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DESIGN OF ROBUST DELAY-DEPENDENT GUARANTEED COST CONTROLLER FOR UNCERTAIN NONLINEAR NEUTRAL SYSTEMS: AN LMI DESCRIPTOR APPROACH

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Abstract: In this paper, a novel robust delay-dependent guaranteed cost controller is introduced for a class of uncertain nonlinear neutral systems with both norm-bounded uncertainties and nonlinear parameter perturbations. A neutral memory state-feedback control law is chosen such that a quadratic cost function is minimized. On the basis of a descriptor type model transformation, an augmented descriptor form Lyapunov-Krasovskii functional is proposed. A linear matrix inequality (LMI) of synthesis condition is derived. Two numerical examples have been introduced to show the application of the theoretical results.

Keywords: Neutral systems, descriptor representation, uncertainty, parameter perturbations, quadratic cost, linear matrix inequality.

1. INTRODUCTION

Time-delay systems is a significantly fruitful field of research for several decades. The main reason for this is that time-delay naturally arises in many physical and dynamical systems leading to bring about many theoretical and practical problems which need to be resolved. Among some of them, one can notice that a system which is subject to time-delay may exhibit a poor performance, it may get into an unstable behavior or it may not endeveour with external disturbances or even it may excite undesired modes which cause to get far away from meeting the required specifications. An important class of delay-differential systems is referred to as time-delay systems of neutral type in which the time-delay may be available in both position and velocity of the system. For a theoretical and practical consideration of neutral systems, one can refer to (Hale, and Lunel, 1993).

In recent years, a number of different guaranteed cost controller design investigations have been taken into consideration for neutral systems with or without uncertainties and/or nonlinear parameter perturbations. See the reference list. Generally, the proposed methods have employed linear matrix inequality (LMI) techniques. It is well known that (Fridman, and Shaked, 2002), (He *et al.*, 2004), Parlakci (2006), the utilization of some free weighting parameters introduce a considerable rate of relaxation into the results in particular for systems under the existence of uncertainties. However, the existing work on the subject has not taken into account this useful phenomenon because of the fact that the involvement of such slack variables destroy the nature of the LMI form of the stability and/or synthesis conditions. Therefore, the motivation of the present research is to develop improved LMI delay-dependent guaranteed cost controller synthesis through the use of free weighting matrices.

In this paper, the design problem of a robust delay-dependent guaranteed cost control of a class of uncertain nonlinear systems with both norm-bounded uncertainties and nonlinear parameter perturbations has been studied on the basis of a descriptor form of system representation along with an augmented descriptor type of Lyapunov-Krasovskii functional which has been recently proposed by Parlakci (2006). The proposed approach allows to introduce free weighting matrices embedded in the Lyapunov-Krasovskii functional. A memory state-feedback controller in the form of neutral structure is introduced to provide integral and delayed state-feedback for a better control performance. The use of descriptor system representation in the quadratic Lyapunov stability analysis has been applied to both delay-differential equation and neutral operator equation which lead to be able to include additional slack variables. A sufficient robust delay-dependent guaranteed cost controller synthesis criterion is derived in terms of relaxed linear matrix inequality. Moreover, a convex optimization problem with LMI constraints is formulated to design the optimal guaranteed cost delayed neutral state-feedback controller which minimizes the upper bound of the guaranteed cost function for the closed-loop uncertain nonlinear neutral system. Two numerical examples show that the novel augmented descriptor LMI approach improves the guaranteed cost performance in comparison to the existing guaranteed cost controllers from the literature.

2. PROBLEM STATEMENT

Let us consider

$$\begin{aligned}\dot{x}(t) &= [A+\Delta A(t)]x(t)+[A_h+\Delta A_h(t)]x(t-h)\\ &+[A_d+\Delta A_d(t)]\dot{x}(t-d)+[B+\Delta B(t)]u(t)\\ &+f(x(t),t)+g(x(t-h),t)+h(\dot{x}(t-d),t)\end{aligned} \quad (1)$$

$$x(t)=\Phi(t),\ \dot{x}(t)=\dot{\Phi}(t),\ \forall t\in[-\tau,0],\ \tau>0 \quad (2)$$

where $x(t)\in\Re^n$ is the state vector of the system, $u(t)\in\Re^m$ is the control input, A, A_h, A_d, B are known real constant system matrices all with appropriate dimensions, $\Delta A(t)$, $\Delta A_h(t)$, $\Delta A_d(t)$, $\Delta B(t)$ are unknown real time-varying matrix functions with appropriate dimensions representing time-varying parametric uncertainties which are assumed to be of the following form

$$\begin{aligned}&[\Delta A(t)\quad \Delta A_h(t)\quad \Delta A_d(t)\quad \Delta B(t)]\\ &\quad = DF(t)[E_a\quad E_h\quad E_d\quad E_b]\end{aligned} \quad (3)$$

where D, E_a, E_h, E_d, E_b are known constant matrices with appropriate dimensions, and $F(t)$ is an unknown real time-varying matrix satisfying

