not simply connected (Theorem 1) or (iii) involutive and the underlying submanifold $\Sigma$ is not compact (Theorem 1) or (iv) the convergence of the joints trajectories to their periodicities is not completed yet, in the sense that the integration time considered is too short. Notice that this is not a numerical inaccuracy in computing the pseudo-inverse as one might initially suspect, but it is a inherent limitation of pulling back the involutive vector fields $v_1$ and $v_2$ along a submersion.

### *4.4 Involutive basis of the nD tangent bundle TQ*

Because a submersion has a kernel, there exist many different $f_1$ and $f_2$ that map to a basis of $S$ in the output space, most of which are not involutive. By preventing the usage of certain directions of the kernel while constructing $f_1$ and $f_2$, involutivity is guaranteed. The following theorem constructs first $f_1$ and $f_2$ in this way, before computing $v_1$ and $v_2$.

*Theorem 2.*

(1) Define $\omega_x = dx$, $\omega_y = dy$, $\omega_z = dz$ where $x$, $y$, $z$ are the end-effector positions.
(2) Let $\omega_4$ and $\omega_5$ be complementary integrable 1-forms in the sense that $\omega_x$, $\omega_y$, $\omega_z$, $\omega_4$, and $\omega_5$ span the cotangent bundle $TQ^*$.
(3) Define $f_1$, $f_2$ as the dual vector fields to the three 1-forms $\omega_1$, $\omega_4$, and $\omega_5$, that is,

$$\begin{pmatrix} \omega_1 \\ \omega_4 \\ \omega_5 \end{pmatrix} (f_1 \quad f_2) = 0,$$

where $\omega_1$ is the 1-form associated with the output manifold $S$.

Under these assumptions, $f_1$ and $f_2$ are involutive and $v_1 = \frac{\partial \phi}{\partial q} f_1 \circ \phi^{-1}(x)$ and $v_2 = \frac{\partial \phi}{\partial q} f_1 \circ \phi^{-1}(x)$ constitute a basis for $TS$.

*Proof*: First notice that $\omega_1$, $\omega_4$, and $\omega_5$ are always independent since $\omega_1$ is a linear combination of $\omega_x$, $\omega_y$, and $\omega_z$, which are, in turn, independent from $\omega_4$ and $\omega_5$ (by construction). This means that $f_1$ and $f_2$ are well defined (no rank loss in the defining 1-forms). Involutivity follows by construction since $\omega_1$ is exact and $\omega_4$ and $\omega_5$ are integrable. The only subtle point is that $v_1$ and $v_2$ do never cancel and span $TS$. On this purpose, notice that $f_1$ and $f_2$ cannot be mapped to a zero vector through $\frac{\partial \phi}{\partial q}$, for if this was the case (say $\frac{\partial \phi}{\partial q} f_1 = 0$), then this would mean that

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} f_1 = 0.$$

Noticing that $\omega_4 f_1 = 0$ and $\omega_5 f_1 = 0$ as well, together with the fact that $\omega_x$, $\omega_y$, $\omega_z$, $\omega_4$, and $\omega_5$ are independent would lead to $f_1 = 0$, which is a contradiction. □

*Remark 2.* Finding the two integrable complementary 1-forms $\omega_4$ and $\omega_5$ can either be done by inspection or in a more systematic way using Cartan's equivalence method (Olver [1995]) which generalizes the canonical form of Darboux to more than a single 1-form. This gives all possible choices of $\omega_4$ and $\omega_5$. This step does not require the knowledge of the output manifold $S$, but depends only on the type of redundant robot used through the specific Lie group structure of the robot. Future research will address this particular construction for redundant robots, and especially the choice of $\omega_4$ and $\omega_5$ so as to guarantee both simple connectedness (using for instance de Rham cohomology (Fulton [1995])) and compacity (using Riemanian geometry) of the integral manifold of $f_1$ and $f_2$.

Considering again the 5R serial-link robot and a spheric output manifold, 2 arbitrary additional 1-forms $\omega_4$ and $\omega_5$ are added to the initial 1-form $\omega_1$ representing the sphere.

$$h : (x(q) - c_x)^2 + (y(q) - c_y)^2 + (z(q) - c_z)^2 = R^2$$
$$\omega_1 = \nabla_q h$$
$$\omega_4 = [\,1\ 0\ 0\ 0\ 1\,]$$
$$\omega_5 = [\,0\ 1\ 0\ 1\ 0\,]$$

Based on these 1-forms, Theorem 2 gives the n-dimensional involutive basis $\{f_1, f_2\}$ of the tangent bundle $T_n Q$.

Using the direct map $\phi$ based on the well-defined jacobian $J$, the local image of the hyper-surface $T\Sigma = span\{f_1, f_2\} \subset \Re^n$ into $\Re^3$ is built, see Figure 1.

$$v_1 = J f_1 \qquad v_2 = J f_2$$

*Remark 3.* The constant complementary 1-forms $\omega_4$ and $\omega_5$, introduced via Theorem 2, added to enforce involutivity of the joint-motion, limit the possible configurations, in the sense that they define a foliation of the hyper-surface $\Sigma$, on which the system remains.

The simulation of the 5DOF serial manipulator realizing a Lissajou curve on a sphere is made (see Figure 6).

