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Robust Impulsive Synchronization for a Class of Unified Chaotic Systems with Parameter Uncertainty ⋆

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Abstract: For a class of unified chaotic systems with parameter uncertainty, a robust impulsive synchronization scheme is proposed. Based on the theory of impulsive differential equations, some new and less conservative sufficient conditions are established in order to guarantee the robust synchronization of the chaotic systems. In particular, some simple and practical conditions are derived in synchronizing the chaotic systems by equal impulsive distances and control gains. Simulation results finally demonstrate the effectiveness of the method.

1. INTRODUCTION

During the last two decades, synchronization of the chaotic systems has attracted considerable attention due to its great potential applications in secure communication, chemical reactions and biological systems, see Boccaletti et al. (2002). The first idea of synchronization of two chaotic systems with different initial conditions was introduced by Pecora et al. (1990) and Carroll et al. (1991). Many different methods are applied theoretically and experimentally to synchronize chaotic systems, such as linear and nonlinear feedback control in Huang et al. (2006) and Chen et al. (2003), backstepping control in Park (2006) and Yassen (2007), variable structure control in Wang et al. (2004), adaptive control in Gao et al. (2007) and Zhang et al. (2006), impulsive control in Chen et al. (2004), Chen et al. (2006), Li et al. (2006) and Ma et al. (2007), active control in Lei et al. (2007) and Zhang et al. (2004), etc. Among these methods, impulsive control may give an efficient method to deal with the dynamical systems which cannot be controlled by continuous control. Additionally, in synchronization process, the response system receives the information from the drive system only at the discrete time instants. This drastically reduces the amount of synchronization information transmitted from the drive system to the response system which makes this method more efficient and thus useful in a great number of real-life applications. Ma et al. (2007) investigates the impulsive synchronization for uncertain unified chaotic systems in the sense of practical stability. Chen et al. (2004) and Chen et al. (2006) discuss the global asymptotic stability of impulsive control and synchronization of the unified chaotic systems. Based on the nonlinear feedback approach, an impulsive synchronization scheme is proposed in Li et al. (2006).

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As is well known, for some special applications, the parameter uncertainties of some systems are inevitable, and the effect of these uncertainties will destroy the synchronization and even break it. In the past few years, the analysis of synchronization for chaotic systems with parameter uncertainties has gained much research attention, e.g., Huang et al. (2006), Ma et al. (2007), and Zhang et al. (2004). Most of the researches mentioned above just assume that the parameter uncertainties of two chaotic systems are the same. However, in practice, the parameter uncertainties of the drive and response systems are always different and time-varying. Therefore, it is essential to investigate the synchronization of two chaotic systems in the presence of different time-varying parameter uncertainties.

Inspired by the above discussion, this paper addresses a practical issue of using impulsive control method to synchronize a class of unified chaotic systems with different time-varying parameter uncertainties. Some new and less conservative conditions are derived for the robust impulsive synchronization criteria, and the synchronization error magnitude can be reduced arbitrarily as long as some specific conditions hold. Finally, some numerical simulations for unified chaotic systems are given to demonstrate the effectiveness and feasibility of the proposed method.

Throughout the paper, the notations R and R^n denote the real number and n-dimensional Euclidean space, respectively. $R^{n\times m}$ is the set of all $n \times m$ real matrices. I denotes the identity matrix with appropriate dimensions. $\|\cdot\|$ refers to Euclidean vector norm or the induced matrix 2-norm.

2. SYSTEM DESCRIPTION AND SYNCHRONIZATION PROBLEM

We consider the drive system as follows:

$$\dot{\mathbf{x}}(t) = (A + \Delta A(t))\mathbf{x}(t) + h(\mathbf{x}(t)), \tag{1}$$

where $\mathbf{x} \in R^n$ is the state variable, $A \in R^{n\times n}$ is a constant matrix, and $h(\mathbf{x}(t))$ is a continuous nonlinear function.

Assume that

$$h(\mathbf{x}(t)) - h(\tilde{\mathbf{x}}(t)) = N(\mathbf{x}(t), \tilde{\mathbf{x}}(t))(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)), \quad (2)$$

where $\tilde{\mathbf{x}} \in R^n$ is the state variable of the response system. $N(\mathbf{x}, \tilde{\mathbf{x}}) \in R^{n\times n}$ is a bounded matrix with elements depending on $\mathbf{x}(t)$ and $\tilde{\mathbf{x}}(t)$. The parameter uncertainty $\Delta A(t)$ satisfies the following assumptions.

