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Stability of Zero Dynamics of Sampled-Data Nonlinear Systems

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Abstract: One of the approaches to sampled-data controller design for nonlinear continuous-time systems consists of obtaining an appropriate model and then proceeding to design a controller for the model. Hence, it is important to derive a good approximate sampled-data model because the exact sampled-data model for nonlinear systems is often unavailable to the controller designers. Recently, Yuz and Goodwin have proposed an accurate approximate model which includes extra zero dynamics corresponding to the relative degree of the continuous-time nonlinear system. Such extra zero dynamics are called sampling zero dynamics. A more accurate sampled-data model is, however, required when the relative degree of a continuous-time nonlinear plant is two. The reason is that the closed-loop system becomes unstable when the more accurate sampled-data model has unstable sampling zero dynamics and a controller design method based on cancellation of the zero dynamics is applied. This paper derives the sampling zero dynamics of the more accurate sampled-data model and shows a condition which assures the stability of the sampling zero dynamics of the model. Further, it is shown that this extends a well-known result for the stability condition of linear systems to the nonlinear case.

1. INTRODUCTION

Advances in digital electronics that occurred in the second half of the 20th century have led to a rapid development in computer technology and this has had a great impact on engineering areas, including control engineering. Since recent control systems usually employ digital technology for controller implementation, the study of sampled-data control systems has become an important issue in control fields. Significant progress has been achieved in this area during this decade.

There are two distinct approaches to sampled-data controller design for nonlinear systems (Laila *et al.* (2006)). The first one, so-called controller emulation, involves digital implementation of a continuous-time stabilizing control law at a sufficient high sampling rate. The second approach consists of obtaining a sampled-data model and then proceeding to design a controller for the model. Emulation is regarded as the simple method, whereas it is typically inferior to the second in terms of stability and achievable performance. On the other hand, the second approach requires a good approximate sampled-data model because the exact sampled-data model for nonlinear systems is often unavailable to the controller designers.

Therefore, the accuracy of the approximate sampled-data model has proven to be a key issue in the context of control design, where a controller designed to stabilize an approximate model may fail to stabilize the exact discrete-time model (Nešić and Teel (2004)).

Recently, Yuz and Goodwin have proposed an accurate approximate model (Yuz and Goodwin (2005)). The resulting model includes extra zero dynamics which are called sampling zero dynamics. It has been shown explicitly that they have no counterpart in the underlying continuous-time system and are the same as those for linear case (Åström *et al.* (1984)), although an implicit characterization has been given in (Monaco and Normand-Cyrot (1988)). It is worth noting here that Yuz and Goodwin's model has a mode corresponding to the sampling zero dynamics just on the unit circle when the relative degree of a continuous-time nonlinear system is two. For linear systems, the sampling zeros correspond to the sampling zero dynamics for nonlinear systems. The properties of the sampling zeros for linear systems have been discussed in many papers (Åström *et al.* (1984), Hagiwara *et al.* (1993), Passino and Antsaklis (1988), Ishitobi (1996), Hayakawa *et al.* (1983), Weller (1999), Ishitobi (2000)).

A previous study shows that to derive a more accurate sampled-data model is required when the relative degree of a continuous-time nonlinear plant is two (Ishitobi and Nishi (2008)). The reason is that the closed-loop system becomes unstable when the more accurate sampled-data model has unstable sampling zero dynamics and a controller design method based on cancellation of the zero dynamics is applied.

This paper derives the sampling zero dynamics of the more accurate sampled-data model than that of Yuz and Goodwin, and shows a condition which assures the

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stability of the sampling zero dynamics of the model. Further, the relationship between the result of this paper and that for linear systems is discussed.

2. SYSTEM DESCRIPTION AND PREVIOUS RESULTS

Consider a class of the following single-input single-output nth-order nonlinear system

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u \\ y = h(\boldsymbol{x}) \end{cases} \tag{1}$$

where $\boldsymbol{x}$ is the state evolving on an open subset $\mathcal{M} \subset R^n$, and where the vector fields $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$, and the output function $h(\boldsymbol{x})$ are analytic on $\mathcal{M}$.

