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A Descriptor Systems Approach to Robust Exponential Stability of Networked Control Systems

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Abstract: In this paper, we consider a class of networked control systems (NCSs) with norm-bounded uncertainties. Using the continuous modeling method, the NCSs can be described as delayed differential equations (DDEs), which can be viewed as a general form of the NCSs model, where the effect of the network-introduced delay and data packet dropout are simultaneously considered. Robust exponential stability criterion are derived based on a delay-dependent method. Robust output feedback controller can also be determined by solving a set of linear matrix inequalities.

1. INTRODUCTION

Networked control systems (NCSs) are spatially distributed systems in which the communication between sensors, actuators, and controllers occurs through a shared bandlimited digital communication network (see Hespanha et al. (2007)). Recently, much attention has been paid to the study of stability analysis and controller design of NCSs due to their low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. However, the insertion of the communication channels raises new interesting and challenging problems such as quantization, sampling, time delays, and packet dropout. Therefore, NCSs have been a hot research topic for the high value of theory and application.

Network introduced time-delay is one of the fundamental problems in NCSs. As far, various methods have been used to deal with the problem of network delay. Based on hybrid systems technique, stability of the NCSs has been investigated in Zhang et al. (2001) under the assumption that the network-introduced delay is less than the sampling period. Optimal stochastic control theory has been used to treat random delays in Nilsson et al. (1998), Hu and Zhu (2003) for short delay and long delay case, respectively. By modeling Random delays as Markov chain, the closed-loop NCSs have been considered as Markovian jump systems in Zhang et al. (2005). The closed-loop NCSs have also been written as discrete-time switched systems under the situation that computation and transmission delays are negligible and access delays serve as the main source of delays (see e.g. Lin and Antsaklis (2004), Lin et al. (2003), Zhai et al. (2002)).

Packet dropout is another main concern in NCSs. In the view of stochastic process, most results available have assumpted that packet dropout be Bernoulli process (see Zhang et al. (2001)), Markov process with two operation modes (see Seiler and Sengupta (2005)) or multiple operation modes (see Xiong and Lam (2007)), and arbitrary packet-loss process which takes values in a finite set arbitrarily (see Yu et al. (2004a)). Therefore, different kinds of systems, such as hybrid systems (see Zhang et al. (2001)), Markovian jump systems (see e.g.Seiler and Sengupta (2005), Xiong and Lam (2007)) and switched systems (see Yu et al. (2004a)) have been settled according to the different modeling method.

Since network-introduced delays and packet dropout are the potential sources to instability and poor performance of NCSs, a general form of the NCS model, where the effect of the network-introduced delay and data packet dropout are considered at the same time, has been presented as DDEs (see e.g. Yu et al. (2004b), Yue et al. (2004), Naghshtabrizi and Hespanha (2005)) . In this paper, our model is similar to (Yu et al. (2004b), Yue et al. (2004), Naghshtabrizi and Hespanha (2005)). However, there are at least two main differences between this paper and Yu et al. (2004b), Yue et al. (2004), Naghshtabrizi and Hespanha (2005). The first one is that we firstly do some research on the NCSs with norm-bounded uncertainties in derivative matrix by descriptor systems approach (see Fridman and Shaked (2002), Fridman et al. (2004)). The second one is that we stabilize the NCSs with static output feedback controller by delay-dependent LMIs. Numerical examples are presented to illustrate our results' correctness and efficiency.