Assumption 1. The parameter uncertainty $\Delta A(t)$ is of the form:

$$\Delta A(t) = EF(t)H, \quad (3)$$

where E, H are known real constant matrices with appropriate dimensions, and the uncertain matrix $F(t)$ satisfies

$$\|F(t)\| \leq 1. \quad (4)$$

Assumption 2. The parameter uncertainty $\Delta A(t)$ is bounded and does not change the chaotic attractor of the chaotic system (1).

Suppose that a discrete instant set $\{t_k,\ k = 1, 2, \ldots\}$ satisfies $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots, \lim_{k\to\infty} t_k = \infty$, $0 \leq t_0 < t_1$.

At discrete time t_k, the state variables of the drive system are transmitted to the response system and the states of the response system are subjected to sudden changes at these instants. Therefore, the response system can be written in the following form

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= (A + \Delta\tilde{A}(t))\tilde{\mathbf{x}}(t) + h(\tilde{\mathbf{x}}(t)), \quad t \neq t_k \\ \Delta\tilde{\mathbf{x}}(t) &= \tilde{\mathbf{x}}(t_k^+) - \tilde{\mathbf{x}}(t_k^-) = \tilde{\mathbf{x}}(t_k^+) - \tilde{\mathbf{x}}(t_k) \\ &= -B_k\mathbf{e}(t), \quad t = t_k,\ k = 1, 2, \ldots, \\ \tilde{\mathbf{x}}(t_0^+) &= \tilde{\mathbf{x}}_0, \end{aligned} \quad (5)$$

where $\mathbf{e}(t) = \mathbf{x}(t) - \tilde{\mathbf{x}}(t) = [x_1(t) - \tilde{x}_1(t), x_2(t) - \tilde{x}_2(t), \ldots, x_n(t) - \tilde{x}_n(t)]^T$ is the synchronization error. Let $\tilde{\mathbf{x}}(t_k^+) = \lim_{t\to t_k^+} \tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{x}}(t_k^-) = \lim_{t\to t_k^-} \tilde{\mathbf{x}}(t)$, i.e., t_k^+ and t_k^- denote the time after and before t_k, respectively. $\tilde{\mathbf{x}}(t_k^-) = \tilde{\mathbf{x}}(t_k)$, which implies that $\tilde{\mathbf{x}}(t)$ is left-continuous at $t = t_k$. $B_k \in R^{n\times n}$ are the impulsive control gains. $\Delta\tilde{A}(t) = \tilde{E}\tilde{F}(t)\tilde{H}$ has the same assumptions as $\Delta A(t)$. Then from (1) and (5), the following error system equation is obtained:

$$\begin{aligned} \dot{\mathbf{e}}(t) &= (A + \Delta A(t) + N(\mathbf{x}(t), \tilde{\mathbf{x}}(t)))\mathbf{e}(t) \\ &\quad + (\Delta A(t) - \Delta\tilde{A}(t))\tilde{\mathbf{x}}(t), \quad t \neq t_k \\ \Delta\mathbf{e}|_{t=t_k} &= \mathbf{e}(t_k^+) - \mathbf{e}(t_k) \\ &= B_k\mathbf{e}(t_k), \quad t = t_k,\ k = 1, 2, \ldots, \\ \mathbf{e}(t_0^+) &= \mathbf{e}_0. \end{aligned} \quad (6)$$

Definition 1. The synchronization of (1) and (5) is said to have been achieved if, for arbitrary initial conditions $\mathbf{x}_0$ and $\tilde{\mathbf{x}}_0$, the trivial solution of the error system (6) converges to a predetermined neighborhood of the origin for any admissible parameter uncertainty that satisfies Assumptions 1 and 2.

Here the objective is to find the conditions on the control gains B_k and the impulsive distances $\delta_k = t_{k+1} - t_k$ $(k = 1, 2, \ldots)$ such that the error magnitude, i.e., $\|\mathbf{e}\|$, reduces to below some constant ξ, which implies that the impulsive controlled response system (5) is synchronized with the drive system (1) for arbitrary initial conditions.

