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Delay-Dependent Robust Control of Time-Delay Systems with Polytopic Uncertainty ⋆

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Abstract: In this paper, the delay-dependent robust control problem is considered for time-delay systems with polytopic uncertainty. Robust stabilizing controller and robust H_∞ controller are designed by using a parameter-dependent Lyapunov function that can reduce the conservativeness when used in robust performance analysis and synthesis problems for polytopic systems. Furthermore, multichannel H_∞ dynamic output-feedback controller is also designed. Numerical examples are included to illustrate the proposed method.

1. INTRODUCTION

Application of a single Lyapunov function to the analysis and design is investigated for systems with polytopic uncertainty and multichannel constraints Boyd et al. (1994). The apparently strict requirement of a single Lyapunov function for all admissible uncertainties, all multichannel and multiobjective constraints imposed on systems can lead to rather conservative results. To reduce the conservativeness, researchers turn to using parameter-dependent Lyapunov functions and have obtained many results. Among these works, a simple but effective idea is to separate the products of Lyapunov matrices and controller matrices in the given linear matrix inequalities (LMIs) by introducing auxiliary slack variables. A significant breakthrough toward this direction is the work for discrete systems in Oliveira et al. (1999) and Oliveira et al. (1999) that is extended to continuous time case in Apkarian et al. (2001), Shaked (2001), Ebihara et al. (2001) by different methods. However, in the above literatures, time delay is not considered, while it is a source of instability in many cases. Therefore the stability and performance analysis of time-delay systems are of theoretical and practical importance. On the other hand, although robust performance analysis and synthesis for time-delay systems with polytopic uncertainty are investigated in Fridman et al. (2002), Fridman et al. (2002), He et al. (2004), multichannel H_∞ dynamic output-feedback synthesis still remains an open problem and the products of Lyapunov matrices and controller matrices are not separated completely that motivates the present paper.

In this paper, we solve the robust controller design problem for time-delay systems with polytopic uncertainty. The obtained robust stabilizing controller and robust H_∞ controller synthesis methods separate the products of controller variables and Lyapunov variables. Therefore they can reduce the conservativeness inherent in the conventional Lyapunov method and the previous literature in solving robust control problems for polytopic systems by providing a parameter-dependent Lyapunov function. Furthermore we design a dynamic H_∞ output-feedback controller for systems with multichannel constraints. The advantages of the results over the other methods are shown by numerical examples.

For simplification, we define $\|T_{zw}(s)\| = C(sI-A)^{-1}B+D$ that is the transfer function from w to z and use the symbol $Sym\{\cdot\}$ to denote $Sym\{X\} \overset{\text{def}}{=} X + X^T$, the symbol $*$ to denote the symmetric part.

2. ROBUST STATE-FEEDBACK CONTROLLER SYNTHESIS

Consider the following time-delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t) \\ z(t) &= col\{Cx(t), Du(t)\} \\ x(t) &= 0, \forall t \in [-h, 0] \end{aligned} \tag{1}$$

where, $x(t) \in R^n$ is the state, $u(t) \in R^{n_u}$ is the control input, $w(t) \in R^{n_w}$ is the disturbance signal of finite energy in the space $L_2[0,\infty)$, $z(t) \in R^{n_z}$ is the exogenous output, and A, B_1, B_2, A_d, C, D are constant matrices of appropriate dimensions. The time-delay $h > 0$ is assumed to be known.

The matrices of the system are uncertain and known to reside within a given polytope. Considering the system (1) and denoting

$$\Omega = \begin{bmatrix} A & A_d & B_1 \\ B_2 & C & D \end{bmatrix}$$

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we assume that

$$\Omega = \sum_{j=1}^{N} f_j \Omega_j, \quad for\ some\ \ 0 \le f_j \le 1, \sum_{j=1}^{N} f_j = 1 \qquad (2)$$

where the N vertices of the polytope are described by

$$\Omega_{(j)} = \begin{bmatrix} A^{(j)} & A_d^{(j)} & B_1^{(j)} \\ B_2^{(j)} & C^{(j)} & D^{(j)} \end{bmatrix}$$

We seek a control law

$$u(t) = Kx(t) \qquad (3)$$

that will asymptotically stabilize the system and guarantee H_∞ performance specification $\|T_{zw}(s)\|_\infty < \gamma$.