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2. PROBLEM FORMULATION

Consider the continuous plant which can be described as

$$(I+\Delta I)\dot{x}(t) = (A+\Delta A)\,x(t) + (B+\Delta B)\,u(t) \quad (1)$$
$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in R^n$, $y(t) \in R^p$and $u(t) \in R^m$ are the state vector, output vector and control input vector respectively, and A, B and C are constant matrices. ΔI, ΔA and ΔB stand for norm-bounded uncertainties with

$$[\,\Delta I\ \ \Delta A\ \ \Delta B\,] = MF(t)\,[\,N_I\ \ N_A\ \ N_B\,] \quad (3)$$

where M, N_I, N_A and N_B are constant matrices with appropriate dimensions, the uncertainty $F(t)$ satisfies

$$F^T(t)F(t) \leq I.$$

Suppose the sensor are clock-driven and controller and actuator are event-driven, the data is transmitted with a single packet and the full state variables are not available for measurements, matrix C is of full row rank, the real input $u(t)$ realized through zero-order hold is a piecewise constant function. Then the real control system can be modeled as

$$(I+\Delta I)\dot{x}(t) = (A+\Delta A)\,x(t) + (B+\Delta B)\,u(t) \quad (4)$$
$$t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) \quad (5)$$

$$u(t^+) = Ky\,(t-\tau_k)\,, t \in \{i_k h + \tau_k, k = 1, 2, \cdots\} \quad (6)$$

where h is the sampling period, i_k, $k = 1, 2, \cdots$, are some integers and $\{i_1, i_2, \cdots\} \subset \{0, 1, 2, \cdots\}$. τ_k is the time delay which denotes the time from the instant i_k when sensor nodes sample from plant to the instant when actuator transmit data to the plant. Obviously, $\cup_{k=1}^{\infty}[i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) = [t_0, \infty), t_0 \geq 0$. In this paper, we assume that $u(t) = 0$ before the first control signal reaches the plant. Notice that it is not required to have $i_{k+1} > i_k$. If $i_{k+1} - i_k = 1$, it means that there is no data packet dropout in the transmission, which includes $\tau_k = \tau_0$ and $\tau_k < h$ as the special cases taken in account (4). If $i_{k+1} > i_k + 1$, there are some data packet dropout and but the data are ordered correctly. If $i_{k+1} < i_k$, it means unordered data arrival sequence occurs. The system (4) can be rewritten as

$$(I+\Delta I)\dot{x}(t) = (A+\Delta A)\,x(t) + (B+\Delta B)\,KCx\,(i_k h) \quad (7)$$
$$t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) \quad (8)$$

Since $x\,(i_k h) = x\,(t-(t-i_k h))$(Fridman et al. (2004)), we define $\tau(t) = t - i_k h$, $t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$, $k = 1, 2, \cdots$, then (7) becomes

$$\begin{aligned}(I+\Delta I)\dot{x}(t) = (B+\Delta B)\,KCx\,(t-\tau(t)) \\ +(A+\Delta A)\,x(t)\end{aligned} \quad (9)$$

$$t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) \quad (10)$$
$$x(t) = \phi(t), t \in [t_0 - \bar{\tau}, t_0] \quad (11)$$

where the time-varying delay $\tau(t)$ satisfies

$$\tau(t) \in [\underline{\tau}, \bar{\tau}], \dot{\tau}(t) = 1, \forall t \geq 0, a.e. \quad (12)$$

where

$$\underline{\tau} = \min_{k\in N}\{\tau_k\}\,, \bar{\tau} = \max_{k\in N}\{i_{k+1}h + \tau_{k+1} - i_k h\}\,.$$

Therefore, (9), (11) and (12) can be viewed as a general form of the NCSs model, where the effect of the network-introduced delay and data packet dropout are simultaneously considered.

Definition 1. The system (9) and (11) with a feedback gain K is said to be exponentially stable if there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$\|x(t)\| \leq \alpha \sup_{t_0-\bar{\tau}\leq s\leq t_0} \|\phi(s)\|\, e^{-\beta t}, t \geq t_0.$$

Also, if there exists a matrix K such that the system (9) and (11) with the feedback gain K is exponentially stable, then the NCS (4) is said to be exponentially stabilized.

Remark 1. For case (12), the Lyapunov-Krasovskii and the Razumikhin theorems are the two main tools available to study the stability of DDEs of the form of (9) and (11). However, the Razumikhin method is generally more conservative than the Lyapunov-Krasovskii method. Therefor, we will adopt the latter to tack with the problem.