For simplicity, in the following sections, $\tilde{\mathbf{x}}(t)$, $\mathbf{x}(t)$, $N(\mathbf{x}(t), \tilde{\mathbf{x}}(t))$, $\Delta A(t)$, $\Delta\tilde{A}(t)$ are denoted by $\tilde{\mathbf{x}}$, $\mathbf{x}$, N, ΔA, $\Delta\tilde{A}$, respectively.

Remark 1. Due to the boundedness of the chaotic signals, there exist constants $\chi > 0$, $M > 0$ such that $\|\mathbf{x}\| \leq \chi$, $\|\tilde{\mathbf{x}}\| \leq \chi$, $|x_i| \leq M, |\tilde{x}_i| \leq M, i = 1, 2, \ldots$.

3. MAIN RESULTS

A specific chaotic system is taken as an example to describe our methodology. This specific chaotic system, referred to as unified chaotic system by Lü et al. (2002), is described by

$$\begin{aligned} \dot{x}_1 &= (25a + 10)(x_2 - x_1), \\ \dot{x}_2 &= (28 - 35a)x_1 - x_1x_3 + (29a - 1)x_2, \\ \dot{x}_3 &= x_1x_2 - \frac{1}{3}(a + 8)x_3. \end{aligned} \quad (7)$$

The system (7) is chaotic for $a \in [0, 1]$, and from (1) and (2), we can get

$$A = \begin{pmatrix} -(25a+10) & 25a+10 & 0 \\ 28-35a & 29a-1 & 0 \\ 0 & 0 & -\dfrac{a+8}{3} \end{pmatrix},$$

$$h(\mathbf{x}) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix},\ N = \begin{pmatrix} 0 & 0 & 0 \\ -x_3 & 0 & -\tilde{x}_1 \\ x_2 & \tilde{x}_1 & 0 \end{pmatrix}.$$

The following theorem will give sufficient conditions for robust stability of the error system (6), which implies the impulsive controlled response system (5) is synchronized with the drive system (1).

Theorem 1. Let β_k and λ_A be the largest eigenvalue of $(I + B_k)^T(I + B_k)$ and $A + A^T$, respectively. If there exists a constant $\rho > 1$ such that

$$\begin{aligned} &\ln(\rho\beta_{2k-1}\beta_{2k}) + (\lambda_A + \sqrt{2}M + 2\|E\|\|H\| \\ &+ \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|))(t_{2k+1} - t_{2k-1}) \leq 0, \quad (8) \\ &\qquad k = 1, 2, \ldots, \end{aligned}$$

and

$$\begin{aligned} &\sup_k\{\beta_k \exp(\lambda_A + \sqrt{2}M + 2\|E\|\|H\| \\ &+ \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)(t_{k+1} - t_k))\} = \Gamma < \infty, \end{aligned} \quad (9)$$

then the response system (5) is synchronized with the drive system (1), where $\xi > 0$ is the bound of the error magnitude $\|\mathbf{e}\|$ and can be chosen small enough.

Proof. Let the Lyapunov function be in the form of

$$V(\mathbf{e}) = \mathbf{e}^T\mathbf{e}. \quad (10)$$

For $t \in (t_{k-1}, t_k],\ k = 1, 2, \ldots$, the time derivative of $V(\mathbf{e})$ along the solution of (6) is

$$\begin{aligned} \dot{V}(\mathbf{e}(t)) &= \dot{\mathbf{e}}^T\mathbf{e} + \mathbf{e}^T\dot{\mathbf{e}} \\ &= ((A + \Delta A + N)\mathbf{e} + (\Delta A - \Delta\tilde{A})\tilde{\mathbf{x}})^T\mathbf{e} \\ &\quad + \mathbf{e}^T((A + \Delta A + N)\mathbf{e} + (\Delta A - \Delta\tilde{A})\tilde{\mathbf{x}}) \quad (11) \\ &= \mathbf{e}^T(A + A^T)\mathbf{e} + \mathbf{e}^T(N + N^T)\mathbf{e} \\ &\quad + \mathbf{e}^T(\Delta A + \Delta A^T)\mathbf{e} + 2\mathbf{e}^T(\Delta A - \Delta\tilde{A})\tilde{\mathbf{x}}. \end{aligned}$$

Since

$$A + A^T = \begin{pmatrix} -(50a+20) & 38-10a & 0 \\ 38-10a & 58a-2 & 0 \\ 0 & 0 & -\dfrac{2a+16}{3} \end{pmatrix},$$

it is easy to see that $\lambda_A > 0$ for $a \in [0,1]$.