Lemma 1. Saadni et al. (2004) Let ϕ, a and b be given matrices of appropriate dimension. Then the two statements are equivalent:

(i) ϕ, a and b satisfy $\phi < 0$ and $\phi + ab^T + ba^T < 0$.
(ii) ϕ, a and b are such as the LMI

$$\begin{bmatrix} \phi & a + bG^T \\ * & -G - G^T \end{bmatrix} = \begin{bmatrix} \phi & a \\ * & 0 \end{bmatrix} + Sym\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} b^T & -I \end{bmatrix} \right\} < 0$$

is feasible in the variable G.

Lemma 2. Fridman et al. (2002) Let $w(t) = 0$. The control law (3) asymptotically stabilizes the system (1) for all the system parameters that reside in the uncertainty polytope, if there exists a symmetric and positive definite matrix $Q_1 > 0$ and matrices $Q_2^{(j)}$, $Q_3^{(j)}$, R, $Z_i^{(j)}$, $i = 1, 2, 3$ and Y such that the following LMIs hold for $j = 1, \cdots, N$. The state-feedback gain is then given by $K = YQ_1^{-1}$.

$$\begin{bmatrix} Sym\{Q_2^{(j)}\} + hZ_1^{(j)} & \Xi^{(j)} & hQ_2^{(j)T} \\ * & hZ_3^{(j)} - Sym\{Q_3^{(j)}\} & hQ_3^{(j)T} \\ * & * & -hR \end{bmatrix} < 0 \quad (4)$$

$$\begin{bmatrix} R & 0 & RA_d^{(j)T} \\ * & Z_1^{(j)} & Z_2^{(j)} \\ * & * & Z_3^{(j)} \end{bmatrix} \ge 0 \qquad (5)$$

where

$$\Xi^{(j)} = Q_3^{(j)} - Q_2^{(j)T} + Q_1(A^{(j)T} + A_d^{(j)T}) + hZ_2^{(j)} + Y^T B_2^{(j)T}$$

Lemma 3. Fridman et al. (2002) Consider the system (1), where the system matrices reside within the polytope Ω. For a prescribed $\gamma > 0$, the state-feedback law (3) achieves $\|T_{zw}\|_\infty < \gamma$ for all nonzero $w \in L_2[0, \infty)$ and for all the matrices in Ω if for a prescribed scalar $\lambda \in R$ there exists a symmetric and positive definite matrix $Q_1 > 0$ and matrices S, Q_2, Q_3, $R_i^{(j)}$, $i = 1, 2, 3$ and Y such that the LMI (6) holds as shown at the top of the next page for $j = 1, \cdots, N$. The state-feedback gain is then given by $K = YQ_1^{-1}$.
where

$$\Pi^{(j)} = Q_3 - Q_2^T + Q_1(A^{(j)} + (\lambda + 1)A_d^{(j)}) + Y^T B_2^{(j)T}$$

Lemma 4. Xu et al. (2005) Given any positive number $\gamma > 0$, the system (1) is asymptotically stable and $\|T_{zw}(s)\|_\infty < \gamma$, if there exist symmetric and positive-definite matrices $P_1 > 0$, $R > 0$ and matrices P_2, P_3, such that the following LMI holds:

$$\begin{bmatrix} (1,1) & (1,2) & -hP_2^T A_d & P_2^T B_1 & C^T \\ * & -P_3^T - P_3 + hR & -hP_3^T A_d & P_3^T B_1 & 0 \\ * & * & -hR & 0 & 0 \\ * & * & * & -\gamma^2 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (7)$$

where $(1,1) = P_2^T(A + A_d) + (A + A_d)^T P_2$, $(1,2) = P_1 - P_2^T + (A + A_d)^T P_3$.