$$F^T(t)F(t)\le \mathrm{I} \quad (4)$$

and $f(x(t),t)$, $g(x(t-h),t)$, $h(\dot{x}(t-d),t)$ represent nonlinear parameter perturbations satisfying

$$f(0,t)=g(0,t)=h(0,t)=0 \quad (5)$$

and $\Phi(\cdot)$ is a vector valued initial condition function. Note that the parameter uncertainties are said to be admissible if both (3) and (4) hold. Moreover, it is assumed that the nonlinear perturbations satisfy

$$\|f(x(t),t)\|\le\alpha\|x(t)\|,\quad \forall t>0 \quad (6)$$

$$\|g(x(t-h),t)\|\le\beta\|x(t-h)\|,\quad \forall t>0 \quad (7)$$

$$\|h(\dot{x}(t-d),t)\|\le\gamma\|\dot{x}(t-d)\|,\quad \forall t>0 \quad (8)$$

where α, β, and γ are known positive scalars. The discrete, and neutral delays are scalar constants such that $\tau=\max(h,d)$. A difference operator can be defined as $\nabla:\Im(-[\tau,0],\Re^n)\to\Re^n$ such that

$$\nabla(x_t)=x(t)+A_h\int_{t-h}^{t}x(s)ds-A_dx(t-d) \quad (9)$$

Definition 1: (Hale, and Lunel, 1993) The difference operator ∇ is said to be stable if the zero solution of the homogeneous difference equation $\nabla x_t=0$, $t\ge 0$, $x_0=\psi\in\{\phi\in C(-[\tau,0]):\nabla\phi=0\}$ is uniformly asymptotically stable. The stability of ∇ is necessary for the stability of system (1).

Assumption 1: (Park, 2005b) Given positive scalars h, d, and any constant matrices A_h, $A_d\in\Re^{n\times n}$, the operator $\nabla(x_t)$ is asymptotically stable if there exist a symmetric and positive definite matrix, Λ_0 and positive scalars ρ_1, ρ_2 such that

$$\rho_1+\rho_2<1$$

$$\begin{bmatrix}A_d^T\Lambda_0A_d-\rho_1\Lambda_0 & hA_d^T\Lambda_0A_h\\ * & -\rho_2\Lambda_0+h^2A_h^T\Lambda_0A_h\end{bmatrix}<0 \quad (10)$$

A quadratic cost function associated with the uncertain nonlinear neutral system (1) is defined as

$$J=\int_0^\infty[x^T(t)S_1x(t)+u^T(t)S_2u(t)]dt \quad (11)$$

where S_1, and S_2 are given constant gain matrices. A linear memory type neutral state feedback control law is chosen as in (Park, 2005b) as follows

$$u(t)=K\nabla(x_t) \quad (12)$$

where $K\in\Re^{m\times n}$ denotes the feedback gain.

3. MAIN RESULTS

Let us assume that system (1) is not subject to any norm-bounded uncertainty, then one can obtain

$$\begin{aligned}\dot{x}(t)&=Ax(t)+A_hx(t-h)+A_d\dot{x}(t-d)\\ &+BK\nabla(x_t)+f(x(t),t)+g(x(t-h),t)\\ &+h(\dot{x}(t-d),t)\end{aligned} \quad (13)$$

Differentiating $\nabla(x_t)$ along trajectory of (13) gives

$$\begin{aligned}\dot{\nabla}(x_t)&=A_0x(t)+BK\nabla(x_t)+f(x(t),t)\\ &+g(x(t-h),t)+h(\dot{x}(t-d),t)\end{aligned} \quad (14)$$

where $A_0=A+A_h$, or as a descriptor form, we get

$$\begin{aligned}\dot{\nabla}(x_t)&=y(t),\ \ y(t)=A_0x(t)+BK\nabla(x_t)+f(x(t),t)\\ &+g(x(t-h),t)+h(\dot{x}(t-d),t)\end{aligned} \quad (15)$$

where $y(t)$ is the descriptor variable.

Theorem 1: Given the positive scalars, h, d, and ρ_1, ρ_2 such that $\rho_1+\rho_2<1$, if there exist symmetric positive definite matrices Λ_0, X_1, $\bar{Q}$, $\bar{R}$, $\bar{S}$, $\bar{Z}$, and matrices X_i,

Y, $i = 2,\ldots,6$ and scalars λ, $\bar{\varepsilon}_j$, $j = 1,2,3$, ε_k, $k = 4,5,6$ satisfying

$$\begin{bmatrix} -\rho_1\Lambda_0 & \lambda h E_d^T E_h & A_d^T\Lambda_0 & 0 \\ * & \begin{matrix} -\rho_2\Lambda_0 \\ +\lambda h^2 E_h^T E_h \end{matrix} & hA_h^T\Lambda_0 & 0 \\ * & * & -\Lambda_0 & \Lambda_0 D \\ * & * & * & -\lambda\mathrm{I} \end{bmatrix} < 0 \quad (16)$$