3. ROBUST STABILIZATION

3.1 Stability analysis

For convenience, we first define some new notations:

$$\bar{I} = I + \Delta I, \bar{A} = A + \Delta A, \bar{B} = B + \Delta B \quad (13)$$

Theorem 1. For given scalars $\underline{\tau} \geq 0$, $\bar{\tau} > 0$ and matrix K, if there exist matrices $P_1 > 0$, P_2, P_3, S, R, Z_1, Z_2 and T_1 of appropriate dimensions such that

$$\Omega = \begin{bmatrix} \bar{\Psi} & P^T\begin{bmatrix}0\\ \bar{B}KC\end{bmatrix} - T_1^T \\ * & -S \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} R & T_1 \\ * & Z_1 \end{bmatrix} \geq 0 \quad (15)$$

$$\begin{bmatrix} R & [\,0\ \ C^TK^T\bar{B}^T\,]\,P \\ * & Z_2 \end{bmatrix} \geq 0 \quad (16)$$

where "$*$" denotes the symmetric block, and $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, $\bar{\Psi} = P^T\begin{bmatrix} 0 & I \\ \bar{A} & -\bar{I}\end{bmatrix} + \begin{bmatrix} 0 & I \\ \bar{A} & -\bar{I}\end{bmatrix}^T P + \begin{bmatrix} S & 0 \\ 0 & \bar{\tau}R\end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix}^T + \underline{\tau}Z_1 + (\bar{\tau} - \underline{\tau})Z_2$, then the system (9) and (11), namely the NCS (4) and (6) is exponential stable.

Proof. Based on descriptor system approach, we represent (9) in an equivalent form

$$E\dot{\xi}(t) = \begin{bmatrix} 0 & I \\ \bar{A} & -\bar{I}\end{bmatrix}\xi(t) + \begin{bmatrix} 0 \\ \bar{B}\end{bmatrix}[\,KC\ \ 0\,]\,\xi(t-\tau(t)) \quad (17)$$

$$= \begin{bmatrix} 0 & I \\ \bar{A} + \bar{B}KC & -\bar{I}\end{bmatrix}\xi(t) - \begin{bmatrix} 0 \\ \bar{B}KC\end{bmatrix}\int_{t-\tau(t)}^{t} \dot{x}(s)ds$$

where $\xi^T(t) = [\,x^T(t)\ \ \dot{x}^T(t)\,]\,, E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

Construct a Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{18}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, P_1 > 0, EP = P^T E \geq 0$$

$$V_1(t) = \xi^T(t)EP\xi(t), V_3(t) = \int_{t-\underline{\tau}}^{t} x^T(s)Sx(s)ds$$

$$V_2(t) = \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta$$

Differentiating the first term of (18) with respect to t gives

$$\dot{V}_1(t) = 2\xi^T(t)P^T \begin{bmatrix} 0 & I \\ \bar{A} + \bar{B}KC & -\bar{I} \end{bmatrix} \xi(t) - \eta \tag{19}$$

where

$$\eta = 2\xi^T(t)P^T \begin{bmatrix} 0 \\ \bar{B}KC \end{bmatrix} \int_{t-\tau(t)}^{t} \dot{x}(s)ds.$$

By Moon et al. (2001), we get

$$\begin{aligned} -\eta \leq & \int_{t-\underline{\tau}}^{t} \zeta_{ts}^T \begin{bmatrix} R & T_1 - \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P \\ * & Z_1 \end{bmatrix} \zeta_{ts} ds + \\ & \int_{t-\tau(t)}^{t-\underline{\tau}} \zeta_{ts}^T \begin{bmatrix} R & T_2 - \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P \\ * & Z_2 \end{bmatrix} \zeta_{ts} ds \\ = & \int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + \xi^T(t)\left(\underline{\tau}Z_1 + \left(\tau(t) - \underline{\tau}\right)Z_2\right)\xi(t) \\ & +2\int_{t-\tau(t)}^{t-\underline{\tau}} \dot{x}^T(s)\left(T_2 - \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P\right)\xi(t)ds \\ & +2\int_{t-\underline{\tau}}^{t} \dot{x}^T(s)\left(T_1 - \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P\right)\xi(t)ds \end{aligned}$$