Theorem 1. Let $w(t) = 0$. The control law (3) asymptotically stabilizes the system (1) for all the system parameters that reside in the uncertainty polytope, if for prescribed positive scalars $\varepsilon > 0$, $h > 0$, there exist symmetric and positive definite matrix $Q_1^{(j)} > 0$ and matrices $Q_2^{(j)}$, $Q_3^{(j)}$, R, $Z_i^{(j)}$, $i = 1, 2, 3$, V and U that satisfy the following LMIs for $j = 1, \cdots, N$. The state-feedback gain is then given by $K = UV^{-1}$.

$$\begin{bmatrix} (1,1) & (1,2) & hQ_2^{(j)T} & Q_1^{(j)} + \frac{1}{2}\varepsilon V \\ * & (2,2) & hQ_3^{(j)T} & (2,4) \\ * & * & -hR & 0 \\ * & * & * & -V - V^T \end{bmatrix} < 0 \qquad (8)$$

$$\begin{bmatrix} R & 0 & RA_d^{(j)T} \\ * & Z_1^{(j)} & Z_2^{(j)} \\ * & * & Z_3^{(j)} \end{bmatrix} \ge 0 \qquad (9)$$

where

$$(1,1) = -\varepsilon Q_1^{(j)} + Q_2^{(j)} + Q_2^{(j)T} + hZ_1^{(j)}$$
$$(1,2) = Q_3^{(j)} - Q_2^{(j)T} + hZ_2^{(j)}$$
$$(2,2) = -Q_3^{(j)} - Q_3^{(j)T} + hZ_3^{(j)}$$
$$(2,4) = (A^{(j)} + A_d^{(j)})V + B_2^{(j)}U$$

Proof: Firstly, we consider the system (1) without polytopic uncertainty. It is obvious that there exists a large enough positive scalar ε such that the following equation holds.

$$\Gamma = \begin{bmatrix} (1,1) & Q_3 - Q_2^T + hZ_2 & hQ_2^T \\ * & -Q_3 - Q_3^T + hZ_3 & hQ_3^T \\ * & * & -hR \end{bmatrix} < 0 \qquad (10)$$

where $(1,1) = -\varepsilon Q_1 + Q_2 + Q_2^T + hZ_1$.

When the matrices of the system (1) are exactly known that is we consider the system (1) without uncertainty, the equation (4) can be rewritten

$$\Gamma + ab^T + ba^T < 0 \qquad (11)$$

where $b = \begin{bmatrix} \frac{1}{2}\varepsilon & A + A_d + B_2K & 0 \end{bmatrix}^T$, $a = \begin{bmatrix} Q_1 & 0 & 0 \end{bmatrix}^T$.
From Lemma 1, we can get the equations (10) and (11) are equivalent to the following equation.

$$\begin{bmatrix} (1,1) & (1,2) & hQ_2^T & Q_1 + \frac{1}{2}\varepsilon V \\ * & (2,2) & hQ_3^T & (A + A_d)V + B_2U \\ * & * & -hR & 0 \\ * & * & * & -V - V^T \end{bmatrix} < 0 \qquad (12)$$

$$\begin{bmatrix} Q_2+Q_2^T & \Pi^{(j)} & 0 & h(\lambda+1)R_1^{(j)} & h(\lambda+1)R_2^{(j)} & 0 & Q_1^{(j)} & Q_1^{(j)}C^{(j)T} & Y^TD^{(j)T} & 0 & hQ_2^TA_d^{(j)T} \\ * & -Q_3-Q_3^T & B_1^{(j)} & h(\lambda+1)R_2^{(j)T} & h(\lambda+1)R_3^{(j)} & \lambda A_d^{(j)}S & 0 & 0 & 0 & 0 & hQ_3^TA_d^{(j)T} \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -hR_1^{(j)} & -hR_2^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -hR_3^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -hR_1^{(j)} & -hR_2^{(j)} \\ * & * & * & * & * & * & * & * & * & * & -hR_3^{(j)} \end{bmatrix} < 0 \qquad (6)$$

where

$$(1,1) = -\varepsilon Q_1 + Q_2 + Q_2^T + hZ_1$$
$$(1,2) = Q_3 - Q_2^T + hZ_2$$
$$(2,2) = -Q_3 - Q_3^T + hZ_3$$

