Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

High Performance Control of an Active Heave Compensation System ⋆

K. D. Do * **J. Pan** **

* *School of Mechanical Engineering, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia, Tel: +61864883125, Fax: +61864881024, Email: duc@mech.uwa.edu.au*
** *School of Mechanical Engineering, The University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia, Email: pan@mech.uwa.edu.au*

Abstract: To compensate for heave motion, which has an adverse impact on the response of a drill-string or a riser, passive and active devices are usually used. Active heave compensators, whose control system is an essential part, allow conducting operations under more extreme weather conditions than passive ones. This paper presents a constructive method to design a nonlinear controller for an active heave compensation system using an electro-hydraulic system driven by a double rod actuator. The control development is based on Lyapunov's direct method and disturbance observers.

Keywords: Heave compensation, Nonlinear control, Riser.

1. INTRODUCTION

Both vertical and horizontal motions of a drilling vessel/rig have adverse effects on the riser(s) connecting between the rig and the well at the sea bed, Sarpkaya [1981]. The horizontal motion of the rig (hence the upper end of the risers) are often controlled by a dynamic positioning system in deep-water applications. Based on linear and nonlinear control theories, a number of control techniques for dynamic positioning systems has been proposed in literature, see for example Fossen and Strand [2001], Do et al. [2002] and references therein. However, in these papers the vertical (heave) motion of the rig, which directly changes the tension in the riser, is completely ignored. To reduce the effect of the heave motion of the rig, an active heave compensation system is often used in combination with a passive riser compensator to provide a stable position of the crown block referred to seabed. While controlling dynamic positioning systems receives a lot of attention, control of active heave compensation systems has received much less attention from researchers. In Korde [1998], a control system based on linear control techniques is proposed for an active heave compensation system on drill-ships. Recently, active control of heave compensated cranes or module handling systems during the water entry phase of a subsea installation is studied in Johansen et al. [2003]. In this work, a concept referred to as wave synchronization is introduced, where the main idea is to utilize a wave amplitude measurement in order to compensate directly for the water motion due to waves inside the moonpool. In Skaare and Egeland [2006], the authors proposed a parallel force/position controller for the control of loads through the wave zone in marine operations. Their controller achieved a significant improvement of the minimum value of the wire tension over the wave synchronization approach which was used in Johansen et al. [2003]. Designing a high performance controller for an active heave compensation system is a challenging task due to the fact that the force acting between the riser/drill-string and the active heave compensation unit is difficult to model accurately.

This paper proposes a nonlinear controller for an active heave compensation system using an electro-hydraulic system driven by a double rod actuator to minimize the effect of the heave motion of the vessel on the response of the riser. A disturbance observer is developed to estimate the force acting on the piston of the hydraulic system, and the heave acceleration of the vessel. This disturbance observer is then implemented in the control design. The control development and stability analysis are based on Lyapunov's direct method. This paper is a short version of Do and Pan [2007].

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Problem formulation

We consider an active heave compensation system depicted in Figure 1. It consists of an electro-hydraulic system driven by a double rod actuator. The riser connects to the piston of the hydraulic system via a ball joint. Hence there is no bending moment. The hydraulic system's house is fixed to the vessel/rig. The heave motion of the vessel/rig is denoted by $z(t)$, which is coordinated in the earth-fixed frame, $O_E X_E Y_E Z_E$. Let x_H, which is coordinated in the vessel body-fixed frame, be the displacement of the cylinder rod (piston) of the hydraulic system, i.e. the motion of the piston with respect to the vessel in the vertical direction. The mathematical formulation and

⋆ This work was supported in part by the the ARC Discovery grant No. DP0774645.

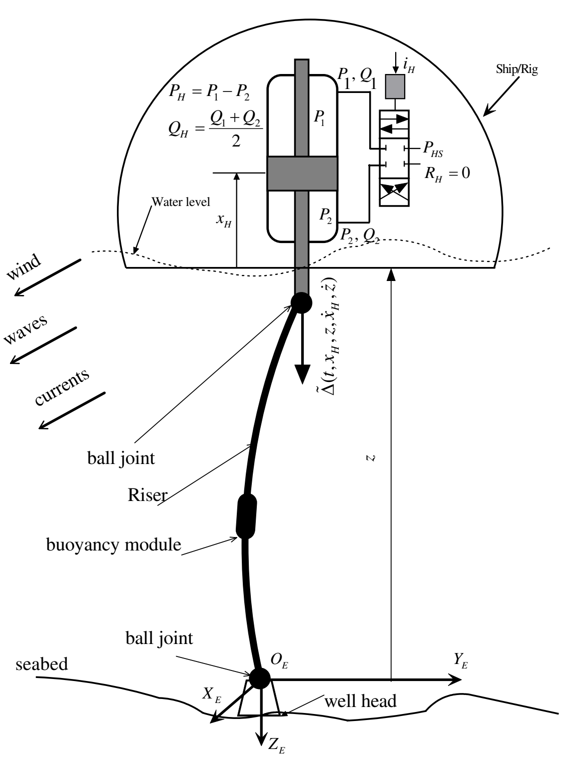

Fig. 1. Representative structure of an active heave compensation system

control design in this paper is generic. Hence they also work for one or more hydraulic systems.

