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Control of wave motion in the chain of pendulums ⋆

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Abstract: The problem of controlled excitation of oscillations in a chain of N coupled pendulums is considered. Such a problem may arise when studying different physical and mechanical systems and is also of interest for design of prospective laboratory equipment for control education. The simulation results for the chain of 50 pendulums are given. The multi-pendulum mechatronic setup from 50 computer-controlled coupled pendulums, developed in the IPME RAS, Saint Petersburg, is presented.

Keywords: chain of pendulums, energy control, wave motion, speed-gradient control

1. INTRODUCTION

Control of complex motion in interconnected and spatially distributed systems (networks) has become an area of growing research interest recently. It has numerous applications in many disciplines, especially in physics and engineering. In engineering there is a strong demand for control of power systems, traffic control, control of communication networks, etc., see special issues of engineering journals (IEEE CSM, 2001; IEEE CSM, 2002; IEEE CSM, 2007; IEEE Tr. AC. 2004; IEEE Proc., 2007). In physics a growing interest in application of control theory for studying dynamics of complex systems has lead to appearance of a new area gradually becoming known as "Cybernetical Physics" (Fradkov, 2007).

The implementation of control strategies to manipulate complex oscillations and spatiotemporal patterns has become a central issue of nonlinear dynamics (Lebiedz and Brandt-Pollmann, 2003; Kheowan et al., 2004b). Feedback methods provide one of the possible control techniques that yield new modes of spatiotemporal behavior (Schuster, 1999; Ott et al., 1990). These techniques may be designed in different ways. A feedback is global or nonlocal, in contrast to local techniques, if the control signal represents a sum of contributions from all or many parts of the system. Such feedbacks have been used, for instance, to control spatiotemporal activity in the Pt-catalyzed oxidation of CO (Jakubith et al., 1990), suggesting a means for enhancing catalytic efficiency (Wolff et al., 2001), in gas discharges to suppress plasma instabilities (Pierre et al., 1996), in electrochemical systems to influence spatial coupling among different active sites (Krischer and Electroanal, 2001), and in semiconductors in connection with charge transport phenomena (Franceschini et al., 1999). Propagating waves (Sakurai et al., 2002) and, in particular, spiral waves (Kheowan et al., 2001, 2002, 2004a; Zykov et al., 2004), in the Belousov–Zhabotinsky (BZ) reaction (Winfree, 1972; Müller et al., 1985) have also been controlled by using these feedback methods, which points to the possibility of manipulating dynamical patterns in excitable media including excitable biological tissues (Davidenko et al., 1992; Dahlem and Müller, 1997; Falcke et al., 2003). A recent advance in this direction is the control of seizure-like events in hippocampal brain slices with adaptive electric fields (Gluckman et al., 2001). Thus, the ability to regulate spatiotemporal behavior provides both a means of generating desired dynamical patterns and the tools for probing underlying mechanisms. Design and analysis of an active device capable of controlling mechanical waves based on feedforward control of waves in a structural member is considered in (Nauclér et al., 2007), where the concept of a "mechanical wave diode" is presented. However, none of these studies allow for a systematic way of influencing the systems with respect to general control aims. A promising approach to systematic development of control methods for complex networks is to start with the systems of intermediate complexity, e.g. one-dimension chains of simple subsystems.

In the paper the problem of controlled excitation of oscillations in the chain of N coupled mathematical pendulums is considered. Such a problem may arise when studying different physical and mechanical systems, see e.g. (Jackson, 1990; Pogromsky et al., 1996; Landa, 1996; Aero et al., 2005, 2006). It is also of interest for design of prospective laboratory equipment for control education. The example of the such an equipment – the "*Chain of Pendulums*" mechatronic set-up, created in the IPME RAS, Saint Petersburg (Fradkov et al., 2008) is briefly described in Sec. 3.