Therefore, given a large enough ε, (4) is equivalent to (12). If matrices of the system are known to reside within a given polytope, take the matrices A, A_d, B_2, Q_i, Z_i with the upper index j. Then we can obtain the equation (8). This completes the proof.

Example 1: We consider the following time-delay system

$$\dot{x}(t) = Ax(t) + A_d x(t-h) + B_2 u(t) \qquad (13)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -e & f \end{bmatrix}, A_d = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad (14)$$

where $0.8 \le e \le 1.2$, $0.6 \le f \le 0.8$, therefore, the plant can be described as a polytope with four vertices. The problem here is to find a state-feedback gain K stabilizing the system (13) for all possible values of the parameters e and f. Applying Lemma 2 and the method in this paper to this problem, we get the maximum bound of time delay h and the corresponding state-feedback gain.

TABLE I
THE MAXIMUM BOUND OF DELAY AND FEEDBACK GAIN

Method	Maximum bound	The state-feedback gain
Lemma 2	1.42	$K = [\,-897.0114\;\; 432.2629\,]$
This paper	1.42	$K = [\,-45.4262\;\; 16.6890\,]$

We can see that although the maximum bound of time delay obtained by Fridman et al. (2002) and the present method is the same, the latter reaches much lower gain than the former. Therefore it is less conservative.

Theorem 2. Consider the system (1), where the system matrices reside within the polytope Ω. For prescribed scalars $\gamma > 0$, $h > 0$, the state-feedback law (3) achieves $\|T_{zw}\|_\infty < 0$ for all nonzero $w \in L_2[0,\infty)$ and for all the matrices in Ω if for a prescribed positive scalar $\varepsilon > 0$ and scalar $\lambda \in R$ there exist symmetric and positive definite matrices $Q_1^{(j)} > 0$ and matrices S, Q_2, Q_3, $R_i^{(j)}$, $i = 1,2,3$, V and U that satisfy the LMI (15) as shown at the top of the next page for $j = 1,\cdots,N$. The state-feedback gain is then given by $K = UV^{-1}$.
where

$$(1,1) = -\varepsilon Q_1^{(j)} + Q_2 + Q_2^T$$
$$(1,12) = Q_1^{(j)} + \frac{1}{2}\varepsilon V$$
$$(2,12) = [A^{(j)} + (\lambda+1)A_d^{(j)}]V + B_2^{(j)}U$$

Proof: Along similar lines as in the proof of Theorem 1, the desired result follows immediately.

Remark 1. From Lemma 2 and Lemma 3, we can see that the controller matrix is coupled with a symmetric and positive definite Q_1. Therefore, the products of controller matrices and Lyapunov matrices are not separated completely. However, with our method, the weakness is eliminated. Therefore, it is expected to obtain less conservative results.

Example 2: We consider the following time-delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t) \\ z(t) &= col\{Cx(t), Du(t)\} \end{aligned} \qquad (16)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [\,0\;\;1\,], D = 0.1 \\ A_d &= \begin{bmatrix} -1 & -1 \\ 0 & -0.9+g \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1-g \end{bmatrix} \\ g &\in [-0.2\;\;0.7] \end{aligned} \qquad (17)$$

Applying Lemma 3 we obtain for $h = 0.99$ and $\lambda = -0.3$ that $K = [\,0\; -1.3962\,] \times 10^7$ stabilizes the system and achieves $\gamma = 0.34$ for all $g \in [-0.2\;\;0.7]$. For $\gamma = 0.4$, a gain of $K = [\,0\; -215.53\,]$ is achieved.