To regulate the distance from the upper end of the riser to the seabed, it is obvious that we want to keep the sum $|x_H(t)+z(t)-L|$ as small as possible, where L is a constant, which is preset to achieve the desired pretension of the riser. This is done in the calibration process when the riser system is installed. Let i_H be the current input to the hydraulic system. Then the control objective can be described as designing the control input i_H to drive the cylinder rod in such a way that $|x_H(t) + z(t) - L|$ is kept as small as possible. The mathematical model consisting of the piston/rod, the actuator or the cylinder, and the servovalve dynamics can be written as

$$\begin{aligned}
\ddot{x}_H &= \frac{A_H C_{H3}}{m_H}\bar{P}_H - \frac{b_H}{m_H}\dot{x}_H + \frac{1}{m_H}\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z}),\\
\dot{\bar{P}}_H &= -\frac{4\beta_{He}A_H}{V_H C_{H3}}\dot{x}_H - \frac{4\beta_{He}C_{HT}}{V_H}\bar{P}_H + \\
&\quad \frac{4\beta_{He}C_{HD}W_H C_{H4}\bar{x}_{Hv}}{V_H\sqrt{\rho_H}\sqrt{C_{H3}}}\sqrt{\bar{P}_{HS} - \mathrm{sgn}(\bar{x}_{Hv})\bar{P}_H},\\
\dot{\bar{x}}_{Hv} &= -\frac{1}{\tau_{Hv}}\bar{x}_{Hv} + \frac{k_{Hv}}{\tau_{Hv}C_{H4}}i_H
\end{aligned} \tag{1}$$

where m_H is the mass of the rod of the hydraulic system, $P_H = P_1 - P_2$ is the load pressure of the cylinder with P_1 and P_2 being the pressures in the upper and lower compartments of the cylinder, see Figure 1, A_H is the ram area of the cylinder, b_H represents the combined coefficient of the modeled damping and viscous friction forces on the cylinder rod, $\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z})$ denotes the force acting on the rod from the riser or drill-string, V_H is the total volume of the cylinder and the hoses between the cylinder and the servovalve, β_{He} is the effective bulk modulus, C_{HT} is the coefficient of the total internal leakage of the cylinder due to pressure, Q_H is the load flow, Q_H is the load flow, x_{Hv} is the spool displacement of the servovalve, C_{HD} is the discharge coefficient, W_H is the spool valve area gradient, P_{HS} is the supply pressure of the fluid, sgn denotes the standard signum function, ρ_H is density of the oil, τ_{Hv} and k_{Hv} are the time constant and gain of the servovalve respectively, i_H is the current input to the hydraulic system, $\bar{P}_H = \frac{P_H}{C_{H3}}$ and $\bar{x}_{Hv} = \frac{x_{Hv}}{C_{H4}}$ where C_{H3} and C_{H4} are scaling constants to avoid numerical error and facilitating the control gain tuning process since P_H is usually very large and τ_{Hv} is usually very small, and $\bar{P}_{HS} = \frac{P_{HS}}{C_{H3}}$. It is noted that there would be a force from the riser acting on the cylinder rod in the horizontal plane. This force will affect the motions in sway and surge. However, in this paper we do not consider the sway and surge motions, which are considered in a dynamic positioning system. For stabilization/tracking control of the hydraulic system (1) without using disturbance observers but the "concept of dominating the size of disturbances", the reader is referred to Yao et al. [2001] and references therein. We now restate the control objective as follows:

Control objective. Under the assumption that $z(t)$ and $\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z})$ and their derivatives are bounded, design the control input i_H to regulate the distance from the upper end of the riser to seabed, i.e. to keep the sum $|z(t)+x_H(t)-L|$ as small as possible, where L is a constant which is preset to achieve a desired tension in the riser.

2.2 Preliminaries

Since we assume that $\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z})$ and $\dot{z}(t)$ are not available for the control design, we present in this subsection a disturbance observer, which will be used in the control design in the next section. In addition, discontinunity of the term $\sqrt{\bar{P}_{HS} - \mathrm{sgn}(\bar{x}_{Hv})\bar{P}_H}$ in (1) causes difficulty in the control design. Therefore, we also present a p-times differentiable signum function to approximate the signum function in (1).

Disturbance observer Consider the following system

$$\dot{x} = f(x) + u + d(t, x) \tag{2}$$

where $x \in \mathbb{R}^n$, $f(x)$ is a vector of known functions of x, u is the control input vector, and $d(t, x)$ is a vector of unknown disturbances. We assume that there exists a nonnegative constant C_d such that $\|\dot{d}(t, x)\| \leq C_d$. Now we want to design the control input u to stabilize the system (2) at the origin. It is obvious that if we can design a disturbance observer, $\hat{d}(t, x)$, that estimates $d(t, x)$ sufficiently accurately, then the control input u is straightforwardly designed as $u = -\kappa x - f(x) - \hat{d}(t, x)$ with κ a positive definite matrix. The disturbance observer is given in the following lemma.

Lemma 1. Consider the following disturbance observer

$$\begin{aligned}\hat{d}(t,x) &= \xi + \rho(x)\\ \dot{\xi} &= -K(x)\xi - K(x)(f(x) + u + \rho(x))\end{aligned} \tag{3}$$

where $K(x) = \frac{\partial \rho(x)}{\partial x}$, $\rho(x)$ is chosen such that the matrix $K(x)$ is positive definite for all $x \in \mathbb{R}^n$. The disturbance observer (3) guarantees that the disturbance observer error $d_e(t,x) = d(t,x) - \hat{d}(t,x)$ exponentially converges to a ball centered at the origin. The radius of this ball can be made arbitrarily small by adjusting the function $\rho(x)$. In the case $C_d = 0$, the disturbance observer error $d_e(t,x)$ exponentially converges to zero.