where $\zeta_{ts} = \begin{bmatrix} \dot{x}(s) \\ \xi(t) \end{bmatrix}, \begin{bmatrix} R & T_1 \\ * & Z_1 \end{bmatrix} \geq 0, \begin{bmatrix} R & T_2 \\ * & Z_2 \end{bmatrix} \geq 0.$

By choosing $T_2 = \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P$,

$$\begin{aligned} -\eta \leq & \int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + \xi^T(t)\left(\underline{\tau}Z_1 + (\bar{\tau} - \underline{\tau})Z_2\right)\xi(t) \\ & +2x^T(t)\left(T_1 - \left[\, 0 \;\; C^T K^T \bar{B}^T \,\right] P\right)\xi(t) \\ & -2x^T(t-\underline{\tau})\left(T_1 - \;0\;\; C^T K^T \bar{B}^T \; P\right)\xi(t) \end{aligned}$$

Differentiating the second and third term of (18) with respect to t gives

$$\dot{V}_2(t) = \bar{\tau}\dot{x}^T(t)R\dot{x}(t) - \int_{t-\bar{\tau}}^{t} \dot{x}^T(s)R\dot{x}(s)ds \tag{20}$$

$$\dot{V}_3(t) = x^T(t)Sx(t) - x^T(t-\underline{\tau})Sx(t-\underline{\tau}) \tag{21}$$

Combining (19) to (21)

$$\dot{V}(t) \leq \chi^T(t)\Omega\chi(t) \tag{22}$$

where $\chi^T(t) = \left[\, \xi^T(t) \;\; x^T(t-\underline{\tau}) \,\right]$.

According to (14), $\Omega < 0$, then

$$\dot{V}(t) \leq -\lambda \left\|x(t)\right\|^2 - \lambda \left\|\dot{x}(t)\right\|^2 \tag{23}$$

where $\lambda = \lambda_{\min}(-\Omega)$. Defining a new function $W(t) = e^{\varepsilon t}V(t)$ and using the similar analysis method in Yue et al. (2004), it can be seen that there exist a small enough constant $\varepsilon > 0$ and a constant $\rho > 0$ such that

$$V(t) \leq \rho \sup_{t_0-\bar{\tau}\leq s\leq t_0} \left\|\phi(s)\right\|^2 e^{-\varepsilon t}, t \geq t_0$$

Due to the fact that $V_1(t) = \xi^T(t)EP\xi(t) = x^T(t)P_1x(t) \leq V(t)$, we can obtain

$$\left\|x(t)\right\| \leq \sqrt{\lambda_{\min}^{-1}\left(P_1\right)\rho} \sup_{t_0-\bar{\tau}\leq s\leq t_0} \left\|\phi(s)\right\| e^{\frac{-\varepsilon t}{2}}, t \geq t_0 \tag{24}$$

Then, by *Definition 1*, we can complete the proof.

Due to the norm-bounded uncertainties contained in $\bar{I}$, $\bar{A}$ and $\bar{B}$, we have to transform the (14) and (16) into another equivalent form using a standard technique for dealing with norm-bounded uncertain systems. Now, combining (3) and (14), we obtain

$$Y + \bar{M}F(t)\bar{N} + \bar{N}^T F^T(t)\bar{M}^T < 0 \tag{25}$$

where

$$\bar{M} = \begin{bmatrix} P^T \begin{bmatrix} 0 \\ M \end{bmatrix} \\ 0 \end{bmatrix}, Y = \begin{bmatrix} \Psi & P^T \begin{bmatrix} 0 \\ BKC \end{bmatrix} - T_1^T \\ * & -S \end{bmatrix}$$

$$\bar{N}^T = \begin{bmatrix} \begin{bmatrix} N_A^T \\ -N_I^T \end{bmatrix} \\ C^T K^T N_B^T \end{bmatrix}, \Psi = P^T \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P$$

$$+ \begin{bmatrix} S & 0 \\ 0 & \bar{\tau}R \end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix}^T + \underline{\tau}Z_1 + (\bar{\tau} - \underline{\tau})Z_2$$