Applying the method in this paper for the same h and $\lambda = -0.32$, $\varepsilon = 3350$, we can get that $K = [\,0\; -498.0365\,]$ stabilizes the system and achieves $\gamma = 0.34$ for all $g \in [-0.2\;\;0.7]$. For $\gamma = 0.4$, a gain of $K = [\,0\; -48.2561\,]$ can be achieved.

We can see that the present method achieves much lower gain than Lemma 3. Therefore it is less conservative.

3. MULTICHANNEL H_∞ OUTPUT-FEEDBACK SYNTHESIS

In this section, we provide a method for output-feedback synthesis with multichannel constraints. Consider the following time-delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) \end{aligned} \qquad (18)$$

$$\begin{bmatrix} (1,1) & Q_3-Q_2^T & 0 & h(\lambda+1)R_1^{(j)} & h(\lambda+1)R_2^{(j)} & 0 & Q_1^{(j)} & Q_1^{(j)}C^{(j)T} & Y^TD^{(j)T} & 0 & hQ_2^TA_d^{(j)T} & (1,12) \\ * & -Q_3-Q_3^T & B_1^{(j)} & h(\lambda+1)R_2^{(j)T} & h(\lambda+1)R_3^{(j)} & \lambda A_d^{(j)}S & 0 & 0 & 0 & 0 & hQ_3^TA_d^{(j)T} & (2,12) \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -hR_1^{(j)} & -hR_2^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -hR_3^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -S & 0 & 0 & 0 & 0 & V \\ * & * & * & * & * & * & * & -I & 0 & 0 & 0 & CV \\ * & * & * & * & * & * & * & * & -I & 0 & 0 & DU \\ * & * & * & * & * & * & * & * & * & -hR_1^{(j)} & -hR_2^{(j)} & 0 \\ * & * & * & * & * & * & * & * & * & * & -hR_3^{(j)} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -V-V^T \end{bmatrix} < 0 \quad (15)$$

where $y(t) \in R^{n_y}$ is the measured output and the other signals are the same with the system (1).

The goal is to design a full-order strictly proper output-feedback controller K given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A_K\hat{x}(t) + B_K y(t) \\ u(t) &= C_K\hat{x}(t) \end{aligned} \quad (19)$$

which meets a family of input-output specifications. One such set of specifications is for instance $\|L_1T_{zw}(s)R_1\|_\infty < \gamma_1, \|L_2T_{zw}(s)R_2\|_\infty < \gamma_2$. Matrices L_i, R_i are selected matrices that specify which channel is involved in the corresponding constraint.

Applying the controller (19) to (18) will result in the closed-loop system

$$\begin{aligned} \dot{\hat{x}}_c(t) &= A_c\hat{x}_c(t) + A_{dc}\hat{x}_c(t-h) + B_{1c}w(t) \\ \hat{z}_c(t) &= C_c\hat{x}_c(t) \end{aligned} \quad (20)$$

where

$$\begin{aligned} &\hat{x}_c = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, A_c = \begin{bmatrix} A & B_2C_K \\ B_KC_2 & A_K \end{bmatrix}, A_{dc} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix} \\ &B_{1c} = \begin{bmatrix} B_1 \\ B_KD_{21} \end{bmatrix}, C_c = [\, C_1 \;\; D_{12}C_K \,] \end{aligned} \quad (21)$$

The desired characterization for output-feedback synthesis with multichannel specifications can be derived in the following three steps: 1) introduce different Lyapunov matrices $T_1^{(j)}$ for each channel; 2) introduce a variable V common to all channels; 3) perform adequate congruence transformations and use linear transformations of variables to end up with LMI synthesis condition.

Then we design a dynamic output-feedback H_∞ controller with form (19) for the closed-loop system (20).