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{11} & h\bar{\Sigma}_{12} & \bar{\Sigma}_{12} & \bar{\Sigma}_{12} & \bar{\Sigma}_{15} & \bar{\Sigma}_{16} \\ * & -\bar{Q} & 0 & 0 & 0 & 0 \\ * & * & -\bar{R} & 0 & 0 & 0 \\ * & * & * & -S_1^{-1} & 0 & 0 \\ * & * & * & * & -S_2^{-1} & 0 \\ * & * & * & * & * & \bar{\Sigma}_{66} \end{bmatrix} < 0 \quad (17)$$

$$\bar{\Sigma}_{11} = \begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 & 0 & (1,6) \\ * & (2,2) & (2,3) & 0 & 0 & (2,6) \\ * & * & (3,3) & A_h\bar{Q} & -A_d\bar{R} & (1,6) \\ * & * & * & -\bar{Q} & 0 & (1,6) \\ * & * & * & * & -\bar{R} & (1,6) \\ * & * & * & * & * & (6,6) \end{bmatrix},$$

$(1,1) = X_2 + X_2^T, (1,2) = X_3 + Y^T B^T - X_2^T + X_4^T A_0^T$,

$(1,3) = -X_1 + X_4^T, (1,6) = [0 \quad 0 \quad 0 \quad 0 \quad 0]$,

$(2,2) = -X_3 - X_3^T + A_0 X_5 + X_5^T A_0^T + \varepsilon_4 DD^T$,

$(2,3) = A_0 X_6 + X_5^T, (2,6) = [\bar{\varepsilon}_1\mathrm{I} \quad 0 \quad \bar{\varepsilon}_2\mathrm{I} \quad 0 \quad \bar{\varepsilon}_3\mathrm{I}]$,

$(3,3) = X_6 + X_6^T + \varepsilon_5 DD^T$,

$(6,6) = \mathrm{diag}\{-\bar{\varepsilon}_1\mathrm{I}, -\bar{S}, -\bar{\varepsilon}_2\mathrm{I}, -\bar{Z}, -\bar{\varepsilon}_3\mathrm{I}\}$,

$\bar{\Sigma}_{12} = [hX_4 \quad hX_5 \quad hX_6 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$,

$\bar{\Sigma}_{15} = [Y \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$,

$\bar{\Sigma}_{16} = [\alpha\bar{\Sigma}_{12} \quad \bar{\Sigma}_{12} \quad \bar{\Sigma}_{18} \quad \bar{\Sigma}_{19} \quad \bar{\Sigma}_{110} \quad \bar{\Sigma}_{111}]$,

$\bar{\Sigma}_{18} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \beta\bar{S} \quad 0 \quad 0 \quad 0]^T$,

$$\bar{\Sigma}_{19} = \begin{bmatrix} \begin{matrix} BY \\ +AX_4 \end{matrix} & AX_5 & AX_6 & 0 & 0 & \bar{\varepsilon}_1\mathrm{I} & A_h\bar{S} & \bar{\varepsilon}_2\mathrm{I} & A_d\bar{Z} & \bar{\varepsilon}_3\mathrm{I} \end{bmatrix}^T,$$

$\bar{\Sigma}_{110} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \gamma\bar{Z} \quad 0]$,

$$\bar{\Sigma}_{111} = \begin{bmatrix} \begin{matrix} E_bY \\ +E_0X_4 \end{matrix} & E_0X_5 & E_0X_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_hQ & -E_dR & 0 & 0 & 0 & 0 & 0 \\ \begin{matrix} E_bY \\ +E_aX_4 \end{matrix} & E_aX_5 & E_aX_6 & 0 & 0 & 0 & E_h\bar{S} & 0 & E_d\bar{Z} & 0 \end{bmatrix}$$

$\bar{\Sigma}_{66} = \mathrm{diag}\{-\bar{\varepsilon}_1\mathrm{I}, -\bar{S}, -\bar{\varepsilon}_2\mathrm{I}, -\bar{Z} + \varepsilon_6 DD^T, -\bar{\varepsilon}_3\mathrm{I}, -\varepsilon_0\mathrm{I}\}$,

and $\varepsilon_0 = \begin{bmatrix} \varepsilon_4\mathrm{I} & 0 & 0 \\ 0 & \varepsilon_5\mathrm{I} & 0 \\ 0 & 0 & \varepsilon_6\mathrm{I} \end{bmatrix}$, and (*) denotes the symmetricity, then a robust guaranteed cost neutral state-feedback controller with $K = YX^{-1}$ robustly asymptotically stabilizes system (1) via ensuring an upper bound for the cost function computed as

$$J^* = \nabla^T(\Phi(0))X^{-1}\nabla(\Phi(0)) + h\int_{-h}^{0}(s+h)\Phi^T(s)\bar{Q}^{-1}\Phi(s)ds + \int_{-d}^{0}\Phi^T(s)\bar{R}^{-1}\Phi(s)ds + \int_{-h}^{0}\Phi^T(s)\bar{S}^{-1}\Phi(s)ds + \int_{-d}^{0}\dot{\Phi}^T(s)\bar{Z}^{-1}\dot{\Phi}(s)ds \quad (18)$$