2. CONTROL OF WAVE MOTION IN THE CHAIN OF PENDULUMS

2.1 Modeling the chain of the pendulums

The system of coupled pendulums is described by the equations

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$$\begin{cases} \ddot{\varphi}_1 + \rho\,\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 = k(\varphi_2 - \varphi_1) + u, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \ddot{\varphi}_i + \rho\,\dot{\varphi}_i + \omega_0^2 \sin\varphi_i = k(\varphi_{i-1} - 2\varphi_i + \varphi_{i+1}), \\ \qquad\qquad (i = 2, 3, \dots, N-1), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \ddot{\varphi}_N + \rho\,\dot{\varphi}_N + \omega_0^2 \sin\varphi_N = k(\varphi_{N-1} - \varphi_N), \end{cases} \qquad (1)$$

where $\varphi_i = \varphi_i(t)$ $(i = 1, 2, \dots, N)$ are the pendulum deflection angles; $u = u(t)$ is the controlling action: external torque, applied to the first pendulum. It is assumed that the torque is measured in the units of angular acceleration. The values ρ, ω_0, k are parameters of the system: ρ is the viscous friction parameter; ω_0 is the natural frequency of small oscillations of isolated pendulums; k is the parameter of coupling strength, for example, the stiffness of the spring connecting the pendulums. Schematic diagram of the chain is shown in Fig. 1.

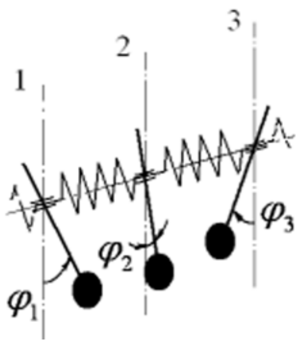

Fig. 1. Schematic diagram of the chain of pendulums

Introduce the state vector of the system $x(t) \in \mathbb{R}^{2N}$ as $x(t) = \mathrm{col}\,\{\varphi_1, \dot{\varphi}_1, \dots, \varphi_N, \dot{\varphi}_N\}$. The total energy of the system (1) $H(x)$ is defined by the expression

$$H(x) = \sum_{i=1}^{N} H_i(x), \quad \text{where}$$

$$\begin{cases} H_i(x) = 0.5\dot{\varphi}_i^2 + \omega_0^2(1 - \cos\varphi_i) + 0.5k(\varphi_{i+1} - \varphi_i)^2, \\ \qquad\qquad (i = 1, 2, \dots, N-1), \\ H_N(x) = 0.5\dot{\varphi}_N^2 + \omega_0^2(1 - \cos\varphi_N). \end{cases} \qquad (2)$$

In the absence of control the model in question coincides with the classical Frenkel–Kontorova model, (Frenkel and Kontorova, 1938; Landa, 1996). However, it is distinct from the Frenkel–Kontorova model in the type of the control appearance: here control is localized and affects only one pendulum. In terms of control of distributed systems it corresponds to the boundary control.

In addition, we will consider the system of cyclically coupled pendulums, similar to (1), except the presence of the elastic link between the first and the last pendulums:

$$\begin{cases} \ddot{\varphi}_1 + \rho\,\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 = k(\varphi_2 - 2\varphi_1 + \varphi_N) + u(t), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \ddot{\varphi}_i + \rho\,\dot{\varphi}_i + \omega_0^2 \sin\varphi_i = k(\varphi_{i-1} - 2\varphi_i + \varphi_{i+1}) \\ \qquad\qquad (i = 2, 3, \dots, N-1), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \ddot{\varphi}_N + \rho\,\dot{\varphi}_N + \omega_0^2 \sin\varphi_N = k(\varphi_{N-1} - 2\varphi_N + \varphi_1). \end{cases} \qquad (3)$$

The expression for the total energy changes correspondingly:

$$H(x) = \sum_{i=1}^{N} H_i(x), \quad \text{where}$$

$$\begin{cases} H_i(x) = 0.5\,\dot{\varphi}_i^2 + \omega_0^2(1 - \cos\varphi_i) + 0.5k(\varphi_{i+1} - \varphi_i)^2, \\ \qquad\qquad (i = 1, 2, \dots, N-1), \\ H_N(x) = 0.5\,\dot{\varphi}_1^2 + \omega_0^2(1 - \cos\varphi_N) + 0.5\,k\big(\varphi_1 - \varphi_N\big)^2. \end{cases} \qquad (4)$$

The equations (3) are symmetrical with respect to the free motion of the pendulums. Such a symmetry allows the achievement of an additional control goal: synchronization of the pendulum motions.