A similar procedure can also be carried out to handle with (16).

Using the fact

$$\begin{aligned} & Y + \bar{M}F(t)\bar{N} + \bar{N}^T F^T(t)\bar{M}^T < 0,\; F^T(t)F(t) \leq I \\ & \Longleftrightarrow Y + \varpi^{-1}\bar{M}\bar{M}^T + \varpi\bar{N}^T\bar{N} < 0,\; \exists\, \varpi > 0 \end{aligned} \tag{26}$$

we achieve the following theorem.

Theorem 2. For given scalars $\underline{\tau} \geq 0$, $\bar{\tau} > 0$ and matrix K, if there exist scalars $\varpi > 0$, $\varpi_1 > 0$ and matrices $P_1 > 0, P_2, P_3, S, R, Z_1, Z_2$ and T_1 of appropriate dimensions such that

$$\begin{bmatrix} \Psi & \varphi^T - T_1^T & P^T \begin{bmatrix} 0 \\ M \end{bmatrix} & \varpi \begin{bmatrix} N_A^T \\ -N_I^T \end{bmatrix} \\ * & -S & 0 & \varpi C^T K^T N_B^T \\ * & * & -\varpi I & 0 \\ * & * & * & -\varpi I \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} R & T_1 \\ * & Z_1 \end{bmatrix} \geq 0 \quad (28)$$

$$\begin{bmatrix} -R & -\varphi & 0 & \varpi_1 C^T K^T N_B^T \\ * & -Z_2 & P^T \begin{bmatrix} 0 \\ M \end{bmatrix} & 0 \\ * & * & -\varpi_1 I & 0 \\ * & * & * & -\varpi_1 I \end{bmatrix} < 0 \quad (29)$$

where "$*$" denotes the symmetric block, and

$$\Psi = P^T \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P + \begin{bmatrix} S & 0 \\ 0 & \bar{\tau} R \end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix} + \begin{bmatrix} T_1 \\ 0 \end{bmatrix}^T + \underline{\tau} Z_1 + (\bar{\tau} - \underline{\tau}) Z_2, \varphi^T = P^T \begin{bmatrix} 0 \\ BKC \end{bmatrix}$$

then the system (9) and (11), namely the NCS (4) and (6) is exponential stable.

When $\underline{\tau} = 0$, the $V_3(t)$ and condition (28) will disappear. *Theorem 2* can be simplified to the following.

Corollary 1. For given scalar $\bar{\tau} > 0$ and matrix K, if there exist scalars $\varpi > 0$, $\varpi_1 > 0$ and matrices $P_1 > 0$, P_2, P_3, R and Z of appropriate dimensions such that

$$\begin{bmatrix} \hat{\Psi} & P^T \begin{bmatrix} 0 \\ M \end{bmatrix} & \varpi \begin{bmatrix} N_A^T + C^T K^T N_B^T \\ -N_I^T \end{bmatrix} \\ * & -\varpi I & 0 \\ * & * & -\varpi I \end{bmatrix} < 0 \quad (30)$$

$$\begin{bmatrix} -R & -\varphi & 0 & \varpi_1 C^T K^T N_B^T \\ * & -Z & P^T \begin{bmatrix} 0 \\ M \end{bmatrix} & 0 \\ * & * & -\varpi_1 I & 0 \\ * & * & * & -\varpi_1 I \end{bmatrix} < 0 \quad (31)$$

where "$*$" denotes the symmetric block, and

$$\hat{\Psi} = P^T \begin{bmatrix} 0 & I \\ A + BKC & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A + BKC & -I \end{bmatrix}^T P + \begin{bmatrix} 0 & 0 \\ 0 & \bar{\tau} R \end{bmatrix} + \bar{\tau} Z, P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \varphi^T = P^T \begin{bmatrix} 0 \\ BKC \end{bmatrix}$$

then the system (9) and (11), namely the NCS (4) and (6) is exponential stable.