Let λ_N is the largest eigenvalue of the matrix $N + N^T$, from $N + N^T = \begin{pmatrix} 0 & -x_3 & x_2 \\ -x_3 & 0 & 0 \\ x_2 & 0 & 0 \end{pmatrix}$, we can get $0 \le \lambda_N = \sqrt{x_2^2 + x_3^2} \le \sqrt{M^2 + M^2} = \sqrt{2}M$.

For $\|\mathbf{e}\| \ge \xi$, one can conclude that $\|\mathbf{e}\| \le \dfrac{\|\mathbf{e}\|^2}{\xi}$ and therefore

$$\begin{aligned} \dot{V}(\mathbf{e}(t)) &\le (\lambda_A + \lambda_N + 2\|E\|\|H\|)\mathbf{e}^T\mathbf{e} \\ &\quad + 2(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)\|\mathbf{e}\|\|\tilde{\mathbf{x}}\| \\ &\le (\lambda_A + \lambda_N + 2\|E\|\|H\|)V(\mathbf{e}(t)) \\ &\quad + 2\chi(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)\|\mathbf{e}\| \\ &\le (\lambda_A + \sqrt{2}M + 2\|E\|\|H\|)V(\mathbf{e}(t)) \qquad (12) \\ &\quad + \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)\|\mathbf{e}\|^2 \\ &= \Big(\lambda_A + \sqrt{2}M + 2\|E\|\|H\| \\ &\quad + \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)\Big)V(\mathbf{e}(t)) \\ &= QV(\mathbf{e}(t)), \end{aligned}$$

where $Q = \lambda_A + \sqrt{2}M + 2\|E\|\|H\| + \dfrac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|) > 0$, and (12) implies that

$$V(\mathbf{e}(t)) \le V(\mathbf{e}(t_{k-1}^+))\exp(Q(t - t_{k-1})), \quad t \in (t_{k-1}, t_k],\ k = 1, 2, \ldots. \qquad (13)$$

On the other hand, from the second equation of (6), we obtain

$$\begin{aligned} V(\mathbf{e}(t_k^+)) &= [(I + B_k)\mathbf{e}(t_k)]^T(I + B_k)\mathbf{e}(t_k) \\ &= \mathbf{e}^T(t_k)(I + B_k)^T(I + B_k)\mathbf{e}(t_k) \qquad (14) \\ &\le \beta_k V(\mathbf{e}(t_k)). \end{aligned}$$

Thus, letting $k = 1$ in the inequality (13), we have

$$V(\mathbf{e}(t)) \le V(\mathbf{e}(t_0^+))\exp(Q(t - t_0)),\ t \in (t_0, t_1],$$

which leads to

$$V(\mathbf{e}(t_1)) \le V(\mathbf{e}(t_0^+))\exp(Q(t_1 - t_0)),$$

and

$$V(\mathbf{e}(t_1^+)) \le \beta_1 V(\mathbf{e}(t_1)) \le V(\mathbf{e}(t_0^+))\beta_1\exp(Q(t_1 - t_0)).$$

Similarly, for $t \in (t_1, t_2]$,

$$\begin{aligned} V(\mathbf{e}(t)) &\le V(\mathbf{e}(t_1^+))\exp(Q(t - t_1)) \\ &\le V(\mathbf{e}(t_0^+))\beta_1\exp(Q(t - t_0)). \end{aligned}$$

In general, for $t \in (t_k, t_{k+1}]$,

$$V(\mathbf{e}(t)) \le V(\mathbf{e}(t_0^+))\beta_1\beta_2\cdots\beta_k\exp(Q(t - t_0)). \qquad (15)$$

It follows from (8), (9), and (15) that,

1) $t \in (t_{2k-1}, t_{2k}]$, we have

$$\begin{aligned} V(\mathbf{e}(t)) &\le V(\mathbf{e}(t_0^+))\prod_{i=1}^{2k-1}\beta_i\exp(Q(t - t_0)) \\ &\le V(\mathbf{e}_0)\prod_{i=1}^{2k-1}\beta_i\exp(Q(t_{2k} - t_0)) \\ &= V(\mathbf{e}_0)\beta_1\beta_2\exp(Q(t_3 - t_1))\cdots \qquad (16) \\ &\quad \times \beta_{2k-3}\beta_{2k-2}\exp(Q(t_{2k-1} - t_{2k-3})) \\ &\quad \times \beta_{2k-1}\exp(Q(t_{2k} - t_{2k-1}))\exp(Q(t_1 - t_0)) \\ &\le \Gamma\frac{V(\mathbf{e}_0)}{\rho^{k-1}}\exp(Q(t_1 - t_0)), \end{aligned}$$