2.2 *Problem statement and control algorithm design*

The key point of our approach is to interpret the problem of the excitation of a wave as achievement of the given level of the system energy with additional requirement that the neighbor pendulums have opposite oscillation phases. It is proposed to use the *Speed-gradient* (SG) method (Fradkov, 1979, 1996) for control algorithm design.

Consider a system described by state-space equations

$$\dot{x} = F(x, u), \qquad (5)$$

where $x = x(t) \in \mathbb{R}^n$ is the state vector, $u = u(t)$ is the scalar input. Let the control goal for the system (5) be expressed as the limit relation

$$Q\big(x(t)\big) \to 0 \quad \text{when } t \to \infty, \qquad (6)$$

where $Q(x)$ is a nonnegative goal function.

In order to achieve the goal (6), the following SG-algorithm in the finite form may be applied:

$$u = -\Psi\big(\nabla_u \dot{Q}(x, u)\big), \qquad (7)$$

where $\dot{Q} = \big(\partial Q/\partial t\big)F(x, u)$ is the speed of changing $Q\big(x(t)\big)$ along the trajectories of (5), vector $\Psi(z)$ forms an acute angle with the vector z, i.e. $\Psi(z)^{\mathrm{T}} z > 0$ when $z \neq 0$. The first step of the speed-gradient procedure is to calculate the speed $\dot{Q}$. The second step is to evaluate the gradient $\nabla_u \dot{Q}(x, u)$ with respect to controlling input u. Finally the vector-function $\Psi(z)$ should be chosen to meet the acute angle condition. The choice $\Psi(z) = \gamma z$, $\gamma > 0$ yields the *proportional* (with respect to speed-gradient) feedback

$$u = -\gamma\big(\nabla_u \dot{Q}(x, u)\big), \qquad (8)$$

while the choice $\Psi(z) = \gamma\,\mathrm{sign}\,z$ yields the *relay* algorithm

$$u = -\gamma\,\mathrm{sign}\,\big(\nabla_u \dot{Q}(x, u)\big), \qquad (9)$$

The underlying idea of the choice (8), (9) is that moving along the antigradient of the speed $\dot{Q}$ provides decrease of $\dot{Q}$. It may eventually lead to negativity of $\dot{Q}$ which, in turn, yields decrease of Q and, eventually, achievement of the primary goal (6). However, to prove (6) some additional assumptions are needed.

To apply the speed-gradient method introduce two auxiliary goal functions.

$$Q_\varphi(\dot{\varphi}_1, \dot{\varphi}_2) = \frac{1}{2}\,\delta_\varphi^2, \quad Q_H(x) = \frac{1}{2}(H(x) - H^*)^2, \qquad (10)$$

where $\delta_\varphi = \dot{\varphi}_1 + \dot{\varphi}_2$; $H(x)$ is *the total energy of the system*; H^* is the desired value of energy.

Apparently, the minimum value of the function Q_φ corresponds to the anti-phase motion of the first and second pendulums since the identity $Q_\varphi(\dot{\varphi}_1, \dot{\varphi}_2) \equiv 0$ holds only if $\dot{\varphi}_1 \equiv -\dot{\varphi}_2$. The minimization of Q_H corresponds to achievement of the desired oscillation amplitude.

Let us introduce the total goal function $Q(x)$ as the weighted sum of Q_φ and Q_H, namely

$$Q(x) = \alpha Q_\varphi(\dot\varphi_1, \dot\varphi_2) + (1-\alpha)Q_H(x), \qquad (11)$$

where α $(0 \le \alpha \le 1)$ is the weighting coefficient (design parameter).