First, the following assumptions are introduced.

Assumption 1: The unique equilibrium point lies on the origin.

Assumption 2: The continuous-time nonlinear system (1) has the uniform relative degree $r(\leq n)$ and is minimum phase in the open subset $\mathcal{M}$, where the state $\boldsymbol{x}$ evolves.

Then, the system can be expressed in its so-called normal form (Isidori (1995), Khalil (2002)).

$$\begin{cases} \dot{\boldsymbol{\zeta}} = \begin{bmatrix} \mathbf{0}_{r-1} & I_{r-1} \\ 0 & \mathbf{0}_{r-1}^T \end{bmatrix} \boldsymbol{\zeta} \\ \qquad + \begin{bmatrix} \mathbf{0}_{r-1} \\ 1 \end{bmatrix} (b(\boldsymbol{\zeta},\ \boldsymbol{\eta}) + a(\boldsymbol{\zeta},\ \boldsymbol{\eta})u) \\ \dot{\boldsymbol{\eta}} = \boldsymbol{c}(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \\ y = z_1 \end{cases} \tag{2}$$

$$\boldsymbol{\zeta} = \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix},\ \boldsymbol{\eta} = \begin{bmatrix} z_{r+1} \\ \vdots \\ z_n \end{bmatrix}, \tag{3}$$

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{\zeta} \\ \boldsymbol{\eta} \end{bmatrix},\ \boldsymbol{c} = \begin{bmatrix} c_{r+1}(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \\ \vdots \\ c_n(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \end{bmatrix} \tag{4}$$

where $a(\mathbf{0},\mathbf{0}) \neq 0$, $b(\mathbf{0},\mathbf{0}) = 0$ and $\boldsymbol{c}(\mathbf{0},\mathbf{0}) = \mathbf{0}$.

Under the assumptions 1 and 2, the zero dynamics of (2) is determined by

$$\dot{\boldsymbol{\eta}} = \boldsymbol{c}(\mathbf{0},\ \boldsymbol{\eta}) \tag{5}$$

and is asymptotically stable in $\mathcal{M}$.

We are interested in the sampled-data model for the nonlinear system (2) when the input is a piecewise constant signal generated by a zero-order hold (ZOH); i.e.,

$$u(t) = u(kT),\ kT \leq t < (k+1)T, \quad k = 0,\ 1, \cdots \tag{6}$$

where T is a sampling period.

For small sampling periods, Yuz and Goodwin have derived an approximate sampled-data model of the following form

$$\begin{cases} \boldsymbol{\zeta}_{k+1} = F_s \boldsymbol{\zeta}_k + \boldsymbol{g}_s (b_k + a_k u_k) \\ \boldsymbol{\eta}_{k+1} = \boldsymbol{\eta}_k + T\boldsymbol{c}(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k) \\ y_k = [1\ \mathbf{0}_{r-1}^T]\boldsymbol{\zeta}_k \end{cases} \tag{7}$$

$$F_s = \begin{bmatrix} 1 & T & \frac{T^2}{2} & \cdots & \frac{T^{r-1}}{(r-1)!} \\ 0 & 1 & T & \cdots & \frac{T^{r-2}}{(r-2)!} \\ & & \ddots & \ddots & \vdots \\ & O & & \ddots & T \\ & & & & 1 \end{bmatrix},\ \boldsymbol{g}_s = \begin{bmatrix} \frac{T^r}{r!} \\ \frac{T^{r-1}}{(r-1)!} \\ \vdots \\ \frac{T^2}{2!} \\ T \end{bmatrix} \tag{8}$$

$$b_k \equiv b(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k),\ a_k \equiv a(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k) \tag{9}$$

where the subscripts k and $k+1$ denote the time instants kT and $(k+1)T$, respectively.