Remark 2. When the lower bound of the time delay $\underline{\tau}$ exists, more slack matrix variables are introduced in our results. Therefore, theorem 2 is usually less conservative than *Corollary 1*. Later, we will continue to illustrate this fact by numerical example.

3.2 Robust stabilization

Based on *Theorem 2*, we are now in a position to determine the feedback gain K which can make systems (4) and (6) is exponential stable.

Theorem 3. For given scalars γ, $\underline{\tau} \geq 0$, $\bar{\tau} > 0$, if there exist scalars $\bar{\varpi} > 0$, $\bar{\varpi}_1 > 0$ and matrices $Q_1 > 0$, Q_2, Q_3, X, V, $\bar{S}$, $\bar{Z}_i^{(ij)} (i, j = 1, 2)$, $\bar{T}_{11}$ and $\bar{T}_{12}$ of appropriate dimensions such that

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & -\bar{T}_{11}^T & 0 & Q_1^T N_A^T - Q_2^T N_I^T & \bar{\tau} Q_2^T \\ * & \Xi_{22} & \Xi_{23} & \bar{\varpi} M & -Q_3^T N_I^T & \bar{\tau} Q_3^T \\ * & * & -\bar{S} & 0 & C^T X^T N_B^T & 0 \\ * & * & * & -\bar{\varpi} I & 0 & 0 \\ * & * & * & * & -\bar{\varpi} I & 0 \\ * & * & * & * & * & -\frac{\bar{\tau}}{\gamma} Q_1 \end{bmatrix} < 0 \quad (32)$$

$$\begin{bmatrix} \gamma Q_1 & \bar{T}_{11} & \bar{T}_{12} \\ * & \bar{Z}_1^{(11)} & \bar{Z}_1^{(12)} \\ * & * & \bar{Z}_1^{(22)} \end{bmatrix} \geq 0 \quad (33)$$

$$\begin{bmatrix} -\gamma Q_1 & 0 & -C^T X^T B^T & 0 & C^T X^T N_B^T \\ * & -\bar{Z}_2^{(11)} & -\bar{Z}_2^{(12)} & \bar{\varpi}_1 M & 0 \\ * & * & -\bar{Z}_2^{(22)} & 0 & 0 \\ * & * & * & -\bar{\varpi}_1 I & 0 \\ * & * & * & * & -\bar{\varpi}_1 I \end{bmatrix} < 0 \quad (34)$$

$$VC = CQ_1 \quad (35)$$

where "$*$" denotes the symmetric block, and

$$\Xi_{11} = Q_2 + Q_2^T + \bar{S} + \underline{\tau} \bar{Z}_1^{(11)} + (\bar{\tau} - \underline{\tau}) \bar{Z}_2^{(11)} + \bar{T}_{11} + \bar{T}_{11}^T,$$
$$\Xi_{12} = Q_3 - Q_2^T - Q_1^T A^T + \underline{\tau} \bar{Z}_1^{(12)} + (\bar{\tau} - \underline{\tau}) \bar{Z}_2^{(12)} + \bar{T}_{12},$$
$$\Xi_{22} = -Q_3 - Q_3^T + \underline{\tau} \bar{Z}_1^{(22)} + (\bar{\tau} - \underline{\tau}) \bar{Z}_2^{(22)},$$
$$\Xi_{23} = BXC - \bar{T}_{12}^T$$

then the system (9) and (11), namely the system (4) and (6) with $K = XV^{-1}$ is exponential stable.