2) $t \in (t_{2k}, t_{2k+1}]$, we have

$$\begin{aligned} V(\mathbf{e}(t)) &\le V(\mathbf{e}(t_0^+))\prod_{i=1}^{2k}\beta_i\exp(Q(t - t_0)) \\ &\le V(\mathbf{e}_0)\prod_{i=1}^{2k}\beta_i\exp(Q(t_{2k+1} - t_0)) \\ &= V(\mathbf{e}_0)\beta_1\beta_2\exp(Q(t_3 - t_1))\cdots \qquad (17) \\ &\quad \times \beta_{2k-1}\beta_{2k}\exp(Q(t_{2k+1} - t_{2k-1})) \\ &\quad \times \exp(Q(t_1 - t_0)) \\ &\le \frac{V(\mathbf{e}_0)}{\rho^k}\exp(Q(t_1 - t_0)). \end{aligned}$$

From (16) and (17), it follows that the error magnitude $\|\mathbf{e}\|$ will converge to below the constant ξ if the error started with $\|\mathbf{e}\| > \xi$. This concludes the proof of Theorem 1. □

Remark 2. When $\Delta A = \Delta\tilde{A} = 0$, we can get "$\sqrt{2}M$" in (8) instead of "$2M$" in Chen et al. (2006), which will obtain larger stable region in estimating the impulsive distances $\Delta_k = t_{2k+1} - t_{2k-1}$ $(k = 1, 2, \ldots)$. Thus, the condition in Theorem 1 is less conservative than that in Chen et al. (2006).

Remark 3. From Theorem 1, it can be seen that we need only to choose the odd switching sequence $\{t_{2k-1}\}$ instead of choosing the whole switching sequence $\{t_k\}$ as in Theorem 1 in Chen et al. (2004).

Remark 4. Equation (8) can be generalized to the following condition.

There exist a finite integer $n_0 > 0$ and a constant $\rho > 1$ such that

$$\begin{aligned} &\ln(\rho\beta_{n_0(k-1)+1}\cdots\beta_{n_0 k}) + (\lambda_A + \sqrt{2}M + 2\|E\|\|H\| \\ &+ \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|))(t_{n_0 k+1} - t_{n_0(k-1)+1}) \le 0, \\ &\qquad k = 1, 2, \ldots. \qquad (18) \end{aligned}$$

The choice of n_0 in (18) depends on the actual system considered. Especially, $n_0 = 1$ corresponds to the case of the whole switching sequence $\{t_k\}$ and $n_0 = 2$ corresponds to the case of the odd switching sequence $\{t_{2k-1}\}$.

Based on the matrix theory, we obtain $\lambda_A \le \|A + A^T\|$. Then we can have the following corollary.

Corollary 1. Let β_k be the largest eigenvalue of $(I + B_k)^T(I + B_k)$. If there exists a constant $\rho > 1$ such that

$$\begin{aligned}&\ln(\rho\beta_{2k-1}\beta_{2k}) + (\|A+A^T\| + \sqrt{2}M + 2\|E\|\|H\| \\ &+ \frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|))(t_{2k+1} - t_{2k-1}) \le 0, \\ &\qquad\qquad\qquad\qquad k = 1, 2, \ldots\end{aligned} \tag{19}$$

and

$$\begin{aligned}&\sup_k\{\beta_k \exp(\|A+A^T\| + \sqrt{2}M + 2\|E\|\|H\| \\ &+\frac{2\chi}{\xi}(\|E\|\|H\| + \|\tilde{E}\|\|\tilde{H}\|)(t_{k+1} - t_k))\} = \Gamma < \infty,\end{aligned} \tag{20}$$

then the response system (5) is synchronized with the drive system (1), where $\xi > 0$ is the bound of the error magnitude $\|\mathbf{e}\|$ and can be chosen small enough.

Remark 5. Though Corollary 1 will be more conservative than Theorem 1, considering that the computation of $\|A + A^T\|$ will be easier than that of λ_A, in practice, the result in Corollary 1 will be more convenient.