Theorem 3. For prescribed positive scalars $\gamma > 0$, $h > 0$, $\varepsilon > 0$ and scalar λ, there exists a dynamical output-feedback controller such that the closed-loop system (20) is asymptotically stable and satisfies $\|T_{zw}(s)\|_\infty < \gamma$ for all nonzero $w \in L_2[0, \infty)$ if there exist symmetric and positive-definite matrices $T_1^{(j)} > 0$, $S > 0$, $R > 0$ and matrices V_{11}, W_{21}, W_{11}, U, $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ such that the LMIs holds, as shown in (22) at the top of the next page, where

$$(1,7) = T_1^{(j)} + \frac{1}{2}\varepsilon \begin{bmatrix} W_{11}^T & U^T \\ I & V_{11} \end{bmatrix}$$

$$(2,7) = \begin{bmatrix} -hA_d & -hA_dV_{11} \\ 0 & 0 \end{bmatrix}$$

$$(3,7) = [\, B_1^T \;\; B_1^TV_{11} + D_{21}^T\mathcal{L}_2^T \,]$$

$$(4,6) = [\, C_1W_{11} + D_{12}\mathcal{L}_3 \;\; C_1 \,]$$

$$(5,8) = h\begin{bmatrix} W_{11}^T & U^T \\ I & V_{11} \end{bmatrix}$$

$$(6,7) = \begin{bmatrix} (A+A_d)W_{11} + B_2\mathcal{L}_3 & A + A_d \\ \mathcal{L}_1 & V_{11}^T(A+A_d) + \mathcal{L}_2C_2 \end{bmatrix}^T$$

$$(7,7) = -\begin{bmatrix} W_{11} + W_{11}^T & U^T + I \\ * & V_{11} + V_{11}^T \end{bmatrix}$$

$$(8,8) = -h\begin{bmatrix} 2I & V_{11} \\ * & V_{21}^T + V_{21} \end{bmatrix} + hR$$

Proof: Applying Lemma 4 to the closed-loop system (20), we can get

$$\begin{bmatrix} (1,1) & (1,2) & -hP_2^TA_{dc} & P_2^TB_{1c} & C_c^T \\ * & -P_3^T - P_3 + hR & -hP_3^TA_{dc} & P_3^TB_{1c} & 0 \\ * & * & -hR & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (23)$$

where

$$\begin{aligned} (1,1) &= P_2^T(A_c + A_{dc}) + (A_c + A_{dc})^TP_2 \\ (1,2) &= P_1 - P_2^T + (A_c + A_{dc})^TP_3 \end{aligned}$$

Pre- and Post-multiply inequality (23) with $diag\{P_2^{-T}, P_3^{-T}, I, I, I\}$ and its inverse, respectively, and follow the Schur complement Lemma, we can have

$$\begin{bmatrix} (1,1) & (1,2) & -hA_{dc} & B_{1c} & P_2^{-T}C_c^T & 0 \\ * & (2,2) & -hA_{dc} & B_{1c} & 0 & hP_3^{-T} \\ * & * & -hR & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -hR^{-1} \end{bmatrix} < 0 \quad (24)$$

where

$$\begin{aligned} (1,1) &= (A_c + A_{dc})P_2^{-1} + P_2^{-T}(A_c + A_{dc})^T \\ (1,2) &= P_2^{-T}P_1P_3^{-1} - P_3^{-1} + P_2^{-T}(A_c + A_{dc})^T \\ (2,2) &= -P_3^{-T} - P_3^{-1} \end{aligned}$$

$$\begin{bmatrix} -\varepsilon T_1^{(j)} - T_3 - T_3^T & 0 & 0 & 0 & hT_3^T & T_3^T + T_2 & (1,7) & 0 & 0 \\ * & -hR & 0 & 0 & 0 & 0 & (2,7) & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & (3,7) & 0 & 0 \\ * & * & * & -I & 0 & (4,6) & 0 & 0 & 0 \\ * & * & * & * & -2hT_1^{(j)} & -hT_2 & 0 & (5,8) & 0 \\ * & * & * & * & * & -T_2 - T_2^T & (6,7) & 0 & 0 \\ * & * & * & * & * & * & (7,7) & 0 & 0 \\ * & * & * & * & * & * & * & (8,8) & hI \\ * & * & * & * & * & * & * & * & -hR \end{bmatrix} < 0 \quad (22)$$