Proof. We calculate the derivative of $d_e(t,x)$ as follows

$$\begin{aligned}\dot{d}_e(t,x) &= \dot{d}(t,x) - (\dot{\xi} + \dot{\rho}(x))\\ &= -K(x)d_e(t,x) + \dot{d}(t,x).\end{aligned} \tag{4}$$

Consider the Lyapunov function $V_e = \frac{1}{2}d_e^T d_e$ whose first time derivative along the solutions of (4) satisfies (we here drop arguments t and x of $d_e(t,x)$ for simplicity of presentation)

$$\dot{V}_e = -0.5 d_e^T (K(x) + K^T(x)) d_e + d_e^T \dot{d}. \tag{5}$$

Since $K(x)$ is a positive definite matrix, there exists a positive constant ϖ such that

$$d_e^T \big(K(x) + K^T(x)\big) d_e \geq \varpi d_e^T d_e, \ \forall x \in \mathbb{R}. \tag{6}$$

Substituting (6) into (5) results in

$$\dot{V}_e \leq -(\varpi - 2\varepsilon)V_e + C_d^2/(4\varepsilon) \tag{7}$$

where we have used $|d_e^T(t,x)\dot{d}(t,x)| \leq \varepsilon \|d_e(t,x)\| + \frac{1}{4\varepsilon}\|\dot{d}(t,x)\|$ with ε a positive constant such that $\varpi - 2\varepsilon$ is strictly positive. From (7) and $V_e = \frac{1}{2}d_e^T d_e$, we have the disturbance error $d_e(t,x)$ exponentially converges to a ball centered at the origin with the radius $R_{de} = \sqrt{\frac{1}{2\varepsilon(\varpi - 2\varepsilon)}}C_d$ as long as the solutions $x(t)$ exist. The existence of the solutions $x(t)$ is to be guaranteed by the design of the control input u. Since ϖ can be chosen arbitrarily large by choosing the function $\rho(x)$, R_{de} can be made arbitrarily small. In the case $C_d = 0$, the radius $R_{de} = 0$ meaning that the disturbance error $d_e(t,x)$ exponentially converges to zero.□

p-times differentiable signum function: A scalar function $h(x,a,b)$ is called a p-times differentiable signum function if it enjoys the following properties

$$\begin{aligned}&1)\ h(x,a,b) = -1 \quad \text{if} \quad -\infty < x \leq a,\\ &2)\ h(x,a,b) = 1 \quad \text{if} \quad x \geq b,\\ &3)\ -1 < h(x,a,b) < 1 \ \text{if}\ a < x < b,\\ &4)\ h(x,a,b)\ \text{is}\ p\ \text{times differentiable with respect to}\ x\end{aligned} \tag{8}$$

where p is a positive integer, $x \in \mathbb{R}_+$, and a and b are constants such that $a < 0 < b$. Moreover, if the function $h(x,a,b)$ is infinite times differentiable with respect to x, then it is called a smooth signum function.

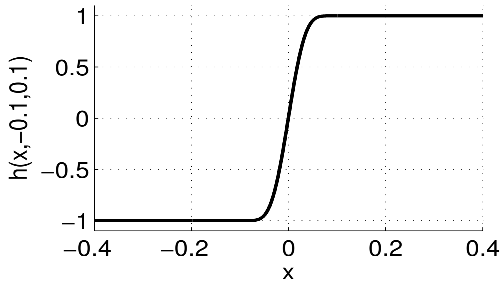

Fig. 2. A twice differentiable signum function.

Lemma 2. Let the scalar function $h(x,a,b)$ be defined as

$$h(x,a,b) = 2\frac{\int_a^x f(\tau - a)f(b-\tau)d\tau}{\int_a^b f(\tau - a)f(b-\tau)d\tau} - 1 \tag{9}$$

where the function $f(y)$ is defined as follows

$$f(y) = 0 \text{ if } y \leq 0 \text{ and } f(y) = y^p \text{ if } y > 0 \tag{10}$$

with p being a positive integer. Then $h(x,a,b)$ is a p times differentiable signum function. Moreover, if $f(y) = y^p$ in (10) is replaced by $f(y) = e^{-1/y}$ then property 4) is replaced by 4') $h(x,a,b)$ is a smooth signum function.

Proof. See Do [2007].