Remark 6. From the boundedness of the chaotic systems, λ_A and $\|A + A^T\|$ are bounded, and the inequalities (8) and (19) can be satisfied by choosing appropriate β_k and $\Delta_k = t_{2k+1} - t_{2k-1}$ $(k = 1, 2, \ldots)$.

In practice, for the purpose of convenience, the gains B_k are always selected as a constant matrix and the impulsive distances Δ_k are set to be a positive constant. Then we have the following corollaries.

Corollary 2. Assume that $t_{2k+1} - t_{2k-1} = \Delta > 0$ and $B_k = B$ $(k = 1, 2, \ldots)$. If there exists a constant $\rho > 1$ such that

$$\begin{aligned}&\ln(\rho\beta^2) + \Delta(\lambda_A + \sqrt{2}M + 2\,\|E\|\,\|H\| \\ &\qquad + \frac{2\chi}{\xi}(\|E\|\,\|H\| + \|\tilde{E}\|\|\tilde{H}\|)) \le 0\end{aligned} \tag{21}$$

and

$$\begin{aligned}&\beta\exp(\Delta(\lambda_A + \sqrt{2}M + 2\,\|E\|\,\|H\| \\ &\qquad + \frac{2\chi}{\xi}(\|E\|\,\|H\| + \|\tilde{E}\|\|\tilde{H}\|))) = \Gamma < \infty,\end{aligned} \tag{22}$$

then the response system (5) is synchronized with the drive system (1), where $\xi > 0$ is the bound of the error magnitude $\|\mathbf{e}\|$ and can be chosen small enough, β and λ_A are the largest eigenvalue of $(I + B)^T(I + B)$ and $A + A^T$, respectively.

Corollary 3. Assume that $t_{2k+1} - t_{2k-1} = \Delta > 0$ and $B_k = B$ $(k = 1, 2, \ldots)$. If there exists a constant $\rho > 1$ such that

$$\begin{aligned}&\ln(\rho\beta^2) + \Delta(\|A+A^T\| + \sqrt{2}M + 2\,\|E\|\,\|H\| \\ &\qquad + \frac{2\chi}{\xi}(\|E\|\,\|H\| + \|\tilde{E}\|\|\tilde{H}\|)) \le 0\end{aligned} \tag{23}$$

and

$$\begin{aligned}&\beta\exp(\Delta(\|A+A^T\| + \sqrt{2}M + 2\,\|E\|\,\|H\| \\ &\qquad + \frac{2\chi}{\xi}(\|E\|\,\|H\| + \|\tilde{E}\|\|\tilde{H}\|))) = \Gamma < \infty,\end{aligned} \tag{24}$$

then the response system (5) is synchronized with the drive system (1), where $\xi > 0$ is the bound of the error magnitude $\|\mathbf{e}\|$ and can be chosen small enough, β is the largest eigenvalue of $(I + B)^T(I + B)$.

4. SIMULATION RESULTS

In simulation, the impulsive distances Δ_k are set to be a positive constant Δ, and the gains B_k are selected as a constant matrix, i.e., $B_k = B = \mathrm{diag}\{d, d, d\}$. Then, we have $\beta = (d + 1)^2$.

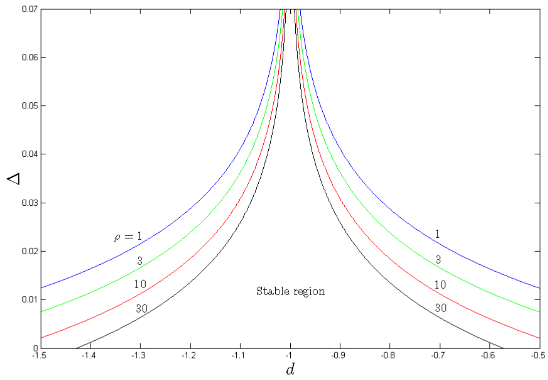

Fig. 1. The boundaries of stable region with different ρ

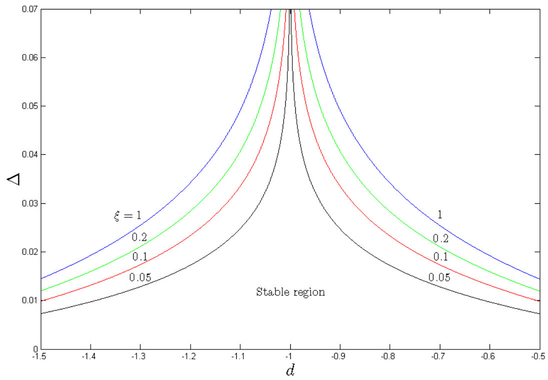

Fig. 2. The boundaries of stable region with different ξ

Let $a = 1$. Then

$$A = \begin{pmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \quad A + A^T = \begin{pmatrix} -70 & 28 & 0 \\ 28 & 56 & 0 \\ 0 & 0 & -6 \end{pmatrix}.$$