Define

$$U = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

Making a congruence transformation on (24) with U, pre- and post-multiplying the obtained equation with $diag\{P_2^T, P_1, I, I, I, P_1\}$, defining $Q_2 = P_1 P_3^{-1} P_2$, $Q_3 = P_1 P_3^{-1} P_1$ and following the Schur complement Lemma yield

$$\begin{bmatrix} (1,1) & -hP_1 A_{dc} & P_1 B_{1c} & 0 & hQ_3^T & (1,6) \\ * & -hR & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -I & 0 & C_c \\ * & * & * & * & (5,5) & -hQ_2 \\ * & * & * & * & * & -Q_2 - Q_2^T \end{bmatrix} < 0 \quad (25)$$

where

$$(1,1) = -Q_3 - Q_3^T$$
$$(1,6) = Q_3^T + Q_2 + P_1(A_c + A_{dc})$$
$$(5,5) = -hP_1 R^{-1} P_1$$

Along similar lines as in the proof of Theorem 1, given a large enough $\varepsilon > 0$, we can get (25) is equivalent to the following equation.

$$\begin{bmatrix} (1,1) & 0 & 0 & 0 & hQ_3^T & (1,6) & (1,7) \\ * & -hR & 0 & 0 & 0 & 0 & -hA_{dc}^T V \\ * & * & -\gamma^2 I & 0 & 0 & 0 & B_{1c}^T V \\ * & * & * & -I & 0 & C_c & 0 \\ * & * & * & * & (5,5) & -hQ_2 & 0 \\ * & * & * & * & * & (6,6) & (6,7) \\ * & * & * & * & * & * & -V - V^T \end{bmatrix} < 0 \quad (26)$$

where

$$(1,1) = -\varepsilon P_1 - Q_3 - Q_3^T$$
$$(1,6) = Q_3^T + Q_2$$
$$(1,7) = P_1 + \frac{1}{2}\varepsilon V$$
$$(5,5) = -hP_1 R^{-1} P_1$$
$$(6,6) = -Q_2 - Q_2^T$$
$$(6,7) = (A_c + A_{dc})^T V$$

Partition V and its inverse V^{-1} in the equation (26) as

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, W = V^{-1} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \quad (27)$$

From $VW = I$, define

$$F_1 = \begin{bmatrix} W_{11} & I \\ W_{21} & 0 \end{bmatrix}, F_2 = \begin{bmatrix} I & V_{11} \\ 0 & V_{21} \end{bmatrix} \quad (28)$$

Pre- and post-multiply the equation (26) by $diag\{F_1^T, I, I, I, F_1^T, F_1^T, F_1^T\}$ and its inverse, respectively, substitute (21), (27), (28) into the obtained equation, and write

$$\begin{aligned} \mathcal{L}_1 &= V_{11}^T(A + A_d)W_{11} + V_{21}^T B_K C_2 W_{11} \\ &\quad + V_{11}^T B_2 C_K W_{21} + V_{21}^T A_K W_{21} \\ \mathcal{L}_2 &= V_{21}^T B_K, \mathcal{L}_3 = C_K W_{21}, U = V_{11}^T W_{11} + V_{21}^T W_{21} \\ T_1 &= F_1^T P_1 F_1, T_2 = F_1^T Q_2 F_1, T_3 = F_1^T Q_3 F_1 \end{aligned} \quad (29)$$

Obviously, $-F_1^T P_1 R^{-1} P_1 F_1 \le -2F_1^T P_1 F_1 + F_1^T R F_1 \le -2F_1^T P_1 F_1 + F_1^T V F_1 (F_1^T V^T + V F_1 - R^{-1})^{-1} F_1^T V^T F_1$. Using the above equations, the equation (22) can be derived. This completes the proof.