With materials presented in this subsection, for the purpose of control design in the next section we rewrite the entire system (1) as follows:

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= \theta_{21}x_3 - \theta_{22}x_2 + \Delta(t, x_1, x_2, z, \dot{z}),\\ \dot{x}_3 &= -\theta_{31}x_2 - \theta_{32}x_3 + \theta_{33}g_3(x_3, x_4),\\ \dot{x}_4 &= -\theta_{41}x_4 + \theta_{42}i_H\end{aligned} \tag{11}$$

with $x_1 = x_H, x_2 = \dot{x}_H, x_3 = \bar{P}_H, x_4 = \bar{x}_{Hv}$, and

$$\begin{aligned}&\theta_{21} = \frac{A_H C_{H3}}{m_H}, \theta_{22} = \frac{b_H}{m_H},\\ &\Delta(t, x_1, x_2, z, \dot{z}) = \frac{1}{m_H}\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z}),\\ &\theta_{31} = \frac{4\beta_{He}A_H}{V_H C_{H3}}, \theta_{32} = \frac{4\beta_{He}C_{HT}}{V_H},\\ &\theta_{33} = \frac{4\beta_{He}C_{HD}W_H C_{H4}}{V_H\sqrt{\rho_H}\sqrt{C_{H3}}}, \theta_{41} = \frac{1}{\tau_{Hv}}, \theta_{42} = \frac{k_{Hv}}{\tau_{Hv}C_{H4}},\\ &g_3(x_3, x_4) = x_4\sqrt{\bar{P}_{HS} - h(x_4, a, b)x_3}\end{aligned} \tag{12}$$

where we have used the p-times differentiable signum function $h(x_4, a, b)$ to replace the signum function $\text{sgn}(x_4)$.

3. CONTROL DESIGN

A close look at (11) shows that (11) is of a strick-feedback form, Krstic et al. [1995]. Therefore, we will use the backstepping technique to design the control input i_H to achieve the control objective. The control design consists of 4 steps. At steps 1 and 2, we will use the disturbance observer presented in Subsection 2.2.1 to design an estimate for $\dot{z}(t)$ and $\Delta(t, x_1, x_2, z, \dot{z})$, respectively.

3.1 Step 1

We consider the first equation of the entire system (11) where the state x_2 is considered as a control. Define

$$x_{1e} = x_1 + z(t) - L, \quad x_{2e} = x_2 - \alpha_1 \tag{13}$$

where α_1 is a virtual control of x_2. Differentiating both sides of the first equation of (13) along the solutions of the second equation of (13) and the first equation of (11) gives

$$\dot{x}_{1e} = \alpha_1 + x_{2e} + w \tag{14}$$

where $w := \dot{z}(t)$. From (14) and the disturbance observer design proposed in Subsection 2.2.1, we design the virtual control α_1 and the estimate $\hat{w}$ of w as follows:

$$\begin{aligned}\alpha_1 &= -k_{11}x_{1e} - \hat{w}, \quad \hat{w} = \xi_1 + k_{12}x_{1e}, \\ \dot{\xi}_1 &= -k_{12}\xi_1 - k_{12}(x_{2e} + \alpha_1 + k_{12}x_{1e})\end{aligned} \tag{15}$$

where k_{11} and k_{12} are positive constants. The above disturbance observer is an application from Subsection 2.2.1 with $\rho(x) = k_{12}x$. Using (14) and (15), we have

$$\dot{x}_{1e} = -k_{11}x_{1e} + x_{2e} + w_e, \quad \dot{w}_e = -k_{12}w_e + \dot{w} \tag{16}$$

where $w_e = w - \hat{w}$. It is of interest to note that the virtual control α_1 is a smooth function of x_{1e} and ξ_1.

3.2 Step 2

At this step, we consider the second equation of the entire system (11) where the state x_3 is considered as a control. As such, we define

$$x_{3e} = x_3 - \alpha_2 \tag{17}$$

where α_2 is a virtual control of x_3. Now differentiating both sides of the second equation of (13) along the solutions of the second equation of (11), (15) and (16) gives

$$\begin{aligned}\dot{x}_{2e} &= \theta_{21}(x_{3e} + \alpha_2) - \theta_{22}(x_{2e} + \alpha_1) + \Delta - \\ &\quad \frac{\partial \alpha_1}{\partial x_{1e}}(-k_{11}x_{1e} + x_{2e} + w_e) - \frac{\partial \alpha_1}{\partial \xi_1}\dot{\xi}_1.\end{aligned} \tag{18}$$

From (18), we choose the virtual control α_2 and an estimate of the disturbance Δ as follows

$$\begin{aligned}\alpha_2 &= \frac{1}{\theta_{21}}\bigg(- x_{1e} - k_{21}x_{2e} + \theta_{22}(x_{2e} + \alpha_1) + \\ &\quad \frac{\partial \alpha_1}{\partial x_{1e}}(-k_{11}x_{1e} + x_{2e}) + \frac{\partial \alpha_1}{\partial \xi_1}\dot{\xi}_1 - \dot{\hat{\Delta}}\bigg), \\ \hat{\Delta} &= \xi_2 + k_{22}x_{2e}, \\ \dot{\xi}_2 &= -k_{22}\xi_2 - k_{22}\bigg(\theta_{21}(x_{3e} + \alpha_2) - \theta_{22}(x_{2e} + \alpha_1) - \\ &\quad \frac{\partial \alpha_1}{\partial x_{1e}}(-k_{11}x_{1e} + x_{2e}) - \frac{\partial \alpha_1}{\partial \xi_1}\dot{\xi}_1\bigg)\end{aligned} \tag{19}$$

where k_{21} and k_{22} are positive constants. It is again noted that the above disturbance observer is an application from Subsection 2.2.1 where we took $\rho(x) = k_{22}x$. Using (18) and (19), we have

$$\begin{aligned}\dot{x}_{2e} &= -x_{1e} - k_{21}x_{2e} + \theta_{21}x_{3e} - \frac{\partial \alpha_1}{\partial x_{1e}}w_e + \Delta_e, \\ \dot{\Delta}_e &= -k_{22}\Delta_e + k_{22}\frac{\partial \alpha_1}{\partial x_{1e}}w_e + \dot{\Delta}\end{aligned} \tag{20}$$

where $\Delta_e = \Delta - \hat{\Delta}$. It is noted that the virtual control α_2 is a smooth function of x_{1e}, ξ_1, x_{2e} and ξ_2.