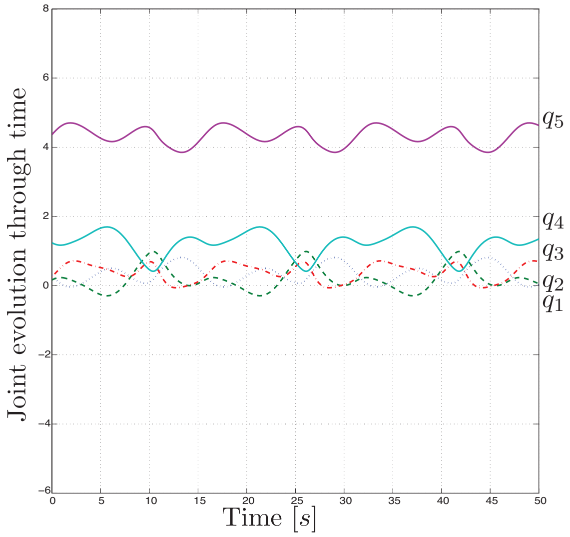



Fig. 6. Periodic motion of the joints. The vector fields $f_1$ and $f_2$ are involutive. (Since the Lissajou task is perfectly realized, as in Figure 4(b), it is not shown again here.)

The integration, using a combination of 1-forms, enforces involutivity: the robot joint-motions are completely predictable and remain in a reduced hyperplane of the coordinate space. The manipulator joint-configurations are constrained and periodic, while realizing perfectly the Lissajou main task. See Appendix A for a link to an illustrative movie.

## 5. CONCLUSION

The importance of the involutivity property of vector fields has been stressed so as to achieve periodic movement of all joints while the end-effector follows a periodic movement.

The results showed the importance of the output manifold $S$. Not only does it invite the definition of overactuation versus the one of redundancy, but also allows both to parameterize the periodic end-effector position and to improve on the direct computation of the joint velocity (using the pseudo inverse of the forward-kinematics jacobian). Indeed, by choosing two involutive vector fields as a basis of the tangent bundle $TS$, the chaotical character of the joint motion could be reduced (Section 4.2).

Nonetheless, achieving exact periodicity relied on a somewhat stringer condition of involutivity, namely finding two involutive vector fields in the joint tangent bundle $TQ$ allowing a surjective correspondance with a basis of $TS$ (the correspondance occuring in a sufficiently large submanifold of $TS$, Section 4.3). The construction of such a basis is successfully illustrated in Section 4.4, where perfect periodicity of the joints trajectories is obtained.

Moreover, the above methodology showed three topological properties that should be considered, namely the Euler characteristic of the surface $S$, the simple connectedness of the submanifold defined by the parameterizing vector fields and its compactedness. These investigations will appear elsewhere.

## REFERENCES

G. I. Boltunov, D. P. Gorelov, and I. V. Miroshnik. Coordinating control of motion of redundant manipulators. *Proceedings of 16th IFAC World Congress*, 2005.

T. Chettibi, H. E. Lehtihet, M. Haddad, and S. Hanchi. Minimum cost trajectory planning for industrial robots. *European Journal of Mechanics*, 23:703–715, 2004.

W. Fulton. *Algebraic topology. A first course.* Graduate Texts in Mathematics, New York: Springer, 1995.

H. Goldstein. *Classical Mechanics.* Reading, MA: Addison-Wesley, 2nd edition, 1980.

E. Y. L. Gu. A configuration manifold embedding model for dynamic control of redundant robots. *The International Journal of Robotics Research*, 19(3):289–304, 2000a.

E. Y. L. Gu. Configuration manifolds and their applications to robot dynamic modeling and control. *IEEE Transactions on Robotics and Automation*, 16(5):517–527, 2000b.

E. Y. L. Gu, Y. Zhou, and D. M. Martin. Posture optimization in a dual-redundant robotic control system with application to automation. *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 146–151, 1996.

S. Harmeyer and A. Bowling. Dynamic performance as a criterion for redundant manipulator control. *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 3:3601–3606, 2004.

J. M. Hollerbach. Optimum kinematic design for a seven degree of freedom manipulator. *Proceedings of 2nd International Symposium on Robotics Research, Kyoto*, pages 215–222, 1984.

R. Hooper and D. Tesar. Motion coordination based on multiple performance criteria with a hyper-redundant serial robot example. *Proceedings of the IEEE International Conference on Intelligent Control*, pages 133–138, 1995.

A. Liegeois. Automatic supervisory control of the configuration and behavior of multibody mechanisms. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-7 (12):868–871, 1977.

D. N. Nenchev. Redundancy resolution through local optimization: A review. *Journal of Robotic Systems*, 6: 769–798, 1989.

F. Nicolato and M. K. Madrid. Recursive algorithm for the inverse kinematics of redundant robotic manipulators. *Proceedings of 16th IFAC World Congress*, 2005.

P. J. Olver. *Equivalence, invariants, and symmetry.* Cambridge University Press, 1995.

E. Tatlicioglu, M. McIntyre, D. Dawson, and I. Walker. Adaptative nonlinear tracking control of kinematically redundant robot manipulators with sub-task extensions. *Proceedings of 44th IEEE Conference on Decision and Control, and the European Control Conference*, pages 4373–4378, 2005.

D. E. Whitney. Resolved motion rate control of manipulators and human prostheses. *IEEE Transactions on Man-Machine Systems*, MMS-10(2):47–53, 1969.

## Appendix A. ILLUSTRATIVE MOVIES

`-http://lawww.epfl.ch/page24555.html`