Evaluation of the speed of changing the goal function $Q(x)$ along trajectories of the controlled system and then its partial derivative in control leads to the following speed-gradient algorithm in the finite form

$$u(t) = -\gamma\big(\alpha\delta_\varphi(t) + (1-\alpha)\delta_H(t)\dot\varphi_1(t)\big),$$
$$\delta_\varphi(t) = \dot\varphi_1(t) + \dot\varphi_2(t), \;\; \delta_H(t) = H_t - H^*. \qquad (12)$$

where $h(t) = H\big(x(t)\big)$.

Note that calculation of the control action according to the equation (12) requires the possibility of the measurement of the angular velocities of the first and second pendulums, as well as the measurement of the total energy of the system.

The method is easily extended to the case when each section in the chain is modeled as a nonlinear 1-DOF (one degree-of-freedom) oscillator. For the case $N = 2$ such a control strategy was proposed by Andrievsky and Fradkov (1999).

2.3 Simulation results

In Fig. 2 – 7 the results of computer simulations for the process of oscillations excitation by means of control law (12) for the chain of $N = 50$ pendulums are presented.

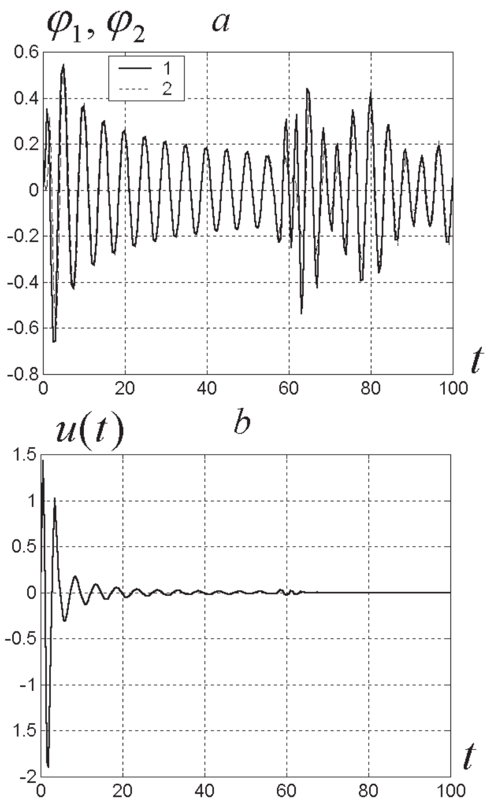

Fig. 2. Transients of φ_1, φ_2 and the control signal. $N=50$, $\alpha=0$.

Fig. 2 – 4 are related to the control of the system (1) by the control algorithm (12) for $\gamma = 1$, $\omega_0 = 0.4\pi$, $\rho = 0.05$ and various values of weighting coefficient α. Fig 2, 3 correspond to the value $\alpha = 0$, when the control goal is stabilization of the given total energy level.

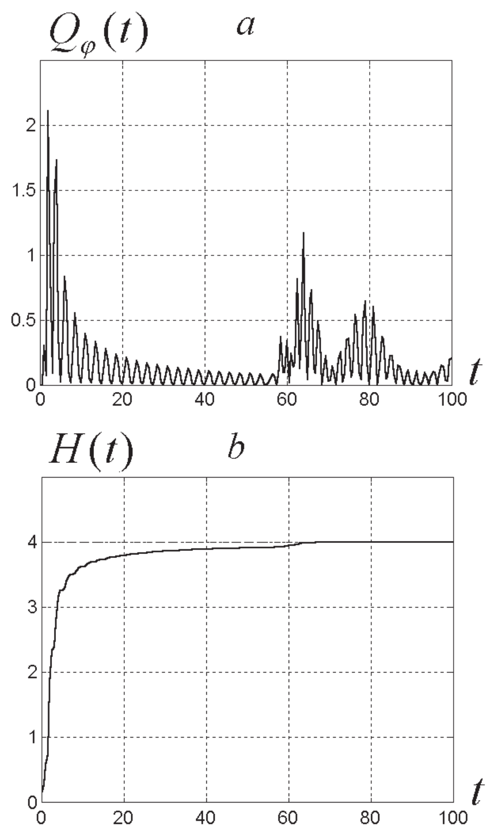

Fig. 3. Time histories of the goal functions $Q_\varphi(t)$ and $H(t)$. $N=50$, $\alpha=0$.