Then, the zero dynamics of the sampled-data model (7) consist of the sampled counterpart of the continuous-time zero dynamics and the additional zero dynamics produced by the sampling process (Yuz and Goodwin (2005)). The latter are called the sampling zero dynamics, and equivalent to the same as those which appear asymptotically for the linear case when the sampling period tends to zero, namely, the roots of the following equations.

$$\begin{aligned} & z + 1 = 0,\ r = 2 \\ & z^2 + 4z + 1 = 0,\ r = 3 \\ & (z+1)(z^2 + 10z + 1) = 0,\ r = 4 \\ & \vdots \end{aligned}$$

It thus follows that a continuous-time nonlinear system with the relative degree r larger that three will always give a sampled-data model with the sampling zero dynamics strictly outside the unit disc. The question is what happens in the case when the relative degree r is two and the sampled-data model (7) has the sampling zero dynamics just on the unit circle. It is an important issue because the relative degree of many nonlinear systems such as mechanical systems in the practical field is two.

For example, consider a controlled Van der Pol system with the following equation (Khalil (2002)).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + \epsilon(1 - x_1^2)x_2 + u,\ \epsilon > 0 \\ y = x_1 \end{cases} \tag{10}$$

It is obvious that the relative degree of the system (10) is two, and that the system does not have zero dynamics.

A sampled-data model by Yuz and Goodwin for (10) is represented as

$$\begin{cases} x_{1,k+1} = x_{1,k} + Tx_{2,k} \\ \qquad + \frac{T^2}{2}\left[-x_{1,k} + \epsilon(1 - x_{1,k}^2)x_{2,k} + u_k\right] \\ x_{2,k+1} = x_{2,k} + T\left[-x_{1,k} + \epsilon(1 - x_{1,k}^2)x_{2,k} + u_k\right] \\ y_k = x_{1,k} \end{cases} \tag{11}$$

When we design a discrete-time model following controller on the basis of the sampled-data model (11), and apply it to the original continuous-time system through a ZOH, then the input and the output result in divergence in simulation (Ishitobi and Nishi (2008)). The reason is that the sampling zero dynamics of the more accurate sampled-data model is unstable. Hence, it is important to study the stability condition of the sampling zero dynamics of the more accurate sampled-data model for nonlinear systems with the relative degree two.

3. MAIN RESULTS

This section studies when a more accurate model for a continuous-time system with the relative degree two has the sampling zero dynamics inside or outside of the unit circle for sufficiently small sampling periods.

Before proceeding, the following assumptions are needed here.

Assumption 3:

$$\frac{\partial a(\boldsymbol{\zeta},\ \boldsymbol{\eta})}{\partial z_2} = 0 \tag{12}$$

Assumption 4:

$$\frac{\partial \boldsymbol{c}(\boldsymbol{\zeta},\ \boldsymbol{\eta})}{\partial z_2} = \boldsymbol{0} \tag{13}$$

The assumption 3 ensures that a new sampled-data system is also an affine one. The assumption 4 implies that the vector $\boldsymbol{c}(\boldsymbol{\zeta},\ \boldsymbol{\eta})$ does not include a term of z_2.

Under the use of a ZOH and the assumptions 3 and 4, it is easy to obtain the relations

$$\dot{y} = \dot{z}_1 = z_2 \tag{14}$$

$$\ddot{y} = \dot{z}_2 = b + au \tag{15}$$

$$y^{(3)} = \overline{b} + \overline{a}u \tag{16}$$

where

$$\overline{b} = \overline{b}(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \equiv \frac{\partial b}{\partial z_1}z_2 + \frac{\partial b}{\partial z_2}b + \sum_{i=3}^{n}\frac{\partial b}{\partial z_i}c_i \tag{17}$$

$$\overline{a} = \overline{a}(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \equiv \frac{\partial b}{\partial z_2}a + \frac{\partial a}{\partial z_1}z_2 + \sum_{i=3}^{n}\frac{\partial a}{\partial z_i}c_i \tag{18}$$