Proof: Applying Schur complement to (10) gives

$$\Psi = \begin{bmatrix} -\rho_1\Lambda_0 & 0 & A_d^T\Lambda_0 \\ * & -\rho_2\Lambda_0 & hA_h^T\Lambda_0 \\ * & * & -\Lambda_0 \end{bmatrix} < 0 \quad (19)$$

then replacing A_h, A_d with $A_h + \Delta A_h(t)$, $A_d + \Delta A_d(t)$, respectively in (19) gives

$$\Psi + \Delta\Psi(t) + \Delta\Psi^T(t) < 0 \quad (20)$$

where

$$\Delta\Psi(t) = \Pi_0^T F(t)\Theta_0$$

with $\Pi_0 = [0 \quad 0 \quad D^T\Lambda_0]$, $\Theta_0 = [E_d \quad hE_h \quad 0]$. Applying norm bounding inequality yields

$$\Psi + \lambda^{-1}\Pi_0^T\Pi_0 + \lambda\Theta_0^T\Theta_0 < 0 \quad (21)$$

which then by Schur complement verifies the condition (16). Let us choose an augmented Lyapunov-Krasovskii functional (Parlakci, 2006) as

$$V(x(t),t) = \sum_{i=1}^{5} V_i \quad (22)$$

where $V_1 = \eta^T(t)EP\eta(t), V_2 = h\int_{t-h}^{t}(s-t+h)x^T(s)Qx(s)ds$,

$V_3 = \int_{t-d}^{t}x^T(s)Rx(s)ds$, $V_4 = \int_{t-h}^{t}x^T(s)Sx(s)ds$,

$V_5 = \int_{t-d}^{t}\dot{x}^T(s)Z\dot{x}(s)ds, \eta(t) = [\nabla^T(x_t) \quad y^T(t) \quad x^T(t)]^T$, and

$E = \begin{bmatrix} \mathrm{I} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $P = \begin{bmatrix} P_1 & 0 & 0 \\ P_2 & P_3 & 0 \\ P_4 & P_5 & P_6 \end{bmatrix}$, $P_1^T = P_1 > 0$,

$Q^T = Q > 0$, $R^T = R > 0$, $S^T = S > 0$, $Z^T = Z > 0$. The time-derivative of $V(x(t),t)$ along the state trajectory of system (1), (12), (15) gives

$$\dot{V}(x(t),t) = \sum_{i=1}^{5}\dot{V}_i \quad (23)$$

One can compute $\dot{V}_1$ in the following form

$$\dot{V}_1(x(t),t) = 2\eta^T(t)P^TE^T\frac{d}{dt}\eta(t) \quad (24)$$

It follows from (15) that one can write

$$\begin{aligned} 0 &= -y(t) + A_0x(t) + BK\nabla(x_t) + f(x(t),t) \\ &\quad + g(x(t-h),t) + h(\dot{x}(t-d),t) \end{aligned} \quad (25)$$

Utilizing the neutral difference operator gives

$$0 = -\nabla(x_t) + x(t) + A_h\int_{t-h}^{t}x(s)ds - A_dx(t-d) \quad (26)$$

In view of (25), (26), one can compute

$$E^T\frac{d}{dt}\eta(t) = \bar{A}\,\eta(t) + \Gamma_1\int_{t-h}^{t}x(s)ds + \Gamma_2x(t-d) + \Gamma_3[f(x(t),t) + g(x(t-h),t) + h(\dot{x}(t-d),t)] \quad (27)$$

where $\overline{A} = \begin{bmatrix} 0 & \mathrm{I} & 0 \\ BK & -\mathrm{I} & A_0 \\ -\mathrm{I} & 0 & \mathrm{I} \end{bmatrix}$, $\Gamma_1 = \begin{bmatrix} 0 & 0 & A_h^T \end{bmatrix}^T$,

$\Gamma_2 = \begin{bmatrix} 0 & 0 & -A_d^T \end{bmatrix}^T$, $\Gamma_3 = \begin{bmatrix} 0 & \mathrm{I} & 0 \end{bmatrix}^T$. Substituting (27) into (24) gives

$$\begin{aligned} \dot{V}_1 = 2\eta^T(t)P^T\Big\{&\overline{A}\,\eta(t) + \Gamma_1\int_{t-h}^{t} x(s)ds + \Gamma_2 x(t-d) \\ &+ \Gamma_3\big[f(x(t),t) + g(x(t-h),t) \\ &+ h(\dot{x}(t-d),t)\big]\Big\} = \chi^T(t)\Omega_1\chi(t) \end{aligned} \quad (28)$$

where $\chi(t) = \Big[\eta^T(t) \quad \Big(\int_{t-h}^{t} x(s)ds\Big)^T \quad x^T(t-d) \quad f^T(x(t),t)$

$x^T(t-h) \quad g^T(x(t-h),t) \quad \dot{x}^T(t-d) \quad h^T(\dot{x}(t-d),t)\Big]^T$

$\Omega_1 = \begin{bmatrix} \Omega_1(1,1) & \Omega_1(1,2) \\ * & \Omega_1(2,2) \end{bmatrix}$, $\Omega_1(1,1) = P^T\overline{A} + \overline{A}^T P$,