It is seen that the control goal $H_t \to H^*$ $(H^* = 4)$ is achieved but the motion of the pendulums is irregular (chaotic) and the second goal (synchronization) is not achieved, see Fig. 3. It is seen also that the control intensity decays as soon as the goal is achieved.

The reason of the irregular behavior is clear from Fig. 3. It is seen that the forward wave of oscillations propagates ahead and reaches the last pendulum at the time $t \approx 30$. Then the backward wave arises and after some time the picture of oscillations becomes complicated owing to interference of the waves. Excitation of oscillations for $\alpha = 1$ is shown in Figs. 4, 5. Again, it is seen that the anti-phase motion of the pendulum is not achieved. For the choices of intermediate values of the weighting coefficient $0 < \alpha < 1$, the behavior of the process is qualitatively the same as in Figs. 2, 3 and the goal is not achieved.

A possible explanation of the control failure with respect to the goal function Q_φ is in that the anti-phase free motion is not invariant, i.e. for $u(t) \equiv 0$ such a motion, if it arises once, may be destroyed in the future.

The reason of appearance of the reflected wave is in the symmetry breaking of equations (1): the elasticity forces acting on the first and the last pendulums differ from the force acting on the internal pendulum in the chain.

Let us examine the system of cyclically coupled pendulums (3). The previous considerations justify that the controlled system of cyclically coupled pendulums may possess anti-phase synchronous behavior at the specified energy level. The results of the simulation confirm such a conjecture. Figs. 6, 7 demonstrate the results of application of the algorithm (12) to the system (3). The algorithm design parameters are as follows: $\gamma = 0.8$, $\alpha = 0.7$. It is seen, that the combined goal function tends to its minimum value.

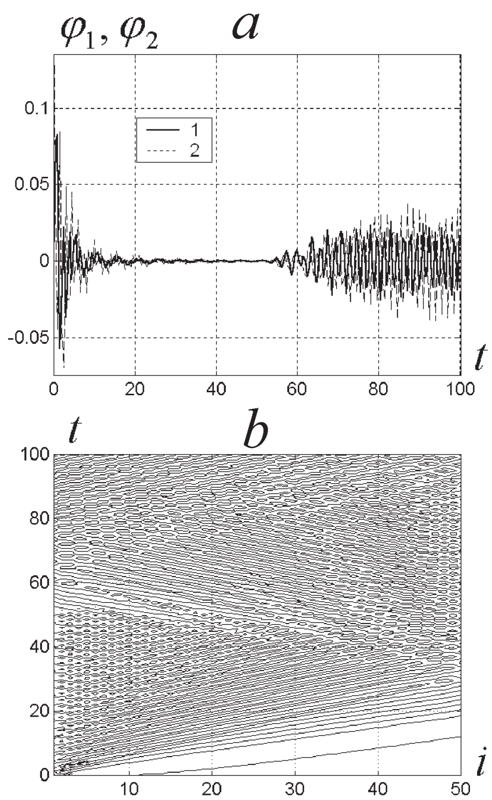

Fig. 4. Excitaton of oscillations. $N = 50$, $\alpha = 1$.

3. LABORATORY SET-UP “CHAIN OF PENDULUMS”

The laboratory set-up “Chain of Pendulums” is created in the IPME RAS, Saint Petersburg (Fradkov et al., 2008) and succeeded the *Coupled Pendulum Mechatronic Set-up* of IPME RAS, presented in (Andrievsky et al., 1998, 2002; Andrievsky and Boykov, 2001; Fradkov et al., 2005). The total amount of pendulums in the “Chain of Pendulums” is 50. The idea of this set-up may be grasped from Fig. 8, where ten pendulum sections are shown.