Here, substitute the relations (14)-(16) to the Taylor's expansion forms of $y((k+1)T)$ and $\dot{y}((k+1)T)$ and neglect higher order terms, then for sufficiently small sampling periods we obtain a more accurate model (Ishitobi and Nishi (2008)) than that of Yuz and Goodwin as

$$y_{k+1} = y_k + T\dot{y}_k + \frac{T^2}{2}b_k + \frac{T^3}{6}\overline{b}_k + \left(\frac{T^2}{2}a_k + \frac{T^3}{6}\overline{a}_k\right)u_k \tag{19}$$

$$\dot{y}_{k+1} = \dot{y}_k + Tb_k + \frac{T^2}{2}\overline{b}_k + \left(Ta_k + \frac{T^2}{2}\overline{a}_k\right)u_k \tag{20}$$

$$\boldsymbol{\eta}_{k+1} = \boldsymbol{\eta}_k + T\boldsymbol{c}(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k) \tag{21}$$

where

$$\overline{b}_k \equiv \overline{b}(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k),\ \overline{a}_k \equiv \overline{a}(\boldsymbol{\zeta}_k,\ \boldsymbol{\eta}_k) \tag{22}$$

Further, under the assumption 4, the sampled-data system (19)-(21) has the sampled counterpart of the continuous-time zero dynamics given by

$$\boldsymbol{\eta}_{k+1} = \boldsymbol{\eta}_k + T\boldsymbol{c}(\boldsymbol{0},\ \boldsymbol{\eta}_{\mathbf{k}}) \tag{23}$$

since $\boldsymbol{c}(\boldsymbol{\zeta},\ \boldsymbol{\eta})$ does not include a term of z_2.

The main result of this paper is given by the following theorem.

Theorem 1. Consider an affine nonlinear system (2) with the relative degree two. Then, for sufficiently small sampling periods the sampling zero dynamics of the sampled-data model (19)-(21) are given approximately by

$$\dot{y}_{k+1} + \dot{y}_k + \frac{\overline{a}_{k0}}{3a_{k0}}T\dot{y}_k = 0 \tag{24}$$

where a_{k0} and $\overline{a}_{k0}$ are the values of a_k and $\overline{a}_k$, respectively, with $y_k = 0$ and $\boldsymbol{\eta} = \boldsymbol{\eta}^S$ where $\boldsymbol{\eta}^S$ is the state vector of the sampled counterpart of the continuous-time zero dynamics. Further, a_k and $\overline{a}_k$ are defined by (9) and (22), respectively.

Proof. On the basis of the result in (Yuz and Goodwin (2005)), the sampling zero dynamics of the model (19)-(21) are calculated below. First, when we set $y_{k+1} = y_k = 0$, then (19) leads to

$$T\dot{y}_k + \frac{T^2}{2}b_{k0} + \frac{T^3}{6}\overline{b}_{k0} + \left(\frac{T^2}{2}a_{k0} + \frac{T^3}{6}\overline{a}_{k0}\right)u_k = 0 \tag{25}$$

where $b_{k0},\ a_{k0},\ \overline{b}_{k0},\ \overline{a}_{k0}$ denote the values of $b_k,\ a_k,\ \overline{b}_k,\ \overline{a}_k$ with $y_k = 0$ and $\boldsymbol{\eta}_k = \boldsymbol{\eta}^S$ where $\boldsymbol{\eta}^S$ is the state vector of the sampled counterpart (23) of the continuous-time zero dynamics (Yuz and Goodwin (2005)). Deleting u_k in (20) by (25) yields

$$\begin{aligned} \dot{y}_{k+1} &= \dot{y}_k + Tb_{k0} + \frac{T^2}{2}\overline{b}_{k0} \\ &\quad - \frac{6}{(3a_{k0} + T\overline{a}_{k0})T^2}\left(Ta_{k0} + \frac{T^2}{2}\overline{a}_{k0}\right) \\ &\quad \times \left(T\dot{y}_k + \frac{T^2}{2}b_{k0} + \frac{T^3}{6}\overline{b}_{k0}\right) \\ &= \dot{y}_k + \frac{T}{2}\left(2b_{k0} + T\overline{b}_{k0}\right) \end{aligned}$$

$$-\frac{2a_{k0}+T\overline{a}_{k0}}{2(3a_{k0}+T\overline{a}_{k0})}\left(6\dot{y}_k+3Tb_{k0}+T^2\overline{b}_{k0}\right)\quad(26)$$