Proof. As we know $P_1 > 0$ and the fact that $-I^T P_3 - P_3^T I$ must be negative definite in (27), we can conclude that P is nonsingular. Defining

$$P^{-1} = Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix}, \Delta_1 = diag \left\{ Q, Q_1, \varpi^{-1} I, \varpi^{-1} I \right\}$$
$$\Delta_2 = diag \left\{ Q_1, Q \right\}, \Delta_3 = diag \left\{ Q_1, Q, \varpi_1^{-1} I, \varpi_1^{-1} I \right\}$$

In order to obtain an LMI, we have to restrict our result to the case $R = \gamma Q_1^{-1}$, where γ is scalar parameter.

Multiply (27) both sides by Δ_1^T and Δ_1 respectively. Applying Schur formular to the emerging quadratic term in Q, denote

$$\bar{S} = Q_1^T S Q_1, \bar{T}_1 = Q_1^T T_1 Q = \left[\bar{T}_{11} \ \bar{T}_{12} \right], \bar{\varpi} = \varpi^{-1}$$
$$\bar{\varpi}_1 = \varpi_1^{-1}, \bar{Z}_i = Q^T Z_i Q = \begin{bmatrix} \bar{Z}_i^{(11)} & \bar{Z}_i^{(12)} \\ * & \bar{Z}_i^{(22)} \end{bmatrix}, i = 1, 2.$$

By (35), we denote $XC = KVC = KCQ_1$.

Using the same way, multiply (28) both sides by diag Δ_2^T and Δ_2 respectively, and (29) both sides by Δ_3^T and Δ_3 respectively, we obtain (33) and (34).

According to *Corollary 1*, the similar method to the proof in *Theorem 3* is adopted to determine the feedback gain K which can make systems (4) and (6) for $\underline{\tau} = 0$ is exponential stable.

Corollary 2. For given scalar $\gamma, \bar{\tau} > 0$, if there exist scalars $\bar{\varpi} > 0$, $\bar{\varpi}_1 > 0$ and matrices $Q_1 > 0$, Q_2, Q_3, X, V and $Z_{ij}(i, j = 1, 2)$ of appropriate dimensions such that

$$\begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} & 0 & Q_1^T N_A^T - Q_2^T N_I^T & \bar{\tau} Q_2^T \\ * & \bar{\Xi}_{22} & \bar{\varpi} M & -Q_3^T N_I^T & \bar{\tau} Q_3^T \\ * & * & -\bar{\varpi} I & 0 & 0 \\ * & * & * & -\bar{\varpi} I & 0 \\ * & * & * & * & -\frac{\bar{\tau}}{\gamma} Q_1 \end{bmatrix} < 0 \quad (36)$$

$$\begin{bmatrix} -\gamma Q_1 & 0 & -C^T X^T B^T & 0 & C^T X^T N_B^T \\ * & -Z_{11} & -Z_{12} & \bar{\varpi}_1 M & 0 \\ * & * & -Z_{22} & 0 & 0 \\ * & * & * & -\bar{\varpi}_1 I & 0 \\ * & * & * & * & -\bar{\varpi}_1 I \end{bmatrix} < 0 \quad (37)$$

$$VC = CQ_1 \quad (38)$$

where "$*$" denotes the symmetric block, and

$$\bar{\Xi}_{11} = Q_2 + Q_2^T + \bar{\tau} Z_{11}, \bar{\Xi}_{22} = -Q_3 - Q_3^T + \bar{\tau} Z_{22}$$
$$\bar{\Xi}_{12} = Q_3 - Q_2^T - Q_1^T A^T + C^T X^T B^T + \bar{\tau} Z_{12}$$

then the system (9) and (11), namely the NCS (4) and (6) with $K = XV^{-1}$ is exponential stable.