The “Chain of Pendulums” set-up includes: mechanical oscillating system; electrical equipment (with computer interface facilities), and the personal computer for experimental data processing, representation of the results the real-time control. For data exchange via standard In-Out ports of the computer, the special exchange routine is written. The mechanical oscillating system is composed from up to 50 sections. Each section contains the 1-DOF pendulum. The pendulums of the adjoining sections are connected in sequence via the torsion springs. The sections may form both linear and ring structures. The computer-controlled electric motors, applying the control torques to the boundary sections, may be also connected. Each section includes a pendulum with the permanent magnet in the

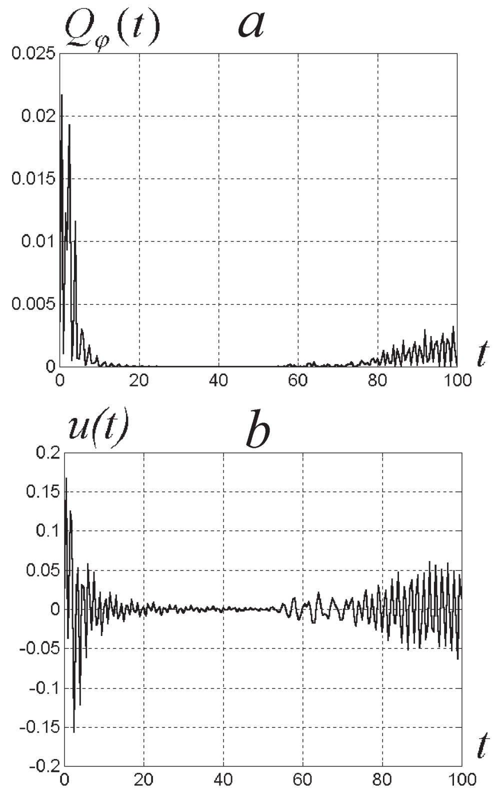

Fig. 5. Synchronization goal function Q_φ and control signal. $N = 50$, $\alpha = 1$.

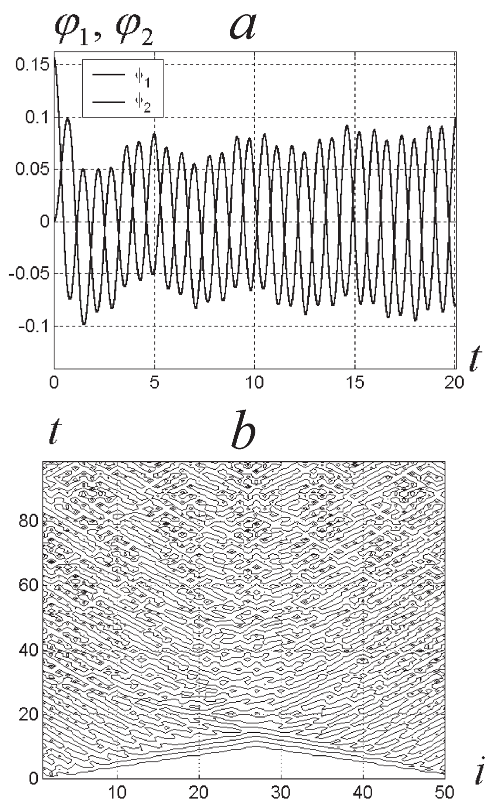

Fig. 6. Control of oscillation excitation for cyclic chain.

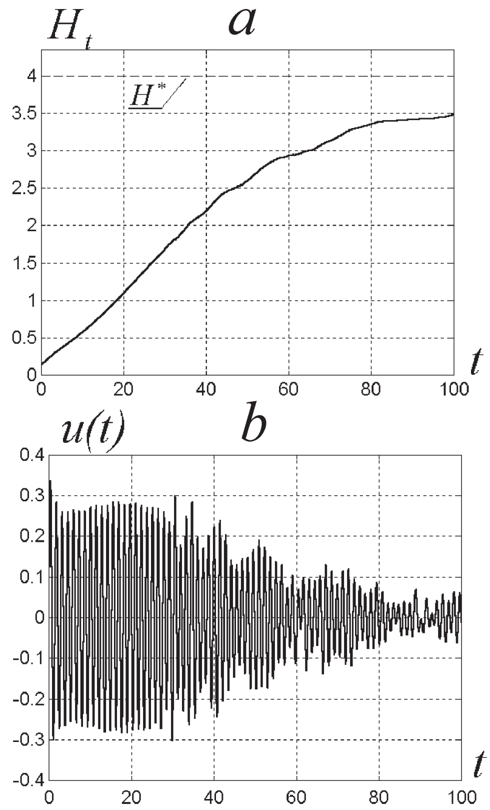

Fig. 7. Total energy and control action for cyclic chain.