$\Omega_1(1,2) = \begin{bmatrix} P^T\Gamma_1 & P^T\Gamma_2 & P^T\Gamma_3 & 0 & P^T\Gamma_3 & 0 & P^T\Gamma_3 \end{bmatrix}$

$\Omega_1(2,2) = \mathrm{diag}\{0,0,0,0,0,0,0\}$. Calculating $\dot{V}_2$ yields

$$\begin{aligned} \dot{V}_2 &= h^2 x^T(t)Qx(t) - h\int_{t-h}^{t} x^T(s)Qx(s)ds \\ &\le \chi^T(t)\big(\Omega_2 + h^2\Gamma_4^T Q\Gamma_4\big)\chi(t) \end{aligned} \quad (29)$$

where $\Omega_2 = \begin{bmatrix} 0 & \Omega_2(1,2) \\ * & \Omega_2(2,2) \end{bmatrix}$, $\Lambda_1 = \begin{bmatrix} 0 & 0 & \mathrm{I} \end{bmatrix}$,

$\Omega_2(1,2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$\Omega_2(2,2) = \mathrm{diag}\{-Q,0,0,0,0,0,0\}$,

$\Gamma_4 = \begin{bmatrix} \Lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Moreover, the time-derivative of V_3 is computed as follows

$$\dot{V}_3 = \chi^T(t)\big(\Omega_3 + \Gamma_4^T R\Gamma_4\big)\chi(t) \quad (30)$$

where $\Omega_3 = \begin{bmatrix} 0 & \Omega_2(1,2) \\ * & \Omega_3(2,2) \end{bmatrix}$,

$\Omega_3(2,2) = \mathrm{diag}\{0,-R,0,0,0,0,0\}$. Computing the time-derivative of V_4 yields

$$\dot{V}_4(x(t),t) = \chi^T(t)\big(\Omega_4 + \Gamma_4^T S\Gamma_4\big)\chi(t) \quad (31)$$

where $\Omega_4 = \begin{bmatrix} 0 & \Omega_2(1,2) \\ * & \Omega_4(2,2) \end{bmatrix}$, $\Omega_4(2,2) = \mathrm{diag}\{0,0,0,-S,0,0,0\}$.

Finally, one can compute $\dot{V}_5(x(t),t)$ as follows

$$\begin{aligned} \dot{V}_5(x(t),t) &= \dot{x}^T(t)Z\dot{x}(t) - \dot{x}^T(t-d)Z\dot{x}(t-d) \\ &= \chi^T(t)\big(\Omega_5 + \Gamma_5^T Z\Gamma_5\big)\chi(t) \end{aligned} \quad (32)$$

where $\Omega_5 = \begin{bmatrix} 0 & \Omega_2(1,2) \\ * & \Omega_5(2,2) \end{bmatrix}$, $\Lambda_2 = \begin{bmatrix} BK & 0 & A \end{bmatrix}$,

$\Omega_5(2,2) = \mathrm{diag}\{0,0,0,0,0,-Z,0\}$,

$\Gamma_5 = \begin{bmatrix} \Lambda_2 & 0 & 0 & \mathrm{I} & A_h & \mathrm{I} & A_d & \mathrm{I} \end{bmatrix}$. It follows from (6)-(8) that one can write the following inequality

$$\begin{aligned} &\chi^T(t)\big(\Omega_6 + \varepsilon_1\alpha^2\Gamma_4^T\Gamma_4 + \varepsilon_2\beta^2\Gamma_6^T\Gamma_6 \\ &+ \varepsilon_3\gamma^2\Gamma_7^T\Gamma_7\big)\chi(t) \ge 0 \end{aligned} \quad (33)$$

where $\Omega_6 = \begin{bmatrix} 0 & \Omega_2(1,2) \\ * & \Omega_6(2,2) \end{bmatrix}$, $\Lambda_3 = \begin{bmatrix} K & 0 & 0 \end{bmatrix}$

$\Omega_6(2,2) = \mathrm{diag}\{0,0,-\varepsilon_1\mathrm{I},0,-\varepsilon_2\mathrm{I},0,-\varepsilon_3\mathrm{I}\}$, and $\Gamma_8 = \begin{bmatrix} \Lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,. Therefore, substituting (29)-(32) into (23) and adding (33) yields

$$\begin{aligned} &\dot{V}(x(t),t) + x^T(t)S_1x(t) \\ &+ u^T(t)S_2u(t) \le \chi^T(t)\Omega_0\chi(t) \end{aligned} \quad (34)$$

where $\Omega_0 = \sum_{i=1}^{6}\Omega_i + \Gamma_4^T\big(h^2Q + R + S_1 + S + \varepsilon_1\alpha^2\mathrm{I}\big)\Gamma_4$

$+ \Gamma_5^T Z\Gamma_5 + \varepsilon_2\beta^2\Gamma_6^T\Gamma_6 + \varepsilon_3\gamma^2\Gamma_7^T\Gamma_7 + \Gamma_8^T S_2\Gamma_8$.