Here, the coefficient of (26) is approximated for sufficiently small sampling periods as

$$\begin{aligned}\frac{2a_{k0}+T\overline{a}_{k0}}{3a_{k0}+T\overline{a}_{k0}} &= \frac{2a_{k0}\left(1+\dfrac{T\overline{a}_{k0}}{2a_{k0}}\right)}{3a_{k0}\left(1+\dfrac{T\overline{a}_{k0}}{3a_{k0}}\right)} \\ &\approx \frac{2}{3}\left(1+\frac{T\overline{a}_{k0}}{2a_{k0}}\right)\left(1-\frac{T\overline{a}_{k0}}{3a_{k0}}\right) \\ &\approx \frac{2}{3}\left\{1+\left(\frac{\overline{a}_{k0}}{2a_{k0}}-\frac{\overline{a}_{k0}}{3a_{k0}}\right)T\right\} \\ &= \frac{2}{3}\left(1+\frac{\overline{a}_{k0}}{6a_{k0}}T\right) \qquad (27)\end{aligned}$$

Hence, the relation (26) is rewritten as

$$\begin{aligned}\dot{y}_{k+1} &\approx \dot{y}_k+\frac{T}{2}\left(2b_{k0}+T\overline{b}_{k0}\right) \\ &\quad -\frac{1}{3}\left(1+\frac{\overline{a}_{k0}}{6a_{k0}}T\right)\left(6\dot{y}_k+3Tb_{k0}\right) \\ &\approx -\dot{y}_k+\left\{b_{k0}-\frac{1}{3}\left(\frac{\overline{a}_{k0}}{a_{k0}}\dot{y}_k+3b_{k0}\right)\right\}T \\ &= -\dot{y}_k-\frac{\overline{a}_{k0}}{3a_{k0}}T\dot{y}_k \qquad (28)\end{aligned}$$

As a result, the sampling zero dynamics are given by (28) for sufficiently small sampling periods. This completes the proof.

It is easy to obtain the following corollary from the theorem 1.

Corollary 2. For sufficiently small sampling periods, the sampling zero dynamics of the sampled-data model (19)-(21) are stable if

$$\frac{\overline{a}_{k0}}{3a_{k0}}<0 \qquad (29)$$

and they are unstable if

$$\frac{\overline{a}_{k0}}{3a_{k0}}>0 \qquad (30)$$

Remark 1. When the assumption 3 is not fulfilled, the term $(b+au)\partial a/\partial z_2$ is included in $\overline{b}$ of (17) and there may appear the term u^2 in (16).

Remark 2. From (21) and (24), it is found that the sampled counterpart of the continuous-time zero dynamics and the sampling zero dynamics cannot be determined separately when the assumption 4 is not satisfied.

4. RELATIONSHIP WITH THE RESULT OF THE LINEAR CASE

Consider the following nth order minimum phase transfer function with the relative degree two

$$\begin{aligned}G(s) &= \frac{N(s)}{D(s)}, \\ N(s) &= b_{n-2}s^{n-2}+\cdots+b_0,\ b_{n-2}\neq 0 \\ D(s) &= s^n+a_{n-1}s^{n-1}+\cdots+a_0 \qquad (31)\end{aligned}$$

Then, the normal form corresponding to (31) is represented (Khalil (2002)) as

$$\begin{cases}\dot{\boldsymbol{\zeta}} = \begin{bmatrix}0&1\\0&0\end{bmatrix}\boldsymbol{\zeta} \\ \qquad +\begin{bmatrix}0\\1\end{bmatrix}(b(\boldsymbol{\zeta},\ \boldsymbol{\eta})+a(\boldsymbol{\zeta},\ \boldsymbol{\eta})u) \\ \dot{\boldsymbol{\eta}} = A_0\boldsymbol{\eta}+B_0C_c^T\boldsymbol{\zeta} \\ y = C_c^T\boldsymbol{\zeta}=z_1,\ C_c^T=[1\ 0]\end{cases} \qquad (32)$$

$$\boldsymbol{\zeta}=\begin{bmatrix}z_1\\z_2\end{bmatrix},\ \boldsymbol{\eta}=\begin{bmatrix}z_3\\ \vdots\\ z_n\end{bmatrix},\ \boldsymbol{z}=\begin{bmatrix}\boldsymbol{\zeta}\\ \boldsymbol{\eta}\end{bmatrix} \qquad (33)$$