bob, the computer-controlled electromagnet inside the basement of the pendulum, the optical angle-data sensor and the interface unit. Data exchange between the personal computer and each section is fulfilled by means of the common controller and the device bus. The communications protocol is based on the sub-addressing method, arrays are considered as data units for transfer.

Control of oscillation is provided on the basis of combined hardware/software method. The energy for excitation is transmitted by the pulse-width modulated (PWM) signal with the constant level and variable duty cycle. From the programming point of view, hardware is represented by the write-only registers (WO) for putting in the prescribed duty cycle of control signal from the computer, and the read-only registers (RO) for transfer oscillation half period duration values to the computer. The PWM based method provides more precise control than the number-pulse one, because of averaging the high frequency pulses by the mechanical subsystem.

The control unit generates the exciting action applied to the pendulums via the opposite magnetic fields. It includes bi-channel asynchronous pulse-width modulator (APWM), logical command interpreting automaton and the power amplifiers to drive the electromagnets. The measured variables are the pendulums rotation angle with accuracy 2.5^o. Additionally, the time intervals between passages the lowest position by the pendulum's bob may be measured. These events are determined by means of the induction coil, sharing the core with the excitation electromagnet. The measuring unit of the section is built using the quartz oscillator to calibrate the main clock pulses. These pulses after the frequency division to 1000 Hz determine the sampling time (1 ms). The measured time interval, provided by 12-bit counter varies from 0.001 to 4.095 s. The beginning and the end of count interval is determined by the Hall sensors and zero-crossing detector. For data transfer the peripheral controller uses the Standard Parallel Port (SPP) in byte bi-directional mode.

Fig. 8. The "Chain of Pendulums" mechatronic set-up (photo).

Preliminary results for testing the described method for the chains of two or three sections (see Fig. 9), were obtained. The experimental results agree with the theoretical statements.

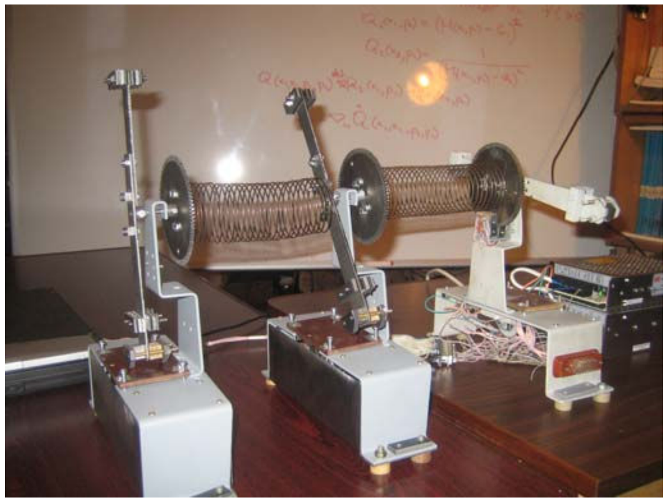

Fig. 9. Oscillation wave in the chain of three pendulums.

4. CONCLUSIONS

In the paper an approach to control of wave motion in the chain of 1-DOF nonlinear oscillators is proposed. The approach is based on speed-gradient method with the goal function including the energy term and the "synchronization" term. Vanishing of the "synchronization" term corresponds to oscillation of the neighbor pendulums in the opposite phases.

Simulation has demonstrated significant influence of the chain configuration and initial conditions on the behavior of the system. For ring configuration the behavior of the control system is much more regular than that of the broken ring. Symmetry of the ring configuration allows the achievement of an additional control goal: synchronization of the pendulum motions.

Future research is aimed at implementing the proposed algorithms in the multi-pendulum mechatronic laboratory set-up developed in IPME RAS, Saint Petersburg.

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