Remark 2. With W_{11}, we can define the nonsingular matrix $\Pi = \begin{bmatrix} I & I \\ 0 & -W_{11} \end{bmatrix}$. The product of the two matrices $F_1^T V F_1$ and Π is

$$F_1^T V^T F_1 \Pi = \begin{bmatrix} W_{11} & 0 \\ U & U - V_{11}^T W_{11} \end{bmatrix}$$

which assures the nonsingularity of $U - V_{11}^T W_{11}$. And it can be seen from the block (8,8) in the equation (22) that V_{21} is nonsingularity. Then we can get W_{21} is also nonsingularity.

Remark 3. Given any solution of the LMIs in Theorem 3, a corresponding controller with form (19) will be constructed as follows:
1. compute W_{21} from $W_{21} = V_{21}^{-T}(U - V_{11}^T W_{11})$; 2. utilizing the matrices W_{21} obtained above and V_{21}, compute the controller data B_K, C_K and A_K (in that order).

Remark 4. In contrast with earlier results, a different Lyapunov function is employed for each channel. Hence far better results can generally be expected.

4. CONCLUSIONS

The problem of robust control is investigated for a class of time-delay systems with polytopic uncertainty. Robust stabilizing controller and robust H_∞ controller are designed and the corresponding conditions are given in terms of LMIs which decouple Lyapunov matrices and controller matrices. Therefore, they are less conservative when used in robust synthesis problems for time-delay systems with polytopic uncertainty and the multichannel H_∞ dynamic output-feedback synthesis problems for time-delay systems. Numerical examples show that the proposed method does provide a further improvement in reducing conservativeness for systems with polytopic uncertainty. Unfortunately, the scalar ε nonlinearly appears in the LMI conditions and thus causes troubles to optimization of the robust disturbance attenuation level. Further research, therefore, includes how to eliminate the nonlinear influence.

REFERENCES

S. Boyd, L.E. Ghaoui, E. Feron, and V. Balakrishnan. Linear Matrix Inequality in Systems and Control Theory. *Philadelphia: SIAM*, 1994.

M.C.de Oliveira, J.Bernussou, and J.C.Geromel. A new discrete-time robust stability condition. *Syst. Contr. Lett.*, volume 37, pages 261-265, 1999.

M.C. de Oliveira, J.C. Geromel, and J. Bernussou. An LMI optimization approach to multiobjective controller design for discrete-time systems. *Proc. 38th Conf. Decision Control*, Phoenix, Arizona USA, pages 3611-3616, 1999.

P.Apkarian, H.D.Tuan, and J.Bernussou. Continuous-time analysis, eigenstructure assignment, and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations. *IEEE Trans. Automat. Contr.*, volume 46, no 12, pages 1941-1946, 2001.

U.Shaked. Improved LMI representation for the analysis and the design of continuous-time systems with polytopic type uncertainty. *IEEE Trans. Automat. Contr.*, volume 46, no 4, pages 652-656, 2001.

Y. Ebihara and T. Hagiwara. Mixed $H_2/D-$stability synthesis with new dilated LMI characterizations. *Proc. SICE 1st Annual Conference on Control Systems*, pages 619-624, 2001.

E. Fridman and U. Shaked. An improved stabilization method for linear time-delay systems. *IEEE Transactions on Automatic Control*, volume 47, no 11, pages 1931-1937, 2002.

E. Fridman and U. Shaked. A descriptor system approach to H_∞ control of linear time-delay systems. *IEEE Transactions on Automatic Control*, volume 47, no 2, pages 253-270, 2002.

Y. He, M. Wu, J. She, and G. Liu. Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties. *IEEE Trans. Automat. Contr.*, volume 49, no 5, pages 828-832, 2004.

M.S.Saadni and D.Mehdi. Stability and stabilizability of a class of state retarded uncertain polytopic systems. *IEEE International Conference on Systems, Man and Cybernetics*, pages 4244-4248, 2004.

S. Xu, J. Lam, and Y. Zou. Simplified descriptor system approach to delay-dependent stability and performance analyses for time-delay systems. *IEE Proc. Control Theory Appl.*, volume 152, no 2, pages 147-151, 2005.