In order to guarantee $\dot{V}(x(t),t) + x^T(t)S_1x(t) + u^T(t)S_2u(t) < 0$, one needs to satisfy

$$\Omega_0 < 0 \quad (35)$$

then applying Schur complement to (35) gives

$$\Omega < 0 \quad (36)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & h\Lambda_1^T & \Lambda_1^T & \Lambda_1^T & \Lambda_3^T & \Omega_{16} \\ * & -Q^{-1} & 0 & 0 & 0 & 0 \\ * & * & -R^{-1} & 0 & 0 & 0 \\ * & * & * & -S_1^{-1} & 0 & 0 \\ * & * & * & * & -S_2^{-1} & 0 \\ * & * & * & * & * & \Omega_{66} \end{bmatrix},$$

$\Omega_{16} = \begin{bmatrix} \alpha\Lambda_1^T & \Lambda_1^T & \Omega(1,8) & \Omega(1,9) & \Omega(1,10) \end{bmatrix}$,

$\Omega(1,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & \beta\mathrm{I} & 0 & 0 & 0 \end{bmatrix}^T$,

$\Omega(1,9) = \begin{bmatrix} \Lambda_2 & 0 & 0 & \mathrm{I} & A_h & \mathrm{I} & A_d & \mathrm{I} \end{bmatrix}^T$,

$\Omega(1,10) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \gamma\mathrm{I} & 0 \end{bmatrix}^T$,

$\Omega_{66} = \mathrm{diag}\{-\varepsilon_1^{-1}\mathrm{I},-S^{-1},-\varepsilon_2^{-1}\mathrm{I},-Z^{-1},-\varepsilon_3^{-1}\mathrm{I}\}$,

$\Omega_{11} = \begin{bmatrix} P^T\overline{A} + \overline{A}^T P & \Omega_{11}(1,2) \\ * & \Omega_{11}(2,2) \end{bmatrix}$,

$\Omega_{11}(1,2) = \begin{bmatrix} P^T\Gamma_1 & P^T\Gamma_2 & P^T\Gamma_3 & 0 & P^T\Gamma_3 & 0 & P^T\Gamma_3 \end{bmatrix}$

$\Omega_{11}(2,2) = \mathrm{diag}\{-Q,-R,-\varepsilon_1\mathrm{I},-S,-\varepsilon_2\mathrm{I},-Z,-\varepsilon_3\mathrm{I}\}$. Then pre- and post- multiplying $\Omega < 0$ in (36) with M^T and M, respectively, where $M = \mathrm{diag}\{X,\overline{Q},\overline{R},\overline{\varepsilon}_1,\overline{S},\overline{\varepsilon}_2,\overline{Z},\overline{\varepsilon}_3,\mathrm{I},\mathrm{I},\mathrm{I},\mathrm{I},\mathrm{I},\mathrm{I},\mathrm{I},\mathrm{I}\}$, $X = P^{-1}$, $\overline{Q} = Q^{-1}$, $\overline{R} = R^{-1}$, $\overline{S} = S^{-1}$, $\overline{Z} = Z^{-1}$ $\overline{\varepsilon}_i = \varepsilon_i^{-1}$, $i = 1,2,3$, and choosing $K = YX^{-1}$ give the following linear matrix inequality

$$\Sigma < 0 \quad (37)$$

where

$$\Sigma = \begin{bmatrix} \Sigma_{11} & h\overline{\Sigma}_{12} & \overline{\Sigma}_{12} & \overline{\Sigma}_{12} & \overline{\Sigma}_{15} & \Sigma_{16} \\ * & -\overline{Q} & 0 & 0 & 0 & 0 \\ * & * & -\overline{R} & 0 & 0 & 0 \\ * & * & * & -S_1^{-1} & 0 & 0 \\ * & * & * & * & -S_2^{-1} & 0 \\ * & * & * & * & * & \Sigma_{66} \end{bmatrix},$$

$$\Sigma_{11} = \begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 & 0 & (1,6) \\ * & (2,2) - \varepsilon_4 DD^T & (2,3) & 0 & 0 & (2,6) \\ * & * & (3,3) - \varepsilon_5 DD^T & A_h\overline{Q} & -A_d\overline{R} & (1,6) \\ * & * & * & -\overline{Q} & 0 & (1,6) \\ * & * & * & * & -\overline{R} & (1,6) \\ * & * & * & * & * & (6,6) \end{bmatrix}$$

Now, one can replace A, A_h, A_d, B with $A + \Delta A(t)$, $A_h + \Delta A_h(t)$, $A_d + \Delta A_d(t)$, $B + \Delta B(t)$, respectively in (37) to get

$$\Sigma + \Delta\Sigma(t) + \Delta\Sigma^T(t) < 0 \qquad (38)$$

where $\Delta\Sigma(t) = \Pi^T F(t)\Theta$,

$$\Pi = \begin{bmatrix} 0 & D^T & 0 & (1,4) & 0 & 0 \\ 0 & 0 & D^T & (1,4) & 0 & 0 \\ 0 & 0 & 0 & (1,4) & D^T & 0 \end{bmatrix}^T,$$

$$(1,4) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} (1,1) & E_0X_5 & E_0X_6 & 0 & 0 & 0 & 0 & 0 & 0 & (1,10) \\ 0 & 0 & 0 & E_h\overline{Q} & -E_d\overline{R} & 0 & 0 & 0 & 0 & (1,10) \\ (3,1) & E_aX_5 & E_aX_6 & 0 & 0 & 0 & E_h\overline{S} & 0 & E_d\overline{Z} & (1,10) \end{bmatrix}^T$$