3.3 Step 3

At this step, we consider the third equation of the entire system (11) where $g_3(x_3, x_4)$ is considered as a control. As such, we define

$$x_{4e} = g_3(x_3, x_4) - \alpha_3 \tag{21}$$

where α_3 is a virtual control of $g_3(x_3, x_4)$. Now differentiating both sides of (17) along the solutions of the third equation of (11), (15), (16) and (20), and choosing the virtual control α_3 as

$$\begin{aligned}\alpha_3 &= \frac{1}{\theta_{33}}\bigg(- \theta_{21}x_{2e} - k_{31}x_{3e} + \theta_{31}x_2 + \theta_{32}x_3 + \frac{\partial \alpha_2}{\partial x_{1e}} \\ &\quad \times(-k_{11}x_{1e} + x_{2e}) + \frac{\partial \alpha_2}{\partial \xi_1}\dot{\xi}_1 + \\ &\quad \frac{\partial \alpha_2}{\partial x_{2e}}(-x_{1e} - k_{21}x_{2e} + \theta_{21}x_{3e}) + \frac{\partial \alpha_2}{\partial \xi_2}\dot{\xi}_2\bigg)\end{aligned} \tag{22}$$

where k_{31} is a positive constant, result in

$$\begin{aligned}\dot{x}_{3e} &= -\theta_{21}x_{2e} - k_{31}x_{3e} + \theta_{33}x_{4e} - \\ &\quad \bigg(\frac{\partial \alpha_2}{\partial x_{1e}} - \frac{\partial \alpha_2}{\partial x_{2e}}\frac{\partial \alpha_1}{\partial x_{1e}}\bigg)w_e - \frac{\partial \alpha_2}{\partial x_{2e}}\Delta_e.\end{aligned} \tag{23}$$

It is noted that the virtual control α_3 is a smooth function of $x_{1e}, \xi_1, x_{2e}, \xi_2, x_{3e}$ since $x_2 = x_{2e} + \alpha_1$ and $x_3 = x_{3e} + \alpha_2$.

3.4 Step 4

The actual control input i_H will be designed. Differentiating both sides of (21) along the solutions of the last equation of the entire system (11), and choosing the actual control i_H as follows

$$\begin{aligned}i_H &= \frac{1}{\theta_{42}\frac{\partial g_3}{\partial x_4}}\bigg(- \theta_{33}x_{3e} - k_{41}x_{4e} + \theta_{41}x_4 - \frac{\partial g_3}{\partial x_3}(-\theta_{31}x_2 \\ &\quad -\theta_{32}x_3 + \theta_{33}g_3(x_3, x_4)) + \frac{\partial \alpha_3}{\partial \xi_1}\dot{\xi}_1 + \frac{\partial \alpha_3}{\partial \xi_2}\dot{\xi}_2 \\ &\quad + \frac{\partial \alpha_3}{\partial x_{1e}}(-k_{11}x_{1e} + x_{2e}) + \frac{\partial \alpha_3}{\partial x_{2e}}(-x_{1e} - k_{21}x_{2e} + \\ &\quad \theta_{21}x_{3e}) + \frac{\partial \alpha_3}{\partial x_{3e}}(-\theta_{21}x_{2e} - k_{31}x_{3e} + \theta_{33}x_{4e})\bigg)\end{aligned} \tag{24}$$

where k_{41} is a positive constant, result in

$$\begin{aligned}\dot{x}_{4e} &= -\theta_{33}x_{3e} - k_{41}x_{4e} - \bigg(\frac{\partial \alpha_3}{\partial x_{1e}} - \frac{\partial \alpha_3}{\partial x_{2e}}\frac{\partial \alpha_1}{\partial x_{1e}} - \frac{\partial \alpha_3}{\partial x_{3e}} \times \\ &\quad \frac{\partial \alpha_2}{\partial x_{1e}} + \frac{\partial \alpha_3}{\partial x_{3e}}\frac{\partial \alpha_2}{\partial x_{2e}}\frac{\partial \alpha_1}{\partial x_{1e}}\bigg)w_e - \bigg(\frac{\partial \alpha_3}{\partial x_{2e}} - \frac{\partial \alpha_3}{\partial x_{3e}}\frac{\partial \alpha_3}{\partial x_{2e}}\bigg)\Delta_e.\end{aligned} \tag{25}$$

Since $P_{HS} - |P_H| = C_{H3}(\bar{P}_{Hs} - |x_3|) \geq \epsilon_1$ and $\frac{\partial g_3}{\partial x_4} = \sqrt{\bar{P}_{HS} - x_3h(x_4a, b)} - \frac{x_3x_4\frac{\partial h(x_4, a, b)}{\partial x_4}}{2\sqrt{\bar{P}_{HS} - x_3h(x_4a, b)}}$, from properties of the p-times differentiable signum function $h(x_4, a, b)$, we can see that there exists a strictly positive constant

ϵ_2 such that $|\frac{\partial g_3}{\partial x_4}| \geq \epsilon_2$. This means that the control i_H given in (24) is well-defined. The control design has been completed. We now state the main result of the paper in the following theorem.