where A_0 is an $(n-2)\times(n-2)$ stable matrix and B_0 is an $(n-2)$th vector. Further, we have

$$a(\boldsymbol{\zeta},\ \boldsymbol{\eta})=b_{n-2},\ b(\boldsymbol{\zeta},\ \boldsymbol{\eta})=\boldsymbol{p}^T\boldsymbol{\zeta}-b_{n-2}C_0^T\boldsymbol{\eta} \qquad (34)$$

where $\boldsymbol{p}=[p_1,\ p_2]^T$ and C_0 is an $(n-2)$th vector. It is straightforward to see that all the assumptions in the previous section are satisfied.

Hence, it follows that

$$\begin{aligned}\ddot{y} &= a(\boldsymbol{\zeta},\ \boldsymbol{\eta})u+b(\boldsymbol{\zeta},\ \boldsymbol{\eta}) \\ &= b_{n-2}u+\boldsymbol{p}^T\boldsymbol{\zeta}-b_{n-2}C_0^T\boldsymbol{\eta} \qquad (35)\end{aligned}$$

Here, the function $1/G(s)$ can be expressed as

$$\begin{aligned}\frac{D(s)}{N(s)} &= \frac{s^2}{b_{n-2}}+\left(a_{n-1}-\frac{b_{n-3}}{b_{n-2}}\right)\frac{s}{b_{n-2}} \\ &\quad +\frac{1}{b_{n-2}}\left\{a_{n-2}-\frac{b_{n-4}}{b_{n-2}}\right. \\ &\quad \left.-\frac{b_{n-3}}{b_{n-2}}\left(a_{n-1}-\frac{b_{n-3}}{b_{n-2}}\right)\right\}+\frac{R(s)}{N(s)}, \\ &\deg R(s)<\deg N(s) \qquad (36)\end{aligned}$$

Combining (35) and (36) yields

$$p_2=-\left(a_{n-1}-\frac{b_{n-3}}{b_{n-2}}\right) \qquad (37)$$

Therefore, it is obtained that

$$\begin{aligned}\overline{a}(\boldsymbol{\zeta},\ \boldsymbol{\eta}) &= \frac{\partial b}{\partial z_2}a+\frac{\partial a}{\partial z_1}z_2+\sum_{i=3}^{n}\frac{\partial a}{\partial z_i}c_i \\ &= \frac{\partial b}{\partial z_2}a \\ &= p_2b_{n-2} \\ &= -\left(a_{n-1}-\frac{b_{n-3}}{b_{n-2}}\right)b_{n-2} \qquad (38)\end{aligned}$$

From the corollary 2 shown in the previous section, the stability condition of the sampling zero dynamics for sufficiently small sampling periods is reduced to

$$\frac{\overline{a}_{k0}}{3a_{k0}} = -\frac{1}{3}\left(a_{n-1} - \frac{b_{n-3}}{b_{n-2}}\right) < 0 \qquad (39)$$

It is obvious that the condition above is equivalent to the result for linear systems (Hagiwara *et al.* (1993)). As a result, the corollary 2 is a natural extension of a well-known result for the stability condition of linear systems to the nonlinear case.

5. CONCLUSIONS

It is important to derive a good approximate sampled-data model for nonlinear continuous-time systems because the exact sampled-data model for nonlinear systems is often unavailable to the controller designers. Recently, Yuz and Goodwin have proposed an accurate approximate model which includes extra zero dynamics, so-called the sampling zero dynamics, corresponding to the relative degree of the continuous-time nonlinear system. A more accurate sampled-data model is, however, required when the relative degree of a continuous-time nonlinear plant is two. The reason is that the closed-loop system becomes unstable when the more accurate sampled-data model has unstable sampling zero dynamics and a controller design method based on cancellation of the zero dynamics is applied. For sufficiently small sampling periods, this paper derives the zero dynamics of the more accurate sampled-data model and shows a condition which assures the stability of the sampling zero dynamics of the model. Further, it is shown that this extends a well-known result for the stability condition of linear systems to the nonlinear case.

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