$(1,1) = E_bY + E_0X_4$, $(3,1) = E_bY + E_aX_4$,

$(1,10) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

with $E_0 = E_a + E_h$. One can rewrite (38) as follows

$$\Sigma + \Delta\Sigma(t) + \Delta\Sigma^T(t) < \Sigma + \varepsilon_0\Pi^T\Pi + \varepsilon_0^{-1}\Theta^T\Theta < 0 \qquad (39)$$

Then, by Schur complement, one obtains the linear matrix inequality (17). This completes the proof.

Next, one can show that by the neutral state-feedback controller (12), the guaranteed cost function in (11) has an upper bound. Note that (34), (37) imply

$$\dot{V}(x(t),t) \le -x^T(t)S_1x(t) - u^T(t)S_2u(t) \qquad (40)$$

Integrating both sides of (40) from 0 to $T > 0$ gives

$$V(x(T),T) - V(x(0),0) \le -\int_0^T \left[x^T(t)S_1x(t) + u^T(t)S_2u(t)\right]dt \qquad (41)$$

As the system is robustly asymptotically stable once the conditions in (16), (17) are satisfied, for $\lim_{T\to\infty} V(x(T),T) \to 0$, one obtains $\lim_{T\to\infty}\int_0^T \left[x^T(t)S_1x(t) + u^T(t)S_2u(t)\right]dt = J \le J^*$.

Theorem 1 describes how to synthesize a robust guaranteed cost neutral state-feedback controller. For the design of an optimal robust guaranteed cost neutral state-feedback controller that minimizes the upper bound of the quadratic cost function (11), the following theorem is presented.

Theorem 2: (Park, 2005b) For the uncertain nonlinear neutral system (1) and the associated quadratic cost function (11), consider the optimization problem defined as

$$\min_{X,\overline{Q},\overline{R},\overline{S},\overline{Z},Y,T_i,i=1,\ldots,4,\bar{\varepsilon}_j,j=1,2,3,\varepsilon_0} \left\{\mu + \sum_{i=1}^{4}\mathrm{trace}(T_i)\right\}$$

subject to (16), (17), and

$$\begin{bmatrix} -\mu & \nabla^T(\Phi(0)) \\ * & -X_1 \end{bmatrix} \le 0, \begin{bmatrix} -T_1 & W_1^T \\ * & -\overline{Q} \end{bmatrix} \le 0$$

$$\begin{bmatrix} -T_2 & W_2^T \\ * & -\overline{R} \end{bmatrix} \le 0, \begin{bmatrix} -T_3 & W_3^T \\ * & -\overline{S} \end{bmatrix} \le 0$$

$$\begin{bmatrix} -T_4 & W_4^T \\ * & -\overline{Z} \end{bmatrix} \le 0 \qquad (42)$$

such that if a feasible solution set of X, $\overline{Q}$, $\overline{R}$, $\overline{S}$, $\overline{Z}$, Y, T_i, $i = 1,\ldots,4$, $\bar{\varepsilon}_j$, $j = 1,2,3$, ε_0 can be achieved then the neutral state-feedback control law (12) is said to be an optimal robust guaranteed cost control law ensuring that the quadratic cost function upper bound is minimized, where $h\int_{-h}^{0}(s+h)\Phi(s)\Phi^T(s)ds = W_1W_1^T$, $\int_{-d}^{0}\Phi(s)\Phi^T(s)ds = W_2W_2^T$, $\int_{-h}^{0}\Phi(s)\Phi^T(s)ds = W_3W_3^T$, $\int_{-d}^{0}\dot{\Phi}(s)\dot{\Phi}^T(s)ds = W_4W_4^T$.

Proof: Similar to the proof of (Park, 2005b), thus it is omitted.

4. NUMERICAL EXAMPLES

This section presents two numerical examples.

Example 1: An uncertain linear neutral system example is considered as follows

$$\dot{x}(t) = [A + DF(t)E_a]x(t) + [A_h + DF(t)E_h]x(t-h) + [A_d + DF(t)E_d]\dot{x}(t-d) + [B + DF(t)E_b]u(t) \qquad (43)$$