Theorem 1. Assume that $z(t)$ and $\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z})$ and their derivatives are bounded, the control input i_H given in (24) and the disturbance observers given in (15) and (19) solve the control objective. In particular, the closed loop system consisting of (16), (20), (23) and (25) is forward complete, and the error $x_{1e}(t) = x_1(t) + z(t) - L$ exponentially converges to a small value. This value can be made arbitrarily small by choosing sufficiently large control and observer gains k_{11}, k_{12}, k_{21}, k_{22}, k_{31}, and k_{41}. If the derivatives of $z(t)$ and $\tilde{\Delta}(t, x_H, z, \dot{x}_H, \dot{z})$ are zero, the active heave compensation error $x_{1e}(t) = x_1(t) + z(t) - L$ exponentially converges to zero.

Proof For convenience, we rewrite the closed loop system consisting of (16), (20), (23) and (25) as follows:

$$\begin{aligned}
\dot{x}_{1e} &= -k_{11}x_{1e} + x_{2e} + w_e, \\
\dot{x}_{2e} &= -x_{1e} - k_{21}x_{2e} + \theta_{21}x_{3e} - \phi_1 w_e + \Delta_e, \\
\dot{x}_{3e} &= -\theta_{21}x_{2e} - k_{31}x_{3e} + \theta_{33}x_{4e} - \phi_2 w_e - \phi_3\Delta_e, \\
\dot{x}_{4e} &= -\theta_{33}x_{3e} - k_{41}x_{4e} - \phi_4 w_e - \phi_5\Delta_e, \\
\dot{w}_e &= -k_{12}w_e + \dot{w}, \\
\dot{\Delta}_e &= -k_{22}\Delta_e + k_{22}\phi_1 w_e + \dot{\Delta}
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\phi_1 &= \frac{\partial\alpha_1}{\partial x_{1e}}, \quad \phi_2 = \Big(\frac{\partial\alpha_2}{\partial x_{1e}} - \frac{\partial\alpha_2}{\partial x_{2e}}\frac{\partial\alpha_1}{\partial x_{1e}}\Big), \quad \phi_3 = \frac{\partial\alpha_2}{\partial x_{2e}} \\
\phi_4 &= \Big(\frac{\partial\alpha_3}{\partial x_{1e}} - \frac{\partial\alpha_3}{\partial x_{2e}}\frac{\partial\alpha_1}{\partial x_{1e}} - \frac{\partial\alpha_3}{\partial x_{3e}}\frac{\partial\alpha_2}{\partial x_{1e}} + \frac{\partial\alpha_3}{\partial x_{3e}}\frac{\partial\alpha_2}{\partial x_{2e}}\frac{\partial\alpha_1}{\partial x_{1e}}\Big) \\
\phi_5 &= \Big(\frac{\partial\alpha_3}{\partial x_{2e}} - \frac{\partial\alpha_3}{\partial x_{3e}}\frac{\partial\alpha_3}{\partial x_{2e}}\Big).
\end{aligned} \tag{27}$$

From the expressions of α_1, α_2 and α_3, see (15), (19) and (22), there exist nonnegative constants A_i, $i = 1, ..., 5$ such that

$$|\phi_i| \leq A_i, \ i = 1, ..., 5. \tag{28}$$

To investigate stability of the closed loop system (26), we consider the following Lyapunov function candidate

$$V = \frac{1}{2}\big(x_{1e}^2 + x_{2e}^2 + x_{3e}^2 + x_{4e}^2 + \delta_1 w_e^2 + \delta_2\Delta_e^2\big) \tag{29}$$

where δ_1 and δ_2 are positive constants to be picked later.

Differentiating both sides of (29) along the solutions of (26), and using (28), we have

$$\begin{aligned}
\dot{V} \leq& -\frac{k_{11}}{2}x_{1e}^2 - \frac{k_{21}}{2}x_{2e}^2 - \frac{k_{31}}{2}x_{3e}^2 - \frac{k_{41}}{2}x_{4e}^2 - \Big(\delta_1 k_{12} - \\
&\frac{1}{2k_{11}} - \frac{A_1^2}{k_{21}} - \frac{A_2^2}{k_{31}} - \frac{A_4^2}{k_{41}} - \delta_1\epsilon_1 - \frac{k_{22}A_1\delta_2}{4\epsilon_2}\Big)w_e^2 \\
&-\Big(\delta_2 k_{22} - \frac{1}{k_{21}} - \frac{A_3^2}{k_{31}} - \frac{A_5^2}{k_{41}} - \epsilon_2 k_{22}A_1\delta_2\Big)\Delta_e^2 + \\
&\frac{\delta_1}{4\epsilon_1}\dot{w}^2 + \frac{\delta_2}{4\epsilon_3}\dot{\Delta}^2
\end{aligned} \tag{30}$$

where ϵ_i, $i = 1, 2, 3$ are positive constants to be picked. Now for chosen control gains k_{11}, k_{21}, k_{31}, k_{41}, k_{12} and k_{22}, we can always pick the positive constants δ_1, δ_2 and ϵ_i, $i = 1, 2, 3$ such that