Let $\Delta A(t) = -\Delta\tilde{A}(t) = 0.04 \begin{pmatrix} \sin t & 0 & 0 \\ 0 & \cos t & 0 \\ 0 & 0 & \sin t \end{pmatrix}$, where $E = H = \tilde{E} = \tilde{H} = 0.2I$, $F(t) = -\tilde{F}(t) = \begin{pmatrix} \sin t & 0 & 0 \\ 0 & \cos t & 0 \\ 0 & 0 & \sin t \end{pmatrix}$. Fig. 1 shows the stable region for different ρ, where $M = 55$, $\chi = 60$. The whole region under the curve of $\rho = 1$ is the stable region. When $\rho \to \infty$, the stable region approaches a vertical line $d = -1$. Similarly, Fig. 2 shows the stable region for different ξ.

We choose $\rho = 1.1, \xi = 0.2, d = -0.9$, the initial conditions of the drive and response systems are taken as $[4, 1, 5]^T$ and $[3.6, 0.8, 5.4]^T$, respectively. Hence, considering $\|E\|\|H\| = \|\tilde{E}\|\|\tilde{H}\| = 0.04$, $\|A + A^T\| = 75.942$, from Corollary 3, the estimates of bounds of stable regions are given by

$$0 < \Delta \le -\frac{\ln\rho + \ln\beta^2}{201.8037} = 0.0452.$$

Using these parameters, conditions in Corollary 3 is satisfied for $\Delta \le 0.0452$. Synchronization errors and synchronization error magnitude with different impulsive distances are given in Figs. 3 and 4. As we can see that the synchronization has been achieved practically and $\|\mathbf{e}\|$ is smaller than $\xi = 0.2$.

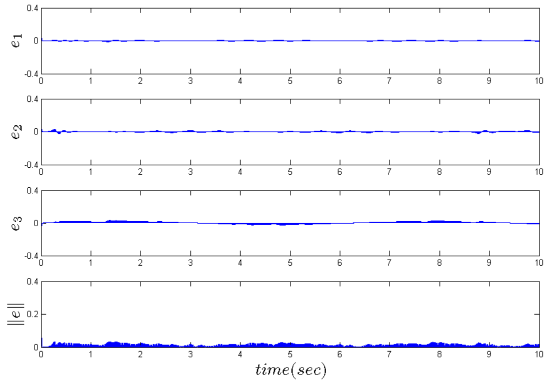

Fig. 3. Synchronization errors and synchronization error magnitude with $\Delta = 0.02$

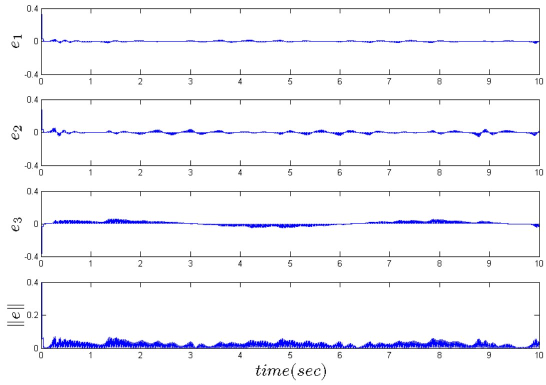

Fig. 4. Synchronization errors and synchronization error magnitude with $\Delta = 0.04$

5. CONCLUSIONS

In this paper, we have investigated problems of synchronizing a class of unified chaotic systems using the impulsive synchronization method. Robust stability of the method in the presence of parameter uncertainties is discussed. Based on the theory of impulsive differential equations, some new and less conservative sufficient conditions are established. Finally, some numerical simulations are given to demonstrate the effectiveness of the method. It should be pointed out that the methods mentioned above can also be applied to most of typical chaotic systems that can be described by (1) and (2), such as Lorenz system, Rössler system, Chen system, Lü system, several variants of Chua's circuits, etc. Due to the similarity and analogy, the details are not further discussed here.

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