where $A = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix}$, $A_h = \begin{bmatrix} 0 & 0.2 \\ 0.2 & -0.5 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$ with $h = 0.5$, $d = 0.2$ and $\phi(t) = \begin{bmatrix} -e^t \\ e^t \end{bmatrix}$, $\forall t \in [-0.5, 0]$. The gain matrices are selected as $S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $S_2 = 0.2$. If $D = 0$, the system in (43) reduces to the example studied in (Park, 2005a). Choosing $\rho_1 = 0.1$, $\rho_2 = 0.2$, the stability of the difference equation is satisfied with $\Lambda_0 = \begin{bmatrix} 9.0812 & -1.3109 \\ -1.3109 & 11.7136 \end{bmatrix}$. A feasible solution set has been obtained for satisfying (17) appropriately. Then computing the cost function upper bound gives $J^* = 1.8478$, $K = [-0.7173 \quad -1.0736]\cdot 10^6$. However, one notices that the achievable cost function bound obtained in (Park, 2005a) is 9.7773 with $K = [-1.8737 \quad -3.3077]$. Hence the proposed methodology is shown to be less conservative than that of (Park, 2005a). Let us now choose $D = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}$, $E_a = [0.1 \quad 0.2]$, $E_h = [0.3 \quad 0.1]$, $E_d = [0.2 \quad 0.1]$, $E_b = 0.1$. Then application of Theorem 1, 2 results in a set of feasible solution. The upper bound of the cost function is calculated as $J^* = 3.8706$ and $K = [-24.8891 \quad -26.5678]$. This result shows that the performance of the proposed guaranteed cost controller is still quite better than that of the method presented in (Park, 2005a) even in the case of norm-bounded uncertainties.

Example 2: An example of uncertain nonlinear neutral system is considered as follows

$$\begin{aligned}\dot{x}(t) &= \left[A + DF(t)E_a\right]x(t) + \left[A_h + DF(t)E_h\right]x(t-h) \\ &+ \left[A_d + DF(t)E_d\right]\dot{x}(t-d) + \left[B + DF(t)E_b\right]u(t) \\ &+ f(x(t),t)) + g(x(t-h),t) + h(\dot{x}(t-d),t) \qquad (44)\end{aligned}$$

where $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, $A_h = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}$, $A_d = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$, $E_a = \begin{bmatrix} 0 & 0.2 \end{bmatrix}$, $E_h = \begin{bmatrix} 1 & 0.3 \end{bmatrix}$, $E_d = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}$, $E_b = 0.5$ and $h = 1.0$, $d = 2.0$. The initial condition is given as $\Phi(t) = \begin{bmatrix} 1 \\ e^t \end{bmatrix}$, $\forall t \in [-2.0, 0]$ with the cost function gain matrices chosen as $S_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$, $S_2 = 1$.

Let us assume that (Xu *et al.*, 2003), $f(x(t),t) = 0$, $g(x(t-h),t) = 0$, $h(\dot{x}(t-d),t) = 0$. The stability of $\nabla(x_t)$ is ensured by choosing $\rho_1 = \rho_2 = 0.3$ and $\Lambda_0 = \begin{bmatrix} 1.4214 & -0.0108 \\ -0.0108 & 1.3121 \end{bmatrix}$, $\lambda = 0.1281$. The linear matrix inequality (17) has given a feasible solution set which indicates that the cost function upper bound is computed as $J^* = 4.7377$ with $K = \begin{bmatrix} -1.5309 & -1.6587 \end{bmatrix}$. However, the achievable cost function bound obtained in (Xu *et al.*, 2003) is 50.0275 with $K = \begin{bmatrix} -0.9943 & -0.9615 \end{bmatrix}$. This shows that the proposed methodology is capable of yielding less conservative cost function bounds. Now let us assume that the numerical example of neutral system given in (44) involves nonlinear parameter perturbations defined such that $f(x(t),t) = \begin{bmatrix} \alpha_1 \cos t |x_1(t)| & \alpha_2 \sin t |x_2(t)| \end{bmatrix}^T$, $|\alpha_i| \le 0.3$, $g(x(t-h(t)),t) = \begin{bmatrix} \beta_1 \cos t |x_1(t-h)| & \beta_2 \sin t |x_2(t-h)| \end{bmatrix}^T$ $|\beta_i| \le 0.3$, $h(\dot{x}(t-d(t)),t) = \begin{bmatrix} \gamma_1 \cos t |\dot{x}_1(t-d)| & \gamma_2 \sin t |\dot{x}_2(t-d)| \end{bmatrix}^T$ $|\gamma_i| \le 0.3$, $i = 1,2$. The simulation work yields a feasible solution set and the upper bound of the cost function is computed as $J^* = 11.9199$ with $K = \begin{bmatrix} -2.8622 & -2.5742 \end{bmatrix}$. Consequently, it can be seen that even in the case of nonlinear parameter perturbations, the proposed robust guaranteed cost neutral state feedback controller performs much better than that given in (Xu *et al.*, 2003).

5. CONCLUSIONS

This paper has investigated the design of a robust delay-dependent guaranteed cost stabilizing controller for uncertain nonlinear neutral systems. A memory type neutral form of state-feedback control law is introduced. On the basis of a descriptor representation, an augmented descriptor form of a candidate Lyapunov-Krasovskii functional is adopted to study the stability of the neutral system. Sufficient robust delay-dependent linear matrix inequality (LMI) synthesis conditions are derived. Two numerical examples have concluded that the proposed method assures a less conservative control cost in comparison to some of the existing approaches.

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