Remark 3. For special case $C = I$, which means that the state variables are available for measurement, the constraint (38) is simplified to $V = Q_1$. However, the state variables are not available for measurement most of time. Due to the equality constrain (38), *Theorem 3* and *Corollary 2* can not be carried out by LMI toolbox. Since matrix C is full row rank, we have to replace V with $CQ_1C^T \left(CC^T\right)^{-T}$ scarifying conservative property.

Remark 4. γ is a tuning scalar. A way to address γ is to choose a cost function the parameter $t_{\min}$ that is obtained while solving the feasibility problem using LMI toolbox. If the above cost function $t_{\min}$ is negative, the tuning scalar that solves the problem is found.

3.3 Numerical example

Example. Consider the following example which was used in Zhang et al. (2001)

$$\begin{aligned} (I + MF(t)N_I)\dot{x}(t) &= (A + MF(t)N_A)\, x(t) \\ &\quad (B + MF(t)N_B)\, u(t), \\ y &= Cx(t) \end{aligned} \quad (39)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 0.5 & 0.4 \end{bmatrix}, M = 0.2I$$

$$N_I = 0.4I, N_A = 0.5I, N_B = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, F^T(t)F(t) \le I.$$

We use the static output feedback controller $u(t) = [\,-3.75\ -11.5\,]\, y(t)$ which is designed without considering the presence of the network. For $\underline{\tau} = 0$, according to *Corollary 1*, it has been computed $\bar{\tau} = 0.3957$. Supposed that we have known the lower bound $\underline{\tau} = 0.1$, according to *Theorem 2*, it has been computed $\bar{\tau} = 0.4186$, which confirms our comment in *Remark 2*.

To obtain a larger $\bar{\tau}$, we also can design a controller considering network impact. Given $\gamma = 2.0$ and $\underline{\tau} = 0.1$, we still use the above invertible matrix C. According to *Theorem 3*, the NCS (39) can be robust exponential stabilized by $u(t) = [\,-10.6534\ 26.9816\,]\, y(t)$ even $\bar{\tau} = 0.5259$. When matrix C is rectangular, choosing $C = [\,1\ 2\,]$, we have to replace V with $CQ_1C^T \left(CC^T\right)^{-T}$ in order to obtain LMI. At this time, the NCS (39) can be robust exponential stabilized by $u(t) = -1.1615y(t)$ even $\bar{\tau} = 0.5215$.

Next, we will analyze the results obtained in last paragraph. No loss of generality, we choose the case $\gamma = 2.0$, $\underline{\tau} = 0.1$ and $C = \begin{bmatrix} 1 & 2 \\ 0.5 & 0.4 \end{bmatrix}$, the NCS (39) can be robust exponential stabilized by $u(t) = [\,-10.6534\ 26.9816\,]\, y(t)$ even $\bar{\tau} = 0.5259$. However, from the fact that $\bar{\tau} = \max_{k \in N} \{i_{k+1}h + \tau_{k+1} - i_k h\}$, we know $(i_{k+1} - i_k)\, h + \tau_{k+1} \le \bar{\tau}$. Therefore, for instance when sampling period $h = 0.12$ without considering data packet dropout, the maximum allowable delay bound (MADB) $\tau_{\max} = 0.4059$. On the other hand, when $0.1 \le \tau_{k+1} \le 0.1659$ considering data packet dropout, the lower bound on transmission rate ε can be calculated by $\varepsilon = \frac{1}{\max_{k \in N}(i_{k+1} - i_k)} = \frac{1}{3}$.

4. CONCLUSION

We have considered a class of NCSs with norm-bounded uncertainties. Using the continuous modeling method, the NCSs can be described as DDEs, which can be viewed as a general form of the NCSs model. Robust exponential stability criterion are derived based on a delay-dependent method. Robust output feedback controller can also be determined by solving a set of linear matrix inequalities.

We will extent our results to the case that some performances are desired. The performance can be H_2, H_∞, H_2 and H_∞ mixed, or guaranteed cost et al.

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