$$\begin{aligned}
&\Big(\delta_1 k_{12} - \frac{1}{2k_{11}} - \frac{A_1^2}{k_{21}} - \frac{A_2^2}{k_{31}} - \frac{A_4^2}{k_{41}} - \delta_1\epsilon_1 - \frac{k_{22}A_1\delta_2}{4\epsilon_2}\Big) \geq \frac{\gamma_1}{2}, \\
&\Big(\delta_2 k_{22} - \frac{1}{k_{21}} - \frac{A_3^2}{k_{31}} - \frac{A_5^2}{k_{41}} - \epsilon_2 k_{22}A_1\delta_2\Big) \geq \frac{\gamma_2}{2}
\end{aligned} \tag{31}$$

where γ_1 and γ_2 are positive constants. Substituting (31) into (30) results in

$$\dot{V} \leq -cV + \lambda \tag{32}$$

where c is some positive constant and λ is a nonnegative constant. Solving the differential inequality (32) shows that V exponentially converges to a ball centered at the origin with the radius $R_V = \lambda/c$. This implies that the closed loop system is forward complete. Convergence of V implies that all the errors x_{ie}, $i = 1, ..., 4$, and w_e and Δ_e exponentially converge to a ball centered at the origin with a radius $R_e = \sqrt{\frac{\lambda}{c\min(1,\delta_1,\delta_2)}}$. Moreover, in the case where the derivatives of the disturbances w and Δ can be ignored, we have $\lambda = 0$. This means that all the state errors x_{ie}, $i = 1, ..., 4$, and the disturbance observer errors w_e and Δ_e exponentially converge zero. □

4. SIMULATIONS

The parameters of the hydraulic system are taken based on Yao et al. [2001] as follows: $m_H = 1000kg, A_H = 0.65m^2, b_H = 40N/(m/s), 4\beta_{He}/V_H = 4.53 \times 10^8 N/m^5$, $C_{HD} = 2.21 \times 10^{-14} m^5/Ns$, $C_{HD}w_H/\sqrt{\rho} = 3.42 \times 10^{-5} m^3\sqrt{N}s, P_{HS} = 10342500Pa, k_{Hv} = 0.0324, \tau_{Hv} = 0.00636$. The scale factors are taken as $C_{H3} = 6 \times 10^5, C_{H4} = 5 \times 10^{-7}$ to scale P_{HS} down and τ_{Hv} up as discussed in Subsection 2.1. The riser parameters are taken from Korde [1998] as follows: length $L = 3832m$, diameter $d_r = 0.14m$, density $\rho_r = 8200kg/m^3$, Young's modulus $E_r = 2\times10^8 kg/m^2$, initial tension $T_{or} = 100KN$. Therefore, the force (disturbance), $\tilde{\Delta}$ acting from the riser to the piston of the hydraulic system is Korde [1998]:

$$\tilde{\Delta} = T_{or} + E_r A_r\Big(\sum_{n=1}^{N_m} \mp n\pi C_n + \delta\Big) \tag{33}$$

with C_n being generated by

$$\ddot{C}_n + c_d\dot{C}_n + \frac{E_r A_r n^2\pi^2}{\rho_r L^2}C_n = \mp\frac{2}{n\pi}\big(c_d\dot{\delta} + \ddot{\delta}\big) \tag{34}$$

where $\delta = z - x_H - L$, $A_r = \frac{\pi d_r^2}{4}$, the damping coefficient $c_d = 0.01m^3/s$, and the notation $\mp$ takes the positive sign if $n = 2, 4, 6, ...$ and the negative sign if $n = 1, 3, 5, ...$, and N_m is number of modes. The vertical (heave) motion of the vessel $z(t)$ can be represented as a sum of sinusoids at different frequencies, amplitudes and phases as follows, see Fossen [1994] and Lloyd [1998]:

$$z(t) = L + \sum_{i=1}^{N_w}\Big(A_{wi}k_{wi}\sin(w_{wi}t - \varphi_{wi})\Big) \tag{35}$$

where L is included since $z(t)$ is coordinated in the earth-fixed frame, the amplitude A_{wi}, coefficient k_{wi}, frequency w_{wi}, phase φ_{wi} of the wave i^{th} are given by

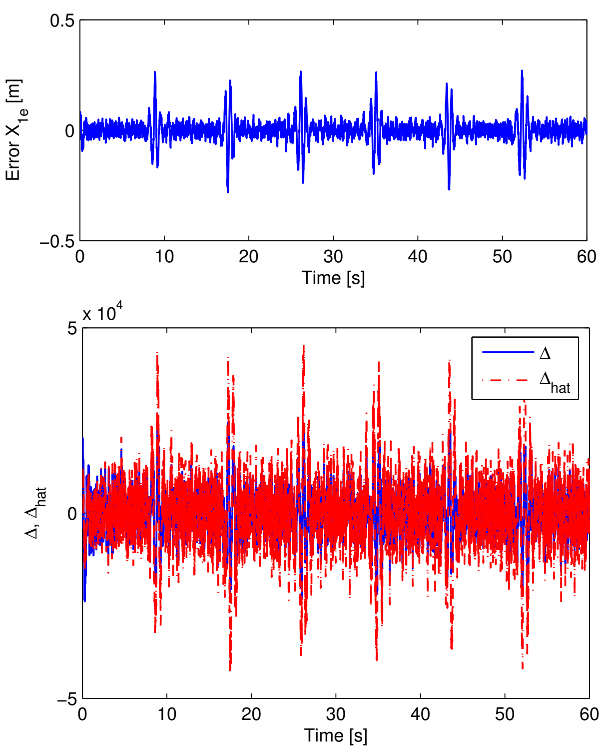

Fig. 3. Simulation result with disturbance observers.

$$w_{wi} = w_m + \frac{w_m - w_M}{N_w} i, \quad S_{wi} = \frac{1.25}{4} \frac{w_o^4}{w_{wi}^5} H_{sw}^2 e^{-1.25\left(\frac{w_o}{w_{wi}}\right)^4}$$
$$A_{wi} = \sqrt{2 S_{wi} \frac{w_{mi} - w_{Mi}}{N_w}}, \; k_{wi} = \frac{w_{wi}^2}{9.8}, \; \varphi_{wi} = 2\pi \mathrm{rand}(). \qquad (36)$$

In (36), the minimum and maximum wave frequencies are $w_m = 0.2$rand/s, $w_M = 2.5$rand/s; the two-parameter Bretschneider spectrum S_{wi} is used with the significant wave height $H_{sw} = 4m$; the modal frequency is $w_o = \frac{2\pi}{T_w}$ with the period $T_w = 7.8$; $N_w = 10$; and rand() is a random number between 0 and 1. The control and disturbance observer gains are chosen as $k_{11} = 2, k_{12} = 15, k_{21} = 4, k_{22} = 25, k_{31} = 8, k_{41} = 12$. In the simulations, we take the number of modes in (33) as $N_m = 5$, and initial values of C_n in (34) are zero. To illustrate the effectiveness of the disturbance observers, we simulate the proposed controller in two cases. In the first case, the disturbance observers are switched on. The simulation result is presented in Figure 3. In Figure 3, the top sub-figure plots the error x_{1e}, the bottom sub-figure plots the disturbance Δ and its estimate $\hat{\Delta}$. From the bottom sub-figure, we can see that the disturbance observer estimates the disturbance Δ quite well. This results in pretty small error x_{1e}. It is noted that the error x_{1e} does not converge to zero as said Theorem 1 because Δ and $\ddot{z}(t)$ are time-varying. In the second case, the disturbance observers are switched-off. The simulation result is presented in Figure 4. From this figure, we can see that when the disturbance observers are switched-off, the error x_{1e} is pretty large. This is because the controller does not compensate for the disturbances via the disturbance observers.

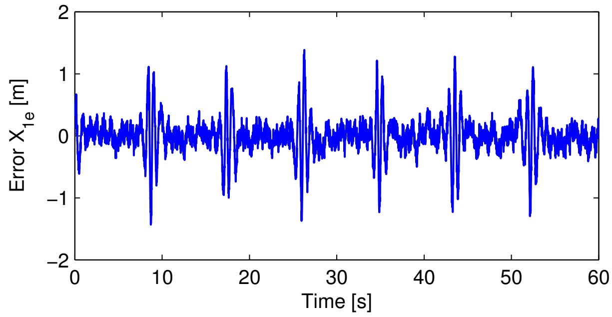

Fig. 4. Simulation result without disturbance observers.

5. CONCLUSION

A nonlinear controller has been designed for the active heave compensation system. One of the keys to success in the proposed method is the use of the disturbance observers, which are then properly embedded in the control design procedure. The heave velocity and the force acting between the riser/drill-string and the active heave compensation unit were well estimated by the disturbances observers.

REFERENCES

K. D. Do. Bounded controllers for formation stabilization of mobile agents with limited sensing ranges. *IEEE Transactions on Automatic Control*, 52(3):569–576, 2007.

K. D. Do and J. Pan. Nonlinear control of an active heave compensation system. *Ocean Engineering*, 2007.

K.D. Do, Z.P. Jiang, and J. Pan. Universal controllers for stabilization and tracking of underactuated ships. *Systems and Control Letters*, 47:299–317, 2002.

T. I. Fossen. *Guidance and Control of Ocean Vehicles*. John Willey and Sons, New York, 1994.

T. I. Fossen and J. P. Strand. Nonlinear passive weather optimal positioning control (wopc) system for ships and rigs: Experimental results. *Automatica*, 37:701–715, 2001.

T.A. Johansen, T. I. Fossen, S. I. Sagatun, and F.G. Nielsen. Wave synchronizing crane control during water entry in offshore moonpool operations - experimental results. *IEEE Journal of Oceanic Engineering*, 28:720 – 728, 2003.

U. A. Korde. Active heave compensation on drill-ships in irregular waves. *Ocean Engineering*, 25(7):541–561, 1998.

M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic. *Nonlinear and adaptive control design*. Wiley, New York, 1995.

A.R.J.M. Lloyd. *Seakeeping Ship Behaviour in Rough Weather*. Ellis Horwood Ltd., 1998.

T. Sarpkaya. *Mechanics of wave forces on offshore structures*. Van Nostrand Reinhold Co, New York, 1981.

B. Skaare and O. Egeland. Parallel force/position crane control in marine operations. *IEEE Journal of Oceanic Engineering*, 31:599 – 613, 2006.

B. Yao, F. Bu, and G. T. C. Chiu. Non-linear adaptive robust control of electro-hydraulic systems driven by double-rod actuators. *International Journal of Control*, 74(8):761–775, 2001.